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Cosmic censorship and the dilaton

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We investigate extremal electrically charged black holes in Einstein-Maxwell-dilaton theory with a cosmological constant inspired by string theory. These solutions are not static, and a timelike singularity eventually appears which is not surrounded by an event horizon. This suggests that cosmic censorship may be violated in this theory.

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I. INTRODUCTION

Perhaps the most important unsolved problem in classical general relativity is cosmic censorship: Penrose's conjecture that generic initial conditions do not evolve to form naked singularities [1]. One way to obtain some insight into this conjecture is to study exact solutions. Since cosmic censorship deals with generic initial data, a particular solution or even a family of solutions will never suffice to prove or disprove this conjecture. However, they can be extremely valuable in illustrating the type of behavior that is possible, and may provide a starting point for a stability analysis that could ultimately lead to a convincing counterexample.

Kastor and Traschen [2] have recently found an exact solution describing colliding black holes. They consider charged black holes in a universe with positive cosmological constant. This solution provides a new arena to test cosmic censorship, and it has been shown that naked singularities form in some of the black hole collisions [3]. In this paper we study a generalization of this solution which includes a scalar dilaton coupled to the metric and Maxwell field in the manner predicted by low energy string theory. There are several motivations for doing so. From the standpoint of general relativity, the dilaton is a physically reasonable matter field, and one can investigate whether cosmic censorship is valid for this choice of matter. From the standpoint of string theory, the dilaton is an essential ingredient in the low energy theory. The analogue of adding a cosmological constant in string theory is to include excess central charge. Solutions with excess central charge are of interest since they arise naturally in several contexts, including gauged Wess-Zumino-Witten models and noncritical string theories. The recently discussed exact black hole solutions in two [4] and three [5] dimensions are examples of solutions with excess central charge.

We will see that the dilaton changes the causal structure of the Kastor-Traschen solution. In particular, it turns out that in the absence of any black holes, the solution with a dilaton describes a collapsing Robertson-Walker universe with a final null singularity. When even a single extremal[1] black hole is added, this null singulari-

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[1]We will show in Sec. II that, unlike the Kastor-Traschen solution, the black holes in this solution are extremal.

ty is replaced by a timelike one which begins far from the black hole and is not hidden by an event horizon. [A more elaborate model based on a generalized Wess-Zumino-Witten (WZW) construction seems to have similar properties [6].] The solution with an extremal black hole also has a null singularity at $r=0$, but we will argue that this will probably be removed in the nonextremal solution. Conformally rescaling to the string metric (the natural metric for string theory) gives a spacetime which is asymptotically flat, and the timelike singularity recedes to infinity.

The Kastor-Traschen solution [2] to the Einstein-Maxwell equations with cosmological constant $\Lambda=3h^2$ describes a number of black holes, each having a charge equal to its mass, situated at arbitrary locations. The metric and gauge field for mass parameters M_i and positions $\mathbf{r}_i$ are given by

$$ds^2=-\frac{d\tau^2}{U^2}+U^2 d\mathbf{r}\cdot d\mathbf{r}\ ,\quad A_\tau=\frac{1}{U}\ , \tag{1.1}$$

where[2]

$$U=-h\tau+\sum_i \frac{2M_i}{|\mathbf{r}-\mathbf{r}_i|}\ . \tag{1.2}$$

We will assume throughout that $h>0$. The special case of a single mass corresponds to the $Q=M$ Reissner–Nordström–de Sitter solution. The general case is quite similar to the Majumdar-Papapetrou solution [7] describing a collection of $Q=M$ black holes in the absence of the cosmological constant. If one simply sets $h=0$ in (1.2), the resulting metric is not asymptotically flat. To obtain the Majumdar-Papapetrou solution, one must first introduce the coordinate t measuring proper time at infinity $\tau=-e^{-ht}/h$, and then take the limit $h\rightarrow 0$ keeping t fixed. The resulting solution takes the form (1.1) with τ replaced by t and U replaced by

$$\tilde{U}=1+\sum_i \frac{2M_i}{|\mathbf{r}-\mathbf{r}_i|}\ . \tag{1.3}$$

The low energy action that arises naturally in string theory with a central charge $-h^2$ is

$$S=\int d^4x\sqrt{-\hat{g}}\,e^{-2\phi}[-h^2+\hat{R}+4(\nabla\phi)^2-F^2]\ , \tag{1.4}$$

where ϕ is the dilaton, F is the Maxwell field, and $\hat{g}_{\mu\nu}$ is the string metric. The $e^{-2\phi}$ factor in front of the scalar curvature can be eliminated by rescaling to the Einstein metric using $g_{\mu\nu}=e^{-2\phi}\hat{g}_{\mu\nu}$ to yield

$$S=\int d^4x\sqrt{-g}\,[R-2(\nabla\phi)^2-h^2e^{2\phi}-e^{-2\phi}F^2]\ . \tag{1.5}$$

This can be viewed as general relativity with somewhat unusual matter. The constant h provides a positive potential for the dilaton, so the local energy density is always positive. When $h=0$, the theory has a duality symmetry which relates electrically charged solutions to magnetically charged solutions and changes the sign of ϕ. When $h\neq 0$, this symmetry is broken and there does not appear to be a simple relation between solutions with different types of charge. We will mainly discuss the electrically charged solution and comment on the magnetically charged case at the end.

It was noticed in [8] that an extremum of (1.5) with $h=0$ could be obtained from the Majumdar-Papapetrou solution by setting

$$e^{-2\phi}=\tilde{U},\quad A_t=\frac{1}{\sqrt{2}\tilde{U}}\ , \tag{1.6}$$

and taking the "square root" of the metric:

$$ds^2=-\frac{dt^2}{\tilde{U}}+\tilde{U}\,d\mathbf{r}\cdot d\mathbf{r}\ . \tag{1.7}$$

A single extremal charged black hole coupled to a dilaton has $Q^2=2M^2$. The solution (1.7) describes a collection of $Q=\sqrt{2}M$ black holes which is static since the electromagnetic repulsion is balanced by both the gravitational and dilatonic attraction. It can be viewed as the dilatonic version of the Majumdar-Papapetrou solution.

Given the connection between these solutions when $h=0$, it is natural to conjecture that the dilatonic version of the Kastor-Traschen solution is

$$ds^2=-\frac{d\tau^2}{U}+U\,d\mathbf{r}\cdot d\mathbf{r},\quad A_\tau=\frac{1}{\sqrt{2}U},\quad e^{-2\phi}=U\ , \tag{1.8}$$

where U is again given by (1.2). One can in fact verify that (1.8) is an extremum of (1.5) for any h. (This solution has been found independently by Maki and Shiraishi [9], in different coordinates that cover only part of the solution.) Like the Kastor-Traschen solution, (1.8) is time dependent and has no symmetries in general.

In the next section we will explore the global properties of this solution for a single mass. In Sec. III, we discuss the multimass solution, as well as generalizations to higher dimensions and magnetic charges. Section IV contains some concluding remarks.

II. SINGLE MASS SOLUTION

We begin by considering the solution (1.8) with no masses. This will clarify the type of universe in which the extremal black holes exist. When $M_i=0$, the metric (1.8) becomes

$$ds^2=\frac{d\tau^2}{h\tau}-h\tau(dr^2+r^2d\Omega^2)\ , \tag{2.1}$$

when $\tau<0$. This can be put into a more familiar form by defining a new coordinate $t=-\sqrt{4\tau/h}$. The metric now takes the form [10]

$$ds^2=-dt^2+\tfrac{1}{4}h^2t^2(dr^2+r^2d\Omega^2)\ . \tag{2.2}$$

This is simply a collapsing Robertson-Walker universe with $k=0$ and $\rho=-3p$. The singularity at $t=0$ (which corresponds to $\tau=0$) appears similar to the "big crunch" singularity of the standard Friedmann solutions, but

[2]Our expression for U differs from that of [3] by a factor of 2 multiplying M_i. This is because we want $M=\sum M_i$ to represent the total mass of the solution *with dilaton* (described below) in the limit when $h\rightarrow 0$.

there is an important difference. The singularity in this solution is not spacelike but null. One way to see this is to note that outgoing radial light rays satisfy $r=-(2/h)\ln|t|+r_0$, so that as $t \to 0$, $r \to \infty$. In other words, observers never lose causal contact as they approach the singularity. Another argument that the singularity must be null is that the metric on the r,t plane describes the interior of the past light cone of the origin in two-dimensional Minkowski spacetime, where t labels the hyperbola. The surface $t=0$ is the light cone itself, which is, of course, null. (If the spatial metric were not flat Euclidean space, but rather the metric on a unit hyperboloid, the spacetime metric would be flat everywhere.) The Penrose diagram for this spacetime is shown in Fig. 1.

We now insert a single extremal electrically charged black hole into this universe. The metric is now given by

$$ds^2=-\frac{d\tau^2}{-h\tau+2M/r}+(-h\tau+2M/r)(dr^2+r^2d\Omega^2) . \tag{2.3}$$

For $\tau<0$, the solution clearly approaches (2.1) at large r. However, as τ approaches zero, the presence of the mass has an increasing effect even in the asymptotic region. Perhaps the most important consequence of the mass is that the surface $\tau=0$ is no longer singular and no longer null. All constant τ surfaces are spacelike, and observers can travel through $\tau=0$ without difficulty. However, there is now a singularity along the curve $r\tau=2M/h$. We will see that this singularity has a completely different character from the null singularity at $\tau=0$. In particular, the new singularity is timelike and visible to observers in the spacetime.

To show this, we consider the motion of radial null geodesics. We first verify that future directed null geodesics reach this singularity in finite affine parameter, so the singularity is not "at infinity." We then show that past-directed null geodesics can also hit the singularity. This establishes that the singularity is timelike and its effects can propagate into the spacetime.

Denote derivatives with respect to an affine parameter λ by a dot, $\dot{x}^\mu=dx^\mu/d\lambda$, and consider only null geodesics with $\dot{\theta}=\dot{\varphi}=0$. The fact that the geodesic is null then requires

$$\dot{\tau}=\pm(-h\tau+2M/r)\dot{r} , \tag{2.4}$$

where the plus (minus) sign is for future- (past-)directed outgoing geodesics. Since the metric is not static, there is

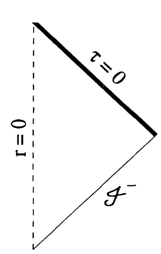

FIG. 1. The Penrose diagram for the solution (1.8) with no masses. The dashed line at $r=0$ represents the origin of spherical coordinates. The thick line at $\tau=0$ is a null singularity.

no conserved energy to use in calculating the geodesics. However, a straightforward calculation yields

$$\ddot{r}+\Gamma^r_{\mu\nu}\dot{x}^\mu\dot{x}^\nu=\ddot{r}\mp h\dot{r}^2=0 , \tag{2.5}$$

where we have used (2.4). This equation can be easily solved to give

$$r=\mp\frac{1}{h}\ln(c_1\lambda+c_0) , \tag{2.6}$$

where c_0 and c_1 are constants. Since the singularity is at a finite value of r, a future-directed null geodesic reaches it in finite affine parameter. Thus, the spacetime described by (2.3) is geodesically incomplete.

Equation (2.6) also shows that past-directed null geodesics reach finite values of r in finite affine parameter, but to show that these curves hit the singularity, we need to explicitly find the geodesics. From (2.4) we see that past-directed radial null curves satisfy

$$\frac{d\tau}{dr}=\left[h\tau-\frac{2M}{r}\right] . \tag{2.7}$$

This can be integrated to yield

$$\tau=e^{hr}\left[v-\int_{r_0}^{r}\frac{2M}{\tilde{r}}e^{-h\tilde{r}}d\tilde{r}\right] , \tag{2.8}$$

where v is a parameter labeling the different null curves, and $r_0>0$ is an arbitrary constant. The integral on the right reaches a finite limit as $r\to\infty$. Denoting this limit v_0, we see that the asymptotic behavior of the null curves depends crucially on whether v is greater than or less than v_0. For $v<v_0$, $\tau\to-\infty$ as $r\to\infty$, and these geodesics reach past null infinity. For $v>v_0$, τ clearly stays positive for all r (and in fact grows at large r). Since the singularity is at $r\tau=2M/h$, these curves always hit the singularity. This shows that past-directed radial null geodesics can reach the singularity.

The fact that future-directed null geodesics are incomplete is not too surprising, since the $M=0$ universe is also geodesically incomplete due to the null singularity at $\tau=0$. However, the fact that past-directed null geodesics are also incomplete is quite unexpected. This appears to be a serious violation of cosmic censorship. The perfectly smooth universe in the far past (2.2) evolves into a spacetime with a naked singularity.

Let us compare this behavior with other known solutions. When there is no dilaton, the Kastor-Traschen solution (1.1) with a single mass still has a timelike singularity at $r\tau=2M/h$. However, in that case the singularity is either inside an event horizon (if M is smaller than an extremal value M_{ext}) or exists for all time (if $M>M_{ext}$) [11]. So one does not have a violation of cosmic censorship of the type found in the solution with a dilaton. In the Reissner-Nordström solution, one can consider a spacelike surface of constant r with $r_-<r<r_+$. Since this surface is homogeneous, it can be viewed as providing initial data for a homogeneous cosmology. If $Q=0$, the universe collapses into a spacelike singularity at $r=0$ (the Schwarzschild singularity). However, if $Q\neq0$, the singularity becomes timelike. Al-

though this seems similar to the solution (2.3), there is a key difference. In the Reissner-Nordström case, if one evolves the homogeneous initial data into the past, one finds a Cauchy horizon. The solution can be extended beyond this horizon to an asymptotically flat spacetime. This shows that the initial surface was inside a black hole. For the solution (2.3), the initial data on a constant $\tau<0$ surface can be evolved infinitely far into the past without reaching a Cauchy horizon.

We have not yet discussed the solution (2.3) near $r=0$. In this region, the $h\tau$ term is negligible. In terms of a proper radial coordinate ρ and a rescaled time coordinate t, the metric (for fixed θ and φ) becomes

$$ds^2=-\rho^2dt^2+d\rho^2 \ . \tag{2.9}$$

This is precisely the metric for Rindler space, i.e., one of the two regions outside the light cone of the origin of two-dimensional Minkowski spacetime, written in coordinates adapted to the boost translation symmetry. It is now clear that $r=0$ (which is $\rho=0$) is a null surface consisting of two parts, corresponding to the future and past Rindler horizon. This surface is also the location of a curvature singularity since the area of the spheres vanishes there. Thus we do not have an ideal counterexample to cosmic censorship. All (nonextendible) spacelike surfaces must hit $r=0$, and hence there are no nonsingular initial data for this spacetime. The Penrose diagram for the entire spacetime is shown in Fig. 2.

We do not believe this is a serious difficulty for the following reason. As we have said, the solution near $r=0$ is similar to the one with $h=0$. The general charged dilatonic black hole with mass M, charge Q, and $h=0$ can be expressed [12, 13]

$$\begin{aligned} ds^2&=-\left[1-\frac{2M}{\hat r}\right]dt^2+\left[1-\frac{2M}{\hat r}\right]^{-1}d\hat r^2\\ &\quad+\hat r\left[\hat r-\frac{Q^2}{M}\right]d\Omega^2\ ,\\ F_{\hat r t}&=\frac{Q}{\hat r^2},\quad e^{2\phi}=1-\frac{Q^2}{M\hat r}\ . \end{aligned} \tag{2.10}$$

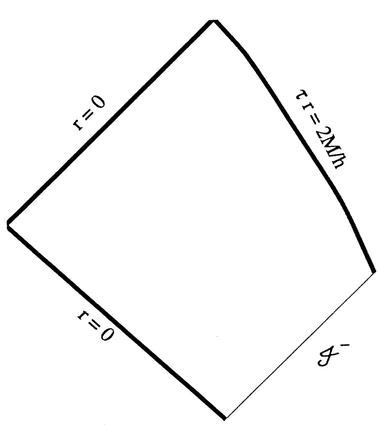

FIG. 2. The Penrose diagram for the single mass solution. The thick line at $r=0$ is the null singularity, and the thick line at $r\tau=2M/h$ is the timelike singularity.

There is an event horizon at $\hat r=2M$ and a spacelike singularity at $\hat r=Q^2/M$. In the extremal limit, $Q^2=2M^2$, the event horizon shrinks down to zero size and becomes singular. Since the casual structure in the $\hat r,t$ plane is independent of the charge, it is clear that this singularity is null and consists of two parts corresponding to the future and past horizon. Setting $\hat r=r+2M$, the extremal metric takes the form (1.7) with $\tilde U=1+2M/r$. So the singularity at $\hat r=2M$ is directly analogous to the one we saw above at $r=0$. This justifies the interpretation of (2.3) as an extremal black hole in the collapsing universe. But it is now clear that if we move slightly away from the extremal limit and have $Q^2<2M^2$, then the surface $r=0$ will become a nonsingular horizon shielding a spacelike singularity inside. Furthermore, the slight change in the charge should not affect the solution asymptotically, so the naked singularity at large r should still be present. If so, one would then have nonsingular initial conditions evolving into a naked singularity. If we then couple the theory to charged matter, we could presumably form a $Q^2<2M^2$ black hole by collapse. This would allow one to find violations of cosmic censorship with a single asymptotic region.

Unfortunately, the usual techniques for adding electric charge [14] do not apply in this case because the known solution is time dependent, and we have been unable to find a closed form expression for the charged black hole in a universe with $h\neq0$ away from the extremal limit. We have also been unable to solve the uncharged case in the presence of h. However, one can still construct nonsingular initial data which evolve into a naked singularity by taking a surface of constant $\tau<0$ in the solution (2.3) and restricting to $r>r_0$ for some constant $r_0>0$. Since the timelike singularity appears first at large r, it will still be obtained by evolving these restricted initial data. Of course, to obtain initial data which are both nonsingular and complete, one needs to find the nonextremal solution. On the other hand, it appears likely that the naked singularity at large r does not depend on the exact nature of the system near $r=0$. If so, the formation of the timelike singularity would be a universal feature for these black holes.

We now consider the string metric, which appears in (1.4). This metric has rather different behavior. The single mass solution is obtained by conformally rescaling the Einstein metric (2.3) by $e^{2\phi}=1/U$ to give

$$d\hat s^2=-\frac{d\tau^2}{(-h\tau+2M/r)^2}+dr^2+r^2d\Omega^2\ . \tag{2.11}$$

Notice that the surfaces of constant τ are now completely flat. The region $\tau<0$ can be put into a somewhat simpler form by introducing the coordinate t measuring proper time at infinity, $\tau=-e^{-ht}/h$. The metric is then

$$d\hat s^2=-\frac{dt^2}{(1+2Me^{ht}/r)^2}+dr^2+r^2d\Omega^2\ . \tag{2.12}$$

This form of the metric was used in [9], but is somewhat misleading since it does not cover the entire spacetime. If $M\neq0$, it is possible to reach $t=\infty$ in finite affine parameter. When $M=0$, the solution is clearly just flat spacetime. The dilaton is $e^{-2\phi_0}=-h\tau=e^{-ht}$ or $\phi_0(t)=ht/2$.

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In other words, the $M=0$ limit is just the familiar linear dilaton solution [15, 10]. Even when $M\neq 0$, (2.12) clearly approaches the linear dilaton solution asymptotically. In fact, the contribution from the mass goes to zero exponentially as $t\rightarrow\infty$. So the spacetime is asymptotically flat in the usual sense at past null infinity and has a complete $\mathcal{I}^-$.

Even though the spacetime is asymptotically flat, the standard methods of defining the total mass do not apply since the asymptotic dilaton is time dependent. The charge is defined as usual to be $Q=(1/4\pi)\oint e^{-2\phi}{}^*F$ where * denotes the dual, and the integral is over a large sphere at infinity. From the solution for the vector potential (1.8), one can verify that $Q=\sqrt{2}M$ where M is the parameter in the solution.

Just like the Einstein metric (2.3), the curvature of the string metric (2.11) diverges at $r=0$ and $r\tau=2M/h$. However, the casual structure is quite different. Timelike and null geodesics never actually reach the timelike singularity at $r\tau=2M/h$. Roughly speaking, this is because the singularity occurs at $U=0$, and the string metric is obtained by multiplying the Einstein metric by U^{-1}. Thus the conformal factor becomes very large in this region and essentially pushes the singularity off to infinity.

We verify this by again considering the geodesics. As before, let λ denote an affine parameter, and set $\dot\theta=\dot\varphi=0$. Then geodesics must satisfy

$$-\kappa=-\frac{\dot\tau^2}{(-h\tau+2M/r)^2}+\dot r^2\ , \qquad (2.13)$$

where $\kappa=1,0,-1$ for timelike, null, and spacelike geodesics. The geodesic equation for r now becomes

$$\ddot r+\frac{2M(\dot r^2+\kappa)}{r^2(-h\tau+2M/r)}=0\ . \qquad (2.14)$$

First consider null geodesics with $\kappa=0$. Unlike the simple solution available for the Einstein geodesics (2.6), it is not possible to solve (2.14) in closed form. We can determine the behavior of null geodesics very near the singularity though. As an outgoing null geodesic gets very near the timelike singularity, (2.13) forces $\dot\tau$ to go to zero. Thus τ approaches a constant near the singularity. In terms of a new coordinate $z=r-2M/h\tau$ which vanishes at the singularity and is negative near it, the geodesic equation (2.14) approaches

$$\ddot z-\frac{\dot z^2}{z}=0\ , \qquad (2.15)$$

where we have dropped terms that go to zero at the singularity. This equation can easily be solved, to get $z=-e^{-\lambda}$. Thus, null geodesics never reach the timelike singularity. Instead, they approach it exponentially slowly. Both $\dot\tau\rightarrow 0$ and $\dot r\rightarrow 0$ for null geodesics near the singularity.

Timelike geodesics are defected before they reach the singularity. Consider (2.14) with $\kappa=1$. Since $-h\tau+2M/r>0$, the κ piece contributes an extra negative acceleration compared to a null geodesic. Thus instead of $\dot r\rightarrow 0$, $\dot r$ becomes negative before reaching the singularity, and the timelike geodesics move away from the timelike singularity. Thus neither timelike nor null geodesics ever reach the timelike singularity, and the spacetime is causally geodesically complete (for outgoing geodesics).

For spacelike geodesics, it follows from (2.13) that $\dot r>1$ everywhere. Since the singularity is at finite r, this implies that all outgoing spacelike geodesics reach the singularity in finite affine parameter, and the spacetime is spacelike geodesically incomplete.

Since the theory (1.4) expressed in terms of the string metric is related to the one expressed in terms of the Einstein metric (1.5) by a field redefinition, they should represent the same physics. Yet, in one metric the $r\tau=2M/h$ singularity is at infinity, and in the other metric it is not. So does this solution have a physical naked singularity? If additional matter is present and minimally coupled to the Einstein metric, the answer is clearly yes. This matter can propagate from the singularity in finite time. Even without additional matter, one can argue that gravitational waves will always propagate with respect to the Einstein metric, and hence one could see the effects of the singularity this way.

III. GENERALIZATIONS OF THE SINGLE MASS RESULTS

The multimass solution in the Einstein metric, Eqs. (1.2) and (1.8), has features similar to the single mass solution discussed in the previous section. For $\tau<0$ the solution describes a number of extremal black holes in a contracting universe. Since U is a decreasing function of τ, the proper distance between the black holes,

$$l=\int_{r_i}^{r_j}\sqrt{U}\,dl\ , \qquad (3.1)$$

is also a decreasing function of τ (in the string metric, the proper distance between the masses remains constant). Thus the black holes are approaching each other. For large r the solution reduces to the single mass solution with $M=\sum M_i$. So as before, when $\tau=0$ a timelike singularity appears at $r=\infty$ and moves in toward smaller r. This singularity is located at $U=0$. So as τ increases, it eventually splits, and surrounds each of the masses separately.

The multimass solution can also be extended to spacetimes with dimension $d>4$ [9]. The solution in the Einstein metric is again given by (1.8), but with a generalized U given by

$$U=-h\tau+\sum_i\frac{2M_i}{|\mathbf{r}-\mathbf{r}_i|^{d-3}}\ . \qquad (3.2)$$

The higher dimension solution is qualitatively similar to the four dimension case. The geodesic equation is again (2.5), so the spacetime again has a naked timelike singularity. In the string metric, null geodesics near the singularity still approach (2.15), so even in higher dimensions, null geodesics in the string metric do not reach the singularity.

We have considered only masses with electric charge. It turns out that adding magnetically charged extremal black holes to the collapsing universe (2.2) does not create naked singularities. As mentioned earlier, the magnetically charged solution cannot be obtained by applying a simple duality transformation to (2.3). However, it can be obtained by recalling that the string metric for the $h=0$ solution is

$$d\tilde{s}^2 = -d\tilde{t}^2 + \tilde{U}^2 d\mathbf{r}\cdot d\mathbf{r} \ , \tag{3.3}$$

where $\tilde{U}$ is given by (1.3), and the dilaton is $e^{2\phi}=\tilde{U}$. In other words it is simply the product of time and a three-dimensional spatial solution. Since the action (1.4) comes from the condition of two-dimensional conformal invariance, and the product of conformal field theories is again a conformal field theory, it follows that a solution with $h\neq 0$ can be obtained by taking the product of the same spatial solution and a linear dilaton solution in $\tilde{t}$. Thus the $h\neq 0$ solution has the same string metric (3.3), and the dilaton given by $e^{2\phi}=e^{h\tilde{t}}\tilde{U}$ [9]. The string metric remains static and nonsingular. Rescaling back to the Einstein metric and introducing the new time coordinate $t=-(2/h)e^{-h\tilde{t}/2}$ yields

$$ds^2 = -\frac{dt^2}{\tilde{U}} + \frac{1}{4}h^2 t^2 \tilde{U}\, d\mathbf{r}\cdot d\mathbf{r} \ . \tag{3.4}$$

For large r, this metric resembles the electrically charged solution with $\tau<0$. However, now the null singularity at $t=0$ remains, and is not converted into a timelike singularity.

IV. CONCLUSIONS

We have discussed a family of solutions to the Einstein-Maxwell-dilaton theory (1.5) which can be viewed as the dilatonic analogue of the recently discovered Kastor-Traschen solution. The solution describes an arbitrary number of extremal electrically charged black holes in a contracting universe. In the absence of the black holes, the universe collapses to a null singularity. But when a single extremal black hole is added, the null singularity turns into a timelike one which is naked. This suggests that cosmic censorship may fail for this theory. To further explore whether this is the case, it would be useful to find the solution describing nonextremal charged black holes in the contracting universe. One could then verify that the naked singularity which arises at large r is independent of the details of the solution at small r. It is also important to study the stability of these solutions. This may depend on the boundary conditions imposed at infinity. Clearly, perturbations of compact support cannot affect the formation of the naked singularity. Physically reasonable boundary conditions should allow one to add black holes of arbitrary mass and charge to the spacetime.

Since the theory we are considering is the low energy limit of string theory (with a central charge), one is led to ask whether cosmic censorship is violated in string theory. Unfortunately, we can say very little at this time. In the context of string theory, the region of the solution near $U=0$ cannot be trusted. Since the curvature is becoming large, higher order α' corrections will be important. Since string theory is in principle a complete quantum theory of gravity, a breakdown of cosmic censorship does not imply a breakdown of the theory, but simply that quantum corrections must be included. Indeed, in our solutions, the string coupling $g=e^{\phi}$ is becoming large near the naked singularity and string loop corrections will be important.

Finally, it is worth remarking that the dilaton may not be necessary to create the naked singularities we have found here. If one considers Einstein's equation with a perfect fluid source having equation of state $p=-\rho/3$, one has the solution (2.2) with its null singularity. If we add a black hole to this solution, does the singularity become timelike?

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