RAPID COMMUNICATIONS

PHYSICAL REVIEW D

PARTICLES AND FIELDS

THIRD SERIES, VOLUME 48, NUMBER 11 1 DECEMBER 1993

RAPID COMMUNICATIONS

Rapid Communications are intended for important new results which deserve accelerated publication, and are therefore given priority in editorial processing and production. A Rapid Communication in **Physical Review D** *should be no longer than five printed pages and must be accompanied by an abstract. Page proofs are sent to authors, but because of the accelerated schedule, publication is generally not delayed for receipt of corrections unless requested by the author.*

QCD fragmentation functions for B_c and B_c^* production

Eric Braaten and Kingman Cheung

Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60208

Tzu Chiang Yuan

Davis Institute for High Energy Physics, Department of Physics, University of California, Davis, California 95616

(Received 29 April 1993)

The dominant production mechanism for $\bar{b}c$ bound states with a large transverse momentum in high energy processes is the production of a high energy $\bar{b}$ antiquark, followed by its fragmentation into the $\bar{b}c$ state. We calculate the fragmentation functions for the production of the S-wave states B_c and B_c^* to leading order in the QCD coupling constant. The fragmentation probabilities for $\bar{b} \to B_c$ and $\bar{b} \to B_c^*$ are approximately 3.8×10^{-4} and 5.4×10^{-4}, while those for $c \to B_c$ and $c \to B_c^*$ are smaller by two orders of magnitude.

PACS number(s): 13.85.Ni, 12.38.Bx, 14.40.Jz

The study of the charmonium and bottomonium systems has played an important role in the development and eventual acceptance of quantum chromodynamics (QCD) as the field theory of the strong interactions. Since bound states containing top quarks may not exist as identifiable states, the final frontier for the study of heavy-quark–antiquark bound states may be the $\bar{b}c$ system. To assess the prospects for the experimental study of $\bar{b}c$ mesons in present and future colliders, it is important to have accurate predictions for their production rates. The dominant production mechanism for $\bar{b}c$ mesons in the large transverse momentum region is the fragmentation of $\bar{b}$ antiquarks. The fragmentation process is described by a universal fragmentation function $D(z)$ that gives the probability for the splitting of the parton into the desired hadron plus other partons. It has recently been shown that the fragmentation functions for heavy quarkonia can be calculated using perturbative QCD [1]. The fragmentation functions for the splitting of gluons and charm quarks into S-wave charmonium states have been calculated to leading order in the QCD coupling constant α_s [1, 2]. In this paper, we calculate the fragmentation functions for the splitting of $\bar{b}$ into the $^1S_0$ ground state B_c and into the $^3S_1$ state B_c^* of the $\bar{b}c$ system. These universal fragmentation functions can be used to calculate the direct production rate of B_c and B_c^* at large transverse momentum in any high-energy process.

We begin by summarizing previous calculations of the production rates for $\bar{b}c$ mesons. A perturbative QCD calculation of the cross section for $e^+e^- \to B_c^* b\bar{c}$ was carried out by Clavelli in 1982 [3]. The cross sections for $e^+e^- \to B_c b\bar{c}$ and $e^+e^- \to B_c^* b\bar{c}$ were subsequently calculated incorrectly by Amiri and Ji [4] and then in the case of the B_c^* correctly by Chang and Chen [5]. Chang and Chen followed Amiri and Ji in expressing their results in terms of fragmentation functions $D(z)$, but they did not realize that these fragmentation functions are universal and can also be applied to the hadronic production of B_c and B_c^*. The inclusive production rates for B_c in e^+e^- and hadron colliders, including those that are produced by cascade decays from B_c^* and other $\bar{b}c$ bound states, have been estimated using a modified version of the parton shower Monte Carlo HERWIG [6]. The modified version included two new parameters describing the probabilities for the production of $c\bar{c}$ pairs, and the results were sensitive to these parameters as well as to the values of the bottom and charm quark masses. The ratio of the cross sections for inclusive B_c production and b-quark production was estimated to be 1.0×10^{-3} for e^+e^- collisions at the CERN e^+e^- collider LEP and 1.3×10^{-3} for $p\bar{p}$ collisions at the Fermilab Tevatron, consistent with a previous crude estimate [7] of the probability for a $\bar{b}$ to form a $\bar{b}c$ meson. Given the adjustable parameters in the Monte Carlo calculation of Ref. [6], it is difficult to estimate the errors in its predictions. It is clear that more reliable calculations of the production rates of $\bar{b}c$ mesons would be valuable in assessing the prospects for the discovery and investigation of these new particles.

Fragmentation dominates the direct production of B_c at sufficiently large transverse momentum p_T. The fragmentation contribution to the differential cross section for the direct production of a B_c with four-momentum p has the general form

$$d\sigma(B_c(p)) = \sum_i \int_0^1 dz\, d\widehat{\sigma}(i(p/z),\mu)\, D_{i\to B_c}(z,\mu) \,, \tag{1}$$

where the sum is over partons of type i, z is the longitudinal momentum fraction of the B_c relative to the parton, and μ is a factorization scale. The physical interpretation of (1) is that a B_c with momentum p can be produced by first producing a parton i of larger momentum p/z which subsequently splits into a B_c carrying a fraction z of the parton energy. The fragmentation functions $D_{i\to B_c}(z,\mu)$ satisfy the evolution equations [8]

$$\mu\frac{\partial}{\partial\mu} D_{i\to B_c}(z,\mu) = \sum_j \int_z^1 \frac{dy}{y}\, P_{i\to j}(z/y,\mu)\, D_{j\to B_c}(y,\mu) \,, \tag{2}$$

where $P_{i\to j}(x,\mu)$ is the Altarelli-Parisi function for the splitting of the parton of type i into a parton of type j with longitudinal momentum fraction x. For example, the $\bar{b}\to\bar{b}$ splitting function for a bottom antiquark with energy much greater than its mass is the usual splitting function for quarks and antiquarks:

$$P_{\bar{b}\to\bar{b}}(x,\mu) = \frac{4\alpha_s(\mu)}{3\pi}\left(\frac{1+x^2}{1-x}\right)_+ \,, \tag{3}$$

where $f(x)_+ = f(x) - \delta(1-x)\int_0^1 f(x')dx'$. The boundary condition on the evolution equation (2) is the initial fragmentation function $D_{i\to B_c}(z,\mu_0)$ at some scale μ_0 of order m_b. The initial fragmentation function can be calculated perturbatively as a series in $\alpha_s(2m_c)$.

The initial fragmentation function $D_{c\to\eta_c}(z,3m_c)$ for production of the ${}^1S_0$ charmonium state η_c was calculated to leading order in α_s in Ref. [2]. The initial fragmentation function for $\bar{b}\to B_c$ can be obtained by extending the calculation of Ref. [2] to the case of unequal quark masses. It is given by the expression

$$D_{\bar{b}\to B_c}(z) = \frac{1}{16\pi^2}\int ds\, \theta\left(s - \frac{(m_b+m_c)^2}{z} - \frac{m_c^2}{1-z}\right) \times \lim_{q_0\to\infty}\frac{|\mathcal{M}|^2}{|\mathcal{M}_0|^2} \,, \tag{4}$$

where $\mathcal{M}$ is the matrix element for the production of a B_c and $\bar{c}$ with total four-momentum q and invariant mass $s=q^2$ and $\mathcal{M}_0$ is the matrix element for the production of a $\bar{b}$ with the same three-momentum $\mathbf{q}$. If the momentum of the B_c is $p=(p_0,p_1,p_2,p_3)$ in a frame where $q=(q_0,0,0,q_3)$, then the longitudinal momentum fraction z is defined by $z=(p_0+p_3)/(q_0+q_3)$. The lower limit on s in (4) follows from the expression $s=[(m_b+m_c)^2+p_\perp^2]/z+(m_c^2+p_\perp^2)/(1-z)$, where $\mathbf{p}_\perp=(p_1,p_2)$ is the transverse momentum. The limit $q_0\to\infty$ in (4) can be taken in any Lorentz frame where $q_3/q_0\to 1$. It is convenient to carry out the calculation in the axial gauge associated with the four-vector $n=(1,0,0,-1)$. In this gauge, the fragmentation contribution comes only from the Feynman diagram for $\bar{b}^*\to\bar{b}c\bar{c}$ shown in Fig. 1. The amplitude for Fig. 1 in this gauge can be reduced to

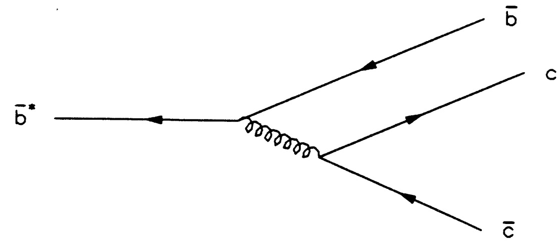

FIG. 1. Feynman diagram for $\bar{b}^*\to\bar{b}c\bar{c}$ which contributes to the fragmentation of $\bar{b}$ into B_c and B_c^*. The outgoing momenta are $(1-r)p$, rp, and p' for the $\bar{b}$, c and $\bar{c}$, respectively.

$$\mathcal{M} = \frac{g_s^2 R(0)}{3\sqrt{3\pi M}}\frac{1}{r(s-m_b^2)^2}\,\bar{\Gamma}\left(2\,(\not{q}+m_b)\,(\not{p}-2M)\,\gamma_5 - \frac{s-m_b^2}{n\cdot[q-(1-r)p]}\,(\not{p}+M)\,\gamma_5\,\not{n}\right) v(p') \,, \tag{5}$$

where $r=m_c/(m_b+m_c)$, $M=m_b+m_c$, and $R(0)$ is the nonrelativistic radial wave function at the origin for the B_c. We have projected the amplitude for the production of $\bar{b}$ and c with four-momenta $(1-r)p$ and rp onto the amplitude for a ${}^1S_0$ state with four-momentum p. The Dirac spinor $\bar{\Gamma}$ is the matrix element for the production of a $\bar{b}$ of momentum $q=p+p'$. Its explicit form is not needed to calculate the fragmentation function (4). Summing over final spins and colors, the square of the matrix element (5) reduces to

$$|\mathcal{M}|^2 = \frac{64\pi\alpha_s^2|R(0)|^2}{27M}\frac{1}{r^2(s-m_b^2)^4}\mathrm{tr}\left(\Gamma\,\bar{\Gamma}\,\Delta\right) \,, \tag{6}$$

where Δ is a Dirac matrix that depends on q and p. We need only keep the terms in Δ that are of order $m_b^4 q_0$. While q and p both have components of order q_0, the invariant mass $s=q^2$ is of order m_b^2 in the fragmentation region. Simplifying the Dirac matrix Δ by dropping terms which are suppressed by powers of m_b/q_0, it reduces to

$$\begin{aligned}\Delta = &\left[s^2 - 2(2-4r+r^2)M^2 s + (1-r)(3-r)(1-4r+r^2)M^4\right] \not{q} - \left[s-(2-r)^2M^2\right]\left[s-(1-r)^2M^2\right] \not{p} \\ &+2\,\frac{n\cdot p}{n\cdot q-(1-r)n\cdot p}\left[s-(1-r)^2M^2\right]^2 (\not{q}-\not{p}) \\ &-2\,\frac{n\cdot p}{n\cdot q-(1-r)n\cdot p} M^2\left[s-(1-r)^2M^2\right]\left[(1-2r)\,\not{q}\; - \;(1-r)\,\not{p}\right] \\ &+\frac{n\cdot p\; n\cdot(q-p)}{[n\cdot q-(1-r)n\cdot p]^2}\left[s-(1-r)^2M^2\right]^2 \not{p}\,. \end{aligned} \tag{7}$$

Since the coefficients of $\not{p}$ and $\not{q}$ in (7) are all manifestly of order m_b^4, we can make the substitution $p = zq$ which is accurate to leading order in m_b/q_0. The Dirac trace in (7) is then proportional to $\mathrm{tr}(\Gamma\bar{\Gamma}\not{q})$. The square of the matrix element for the production of a real $\bar{b}$ of momentum q is $|\mathcal{M}_0|^2 = \mathrm{tr}[\Gamma\bar{\Gamma}(\not{q}-m_b)]$, which reduces in the fragmentation limit to $\mathrm{tr}(\Gamma\bar{\Gamma}\not{q})$. Dividing $|\mathcal{M}|^2$ in (6) by $|\mathcal{M}_0|^2$ and inserting it into (4), we obtain the fragmentation function as an integral over s:

$$\begin{aligned} D_{\bar{b}\to B_c}(z) \;=\; &\frac{4\alpha_s^2|R(0)|^2}{27\pi m_c^3}\int ds\,\theta\!\left(s-\frac{M^2}{z}-\frac{m_c^2}{1-z}\right)\left(\frac{(1-z)(1+rz)^2 rM^2}{[1-(1-r)z]^2(s-m_b^2)^2}\right. \\ &\left. -\;\frac{[2(1-2r)-(3-4r+4r^2)z+(1-r)(1-2r)z^2]rM^4}{[1-(1-r)z](s-m_b^2)^3} \;-\; \frac{4r^2(1-r)M^6}{(s-m_b^2)^4}\right). \end{aligned} \tag{8}$$

Evaluating the integral over s, we obtain our final expression for the initial fragmentation function:

$$\begin{aligned} D_{\bar{b}\to B_c}(z, m_b+2m_c) \;=\; &\frac{2\alpha_s(2m_c)^2|R(0)|^2}{81\pi m_c^3}\,\frac{rz(1-z)^2}{[1-(1-r)z]^6}\Big(6 \;-\; 18(1-2r)z \\ &+(21-74r+68r^2)z^2 \;-\; 2(1-r)(6-19r+18r^2)z^3 \;+\; 3(1-r)^2(1-2r+2r^2)z^4\Big)\,, \end{aligned} \tag{9}$$

where $r = m_c/(m_b+m_c)$. Setting $m_b = m_c$ in (9), we recover the fragmentation function for $c\to\eta_c$ calculated in Ref. [2]. Our result (9) disagrees with that of Chang and Chen [5]. Our result can be obtained up to the overall normalization by changing the sign of $F_1^{(p)}$ in Eq. (13) of Ref. [5]. Chang and Chen normalized their fragmentation functions so that they integrated to 1: $\int_0^1 dz D(z) = 1$. The incorrect result of Amiri and Ji [4] can be obtained by keeping only the term proportional to $1/(s-m_b^2)^2$ in the integrand of (8).

The calculation described above gives the fragmentation function at a scale μ_0 of order m_c. We have therefore set the scale of the running coupling constant in (9) to $2m_c$, which is the minimum value of the invariant mass of the virtual gluon in Fig. 1. It has recently been shown by Jaffe and Randall using heavy quark effective theory methods that the evolution from the scale m_c to the scale m_b is trivial at leading order in $1/m_b$ [11]. In other words, any logarithms of m_b/m_c that are generated by higher order radiative corrections are suppressed by powers of m_c/m_b. We have therefore set the initial scale of the fragmentation function in (9) to m_b+2m_c, which is the minimum value of the invariant mass of the fragmenting $\bar{b}$. This particular choice allows the correct results for the fragmentation of the charm quark into B_c to be obtained from (9) simply by interchanging m_b and m_c. In Ref. [5], Chang and Chen used the value $\alpha_s = 0.15$ for the running coupling constant, which corresponds to a scale μ of order M_Z. Our analysis shows that the scale for the running coupling constant is definitely set by the charm quark mass, and it has the value $\alpha_s(2m_c) = 0.26$. Thus Chang and Chen underestimated the rates for production of B_c and B_c^* in Z^0 decay by about a factor of 3.

The fragmentation function for a $\bar{b}$ to split into the $^3S_1$ state B_c^* can be calculated in the same way as for B_c. The matrix element for the production of $B_c^*\bar{c}$ is

$$\begin{aligned}\mathcal{M} \;=\; &\frac{g_s^2 R(0)}{3\sqrt{3\pi M}}\epsilon_\mu(p)^*\frac{1}{r(s-m_b^2)^2}\,\bar{\Gamma}\left(2M\,(\not{q}+m_b)\,\gamma^\mu\right. \\ &\left.+\;\frac{s-m_b^2}{n\cdot[q-(1-r)p]}\,(\not{p}+M)\,\gamma^\mu\,\not{n}\right) v(p')\,, \end{aligned} \tag{10}$$

where $\epsilon_\mu(p)$ is the polarization vector for B_c^*. Following the same path as in the B_c calculation, we obtain an expression for the fragmentation function as an integral over the invariant mass s:

$$\begin{aligned} D_{\bar{b}\to B_c^*}(z) \;=\; &\frac{4\alpha_s^2|R(0)|^2}{27\pi m_c^3}\int ds\,\theta\!\left(s-\frac{M^2}{z}-\frac{m_c^2}{1-z}\right)\left(\frac{(1-z)(1+2rz+(2+r^2)z^2)rM^2}{[1-(1-r)z]^2(s-m_b^2)^2}\right. \\ &\left. -\;\frac{[2(1+2r)-(1+12r-4r^2)z-(1-r)(1+2r)z^2]rM^4}{[1-(1-r)z](s-m_b^2)^3} \;-\; \frac{12r^2(1-r)M^6}{(s-m_b^2)^4}\right). \end{aligned} \tag{11}$$

Integrating over s, the final result for the fragmentation function for $\bar{b}$ to split into B_c^* is

$$D_{\bar{b}\to B_c^*}(z, m_b+2m_c) = \frac{2\alpha_s(2m_c)^2|R(0)|^2}{27\pi m_c^3}\frac{rz(1-z)^2}{[1-(1-r)z]^6}\Big(2 - 2(3-2r)z +3(3-2r+4r^2)z^2 - 2(1-r)(4-r+2r^2)z^3 + (1-r)^2(3-2r+2r^2)z^4\Big) . \quad (12)$$

This agrees with the result of Chang and Chen [5] up to the overall normalization. Setting $m_b = m_c$ in (12), we recover the fragmentation function for $c \to \psi$ calculated in Ref. [2]. The incorrect result of Amiri and Ji [4] can be obtained by keeping only the term proportional to $1/(s-m_b^2)^2$ in the integrand of (11).

The fragmentation functions for $\bar{b} \to B_c$ and $\bar{b} \to B_c^*$ are shown in Fig. 2. The radial wave function at the origin for the B_c and B_c^* has been estimated from potential models to be $|R(0)|^2 = (1.18\text{ GeV})^3$ [9]. For the quark masses, we use $m_b = 4.9$ GeV and $m_c = 1.5$ GeV. The solid curves in Fig. 2 are the initial fragmentation functions at the scale $\mu = 7.9$ GeV. The dotted curves are the fragmentation functions with the scale μ evolved up to 79 GeV using the Altarelli-Parisi equation (2). The average value of the momentum fraction for $\bar{b} \to B_c$ is $\langle z\rangle = 0.68$ at $\mu = 7.9$ GeV, and it shifts downward to $\langle z\rangle = 0.56$ when the scale is evolved up by a factor of 10. The process $\bar{b} \to B_c^*$ has a slightly harder distribution with $\langle z\rangle = 0.73$ at $\mu = 7.9$ GeV and $\langle z\rangle = 0.60$ at $\mu = 79$ GeV. The evolution of the fragmentation functions would shift the momentum distributions for the B_c and B_c^* produced in Z^0 decay calculated by Chang and Chen [5] to slightly smaller values of z.

Since $\int_0^1 dz P_{\bar{b}\to\bar{b}}(z) = 0$ at leading order in α_s, the evolution equation (2) implies that the fragmentation probability $\int_0^1 dz D(z,\mu)$ does not evolve with the scale μ. The evolution merely shifts the z distribution to smaller values of z. Therefore the fragmentation probabilities can be used to provide simple estimates of production rates. We need only multiply the production rates for bottom

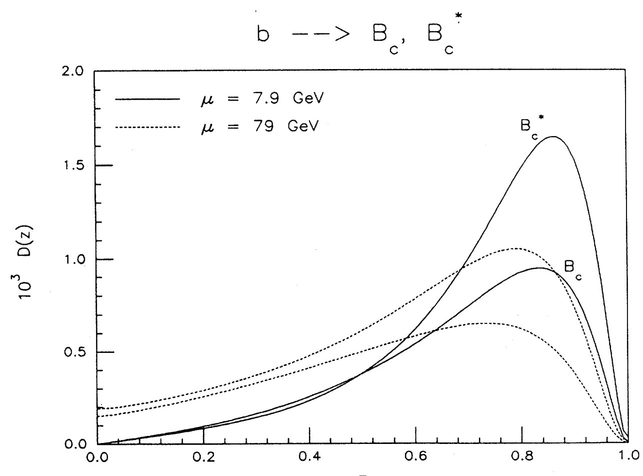

FIG. 2. Fragmentation functions $D_{\bar{b}\to B_c}(z,\mu)$ and $D_{\bar{b}\to B_c^*}(z,\mu)$ as a function of z at $\mu = 7.9$ GeV (solid line) and $\mu = 79$ GeV (dotted line).

and charm quarks with p_T much larger than their masses by the appropriate fragmentation probabilities. The fragmentation probability for the production of the B_c is

$$\int_0^1 dz\, D_{\bar{b}\to B_c}(z, m_b+2m_c) = \frac{2\alpha_s(2m_c)^2|R(0)|^2}{27\pi m_c^3}\, f\left(\frac{m_c}{m_b+m_c}\right) , \quad (13)$$

where the function $f(r)$ is

$$f(r) = \frac{8+13r+228r^2-212r^3+53r^4}{15(1-r)^5} + \frac{r(1+8r+r^2-6r^3+2r^4)}{(1-r)^6}\ln(r) . \quad (14)$$

This function has the value 0.496 at $r = 0.23$. The fragmentation probability for the production of the B_c^* is

$$\int_0^1 dz\, D_{\bar{b}\to B_c^*}(z, m_b+2m_c) = \frac{2\alpha_s(2m_c)^2|R(0)|^2}{27\pi m_c^3}\, g\left(\frac{m_c}{m_b+m_c}\right) , \quad (15)$$

where the function $g(r)$ is

$$g(r) = \frac{24+109r-126r^2+174r^3+89r^4}{15(1-r)^5} + \frac{r(7-4r+3r^2+10r^3+2r^4)}{(1-r)^6}\ln(r) . \quad (16)$$

It has the value 0.702 at $r = 0.23$. The fragmentation probability (15) agrees with the result for $e^+e^- \to B_c^* b\bar{c}$ [3] obtained by Clavelli. The numerical values for the fragmentation probabilities are 3.8×10^{-4} for $\bar{b} \to B_c$ and 5.4×10^{-4} for $\bar{b} \to B_c^*$. The fragmentation probabilities for a charm quark to split into B_c and B_c^* can be obtained from (13) and (15) by interchanging the masses m_b and m_c. Taking $\alpha_s(2m_b) = 0.18$, their numerical values are 5.0×10^{-6} for $c \to B_c$ and 4.3×10^{-6} for $c \to B_c^*$. The fragmentation probabilities for c are smaller than those for $\bar{b}$ by about two orders of magnitude, because they are proportional to $1/m_b^3$ instead of $1/m_c^3$.

It is interesting to compare our perturbative QCD fragmentation functions with the phenomenological Peterson fragmentation function [10] which is often used to describe the fragmentation of a b quark into a hadron H:

$$D_{b\to H}(z) = N_H\frac{z(1-z)^2}{[(1-z)^2+\epsilon z]^2} , \quad (17)$$

where $\epsilon = m_c^2/m_b^2$ for the case of a $\bar{b}c$ hadron H and N_H is a phenomenological parameter. Carefully taking the limit $r \to 0$ in (9), the perturbative QCD fragmentation function reduces to

$$D_{b\to B_c}(z, m_b + 2m_c) \to \frac{2\alpha_s(2m_c)^2|R(0)|^2}{81\pi m_c^3}\frac{(y-1)^2}{r}\left(\frac{8}{y^6}+\frac{4}{y^5}+\frac{3}{y^4}\right), \quad (18)$$

where $y = (1/z - (1-r))/r$. The fragmentation function for $b \to B_c^*$ given in (12) has the same form in this limit except that its normalization is larger by a factor of 3, as required by heavy quark spin symmetry. The width of the z distribution in (18) scales like $1/m_b$ in agreement with the heavy quark effective theory analysis [11]. The width of the Peterson distribution (17) also scales like $1/m_b$, contrary to the statement in Ref. [10].

The only inputs into our perturbative QCD calculations of the fragmentation functions are the QCD coupling constant α_s, the quark masses m_b and m_c, and the nonrelativistic radial wavefunction at the origin $R(0)$. The potential model values for $R(0)$ should be quite reliable, since these models are tuned to reproduce the spectra of the charmonium and bottomonium systems and the $\bar{b}c$ system represents an intermediate case. Potential models should also give accurate results for ratios of quark masses such as $r = m_c/(m_b + m_c)$. The largest uncertainty in the fragmentation functions for $\bar{b} \to B_c, B_c^*$ should therefore come from the overall factor of $1/m_c^3$. Allowing for a variation of 0.2 GeV in the value $m_c = 1.5$ GeV, the uncertainty in the normalization is an overall multiplicative factor of 1.5.

The fragmentation functions $D_{\bar{b}\to B_c}(z,\mu)$ and $D_{c\to B_c}(z,\mu)$ can be used to calculate the rate for the direct production of B_c at large transverse momentum in any high energy process. To obtain the total production rate for B_c, we also need the direct production rates for all the other $\bar{b}c$ states below the BD threshold, since they all decay ultimately into B_c with branching fractions near 100%. We have calculated the fragmentation function for the production of the $^3S_1$ state B_c^*. Our fragmentation functions can also be applied directly to the production of the first radial excitations of the B_c and B_c^* simply by changing the wave function at the origin. From potential model calculations [9], the value for the first radial excitation of the S-wave states is $|R(0)|^2 = (0.99\text{ GeV})^3$. Thus the production rates for these states should be about 60% of those for the B_c and B_c^*. The second radial excitation is probably above the BD threshold, so its branching fraction into B_c will be negligible. Adding up all the S-wave contributions to the B_c production rate, the total fragmentation probabilities are about 1.5×10^{-3} from the splitting of the $\bar{b}$ and 1.5×10^{-5} from the splitting of the charm quark. Our lower bound of 1.5×10^{-3} for the probability for a high-energy b quark to form a $\bar{b}c$ bound state is consistent with the Monte Carlo estimates obtained in Ref. [6].

An accurate calculation of the total production rate for B_c could probably be obtained by including also the two sets of P-wave $\bar{b}c$ states below BD threshold. In calculating the production of the P-wave states, there are two distinct contributions that must be included at leading order in $\alpha_s(m_c)$. The $\bar{b}$ and c which form the P-wave meson can be produced either in a color-singlet P-wave state or in a color-octet S-wave state [12]. There are therefore two nonperturbative parameters that enter into the calculation. In addition to the derivative of the radial wave function at the origin for the P-wave bound state, we require also the probability density at the origin for a $\bar{b}$ and a c in a color-octet S-wave state to make a transition to the P-wave bound state by radiating a soft gluon.

Our results should be useful for assessing the prospects for the discovery of the B_c in Z^0 decay at LEP or in $p\bar{p}$ collisions at the Tevatron. The branching fraction for $Z^0 \to b\bar{b}$ is approximately 15%. Multiplying by the total fragmentation probability for the production of S-wave $\bar{b}c$ states, we obtain a lower bound on the inclusive branching fraction for the production of B_c of 2.3×10^{-4}. The contribution from $Z^0 \to c\bar{c}$ followed by fragmentation of the charm quark or antiquark is smaller by two orders of magnitude. At the Tevatron, the cross section for the production of charm quarks at large p_T is a little larger than for bottom quarks, but not enough to make up for the difference in the fragmentation probabilities. Thus the dominant production mechanism for $\bar{b}c$ states will again be $\bar{b}$ fragmentation. An order of magnitude estimate of the cross section for the inclusive production of B_c with $p_T > 20$ GeV at the Tevatron can be obtained by multiplying the cross section for the production of b quarks with $p_T > 20$ GeV, which is measured to be about 400 nb [13], by the fragmentation probability of 1.5×10^{-3}. Explicit calculations using parton distributions and realistic rapidity cuts give similar results [14].

We have pointed out in this paper that the dominant mechanism for the direct production of high-energy $\bar{b}c$ mesons is fragmentation, the production of a high energy $\bar{b}$ followed by its splitting into the the $\bar{b}c$ state. We calculated the fragmentation functions $D(z)$ for production of the S-wave states B_c and B_c^* to leading order in α_s. These fragmentation functions are universal, so they can be used to calculate the rates for the direct production of B_c and B_c^* at large transverse momentum in any high-energy process.

This work was supported in part by the U.S. Department of Energy, Division of High Energy Physics, under Grants Nos. DE-FG02-91-ER40684 and DE-FG03-91-ER40674, and also by the Texas National Research Laboratory Commission, Grant No. RGFY93-330.

[1] E. Braaten and T.C. Yuan, Phys. Rev. Lett. **71**, 1673 (1993).
[2] E. Braaten, K. Cheung, and T.C. Yuan, Phys. Rev. D **48**, 4230 (1993).
[3] L. Clavelli, Phys. Rev. D **26**, 1610 (1982).
[4] C.-R. Ji and F. Amiri, Phys. Rev. D **35**, 3318 (1987); F. Amiri and C.-R. Ji, Phys. Lett. B **195**, 593 (1987).
[5] C.-H. Chang and Y.-Q. Chen, Phys. Lett. B **284**, 127

(1992); Phys. Rev. D **46**, 3845 (1992).

[6] M. Lusignoli, M. Masetti, and S. Petrarca, Phys. Lett. B **266**, 142 (1991).

[7] S.S. Gershtein, V.V. Kiselev, A.K. Likhoded, S.R. Slabospitsky, and A.V. Tkabladze, Yad Fiz. **48**, 575 (1988) [Sov. J. Nucl. Phys. **48**, 327 (1988)]; V.V. Kiselev, A.K. Likhoded, S.R. Slabospitsky, and A.V. Tkabladze, *ibid.* **49**, 1100 (1988) [**49**, 682 (1988)].

[8] J.C. Collins and G. Sterman, Nucl. Phys. **B185**, 172 (1981); G. Curci, W. Furmanski, and R. Petronzio, *ibid.* **B175**, 27 (1980); J.C. Collins and D.E. Soper, *ibid.* **B194**, 445 (1982).

[9] E. Eichten and C. Quigg, Fermilab report (in preparation).

[10] C. Peterson, D. Schlatter, I. Schmitt, and P.M. Zerwas, Phys. Rev. D **27**, 105 (1983).

[11] R.L. Jaffe and L. Randall, MIT Report No. CTP#2189, 1993 (unpublished).

[12] G.T. Bodwin, E. Braaten, G.P. Lepage, and T.C. Yuan, Phys. Rev. D **46**, R3703 (1992).

[13] CDF Collaboration, F. Abe *et al.*, Phys. Rev. Lett. **68**, 3403 (1992); Phys. Rev. Lett. **69**, 3704 (1992); M.L. Mangano, Z. Phys. C **58**, 651 (1993).

[14] K. Cheung, Phys. Rev. Lett. (to be published).