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Is $f_0(975)$ a narrow resonance?

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We use a new approach to study the $\pi\pi$, $K\bar{K}$ $I=J=0$ interaction from information on the central production and elastic processes with energies from the $\pi\pi$ threshold to 1.7 GeV. Our amplitude analysis separates the pole term for $f_0(975)$ from the background term, meanwhile enforcing unitarity. A second-sheet pole ($988-23i$ MeV) and a third-sheet pole ($797-185i$ MeV) are found for $f_0(975)$. We conclude that if the $f_0(975)$ is a Breit-Wigner resonance, it has a large decay width corresponding to its third-sheet pole, but shows up as a narrow structure in $\pi\pi$ and $K\bar{K}$ invariant mass spectra due to its second-sheet pole. An alternative K-matrix analysis is also briefly reported, which gives a similar conclusion.

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I. INTRODUCTION

After twenty years of controversial arguments, the nature of $f_0(975)$ is still not settled [1]. It has been ascribed as a conventional $q\bar{q}$ meson [2], a "unitarized remnant" of a $q\bar{q}$ state [3], a $K\bar{K}$ molecule [4], a multiquark state [5], a glueball [6], and/or a hybrid [7]. Many arguments favoring or disfavoring these assignments are related to the width or pole position(s) of the $f_0(975)$ [8–10].

In the Particle Data Group (PDG) tables [11] the width is given as 47 ± 9 MeV. But this value is rather misleading. Six analyses [12–17] were used by the PDG to determine both the mass and the width of $f_0(975)$. All of them assumed coupled channel Breit-Wigner formulas [18] for $f_0(975)$. For a two-channel Breit-Wigner (BW) resonance amplitude there is always a main (BW) pole on the third sheet of the complex energy plane and a shadow pole [BW(1)] on the second or fourth sheet (cf. [9,19] for sheet structure and definition). For most resonances the real and imaginary parts of the main (third-sheet) BW pole are taken as their mass and the half width. But for $f_0(975)$ the second-sheet BW(1) shadow pole was used for determining its mass and width [11–17].

It is true that the second-sheet pole is the nearest one to the physical region and determines the width of narrow structures in the $\pi\pi$ or $K\bar{K}$ invariant mass spectra. However, this width is not the real decay width determining the lifetime of the resonance. The reason is simple. If we assume the coupling of the f_0 to the $K\bar{K}$ channel is negligible, then the BW pole and BW(1) pole give the same width which is in fact the $\pi\pi$ decay width. When the coupling of the f_0 to the $K\bar{K}$ channel increases, the width given by the BW(1) pole decreases while the width given by the BW pole increases. Therefore the third-sheet main BW pole reflects the real decay width of the resonance and we should use this main BW pole as the mass and half width of the resonance.

Although the second-sheet pole for $f_0(975)$ is well determined by various analyses to be about the value in the PDG tables [11], the corresponding main BW pole is very model dependent. The analyses [12–17] only presented the second-sheet pole in their papers. Among these analyses some were limited to a data sample from a single experiment with very narrow energy range; the others did not consider the unitarity constraint. The third-sheet pole cannot be reliably determined in these analyses.

The most extensive analysis to date of all high statistics data with $\pi\pi$ and $K\bar{K}$ final states is by the AMP Collaboration [8] with a K-matrix formalism satisfying unitarity. An unexpected outcome of this analysis is the conclusion that the $f_0(975)$ most likely comprises two resonances—a fairly narrow object coupling to $\pi\pi$ and $K\bar{K}$ and a very narrow $K\bar{K}$ bound state coupling weakly to the $\pi\pi$ channel; all this is superposed on a background furnished by a very broad $f_0(\epsilon(1000))$. Recently Morgan and Pennington (MP) [9] have performed a new analysis with a new data sample, concluding that $f_0(975)$ is most probably not a $K\bar{K}$ molecule, nor an amalgam of two resonances, but a conventional Breit-Wigner-like structure with a very narrow width for both the second-sheet and third-sheet poles. But in their formalism, it is difficult to isolate a pole contribution from a slowly varying background, and their background terms are not general enough.

The aim of this paper is to find a general representation of the amplitude which allows the simple separation into a pole term for $f_0(975)$ and a background term, but meanwhile still satisfies unitarity. By using such a formalism we make a combined analysis of $\pi\pi$ scattering phase shifts and the central dipion production in high energy pp collisions for the energies from $\pi\pi$ threshold to 1.7 GeV.

We present our formalism in Sec. II and results and discussion in Sec. III.

II. FORMALISM

The starting point of our formalism is similar to the Dalitz-Tuan representation [19,20]. The Lorentz-invariant amplitudes for elastic scattering of the two coupled channels (1 for $\pi\pi$ and 2 for $K\bar{K}$) can be expressed as [19]

$$\hat{T}_{11}=\frac{e^{2i\phi}-1}{2i\rho_1}+\frac{g_1 e^{2i\phi}}{M_R^2-s-i(\rho_1 g_1+\rho_2 g_2)} , \quad (1)$$

$$\hat{T}_{12}=\frac{\sqrt{g_1 g_2}e^{i\phi}}{M_R^2-s-i(\rho_1 g_1+\rho_2 g_2)} , \quad (2)$$

$$\hat{T}_{22}=\frac{g_2}{M_R^2-s-i(\rho_1 g_1+\rho_2 g_2)} . \quad (3)$$

Here the background term is assumed to be coupled only to the $\pi\pi$ channel. This is roughly true for energies below 1.7 GeV. The $f_0(975)$ is assumed as a Breit-Wigner resonance with three free parameters g_1, g_2, and M_R. Also phase space factors $\rho_1=(1-4m_\pi^2/s)^{1/2}$, $\rho_2=(1-4m_K^2/s)^{1/2}$, and s is the invariant mass squared of the system.

For the background term in $\pi\pi$ elastic scattering, a general form satisfying the unitarity relation is [19]

$$\hat{T}_b=\frac{e^{2i\phi}-1}{2i\rho_1}=\frac{1}{A(s)-i\rho_1} , \quad (4)$$

where $A(s)$ is an arbitrary function real on the real energy axis outside dynamic (left-hand) cuts. To be general enough we assume

$$A(s)=\sum_n a_n s^n \Big/ \sum_n b_n s^n \quad \text{with } n=0,1,2,\ldots . \quad (5)$$

From chiral perturbation calculations [21,22], the lowest order for $\hat{T}_b$ near $\pi\pi$ threshold is proportional to $(s-m_\pi^2/2)$ (related to the so-called Adler zero condition); therefore, we have $b_0/b_1=-m_\pi^2/2$. In practice the terms with $n\geq 3$ in Eq. (5) are negligible. The final form for $A(s)$ in our analysis is taken as

$$A(s)=\frac{1+a_1 s+a_2 s^2}{b_1(s-m_\pi^2/2)+b_2 s^2} , \quad (6)$$

which gives

$$e^{2i\phi}=\frac{1+a_1 s+a_2 s^2+i\rho_1[b_1(s-m_\pi^2/2)+b_2 s^2]}{1+a_1 s+a_2 s^2-i\rho_1[b_1(s-m_\pi^2/2)+b_2 s^2]} . \quad (7)$$

Therefore for $\pi\pi$-$K\bar{K}$ coupled channel elastic scattering, we use the formulas of Eqs. (1)–(3) and (7), with seven free parameters. The relation between $\hat{T}_{11}$ and the $\pi\pi$ S-wave phase shift parameters (δ_0 and η_0) is

$$\hat{T}_{11}=\frac{\eta_0 e^{2i\delta_0}-1}{2i\rho_1} . \quad (8)$$

For the central production process $pp\to pp(\pi\pi)$ or $pp(K\bar{K})$, which may be interpreted as a double Pomeron reaction $PP\to\pi\pi(K\bar{K})$, the invariant amplitudes for $\pi\pi$ and $K\bar{K}$ production can be expressed as (cf. Refs. [8,9])

$$\hat{F}_1=\alpha_1(s)\hat{T}_{11}+\alpha_2(s)\hat{T}_{21} , \quad (9)$$

$$\hat{F}_2=\alpha_1(s)\hat{T}_{12}+\alpha_2(s)\hat{T}_{22} . \quad (10)$$

The functions $\alpha_i(s)$ describe the coupling of the initial state to channel i and should be real along the right-hand cut. Because the functions $\alpha_i(s)$ have only left-hand cuts, they should be smoothly varying functions along the right-hand cut. Since the production amplitudes $\hat{F}_i$ do not necessarily satisfy the Adler zero condition, the Adler zero should be removed by α_i. From these considerations we parametrize the $\alpha_i(s)$ as

$$\alpha_i(s)=\frac{\beta_{i0}}{s-m_\pi^2/2}+\frac{\beta_{i1}}{s+s_{i1}}+\gamma_{i0}+\gamma_{i1}s . \quad (11)$$

Here β_{ij},γ_{ij}, are limited to be real constant parameters and s_{i0} are positive constant parameters. In addition to the first term which is introduced to remove the Adler zero, we include another left-hand pole term. This is a major difference in our treatment of $\alpha_i(s)$ compared with Refs. [8,9] in which polynomials in s were used without a left-hand pole term. In principle we can include more left-hand pole terms and higher order polynomials, but we found that with four terms in Eq. (11) we can already reproduce the experimental data well.

With the above invariant amplitudes, we can calculate the experimental observables in $\pi\pi$-$K\bar{K}$ elastic scattering and central production processes just as in Refs. [8,9].

III. NUMERICAL RESULTS AND DISCUSSION

For the $\pi\pi$-$K\bar{K}$ $I=J=0$ interaction, the most systematic and extensive information comes from $\pi\pi$-$K\bar{K}$ elastic scattering phase shifts (δ_0 and η_0) determined by numerous analyses of dipion production experiments. For the present fit, in addition to the classic energy-independent analysis by the CERN-Munich group of their high-statistics experiment on $\pi^- p\to\pi^-\pi^+ n$ at 17 GeV/c [23] for $\pi\pi$ energies above 0.6 GeV, we extend the data set close to the $\pi\pi$ threshold by adding four data points for 0.32–0.6 GeV by the same CERN-Munich group [24] and five data points from K_{e4} decay near threshold [25].

Other systematic and extensive information is provided by the AFS Collaboration [26] on central dimeson production $pp\to pp\pi\pi(K\bar{K})$.

By using the formalism in Sec. II with 17 free parameters, we made a simultaneous fit to the $\pi\pi$-$K\bar{K}$ phase shifts (δ_0 and η_0) and invariant mass spectra of $\pi\pi$ and $K\bar{K}$ from the AFS central production experiment. Our best fit is shown in Figs. 1–4. As a comparison we also show the result by AMP [8] with 24 free parameters. The quality of the two fits is comparable. The difference of our data selection with that of Ref. [8] is that we extend the data on δ_0 to lower energies but drop the data on cross sections for $\pi\pi\to K\bar{K}$ since different experimental groups give inconsistent results for this process (cf. [9]).

The parameters (the unit for energy is GeV) for the best fit are

$$M_R=0.9535,\quad g_1=0.1108,\quad g_2=0.4229 ,$$

$$a_1=-0.3835,\quad a_2=-0.4237,\quad b_1=3.696 ,$$

$$b_2=-1.462,\quad s_{11}=2.788,\quad \beta_{10}=1.237 ,$$

$$\beta_{11}=-15.16,\quad \gamma_{10}=2.515,\quad \gamma_{11}=52.03 ,$$

$$s_{21}=0.3464,\quad \beta_{20}=6.597,\quad \beta_{21}=42.27 ,$$

$$\gamma_{20}=-12.54,\quad \gamma_{21}=-47.99 .$$

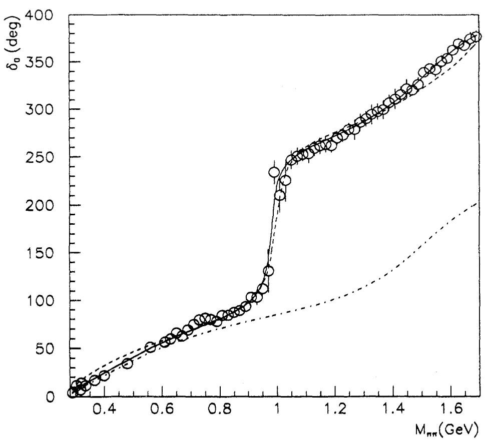

FIG. 1. The $I=0$ S-wave phase shift δ_0 for $\pi\pi$ scattering [23–25]. The solid and dashed curves are our present solution and the AMP solution, respectively. The dot-dashed curve is the background contribution in our solution.

The contribution of the background term to the $\pi\pi$ phase shift δ_0 is shown by the dot-dashed curve in Fig. 1. There are two poles for this $\pi\pi$ elastic scattering background:

$$M_1=0.408-0.342i \text{ (GeV)}$$

and

$$M_2=1.515-0.214i \text{ (GeV)} .$$

The M_1 pole may come from the $\pi\pi$ long range interaction while the M_2 pole may be an effective one coming from two resonances, such as $f_0(1400)$ and $f_0(1590)$.

The Breit-Wigner term for $f_0(975)$ is responsible for the sharp increase of δ_0 around 980 MeV. The corresponding second-sheet and third-sheet poles for $f_0(975)$ are

$$M_0^{\text{II}}=0.988-0.023i \text{ (GeV)} ,$$

$$M_0^{\text{III}}=0.797-0.185i \text{ (GeV)} .$$

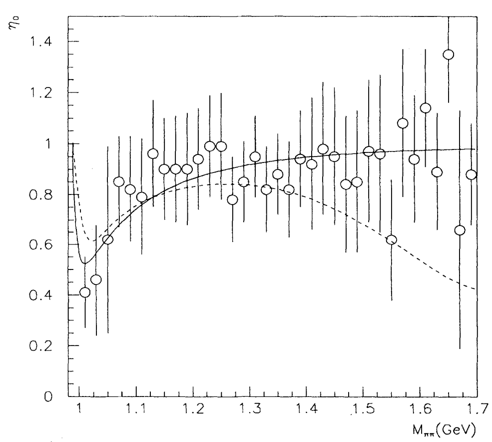

FIG. 2. The $\pi\pi$ $I=0$ S-wave inelasticity [23]. The solid and dashed curves are our present solution and the AMP solution, respectively.

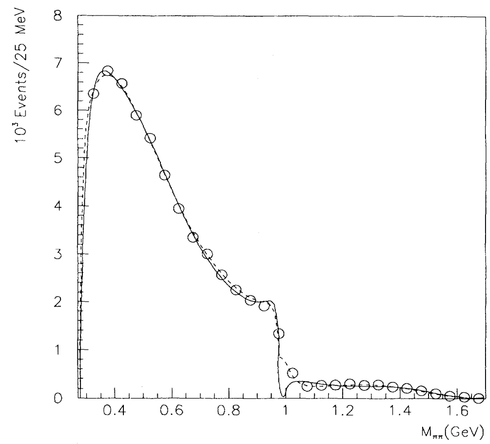

FIG. 3. The mass spectrum of centrally produced S-wave $\pi\pi$ events in $pp\to pp\pi\pi$ from the AFS Collaboration [26]. The solid and dashed curves are our present solution and the AMP solution, respectively.

Our second-sheet pole is quite similar to other analyses [12–17]. We compare our Breit-Wigner term squared with that obtained from [12–16] in Fig. 5. Our result is sitting in the middle of the others. The analyses of [12–17] are on different $\pi\pi$ or $K\bar{K}$ production processes. We believe that with a suitable background term together with our Breit-Wigner term for $f_0(975)$ it should be possible to reproduce the $\pi\pi$ or $K\bar{K}$ invariant mass spectra in these processes.

Our pole topology for $f_0(975)$ is quite different from that of the AMP Collaboration [8] with a K-matrix formalism fitting similar data as in our present analysis. One possible reason may be the different treatment of the background term and $\alpha_i(s)$. They used only polynomials. We include left-hand pole terms in addition, but with fewer free parameters.

We also tried a fit with a K-matrix formalism in which the Lorentz-invariant T matrix for a $\pi\pi$-$K\bar{K}$ elastic

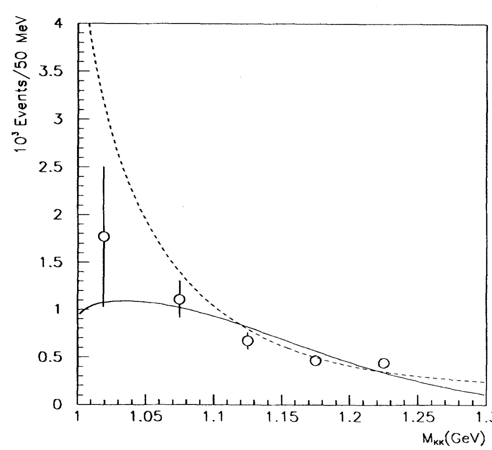

FIG. 4. The mass spectrum of centrally produced S-wave $K\bar{K}$ events in $pp\to ppK\bar{K}$ from the AFS Collaboration [26]. The solid and dashed curves are our present solution and the AMP solution, respectively.

scattering is expressed as

$$\hat{T}=\frac{1}{1-\rho_1\rho_2\hat{D}-i(\rho_1\hat{K}_{11}+\rho_2\hat{K}_{22})}\times\begin{bmatrix}\hat{K}_{11}-i\rho_2\hat{D} & \hat{K}_{12}\\ \hat{K}_{21} & \hat{K}_{22}-i\rho_1\hat{D}\end{bmatrix} \quad (12)$$

with $\hat{D}=\hat{K}_{11}\hat{K}_{22}-\hat{K}_{12}^2$. The real $\hat{K}$ matrix is assumed as

$$\hat{K}_{11}=\frac{a_1^2}{M_R^2-s}+\frac{b_1^2}{s_b-s}+\gamma_{11}\ , \quad (13a)$$

$$\hat{K}_{22}=\frac{a_2^2}{M_R^2-s}+\frac{b_2^2}{s_b-s}+\gamma_{22}\ , \quad (13b)$$

$$\hat{K}_{12}=\hat{K}_{21}=\frac{a_1a_2}{M_R^2-s}+\frac{b_1b_2}{s_b-s}+\gamma_{12}\ . \quad (13c)$$

With this K-matrix formalism we can also get a very similar good fit to the same data sample. The parameters for the best fit are

$$M_R=1.1989,\quad s_b=-0.6945,\quad \gamma_{12}=1.5517\ ,$$

$$a_1=-0.9230,\quad b_1=2.6577,\quad \gamma_{11}=2.8249\ ,$$

$$a_2=0.3469,\quad b_2=1.6448,\quad \gamma_{22}=-0.0626\ .$$

We find that with a left-hand pole term (s_b term) in the K matrix, as part of the background, many fewer free parameters are needed to reproduce the $\pi\pi$ phase shifts as compared with using polynomials only.

For this fit, we find two coupled channel resonances with their second-sheet and third-sheet poles as

$$M_a^{\rm II}=0.989-0.025i,\quad M_b^{\rm II}=1.554-0.213i\ ,$$

$$M_a^{\rm III}=0.914-0.219i,\quad M_b^{\rm III}=1.515-0.217i\ .$$

In the K-matrix formalism the background is now assumed to couple to both $\pi\pi$ and $K\bar{K}$ channels. The M_b pole in the K-matrix formalism is about the same as the M_2 pole in our BW-like formalism. The second-sheet pole for $f_0(975)$ is also nearly the same in the two formalisms. Both formalisms give a large width third-sheet pole ($\sim$400 MeV) for $f_0(975)$. We see that the mass of the third-sheet pole for $f_0(975)$ depends on how one treats the low energy background. We have investigated further K-matrix forms which gives fits to the data of similar quality and similar nearby poles.

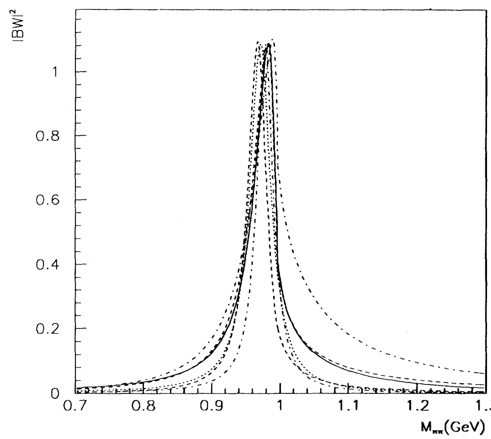

FIG. 5. The Breit-Wigner amplitude squared for $f_0(975)$. The solid curve is our present solution. The dashed and dot-dashed curves are solutions of Refs. [12–16] used by the PDG [11].

From our analysis we conclude that the $f_0(975)$ is most likely a resonance with a large decay width ($\sim$400 MeV) and a narrow peak width ($\sim$47 MeV). This result supports the prediction of Törnqvist's unitarized quark model [3] which interprets the $f_0(975)$ as a $q\bar{q}$ resonance with a large admixture of $K\bar{K}$ virtual state and predicts a decay width of 400 MeV and a peak width of 50 MeV. A recent hadronic loop calculation by Geiger and Isgur [27] also predicts that the lower mass 0^{++} $q\bar{q}$ meson has mass 500–1000 MeV and width 200–500 MeV due to its coupling to meson-meson virtual states. They think it may correspond to the AMP's $f_0(\epsilon(1000))$, but we think it is most probably the $f_0(975)$.

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[1] For general reviews, see L. Montanet, Rep. Prog. Phys. **46**, 337 (1983); F. E. Close, *ibid.* **51**, 833 (1988); M. R. Pennington, in *BNL Workshop on Glueballs, Hybrids, and Exotic Hadrons,* Proceedings, edited by S. U. Chung, AIP Conf. Proc. No. 185 (AIP, New York, 1989), p. 145; N. Isgur, in *Hadron '89,* Proceedings of the International Conference on Hadron Spectroscopy, Corsica, France, 1989, edited by F. Binon *et al.* (Editions Frontières, Gif-sur-Yvette, 1989), p. 709; T. H. Burnett and S. R. Sharpe, Annu. Rev. Nucl. Part. Sci. **40**, 327 (1990).

[2] D. Morgan, Phys. Lett. **51B**, 71 (1974).

[3] N. Törnqvist, Phys. Rev. Lett. **49**, 624 (1982).

[4] J. Weinstein and N. Isgur, Phys. Rev. Lett. **48**, 659 (1982); Phys. Rev. D **27**, 588 (1983); **41**, 2236 (1990).

[5] R. L. Jaffe, Phys. Rev. D **15**, 267 (1977); N. N. Achasov and G. N. Shestakov, Z. Phys. C **41**, 309 (1988).

[6] R. L. Jaffe, Phys. Lett. **41**, 271 (1975); D. Robson, Nucl. Phys. **B130**, 328 (1977).

[7] For general reviews of glueball, hybrids, etc., see T. Barnes, in *Proceedings of the Fourth Workshop on Polarized Targets Materials and Techniques,* Bad Honnef, Germany, 1984, edited by W. Meyer (Bonn University, Bonn, 1984); J. Donoghue, in *Hadron Spectroscopy—1985,* Proceedings of the International Conference on Hadron Spectroscopy, College Park, Maryland, edited by S. Oneda, AIP Conf. Proc. No. 132 (AIP, New York, 1985), p.

460; see also Close [1].
[8] K. L. Au, D. Morgan, and M. R. Pennington, Phys. Rev. D **35**, 1633 (1987).
[9] D. Morgan and M. R. Pennington, Phys. Rev. D **48**, 1185 (1993); Phys. Lett. B **258**, 444 (1991).
[10] F. Cannata, J. P. Dedonder, and L. Leśniak, Z. Phys. A **343**, 451 (1992).
[11] Particle Data Group, K. Hikasa *et al.*, Phys. Rev. D **45**, S1 (1992).
[12] LEBS-EHS Collaboration, M. Aguilar-Benitez *et al.*, Z. Phys. C **50**, 405 (1991).
[13] G. Gidal *et al.*, Phys. Lett. **107B**, 153 (1981).
[14] WA76 Collaboration, T. A. Armstrong *et al.*, Z. Phys. C **51**, 351 (1991).
[15] S. Abach *et al.*, Phys. Rev. Lett. **57**, 1990 (1986).
[16] D. M. Binnie *et al.*, Phys. Rev. Lett. **31**, 1534 (1973).
[17] R. J. Leeper *et al.*, Phys. Rev. D **16**, 2054 (1977).
[18] S. Flatté, Phys. Lett. **63B**, 224 (1976).
[19] A. M. Badalyan, L. P. Kok, M. I. Polikarpov, and Yu. A. Simonov, Phys. Rep. **82**, 31 (1982).
[20] R. H. Dalitz and S. Tuan, Ann. Phys. (N.Y.) **10**, 307 (1960).
[21] S. Weinberg, Phys. Rev. Lett. **17**, 336 (1966); **18**, 1178(E) (1967).
[22] B. R. Martin, D. Morgan, and G. Shaw, *Pion-Pion Interactions in Particle Physics* (Academic, New York, 1974).
[23] G. Grayer *et al.*, Nucl. Phys. **B75**, 189 (1974); W. Ochs, Ph.D. thesis, University of Munich, 1974.
[24] W. Maenner, in *Experimental Meson Spectroscopy—1974 (Boston),* Proceedings of the Fourth International Conference on Meson Spectroscopy, edited by D. A. Garelick, AIP Conf. Proc. No. 21 (AIP, New York, 1974).
[25] L. Rosselet *et al.*, Phys. Rev. D **15**, 574 (1977).
[26] T. Akesson *et al.*, Nucl. Phys. **B264**, 154 (1986); P. C. Cecil, Ph.D. thesis, Cavendish Laboratory, Cambridge, Rutherford Laboratory Report No. RAL-T-004, 1984 (unpublished).
[27] P. Geiger and N. Isgur, Phys. Rev. D **47**, 5050 (1993).