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Logarithmic triviality of scalar quantum electrodynamics

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Using finite size scaling and histogram methods we obtain results from lattice simulations indicating the logarithmic triviality of scalar quantum electrodynamics, even if the bare gauge coupling is large. Simulations of the noncompact formulation of the lattice Abelian Higgs model with fixed length scalar fields on L^4 lattices with L ranging from 6 through 20 indicate a line of second-order critical points. Lengthy runs for each L produce specific-heat peaks which grow logarithmically with L and whose critical couplings shift with L picking out a correlation length exponent of 0.50(2) consistent with mean-field theory.

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Do field theories which are strongly coupled at short distances exist in four dimensions? This is an important question to answer from both a purely theoretical and a phenomenological perspective. Theoretically, one wants to know if the Landau zero [1] (complete screening of interactions) is universal in nonasymptotically free-field theories in four dimensions, as suggested by perturbation theory. In less than four dimensions, theories with nontrivial high-energy interactions are commonplace and perturbation theory in bare coupling parameters is known to be misleading. For example, four Fermi theories in dimensions d, $2<d<4$ have an ultraviolet stable fixed point where chiral symmetry is broken, as indicated by $1/N$ expansions while perturbative expansions in the theory's coupling constant are nonrenormalizable [2]. Phenomenologically, one wants to understand the Higgs mechanism in the successful standard model and build theories where the appropriate form of spontaneous symmetry breaking can occur at short distances.

Issues such as these have rekindled interest in existence questions for various field theories. Considerable work on $\lambda\phi^4$ theories strongly suggest that this theory becomes free as its cutoff is removed [3], although a proof of this property remains elusive. In this paper we shall study scalar electrodynamics at strong gauge couplings with sufficient numerical resources to make quantitative claims about its ultraviolet behavior. We shall see that our numerical results are consistent with the logarithmic triviality of scalar electrodynamics, qualitatively similar to pure $\lambda\phi^4$ theory.

Consider the noncompact formulation of the Abelian Higgs model with a fixed length scalar field [4]:

$$S=\tfrac{1}{2}\beta\sum_{p}\theta_p^2-\lambda\sum_{x,\mu}(\phi_x^* U_{x,\mu}\phi_{x+\mu}+\text{c.c.}) \qquad (1)$$

where p denotes plaquettes, θ_p is the circulation of the noncompact gauge field $\theta_{x,\mu}$ around a plaquette, $\beta=1/e^2$ and $\phi_x=\exp[i\alpha(x)]$ is a phase factor at each site. We choose this action (the electrodynamics of the planar model) because preliminary work has suggested that it has a line of second-order transitions [4], because it does not require fine-tuning and because it is believed to lie in the same universality class as the ordinary lattice Abelian Higgs model with a conventional, variable length scalar field [5]. In Fig. 1 we show the phase diagram of the model in the bare parameter space $\beta-\lambda$. A preliminary investigation has indicated that the line emanating from the $\beta\rightarrow\infty$ limit of Fig. 1 is a line of critical points which potentially could produce a family of interacting, continuum field theories [4]. Note that in the $\beta\rightarrow\infty$ limit the gauge field in Eq. (1) reduces to a pure gauge transformation so the model becomes the four-dimensional planar model which is known to have a second-order phase transition which is trivial, i.e., is described by a free field. The noncompact nature of the gauge field is important in Fig. 1—the compact model has a line of first-order transitions and only at the end point of such a line in the interior of a phase diagram can one hope to have a critical point where a continuum field theory might exist [4]. Since one must fine-tune bare parameters to find such a point, the compact formulation of the model is much harder to use for quantitative work [6]. The fact that Eq. (1) uses fixed length scalar fields avoids another fine-tuning—the variable length scalar-field formulation would possess a quadratically divergent bare mass parameter which would have to be tuned to zero with extraordinary accuracy to search for critical behavior. Conventional wisdom based on the renormalization group states that Eq. (1) should have the same critical behavior as the fine-tuned variable length model [5], so it again emerges as preferable. Note also that in the naive classical limit where the field varies smoothly Eq. (1) reduces to a free

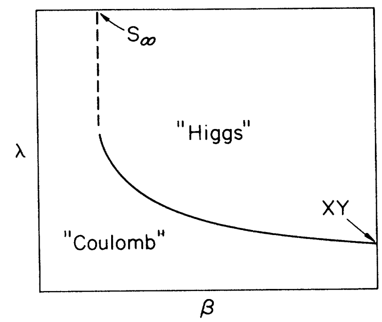

FIG. 1. The phase diagram of noncompact scalar electrodynamics.

massive vector boson. In the vicinity of the strong-coupling critical point we investigate here, the fields are rapidly varying on the scale of the lattice spacing and the specific-heat scaling law is not that of a Gaussian model.

First consider the measurements of the internal energies,

$$E_\gamma=\tfrac{1}{2}\left\langle \sum_p \theta_p^2\right\rangle, \quad E_h=\left\langle \sum_{x,\mu} \phi_x^* U_{x,\mu}\phi_{x+\mu}+\text{c.c.}\right\rangle \tag{2}$$

and their associated specific heats $C_\gamma=\partial E_\gamma/\partial\beta$, and $C_h=\partial E_h/\partial\lambda$. Nonanalytic behavior in the specific heats at critical couplings can be used to find and classify phase transitions. On a L^4 lattice the size dependence of a generic specific heat at a second-order critical point should scale as [7]

$$C_{\max}(L)\sim L^{\alpha/\nu} \tag{3}$$

where α and ν are the usual specific-heat and correlation length critical indices, respectively. Here $C_{\max}$ denotes the peak of the specific heat. A measurement of the index ν can be made from the size dependence of the position of the peak. In a model which depends on just one coupling, call it g, then [7]

$$g_c(L)-g_c\sim L^{-1/\nu} \tag{4}$$

where $g_c(L)$ is the coupling where $C_{\max}(L)$ occurs and g_c is its $L\to\infty$ thermodynamic limit. The scaling laws Eqs. (3) and (4) characterize a critical point with power law singularities. This is a possible behavior for scalar electrodynamics, but there is also the possibility suggested by perturbation theory that the theory is logarithmically trivial. Consider $\lambda\phi^4$ as the simplest, well-studied theory which apparently has this behavior. In this case the theory becomes trivial at a logarithmic rate as the theory's momentum space cutoff Λ is taken to infinity. Then the scaling laws of Eqs. (3) and (4) become [8,9]

$$C_{\max}(L)\sim(\ln L)^p \tag{5}$$

and

$$g_c(L)-g_c\sim\frac{1}{L^2(\ln L)^q} \tag{6}$$

where p and q are powers predictable in one-loop perturbation theory ($p=\frac{1}{3}$ and $q=\frac{1}{6}$ in $\lambda\phi^4$). Note the differences between these scaling laws and those of the usual Gaussian model, obtained from Eqs. (3) and (4) setting $\alpha=0$ and $\nu=0.5$: in the Gaussian model the specific heat should saturate as L grows, and the position of the peaks should approach a limiting value at a rate L^{-2}.

It is particularly interesting in scalar electrodynamics to consider a large value of the bare (lattice) gauge coupling to see if that can induce nontrivial interactions which survive in the continuum limit. So, we ran extensive simulations on lattices ranging from 6^4 through 20^4 at $e^2=5.0$ and searched in parameter space (β,λ) for peaks in C_γ and C_h. We used histogram methods [10,11] to do this as efficiently as possible. For example, on a 6^4 lattice at $\beta=0.2000$ and $\lambda=0.2350$ we found a specific-heat peak near $\lambda_c(6)\approx0.2382$ from the histogram method. The λ value in the lattice action was then tuned to 0.2382 and additional simulations and histograms produced specific heats, found from the variances of E_γ and E_h measurements, at a λ_c very close to 0.2382. Using this strategy, measurements of $\lambda_c(L), C_\gamma(L)$, and $C_h(L)$ could be made without relying on any extrapolation methods. In Table I we show a subset of our results that will be analyzed and discussed here. The columns labeled $\lambda_c(L), C_\gamma^{\max}(L)$, and $C_h^{\max}(L)$ in Table I need no further explanation except to note that the error bars were obtained with standard binning procedures which account for the correlations in the data sets produced by Monte Carlo programs. The Monte Carlo procedure used here was a standard multihit Metropolis for the noncompact gauge degrees of freedom and an over-relaxed plus Metropolis algorithm [12] for the compact matter field. Accuracy and good estimates of error bars are essential in a quantitative study such as this. Unfortunately, cluster and acceleration algorithms have not been developed for gauge theories, so very high statistics of our over-relaxed Metropolis algorithm were essential—tens of

TABLE I. Measurements on noncompact lattice scalar electrodynamics.

L	$\lambda_c(L)$	$C_h^{\max}(L)$	$K_h(L)$	$C_\gamma^{\max}(L)$	$K_\gamma(L)$	Sweeps(millions)
6	0.23815(1)	13.81(2)	0.657668(9)	7.965(9)	0.665784(2)	40
8	0.23375(3)	15.83(2)	0.662954(5)	8.083(3)	0.666374(1)	60
10	0.23173(1)	17.23(4)	0.664892(4)	8.285(6)	0.666544(1)	60
12	0.23070(1)	18.43(7)	0.665713(4)	8.457(9)	0.666606(1)	30
14	0.23004(1)	19.38(9)	0.666110(3)	8.594(15)	0.666633(1)	20
16	0.22962(1)	20.25(13)	0.666319(2)	8.747(17)	0.666647(1)	12
18	0.22933(1)	20.85(15)	0.666441(2)	8.863(26)	0.666654(1)	12
20	0.22912(1)	21.76(20)	0.666510(2)	8.956(20)	0.666658(1)	10

millions of sweeps were accumulated for each lattice size as listed in column 7. Specific heats were measured as the fluctuations in internal energy measurements $[C_h=(\langle E_h^2\rangle-\langle E_h\rangle^2)/4L^4$, etc.], and very high statistics and many L values are needed to distinguish between logarithmic triviality [Eq. (5)] and power law behavior [Eq. (3)]. The other entries in the table, $K_\gamma(L)$ and $K_h(L)$, are the Binder cumulants (kurtosis) [13] for each internal energy. At a continuous phase transition each kurtosis should approach 2/3 with finite size corrections scaling as $1/L^4$.

Consider the kurtosis $K_\gamma(L)$, the specific heat $C_\gamma^{\max}(L)$, and the critical coupling $\lambda_c(L)$ of scalar electrodynamics. The kurtosis $K_\gamma(L)$ is plotted against $10^6/L^4$ in Fig. 2. The size of the symbols include the error bars, but clearly the curve favors a second-order transition. A three parameter fit to the $L=12$, 14, 16, 18, and 20 data using the form $K_\gamma(L)=\alpha L^\rho+b$ is excellent (confidence level=98%) predicting $\rho=-4.1(4)$ and $K_p(\infty)=0.666665(2)$. The hypothesis of a line of second-order transitions in Fig. 1 appears to be very firm, with no evidence for a fluctuation-induced first-order transition. An analysis of $K_h(L)$ gives the same conclusion with somewhat larger error bars. In Fig. 3 we plot our $C_\gamma^{\max}(L)$ data vs L. We attempted power law as well as logarithmic finite size scaling hypotheses. The power law hypothesis did *not* produce a stable fit for any reasonable range of parameters. However, logarithmic fits were quite good. The hypothesis $C_\gamma^{\max}(L)=a\ln^\rho L+b$ for $L=8$, 10, 12, 14, 16, 18, and 20 fit with a confidence level = 90% producing the estimate $\rho=1.4(2)$. If we considered the range $L=8-18$, the same fitting form predicted $\rho=1.5(3)$ with confidence level = 84%, and if the range $L=10-20$ were taken we found $\rho=1.4(5)$ with confidence level = 78%. The solid line in Fig. 3 is the $L=8-20$ fit. An analysis of $C_h^{\max}(L)$ gave consistent results—the same logarithmic dependence should be found in either specific heat—and power law fits to $C_h^{\max}(L)$ were also ruled out. In particular, a fit of the form $C_h^{\max}(L)=a\ln^\rho L+b$ for $L=8-18$ gave $\rho=0.9(3)$ with confidence level = 82% and for $L=8-20$ gave $\rho=1.0(2)$ with confidence level = 85%. Finally, in Fig. 4 we show $\lambda_c(L)$ vs $10^4/L^2$. The error bars again fall within the symbols in the figure. The data are clearly compatible with the correlation length index $\nu=0.5$ expected of a theory which is free in the continuum limit. In the case of $\lambda\phi^4$ it has proven possible to find the logarithm of Eq. (6) under the dominant L^{-2} behavior by using special techniques [14]. We do not quite have the accuracy to do that here: a power law fit to $\lambda_c(L)=\lambda_c+a/L^{1/\nu}$ using $L=12-20$ predicts $1/\nu=2.0(1)$, $\lambda_c=0.22825(8)$ with confidence level = 92% and using $L=14-20$ predicts $1/\nu=1.9(3)$, $\lambda_c=0.2282(2)$ with confidence level = 97%.

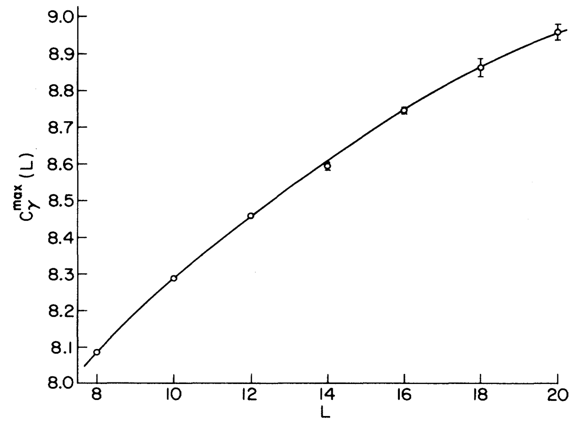

FIG. 3. The specific-heat peaks $C_\gamma^{\max}(L)$ vs L. The solid line is the logarithmic fit discussed in the text.

One of the motivations for this study was the recent finding that the chiral-symmetry-breaking transition in noncompact lattice electrodynamics with dynamical fermions is *not* described by a logarithmically trivial model [15]. Power law critical behavior has been found with nontrivial critical indices satisfying hyperscaling. The present negative result for scalar electrodynamics suggests that the chiral nature of the transition for fermionic electrodynamics is an essential ingredient for its nontriviality. It remains to be seen, however, if the chiral transi-

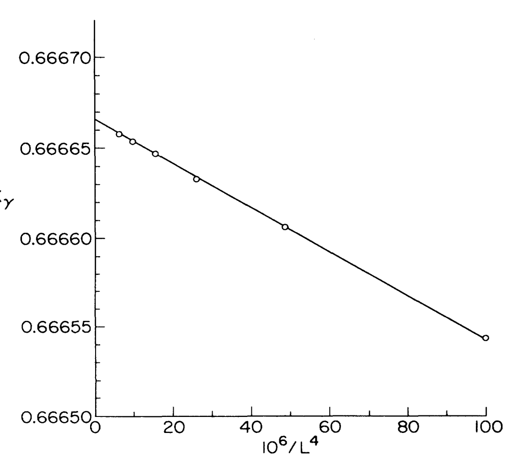

FIG. 2. The kurtosis $K_\gamma(L)$ vs $10^6/L^4$.

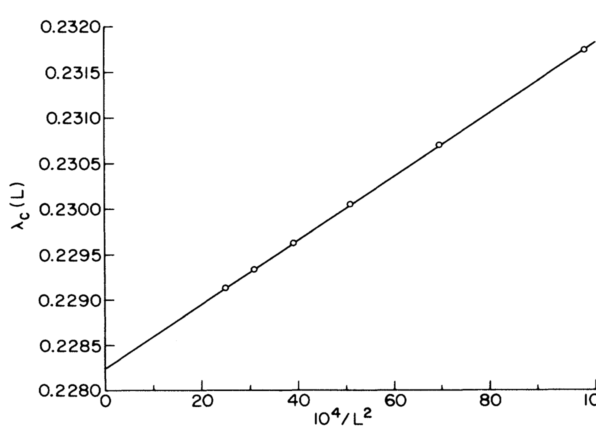

FIG. 4. The critical coupling $\lambda_c(L)$ vs L^{-2}.

tion found on the lattice produces an interesting continuum field theory.

In conclusion, our numerical results support the notion that scalar electrodynamics is a logarithmically trivial theory. We suspect that this result could be made even firmer by additional simulation studies which use more sophisticated techniques such as renormalization groups transformations [5] or partition function methods [14].

The simulation done here used the Cray C90's at PSC and NERSC. We thank these centers for friendly user access. Several thousand CPU hours were needed to accumulate the statistics listed in Table I. J. B. K. was supported in part by the National Science Foundation Grant NSF PHY92-00148. D.K.S. and S.K. were supported by DOE Contract W-31-109-ENG-38. M.B. and H.F. acknowledge the support of CESCA and CICYT.

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