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Proposal for solving the "problem of time" in canonical quantum gravity

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The "problem of time" in canonical quantum gravity refers to the difficulties involved in defining a Hilbert space structure on states, and local observables on this Hilbert space, for a theory in which the spacetime metric is treated as a quantum field, so no classical metrical or causal structure is present on spacetime. We describe an approach, much in the spirit of ideas proposed by Misner, Kuchař, and others, to defining states and local observables in quantum gravity which exploits the analogy between the Hamiltonian formulation of general relativity and that of a relativistic particle. In the case of minisuperspace models, a concrete theory is obtained which appears to be mathematically and physically viable, although it contains some radical features with regard to the presence of an "arrow of time." The viability of this approach in the case of infinitely many degrees of freedom rests on a number of fairly well-defined issues, which, however, remain unresolved. As a by-product of our analysis, the theory of a relativistic particle in curved spacetime is developed.

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A key issue which arises in any attempt to obtain a quantum theory of general relativity, or, more generally, any theory in which the spacetime metric is treated as a quantum observable, is how to formulate a local field theory without the presence of any classical metrical or causal structure on spacetime. This issue is faced most directly in "nonperturbative" approaches to quantization, such as the canonical approach.

As is well known, classical general relativity admits a Hamiltonian formulation, but this formulation possesses constraints which are closely analogous to those occurring in the theory of a relativistic particle. The canonical approach to formulating a quantum theory corresponding to general relativity begins with this Hamiltonian formulation [1], in which the role of configuration variable is played by a Riemannian metric h_{ab} on a three-dimensional manifold Σ and the conjugate momentum π^{ab} has the interpretation of being directly related to the extrinsic curvature of Σ in the classical spacetime obtained by evolving these initial data. The states of the quantum theory are then taken to be wave functionals $\Psi[h_{ab}]$ of the metric on Σ, and the constraints of the classical theory are imposed as conditions on Ψ. The "momentum constraints" imply that Ψ is spatial diffeomorphism invariant (i.e., that it depends only upon the three-geometry), and they can be "solved" by taking Ψ to be a function on "superspace," the manifold of three-geometries. The Hamiltonian constraints give rise to the Wheeler-DeWitt equations on Ψ. Note that I use the plural here to stress that there is an infinite family of Hamiltonian constraints, one for each choice of "lapse function," and a correspondingly infinite family of Wheeler-DeWitt equations.

The difficulties with the canonical approach arise when one attempts to impose a Hilbert space structure on the allowed state vectors, and when one attempts to define operators on this Hilbert space for observables of interest such as h_{ab} and π^{ab}. In particular, it is far from clear what in the theory should play the role of a "Heraclitian time variable" [2], which "sets the conditions" for determining probabilities for the values of the dynamical variables. We refer the reader to [3] (see also Sec. I of [2]) for comprehensive review of the various approaches that have been taken to this "problem of time" and the serious difficulties which these approaches have encountered.

The purpose of this paper is to describe a proposal for defining Hilbert space structure and observables in canonical quantum gravity. The basic ideas involved in this proposal are not new; they appear in the work of Misner [4], Kuchař (see particularly [5]), and others. However, it now appears that it may be possible to overcome two potentially serious obstacles to the implementation of these ideas. We shall focus attention upon the case of minisuperspace models, where regularization issues do not arise, and the proposal can be given a concrete form. Issues related to the generalization of this proposal to the case of infinitely many degrees of freedom will be addressed at the end of this paper.

For definiteness, we focus attention upon "class A" Bianchi cosmological models, whose only matter content is a homogeneous scalar field ϕ. (Class A Bianchi Lie algebras are the three-dimensional ones in which the structure tensor takes the form $c^a{}_{bc} = M^{ad}\epsilon_{dbc}$ with M^{ad} symmetric and ϵ_{dbc} totally antisymmetric; restriction to class A models is made in order to assure the existence of a Hamiltonian formulation of the minisuperspace dynamics.) For such a model, minisuperspace is a four-dimensional manifold, with three parameters characterizing the spatial geometry and one giving the value, ϕ, of the homogeneous scalar field. We choose the parametrization $(\alpha, \beta_+, \beta_-)$ of the spatial metric introduced by Misner (see, e.g., box 30.1 of [6]) where, in essence, α measures the volume of the Universe, and (β_+, β_-) measure the spatial anisotropy. With appropriate rescalings of variables, the super-Hamiltonian takes the form

$$H = -{p_\alpha}^2 + {p_{\beta_+}}^2 + {p_{\beta_-}}^2 + {p_\phi}^2 + \exp(4\alpha) V_\beta(\beta_+, \beta_-) + \exp(6\alpha) V_\phi(\phi) \qquad (1)$$

where $(p_\alpha, p_{\beta_+}, p_{\beta_-}, p_\phi)$ are the momenta canonically conjugate to $(\alpha, \beta_+, \beta_-, \phi)$ and the "potential" V_β depends upon the choice of Lie group.

In the canonical approach to the quantization of this model, it is generally agreed that in the "metric representation" the Hamiltonian constraint should be enforced by requiring the state vector $\Psi(\alpha, \beta_+, \beta_-, \phi)$ to satisfy the Wheeler-DeWitt equation

$$\hat{H}\Psi = 0 \tag{2}$$

where $\hat{H}$ is obtained from H by replacing the classical momenta with the corresponding differentiation operators. Thus, more explicitly, Ψ satisfies

$$[G^{AB}\nabla_A\nabla_B - \exp(4\alpha)V_\beta - \exp(6\alpha)V_\phi]\Psi = 0 \tag{3}$$

where the DeWitt supermetric G_{AB} is just a flat, Lorentz signature metric in the "global, inertial, superspace coordinates" $(\alpha, \beta_+, \beta_-, \phi)$. However, serious difficulties occur when one attempts to define a Hilbert space structure $\mathcal{H}$ on the states and to define self-adjoint operators on $\mathcal{H}$ representing observables of interest. Our aim, now, is to overcome these difficulties.

The super-Hamiltonian (1) has the same mathematical form as that of a relativistic particle in a (time- and space-dependent) potential, and, correspondingly, the Wheeler-DeWitt equation (3) has the mathematical form of a Klein-Gordon equation in an external potential. This analogy is not superficial, nor is it special to the particular class of models explicitly considered here: The presence of a constraint in the theory of a relativistic particle traces its origin to the treatment of the time coordinate of the particle as a dynamical variable, and the origin of the Hamiltonian constraint of general relativity can be given a similar interpretation. Furthermore, the Lorentz signature of G_{AB} is not an artifact of our model; in any homogeneous model, G_{AB} will have a Lorentz signature (with the motion in superspace associated with the conformal scaling of the three-metric being "timelike") provided only that the kinetic energy terms of the matter fields enter with the usual sign. Thus, it seems natural to seek guidance for the definition of $\mathcal{H}$ and observables on $\mathcal{H}$ from the theory of a relativistic particle in a curved spacetime and/or external potential.

However, the theory of a relativistic particle in a nonstationary curved spacetime or external potential is plagued by some well-known difficulties. Historically, these difficulties were cured by passing to a "second quantized" theory, i.e., by changing the nature of the theory to that of a quantum theory of a field (with the value of the field at spacetime events playing the role of the primary observables of the theory) rather than a theory of a particle (where the primary observables are the spatial position and momentum of the particle). This step is well justified physically: It appears that nature truly is described at a fundamental level, or, at least to the level we currently are able to probe, by quantum field theory. However, since canonical quantum gravity already is structured as a field theory, an analogous step here would correspond to a "third quantized" theory, in which the primary observables presumably would become the value of the Wheeler-DeWitt wave function at points of superspace. It seems difficult to imagine how such "observables" might be measured, or how such a theory could be interpreted so as to give predictions about what one ostensibly is interested in trying to describe in quantum gravity, namely local metrical structure as determined by observers in spacetime (as opposed to, say, an S matrix describing the scattering of multiple universes). Thus, I shall not pursue this avenue of approach here.

As I shall now explain, it is possible to give a mathematically consistent, interpretable theory of a relativistic particle in a nonstationary spacetime or external potential. However, this theory suffers from two serious defects. Remarkably, these defects do not appear to be impediments to the viability of an analogous quantum theory for our minisuperspace models. As discussed briefly at the end of this paper, the issue of whether these ideas can be extended to provide a mathematically consistent and physically interpretable quantum theory of gravity in the case of infinitely many degrees of freedom remains open.

Let (M, g_{ab}) be an arbitrary globally hyperbolic spacetime, on which there is prescribed a (possibly time- and space-dependent) "external potential" V, and consider a "relativistic particle" on this spacetime, with the classical Hamiltonian h given by

$$h = g^{ab}p_a p_b + V \tag{4}$$

and where h is constrained to vanish. We wish to construct a quantum theory in which states are represented by (complex) wave functions Φ on spacetime and where the constraint $h = 0$ is imposed by requiring Φ to satisfy the Klein-Gordon equation

$$g^{ab}\nabla_a\nabla_b\Phi - V\Phi = 0. \tag{5}$$

Now, the vector space of complex solutions to (5) possesses the natural conserved, nonpositive, inner product

$$\langle\Phi_1, \Phi_2\rangle_{\text{KG}} = -i\Omega(\bar{\Phi}_1, \Phi_2) \tag{6}$$

where the (real) symplectic product Ω is given by

$$\Omega(\Phi_1, \Phi_2) = \int_\Sigma [\Phi_2\nabla_a\Phi_1 - \Phi_1\nabla_a\Phi_2]d\Sigma^a \tag{7}$$

and Σ is any Cauchy surface. The aim is to define the Hilbert space of states, $\mathcal{H}$, by, in effect, choosing a suitable subspace of complex solutions on which $\langle,\rangle_{\text{KG}}$ is positive definite. As discussed in detail in [7], this can be done by specifying a real inner product μ on the vector space of real, smooth solutions to (5) with initial data of compact support, such that,

$$\mu(\Phi_1, \Phi_1) = \operatorname*{LUB}_{\Phi_2 \neq 0} \frac{|\Omega(\Phi_1, \Phi_2)|^2}{4\mu(\Phi_2, \Phi_2)}. \tag{8}$$

In the case of a stationary spacetime with a time-independent potential $V > 0$, a natural choice of μ exists [8], which corresponds to taking $\mathcal{H}$ to be the subspace of positive frequency solutions. This construction also can be used to define a natural choice of μ if the spacetime and potential are merely asymptotically stationary in the past or future. However, in the absence of time translation symmetry, although a wide class of μ's satsifying Eq. (8) always exists, no μ seems in any way uniquely "distinguished." Indeed, note that in the case where the spacetime and potential are asymptotically stationary in both the past and the future, there will be two "distinguished" choices of μ, which, in general, will differ. The lack of a natural choice of μ, and, thereby, of $\mathcal{H}$, is the first serious deficiency of the theory of a relativistic particle in curved spacetime.

Nevertheless, suppose an inner product μ satisfying Eq. (8) has been chosen. I shall assume, in addition, that the Hilbert space of solutions, $\mathcal{H}$, determined by this μ satisfies the following further properties: Given any Cauchy surface Σ, let $\mathcal{D}$ denote the subspace of $\mathcal{H}$ comprised of C^1 solutions whose restriction to Σ lies in $L^2(\Sigma)$. Now, any $\Phi \in \mathcal{D}$ is uniquely characterized by this restriction, since if Φ and Φ' had the same restriction to Σ, then the norm of $\Phi - \Phi'$ would vanish by Eqs. (6) and (7). Consequently, $\mathcal{D}$ also may be viewed as a subspace of the Hilbert space $L^2(\Sigma)$. I shall assume that $\mathcal{D}$ is dense both as a subspace of $\mathcal{H}$ and as a subspace of $L^2(\Sigma)$. In addition, I shall assume that if $\{\Phi_n\}$ is any sequence in $\mathcal{D}$ which converges in both $\mathcal{H}$ and $L^2(\Sigma)$, then the limit in $\mathcal{H}$ is nonzero if and only if the limit in $L^2(\Sigma)$ is nonzero. I believe that it is likely that these assumptions could be proven to hold in the case (relevant for our considerations below) of a spacetime which is asymptotically stationary in the past, with μ chosen as described above, but I have not attempted to investigate this issue carefully.

Our aim, now, is to define operators on $\mathcal{H}$ corresponding to the position and momentum of the particle at an arbitrary time. More precisely, given any Cauchy surface (i.e., "time"), Σ, for each function $f : \Sigma \to \mathbb{R}$ we wish to obtain a self-adjoint operator $\hat{f} : \mathcal{H} \to \mathcal{H}$ whose spectral resolution yields the probability distribution for the value of f at the position of the particle on Σ. Similarly, given any vector field v^a on Σ with complete orbits, we wish to obtain a self-adjoint operator $\hat{v}$ which can be interpreted as the infinitesimal generator of translations of states by the diffeomorphisms on Σ generated by v^a. For flat spacetime with no potential and for Σ chosen to be a flat hyperplane, such operators were defined by Newton and Wigner [9] for the case where f is a Cartesian coordinate on Σ and v^a is a Euclidean translation. Thus, we wish to generalize this construction to curved spacetimes, to arbitrary choices of Σ, and to general choices of f and v^a.

On the Hilbert space $L^2(\Sigma)$, there is a standard prescription (see, e.g., Appendix C of [10]) for defining position and momentum operators of the sort described above. However, the relevant Hilbert space here is $\mathcal{H}$, and direct application of the L^2 operators to the initial data associated with solutions in $\mathcal{H}$ will not, in general, even yield maps of $\mathcal{H}$ into itself, no less yield self-adjoint operators on $\mathcal{H}$. Nevertheless, we can proceed in the following manner, which generalizes a construction of Kuchař (unpublished) for static hypersurfaces in static spacetimes.

Let Σ be a Cauchy surface, let $\mathcal{D}$ be defined as above, and let $\bar{\mathcal{D}}$ denote the Cauchy completion of $\mathcal{D}$ in the norm $|| \; ||_{\mathcal{H}} + || \; ||_{L^2(\Sigma)}$. By our assumptions above, we may view $\bar{\mathcal{D}}$ as a (dense) subspace of $\mathcal{H}$, on which the L^2 inner product is well defined and positive definite. We view the L^2 inner product as a quadratic form q on $\mathcal{H}$ with form domain $\bar{\mathcal{D}}$. Since q is closed and positive definite, it follows from general results on quadratic forms (see, e.g., section 8.6 of [11]) together with the square root lemma that there exists a unique, positive self-adjoint operator $A : \mathcal{H} \to \mathcal{H}$ with domain $\bar{\mathcal{D}}$ such that for all $\Phi_1, \Phi_2 \in \bar{\mathcal{D}}$, we have

$$\langle \Phi_1, \Phi_2 \rangle_{L^2} = \langle A\Phi_1, A\Phi_2 \rangle_{\mathcal{H}}. \quad (9)$$

Since $\ker(A) = 0$, it follows that A has a dense range in $\mathcal{H}$. Now view A as a linear map from $\bar{\mathcal{D}} \subset L^2(\Sigma)$ into $\mathcal{H}$. By Eq. (9), this map preserves inner products. Since both the range and domain of A are dense, it follows that A uniquely extends to a unitary map $U : L^2(\Sigma) \to \mathcal{H}$. We now simply use this unitary correspondence to "transport" to $\mathcal{H}$ the position and momentum operators defined on $L^2(\Sigma)$. This prescription reproduces the Newton-Wigner operators in the case considered by them [9].

Thus, having chosen a μ which satisfies Eq. (8) and our additional assumptions, we obtain a theory in which for any choice of "time" (i.e., Cauchy surface), we have well-defined operators describing the position and momentum of the particle at that time. These operators satisfy the usual commutation relations, and I would not anticipate any difficulties with the existence of a semiclassical limit of the theory in which the dynamics agrees closely with that of a classical relativistic particle. However, there is a serious deficiency of the theory: The exact quantum dynamics will not respect the causal structure of the underlying spacetime, i.e., a particle which, with unit probability, lies within a region R on the Cauchy surface Σ will not, in general, be localized to within the causal future of R on a later Cauchy surface Σ'. (Indeed, this phenomenon is well known to occur even for the theory obtained with the Newton-Wigner observables in flat spacetime.) Thus, the quantum theory of a relativistic particle constructed above would appear to give rise to a physically unacceptable violation of causality.

Nevertheless, we may carry over the mathematical structure and interpretative framework of the above theory of a relativistic particle to our minisuperspace models for quantum gravity. When we do so, the "causality violation" of the theory no longer poses a physical difficulty, since even classical trajectories in minisuperspace do not respect the light cone structure of the DeWitt metric, G_{AB}, appearing in Eq. (3). Furthermore, it appears that the other serious deficiency of the theory of the relativistic particle, namely, the lack of a natural choice of μ, also can be overcome: The metric G_{AB} is invariant under translations of the timelike coordinate α, and the potential terms in Eq. (3) vanish asymptotically as $\alpha \to -\infty$. Thus, it should be possible [12] to obtain a Hilbert space structure on the solutions to Eq. (3) in a natural way by choosing the μ associated with this asymptotic symmetry, i.e., by choosing $\mathcal{H}$ to be the subspace of solutions which asymptotically oscillate with positive frequency [13] with respect to α as $\alpha \to -\infty$.

It should be emphasized that this asymptotic symmetry of the Wheeler-DeWitt equation used to define $\mathcal{H}$ holds much more generally: The translations in the α direction correspond to scale transformations of the spatial geometry, which is a timelike conformal isometry of all of the inverse DeWitt metrics on full superspace [5]. Similarly, the vanishing of the potential as $\alpha \to -\infty$ also holds on full superspace. Thus, our prescription for the construction of $\mathcal{H}$ is not special to the particular class of models considered here.

The nature of the quantum theory we have just constructed should be noted: Given any state $\Psi \in \mathcal{H}$, one is free to specify any Cauchy surface $\mathcal{C}$ in minisuperspace. (This specification of $\mathcal{C}$ plays precisely the role of a choice of "time" in ordinary quantum theory.) For the given state Ψ at "time" $\mathcal{C}$, one may then predict the probabilities for the remaining metric variables or their conjugate momenta. In particular, the surfaces of constant α are Cauchy surfaces, which, in fact, are naturally picked out as being orthogonal to the Killing field used in the construction of $\mathcal{H}$. If we make this choice, then we are free to specify any value of "volume of the Universe" (i.e., α) which we wish to consider. For any given state $\Psi \in \mathcal{H}$, the theory will then tell us the probabilities for the various possible values of the conformal metric or the trace-free extrinsic curvature at that value of α. Note that, in contrast with most other approaches to quantum cosmology, there is a well-defined Hilbert space structure on states and well-defined rules for calculating probabilities of the above observables, but there does not appear to be any "preferred state" in $\mathcal{H}$.

In order to physically interpret the theory in terms of the perceptions of observers making measurements, it seems necessary to make an identification of the mathematical quantity playing the role of "time" in the theory, namely, the choice of $\mathcal{C}$, with the "time" as perceived by observers. This does not appear to lead to any blatant contradictions in the context of the models considered here, but the issue of how this identification might generalize to the full theory (where spatial homogeneity is not enforced) remains open (see below) and probably poses the most significant challenge to the viability of this approach with regard to obtaining a sensible interpretation of the full theory.

For definiteness in our discussion, let us choose the Cauchy surfaces of constant α as our specification of "time." It should be emphasized that within the context of this theory it does *not* make sense to ask whether the Universe "ever" achieves a given value of α any more than it would make sense in ordinary quantum mechanics to ask if a particle ever achieves a given value of time. In effect, the identification of α with perceived time as proposed above builds into the theory the expansion of the Universe "forever." It is interesting to ask how this theory would describe Bianchi type IX cosmologies, where, in the classical theory, recollapse always occurs [14]. (Classically, in all other Bianchi models the Universe always expands forever.) The answer is that, for any classical Bianchi type IX solution, there appears to be no difficulty in constructing a quantum state Ψ which well approximates this classical solution during the expanding phase in the sense that the probability distributions for (β_+, β_-, ϕ) and their conjugate momenta as functions of α are sharply peaked around the values taken by the given classical solution during its expanding phase. However, when α is chosen of the order of, or larger than, the maximum value $\alpha_{\max}$ achieved by the classical solution, the behavior of the state Ψ becomes highly nonclassical. Further discussion of this nonclassical behavior will be given elsewhere [15].

Note that the momentum p_α conjugate to α, which measures the expansion rate of the Universe in the classical theory, is *not* among the list of quantum observables automatically defined in our theory. ($-p_\alpha$ is, of course, the analog of energy in the theory of the relativistic particle.) However, p_α can be defined by means of the Hamiltonian constraint equation (1), i.e.,

$$p_\alpha{}^2 = p_{\beta_+}{}^2 + p_{\beta_-}{}^2 + p_\phi{}^2 + \exp(4\alpha) V_\beta(\beta_+, \beta_-) + \exp(6\alpha) V_\phi(\phi) \qquad (10)$$

since the operators appearing on the right side of this equation are all well defined at any "time" α. For the Bianchi type IX models, V_β can be negative, so $p_\alpha{}^2$ need not be positive definite. This corresponds to the existence of nonclassical behavior as discussed in the preceding paragraph. The subspace of states associated with negative eigenvalues of $p_\alpha{}^2$ at any "time" α thus may be viewed as the "nonclassical sector" of $\mathcal{H}$ at that "time." On the "classical sector" of $\mathcal{H}$ (which comprises all of $\mathcal{H}$ except in the Bianchi type IX models), we may define $-p_\alpha$ by taking the square root of Eq. (10). There exist many square roots of a positive self-adjoint operator, but it seems most natural (and probably essential for consistency with our interpretative remarks) to choose the positive square root in this case. This choice implies that the Universe must be expanding whenever it can be described classically. Note that $-p_\alpha$ then has the interpretation of yielding the relationship between the rate of change of "Heraclitian time variable" α and that of the "time" registered on physical clocks.

It should be emphasized that the above choice of positive square root for $-p_\alpha$, as well as our interpretative remarks and our construction of $\mathcal{H}$, build "arrows of time" into the theory in a fundamental way. Our particular choice of direction of these "arrows" was based upon the fact that the Universe is observed to be expanding. No mathematical difficulties would arise if we were to reverse the "arrows" by identifying "forward in perceived time" with "decreasing α" and, correspondingly, were to choose the negative square root for $-p_\alpha$. However, it should be emphasized that, even if one wished to do so, it would seem difficult to restructure the theory so as to eliminate the presence of any "arrows." In particular, although other choices of p_α could be made, there does not appear to be any natural choice of "half-positive, half-negative" square root of Eq. (10), as presumably would be needed to obtain a "time-symmetric" theory.

Undoubtedly, the most crucial issue regarding all of the above ideas and proposals is the extent to which they can be generalized to the full superspace case. In order to do this, the following obstacles must be overcome. (1) An analogue of a Cauchy surface for the Wheeler-DeWitt equations on superspace must be found. (2) A symplectic product on the solutions to the Wheeler-DeWitt equations (whose value is independent of choice of Cauchy surface) must be identified. (3) A suitable subspace of solutions must be chosen to serve as the Hilbert space of states, $\mathcal{H}$. (4) Our construction of position and momentum operators must be generalized. (5) Finally, in order to obtain an interpretation of the theory in terms of local measurements made by observers, it would appear necessary to suitably identify the variables defining Cauchy surfaces in superspace with the "perceived time" of local observers.

It will, of course, be necessary to confront the difficult issues of regularization and renormalization of the theory in order to analyze the above issues in a mathematically satisfactory way. However, it would appear that issues (1)–(3) and perhaps (5) can be at least partially investigated [16] without getting deeply involved in regularization issues. The Wheeler-DeWitt equations comprise an infinite family (one for each choice of lapse function N), and one would expect a "Cauchy surface" in superspace to have codimension equal to the number of Wheeler-DeWitt equations. Thus, something of the nature of a cross section of the conformal geometries on superspace would appear to be a good candidate for a Cauchy surface (with the conformal factor thus playing the role of "time," i.e., labeling the Cauchy surfaces). However, it is not at all clear whether this suggestion works in detail and what, if any, additional conditions might need to be imposed upon the cross section. In this regard, it should be noted that the only obvious metrical structure present on superspace is the infinite family of DeWitt inverse supermetrics (which, in general at least, are degenerate [17]), but it is not immediately clear whether or how this structure might be used to obtain an analogue of "Cauchy surfaces." The symplectic product should have the basic structure of the DeWitt product [see Eq. (5.19) of [18]], but it is far from clear exactly what form it would take when expressed as an integral over a Cauchy surface in superspace. (Unfortunately, it also would appear that regularization issues will play a prominent role in defining the symplectic structure.) If the solutions to the Wheeler-DeWitt equation behave in an "ultralocal" manner [19] as the metric is scaled to zero (corresponding to the limit $\alpha \to -\infty$ in our minsuperspace models) then it should be possible to define $\mathcal{H}$ as the subspace of solutions which asymptotically oscillate with positive frequency with respect to independent conformal scalings at each point of space.

The above issues are presently under investigation [15].

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[1] Here we employ the "traditional" Arnowitt-Deser-Misner (ADM) canonical variables. It is essential for our approach that the canonical variables be chosen so that the constraints be at most quadratic in the momenta, but otherwise one should be free to consider other choices, such as the "Ashtekar variables."

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[12] Since the potential terms appearing in Eq. (3) typically will be unbounded at large "spatial coordinates," the manner in which they vanish asymptotically as $\alpha \to -\infty$ is nonuniform, so it is not obvious that the prescription for defining μ always can be applied. Indeed, for vacuum Bianchi type VIII and IX models, the behavior of the V_β as $\alpha \to -\infty$ is well approximated by "confining walls" which move at velocity less than light, and hence V_β cannot be neglected in either the classical or quantum dynamics. However, even in these models, when the ϕ degree of freedom is added, the "walls" are no longer confining (provided that V_ϕ does not grow exponentially rapidly with ϕ), and the asymptotic symmetry needed to define μ should be present (see [15] for further discussion). Thus, the viewpoint adopted here is that the asymptotic dynamics of the vacuum Bianchi type VIII and IX models is of a very special character, and that the generic asymptotic dynamics is "velocity dominated" as $\alpha \to -\infty$.

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