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# Composite gauge fields in renormalizable models

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We construct renormalizable models of gauge theories in four dimensions, with gauge fields composite of bosonic or fermionic constituents. They are obtained by a regularization of $CP^{n-1}$ models where the constraint on constituent fields is replaced by a constraint on expectation values.

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In this paper we present a generalization of $CP^{n-1}$ models [1] in their non-Abelian versions [2], constructing renormalizable models of gauge theories in four dimensions with gauge fields composite of bosonic or fermionic constituents. The latter will be referred to as fermionic $CP^{n-1}$ models.

One obvious motivation is the conceptual economy of reducing the number of fundamental fields. But a complementary motivation is that gauge fields composite of fermionic fields are expected to give rise to softer divergences than fundamental gauge fields, a perspective especially relevant to quantum gravity if in such models a phase of unbroken symmetry exists. We will not investigate this point, but we will keep in mind that in this connection the constituent fields can be just auxiliary fields which need not be identified with physical particles or "physical constituents" such as quarks. They can be, for instance, scalar fermions and/or have *scaling dimensions different from the canonical ones.* We will actually assume scaling dimension zero to have renormalizability by power counting. Obviously this can give rise to problems with unitarity, unless the constituent fields and possible composites of dimension 0 are confined or decoupled, for instance, by a large mass.

Composite models of gauge fields have a long history. Early attempts are based on the generation of gauge field self-interactions at the quantum level [3], a line revived recently with a remarkable follow-up [4]. Such an approach is very different from the present one where self-interactions of composite gauge fields can be introduced from the beginning.

In the case of fermionic constituents our work is close in spirit to that by Amati *et al.* [5]. The latter, however, meets with a difficulty due to the fact that $\delta$ functions of Grassmann variables do not have all the properties of ordinary $\delta$ functions, a difficulty common to another fermionic model recently proposed [6]. This point will be discussed below.

To define composite fields we use a new lattice regularization [7] where the gauge fields are noncompact, a feature which is essential to dealing with fermionic constituents. Since we will need the basic formulas of this regularization in any case, we will briefly outline its construction. Then we will define composite gauge fields and we will discuss the renormalizability by power counting. Finally, we will mention how a proof of reflection positivity can be given. Such a proof, ensuring a quantum mechanical interpretation, is especially important in dealing with composites.

We will confine ourselves to the case of non-Abelian $CP^{n-1}$ models, both bosonic and fermionic, with SU(2) local invariance, but most of our results can be extended to SU($N$), $N>2$, and to the Abelian case. The Lagrangian density of the bosonic models in the continuum can be written

$$\mathcal{L}=\sum_{h=1}^{N_f}\sum_{\mu}(\mathcal{D}_\mu\lambda_h)^\dagger\mathcal{D}_\mu\lambda_h \ , \tag{1}$$

where the covariant derivative

$$\mathcal{D}_\mu=\partial_\mu+iA_\mu, \quad A_\mu=\tfrac{1}{2}\sigma_a A_{\mu a} \tag{2}$$

is defined in terms of the composite gauge field

$$A_{\mu a}=\sum_{h=1}^{N_f}(\lambda_h^\dagger\sigma_a\partial_\mu\lambda_h-\partial_\mu\lambda_h^\dagger\sigma_a\lambda_h) \ . \tag{3}$$

$\lambda_h$ are constituent fields in the fundamental representation of SU(2), subject to the constraint

$$\sum_{h=1}^{N_f}\lambda_h^\dagger\lambda_h=\sum_{h=1}^{N_f}\sum_{\alpha=1}^{2}\lambda_{h\alpha}^*\lambda_{h\alpha}=1 \ , \tag{4}$$

$N_f$ being the number of flavors.

Because of this constraint (i) it is not possible to write a mass term for the constituent fields, (ii) the expression of the Lagrangian in terms of independent fields involves an infinite power series arising from the solution of the constraint, and (iii) it is not possible to have fermionic constituents.

We will be able to avoid such limitations by regularizing the model in such a way that the constraint on the fields is replaced by a constraint on expectation values.

In the noncompact lattice regularization we are going to use, the covariant derivative is

$$\mathcal{D}_\mu(x)\lambda(x)=D_\mu\lambda(x+\mu)-\frac{1}{a}\lambda(x) \ , \tag{5}$$

where $\mu$ is the unit vector with components $\mu_\nu=\delta_{\mu\nu}$ and

$$D_\mu=V_\mu+iA_\mu \tag{6}$$

is the parallel transporter expressed in terms of the gauge field $A_\mu$ and the auxiliary field $V_\mu$. Under gauge transformations

$$\lambda_h(x)\to g(x)\lambda_h(x)\ , \tag{7}$$

$$D_\mu(x)\to g(x)D_\mu(x)g^\dagger(x+\mu) \tag{8}$$

for $g\in SU(2)$.

The transformations of $A_\mu$ and $V_\mu$ are

$$\begin{aligned}\delta A_\mu&=i[A_\mu,\theta]+a\left[V_\mu\Delta_\mu\theta+\frac{i}{2}[A_\mu,\Delta_\mu\theta]\right]\ ,\\ \delta V_\mu&=\tfrac{1}{2}a\,\mathrm{Tr}\,A_\mu\Delta_\mu\theta\ ,\end{aligned} \tag{9}$$

where $\theta_a$ are the gauge parameters. We see that for $a\to 0$, they do not reproduce gauge transformations. These can be recovered if the auxiliary field $V_\mu$ acquires a nonvanishing expectation value

$$\langle V_\mu\rangle=\frac{1}{a}\ , \tag{10}$$

so that defining the shifted field

$$W_\mu=\frac{1}{a}-V_\mu\ , \tag{11}$$

we have

$$\begin{aligned}\delta A_\mu&=\Delta_\mu\theta+i[A_\mu,\theta]-a\left[W_\mu\Delta_\mu\theta-\frac{i}{2}[A_\mu,\Delta_\mu\theta]\right]\ ,\\ \delta W_\mu&=\tfrac{1}{8}a\,\mathrm{Tr}\,A_\mu\Delta_\mu\theta\ .\end{aligned} \tag{12}$$

In conclusion a noncompact regularization can be constructed if (a) spontaneous breaking of GL to IGL can be ensured by a suitable potential and (b) the auxiliary field decouples in the continuum limit.

The strength and the Yang-Mills Lagrangian density can be written in analogy with the continuum

$$F_{\mu\nu}=\frac{1}{i}[D_\mu(x)D_\nu(x+\mu)-D_\nu(x)D_\mu(x+\nu)]\ , \tag{13}$$

$$\mathcal{L}_{\mathrm{YM}}=\tfrac{1}{8}\beta\sum_{\mu\nu}F^\dagger_{\mu\nu}(x)F_{\mu\nu}(x)\ . \tag{14}$$

The possibility of enforcing conditions (a) and (b) is related to the existence of the other invariant

$$t_\mu=\tfrac{1}{2}\mathrm{Tr}\left[D^\dagger_\mu D_\mu-\frac{1}{a^2}\right]=A^2_\mu+W^2_\mu-\frac{2}{a}W_\mu\ . \tag{15}$$

The total action will contain an arbitrary function of this invariant. This function is determined [7] by some requirements including a divergent mass $\sim 1/a$ for $W_\mu$ to ensure its decoupling. The resulting total Lagrangian is

$$\mathcal{L}_G=\mathcal{L}_{\mathrm{YM}}-\tfrac{1}{16}\beta a^2\sum_{\mu\nu}(\Delta_\mu t_\nu-\Delta_\nu t_\mu)^2+\tfrac{1}{2}\gamma^2\sum_\mu t^2_\mu\ . \tag{16}$$

The minimum of the potential occurs at $V_\mu=\pm 1/a$, namely, $W_\mu=0,-2/a$. Since the second root has been shown to add only unessential complications conditions (a) and (b) are satisfied.

It is worthwhile mentioning that in the limit $\gamma\to\infty$ we recover Wilson's definition. The scaling properties of this regularization have been studied perturbatively. It has been found that in the continuum limit $\gamma$ grows proportionally to $\beta$, so that the fixed point is the same as in Wilson scheme. A Monte Carlo calculation has confirmed that the Wilson loop has essentially the same behavior.

This regularization is suitable to put $CP^{n-1}$ models on the lattice. This can be accomplished by the following definition of the parallel transporter:

$$\begin{aligned}D_{\mu\alpha\beta}=\sum_{h,k=1}^{N_f}\frac{1}{2a}[&\bar\lambda^*_{h\alpha}(x)M_{hk}\bar\lambda_{k\beta}(x+\mu)\\ &\pm\lambda_{h\alpha}(x)Q_{hk}\lambda^*_{k\beta}(x+\mu)\\ &+\bar\lambda^*_{h\alpha}(x)N_{hk}\lambda^*_{k\beta}(x+\mu)\\ &\pm\lambda_{h\alpha}(x)P_{hk}\bar\lambda_{k\beta}(x+\mu)]\ ,\end{aligned} \tag{17}$$

where

$$\bar\lambda=\sigma_2\lambda \tag{18}$$

and the $\pm$ sign refers to bosonic/fermionic constituents, respectively. In the latter case if there are spinor indices they are included in the flavor index $h$.

The above is the most general expression compatible with the gauge transformations (8). We must now constrain the matrices $M$, $Q$, $N$ and $P$ in such a way that $A_\mu$ and $W_\mu$ be Hermitian and reflection positivity be satisfied.

The gauge and auxiliary fields resulting from Eq. (17) are

$$\begin{aligned}A_{\mu a}(x)=\frac{i}{2a}[&\lambda^\dagger(x)M\sigma_a\lambda(x+\mu)-\lambda^\dagger(x+\mu)\tilde Q\sigma_a\lambda(x)\\ &+\lambda^\dagger(x)N\sigma_a\sigma_2\lambda^*(x+\mu)\\ &+\tilde\lambda(x+\mu)\tilde P\sigma_2\sigma_a\lambda(x)]\ ,\\ V_\mu(x)=\frac{1}{4a}[&\lambda^\dagger(x)M\lambda(x+\mu)+\lambda^\dagger(x+\mu)\tilde Q\lambda(x)\\ &+\lambda^\dagger(x)N\sigma_2\lambda^*(x+\mu)-\tilde\lambda(x+\mu)\tilde P\sigma_2\lambda(x)]\ ,\end{aligned} \tag{19}$$

where the constituent fields have been ordered in such a way to get rid of the $\pm$ sign.

Hermiticity of $A_\mu$ and $W_\mu$ requires

$$Q=M^*,\quad P=-N^*\ , \tag{20}$$

while to ensure the condition

$$D_{-\mu}(x)=D^\dagger_\mu(x-\mu) \tag{21}$$

related to reflection positivity it is sufficient that

$$M=M^+,\quad N=\pm\tilde N\quad\text{for fermions/bosons}\ . \tag{22}$$

Taking the above conditions into account,

$$\begin{aligned}A_{\mu a}=\frac{i}{2}[&\lambda^\dagger M\sigma_a\Delta_\mu\lambda-\Delta_\mu\lambda^\dagger M\sigma_a\lambda\\ &+\lambda^\dagger N\sigma_a\sigma_2\Delta_\mu\lambda^*-\Delta_\mu\tilde\lambda N^\dagger\sigma_2\sigma_a\lambda]\ ,\\ V_\mu=\frac{r}{4a}+\frac{1}{4}[&\lambda^\dagger M\Delta_\mu\lambda+\Delta_\mu\lambda^\dagger M\lambda\\ &+\lambda^\dagger N\sigma_2\Delta_\mu\lambda^*+\Delta_\mu\tilde\lambda N^\dagger\sigma_2\lambda]\ ,\end{aligned} \tag{23}$$

where

$$r=2\lambda^\dagger M\lambda+\lambda^\dagger N\sigma_2\lambda^*+\tilde{\lambda}N^\dagger\sigma_2\lambda \ . \tag{24}$$

So far we have constructed a parallel transporter [8] in terms of constituent fields, but to have a gauge theory we must impose condition (10). This can obviously be done only in the context of a definite Lagrangian that we now assume to be $\mathcal{L}_G$ (we could as well have chosen the lattice transcription of the $\mathrm{CP}^{n-1}$ Lagrangian). At this point we must distinguish between the bosonic and fermionic case. In the bosonic case condition (10) can be imposed at the semiclassical level. For constant constituent fields it reads

$$r=4 \ . \tag{25}$$

Such a condition follows from the potential of $\mathcal{L}_G$,

$$\frac{2}{a^4}\lambda^2\left[\left[\frac{r}{4}\right]^2-1\right]^2 , \tag{26}$$

which has in fact the degenerate minima $r=\pm4$. We recover a constraint on the fields for $\gamma\to\infty$, where the exponential of the last term of $\mathcal{L}_G$ becomes a $\delta$ function.

Let us now come to the issue of renormalizability. This requires that the Lagrangian have a quadratic term in the constituent fields and that the vertices have dimension zero. Since the vertices are quartic in the gauge fields, these fields must have dimension one and therefore, because of the derivatives in Eq. (23), the constituent fields must have scaling dimension zero. For the Lagrangian to contain a quadratic term it is necessary that $r$ contain a constant. This can be easily obtained. Taking advantage of the Hermiticity of $M$ we can perform on the constituent fields a transformation such that

$$r=\sum_{h=1}^{N_f}\epsilon_h\phi_h^\dagger\phi_h+\phi^\dagger N'\sigma_2\phi^*+\tilde{\phi}N'^\dagger\sigma_2\phi \ , \tag{27}$$

where $\epsilon_h=\pm1$ and $N'$ is the transformed of $N$. Next we introduce the real and imaginary parts of the new constituent fields,

$$\phi_{h\alpha}=\phi_{h\alpha1}+i\phi_{h\alpha2} \ , \tag{28}$$

and fix the gauge according to

$$\phi_{112}=\phi_{121}=\phi_{122}=0 \ . \tag{29}$$

This gauge fixing shows that the case of one single flavor is trivial because $A_\mu=0$. Assuming $\epsilon_1=1$ we shift the field $\phi_{111}$,

$$\phi_{111}=2+\rho \ , \tag{30}$$

getting

$$r=4+4\rho+\rho^2+\sum_{h=2}^{N_f}\epsilon_h\phi_h^\dagger\phi_h+\phi^\dagger N'\sigma_2\phi^*+\tilde{\phi}N'^\dagger\sigma_2\phi \ . \tag{31}$$

It remains to give the constituent fields scaling dimension zero. This can be accomplished by including in the Lagrangian a term with a fourth covariant derivative;

$$\mathcal{L}_{B4}=\sum_{h=1}^{N_f}(\mathcal{D}_2\lambda_h)^\dagger(\mathcal{D}_2\lambda_h) \ , \tag{32}$$

where

$$\mathcal{D}_2\lambda(x)=\frac{1}{a}\sum_\mu\left[D_\mu(x)\lambda(x+\mu)+D_\mu^\dagger(x-\mu)\lambda(x-\mu)-\frac{2}{a}\lambda(x)-\frac{a}{2}[t_\mu(x)+t_\mu(x-\mu)]\lambda(x)\right] . \tag{33}$$

The situation is different for fermions. First of all a condition on the fields cannot be imposed at the semiclassical level, neither by use of a $\delta$ function, because a $\delta$ function of Grassmann variables with all the properties and representations of ordinary $\delta$ functions does not exist. In the case of one single flavor, for instance, the function

$$\delta(\lambda^\dagger\lambda-v)=\frac{1}{2}v^2\exp\left[\frac{1}{v}\lambda^\dagger\lambda+\frac{1}{2v^2}(\lambda^\dagger\lambda)^2\right] \tag{34}$$

acts as a $\delta$ function on powers of $\lambda^\dagger\lambda$ but not on arbitrary functions of this argument nor with respect to integration over $v$. In the presence of more degrees of freedom the right-hand side of the above equation contains higher powers of $\lambda^\dagger\lambda$. This is at the origin of the difficulties in the mentioned attempts [5,6] to relate fermionic bilinears to bosonic fields. In the anticommuting case it is only possible to impose a condition on expectation values and to impose condition (10) we can include in the Lagrangian a term of the form (34).

As far as renormalizability is concerned, an expression corresponding to Eq. (31) does not exist, so that the Lagrangian has no quadratic terms in the constituent fields. A possible way out which, however, remains to be investigated, is to add a term

$$\mathcal{L}_{F4}=\Box\Lambda_h^*\Box\Lambda_h \ , \tag{35}$$

where $\Box$ is the Euclidean Laplacian on the lattice and

$$\Lambda_h=\lambda_h\sigma_2\lambda_h \tag{36}$$

are new gauge-invariant variables. $\mathcal{L}_{F4}$ is quadratic in the $\Lambda$ fields that we now take as fundamental fields according to the integration rule

$$\int d\lambda_{h1}^*d\lambda_{h1}d\lambda_{h2}^*d\lambda_{h2}=\tfrac{1}{4}\int d\Lambda_h^*d\Lambda_h \ . \tag{37}$$

We can adopt such a rule since the only nonvanishing contributions to the integration come from terms $\Lambda_h^*\Lambda_h$. For this reason we can perform the change of variables (36) at any nonvanishing order of perturbation theory even though we cannot express $\mathcal{L}_G$ in terms of the new variables. It should be noted, however, that the propagator arising from the integration rule (37) is not known. In such an integration, in fact, as it will be shown in a separate paper, there appear permanents which do not enjoy the nice properties of determinants.

Let us finally come to the issue of physical positivity. This requires that for any polynomial $\mathcal{P}$ of the $D_\mu$ at positive Euclidean times,

$$\langle \Theta \mathcal{P} \mathcal{P} \rangle \geq 0 \ , \tag{38}$$

where $\Theta$ is an operator which performs complex conjugation and Euclidean time inversion:

$$\Theta D_4(x) = \tilde{D}_4(\theta x - e_4) \ , \quad \Theta D_k(x) = D_k^*(\theta x) \ . \tag{39}$$

In the above equations $e_4$ is the unit vector in the Euclidean time direction and

$$\theta x_4 = x_4, \quad \theta x_k = x_k \ . \tag{40}$$

The proof of Eq. (38) can be split into two parts. The first one gives the proof for the case where the gauge fields are elementary. This is most conveniently done using (with minor changes) the formalism that Mack [9] has developed in the framework of the color-dielectric theory where also nonunitary link variables appear. Then one shows that the action of $\Theta$ on the constituent fields can be defined in such a way that equations (39) are satisfied. It is easy to check that this happens for the natural definition

$$\Theta[\lambda_\alpha(x)\lambda_\beta^*(y)] = \lambda_\beta(\theta y)\lambda_\alpha^*(\theta x) \tag{41}$$

by which also Eq. (21) is satisfied.

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