

Top quark width: Theoretical update

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A critical assessment of the available calculations of the top quark width is presented. QCD corrections, the finite mass of the b quark, and the effect of the W width are included as well as the electroweak corrections. The relative importance of these corrections is demonstrated for a realistic range of top quark masses. For the QCD corrected decay rate we use the formulas from Eilam *et al.* and include the electroweak correction taken from Fujii. Our results differ from those available in the literature because all the later calculations ignored the effect of the W width discussed earlier by Eilam *et al.* This leads to an effect comparable in size to the electroweak correction.

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I. INTRODUCTION AND SUMMARY

The discovery of the top quark has been anticipated for many years at accelerators of increasing energy. Present hopes are based on analyses of high-precision data and the standard theory, see [1]. The top is the first heavy quark whose mass can be measured to better than 1% precision at a future e^+e^- collider. Therefore, measurements of its width will not only test the standard model at the Born level, but also the QCD radiative corrections which are of order 10% [2]. This is in contrast with b and c quarks, where uncertainties in the masses and nonperturbative effects preclude this possibility.

Recently, the complete one-loop electroweak corrections to the total rate have been also calculated [3, 4], and turned out to be rather small (1–2%). Nevertheless, it has been claimed [3, 4] that a precise measurement of the top width may serve as a consistency check for the electroweak sector of the standard model. In fact a number of calculations have been performed studying electroweak effects on the top width in theories extending the standard model [5]. In particular it has been found that the additional corrections from the extended Higgs sector of the minimal supersymmetric standard model are significantly smaller than 1%.

In this article we give the standard model predictions

for the top quark width. Our results are different from those in [3, 4] because we include the effect of W boson width considered in [2] and neglected in later works. This effect is comparable in size to the electroweak corrections. A number of intrinsic uncertainties remain. The present uncertainty in α_s and the ignorance concerning the QCD correction of order $O(\alpha_s^2)$ limit the accuracy of the prediction to about 1–2%. One has to take into account also the errors, both experimental and theoretical, in the determination of the top mass.

At present the best place for a precise determination of Γ_t is believed to be the threshold region for $t\bar{t}$ production in e^+e^- annihilation. The most optimistic current estimate of the relative precision is 5% [6]. Therefore, it is mandatory to give the theoretical prediction which as the one presented in this article is accurate up to order of 1%.

II. QCD CORRECTED DECAY RATE

We assume throughout three families of quarks. Thus the effects of Cabibbo-Kobayashi-Maskawa (CKM) mixing are negligible. The QCD corrected width of the top quark is given by the formula [2]

$$\Gamma^{(1)} = \frac{G_F^2 m_t^5}{192\pi^3} \left(9 + 6\frac{\alpha_s}{\pi}\right) \int_0^{(1-\epsilon)^2} \frac{dy}{(1-y/\bar{y})^2 + \gamma^2} \left[F_0(y, \epsilon) - \frac{2\alpha_s}{3\pi} F_1(y, \epsilon) \right] \quad (1)$$

where

$$\bar{y} = (M_W/m_t)^2, \quad \epsilon = m_b/m_t, \quad \gamma = \Gamma_W/M_W$$

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and

$$\Gamma_W = \frac{G_F M_W^3}{6\sqrt{2}\pi} \left(9 + 6 \frac{\alpha_s}{\pi} \right) . \quad (2)$$

The functions $F_0(y, \epsilon)$ and $F_1(y, \epsilon)$ read¹

$$F_0(y, \epsilon) = \frac{1}{2} \sqrt{\lambda(1, y, \epsilon^2)} C_0(y, \epsilon) \quad (3)$$

where

$$\lambda(u, v, w) = u^2 + v^2 + w^2 - 2(uv + vw + wu) , \quad (4)$$

$$C_0(y, \epsilon) = 4[(1 - \epsilon^2)^2 + y(1 + \epsilon^2) - 2y^2] , \quad (5)$$

and

$$\begin{aligned} F_1(y, \epsilon) = & \frac{1}{2} C_0(y, \epsilon) (1 + \epsilon^2 - y) [2\pi^2/3 + 4\text{Li}_2(u_w) - 4\text{Li}_2(u_q) - 4\text{Li}_2(u_q u_w) - 4 \ln u_q \ln(1 - u_q) \\ & - 2 \ln u_w \ln u_q + \ln y \ln u_q + 2 \ln \epsilon \ln u_w] \\ & - 2F_0(y, \epsilon) [\ln y + 3 \ln \epsilon - 2 \ln \lambda(1, y, \epsilon^2)] + 4(1 - \epsilon^2) [(1 - \epsilon^2)^2 + y(1 + \epsilon^2) - 4y^2] \ln u_w \\ & + [3 - \epsilon^2 + 11\epsilon^4 - \epsilon^6 + y(6 - 12\epsilon^2 + 2\epsilon^4) - y^2(21 + 5\epsilon^2) + 12y^3] \ln u_q \\ & + 6\sqrt{\lambda(1, y, \epsilon^2)}(1 - \epsilon^2)(1 + \epsilon^2 - y) \ln \epsilon + \sqrt{\lambda(1, y, \epsilon^2)} [-5 + 22\epsilon^2 - 5\epsilon^4 - 9y(1 + \epsilon^2) + 6y^2] \end{aligned} \quad (6)$$

where

$$u_q = \frac{1 + \epsilon^2 - y - \sqrt{\lambda(1, y, \epsilon^2)}}{1 + \epsilon^2 - y + \sqrt{\lambda(1, y, \epsilon^2)}} , \quad (7)$$

$$u_w = \frac{1 - \epsilon^2 + y - \sqrt{\lambda(1, y, \epsilon^2)}}{1 - \epsilon^2 + y + \sqrt{\lambda(1, y, \epsilon^2)}} . \quad (8)$$

The inclusion of the W width through Dyson resummation is necessary in a calculation accurate up to a 1% level, see Table I below. Theoretical problems related to unstable particles in renormalizable field theories were discussed a long time ago [7]. In some recent calculations concerning Z^0 boson and W pair production problems were encountered with gauge invariance after Dyson resummation. These problems do not appear for top decays up to the accuracy discussed in the present paper.

In particular the on-shell W width appears in (1), and the resulting expression is manifestly gauge invariant. In higher orders of perturbation theory methods have to be employed analogous to those described in the literature on gauge boson production [8].

Above threshold for real W production the rate (1) can be approximated by

$$\Gamma_{\text{NW}}^{(1)} = \frac{G_F m_t^3}{16\sqrt{2}\pi} \left[F_0(\bar{y}, \epsilon) - \frac{2\alpha_s}{3\pi} F_1(\bar{y}, \epsilon) \right] , \quad (9)$$

a result valid in the narrow width approximation.

Neglecting ϵ one arrives at the relatively compact expressions

$$F_0(y, 0) = 2(1 - y)^2(1 + 2y) \quad (10)$$

and²

$$\begin{aligned} f(y) = & F_1(y, 0)/F_0(y, 0) \\ = & \frac{2\pi^2}{3} - \frac{5}{2} + 2 \ln y \ln(1 - y) + 4\text{Li}_2 y - 2y + \frac{1}{1 + 2y} \left[(5 + 4y) \ln(1 - y) + \frac{2y \ln y}{1 - y} - \frac{4y^3(1 - y + \ln y)}{(1 - y)^2} \right] . \end{aligned} \quad (11)$$

Formula (1) has been derived in [2] and tested in [9, 10]. When applied to charm decays, i.e., in the four fermion limit, it reproduces the numerical results for the total rate [11].

¹We slightly simplify an original formula from [1] using relations between dilogarithms.

²This form clearly exhibits limiting behavior

$$f(y) = \frac{2\pi^2}{3} - \frac{5}{2} - 3y(1 + y \ln y) + \dots$$

for small y , and

$$f(y) = 3 \ln(1 - y) + \frac{4\pi^2}{3} - \frac{9}{2} + \dots$$

for $y \rightarrow 1^-$. Although stated in the text, these limits are not manifest in the original formula given in [2].

TABLE I. Top width as a function of top mass and the comparison of the different approximations.

m_t (GeV)	$\alpha_s(m_t)$	$\Gamma_{\text{NW}}^{(0)}$ (GeV)	$\delta^{(0)}$ (%)	$\delta_{\text{NW}}^{(1)}(0)$ (%)	$\delta_{\text{NW}}^{(1)}$ (%)	$\delta^{(1)}$ (%)	$\Gamma^{(1)}$ (GeV)	δ_{EW} (%)	Γ_t (GeV)
90.0	0.118	0.0234	11.69	7.88	-3.81	6.56	0.0249	0.81	0.0251
100.0	0.116	0.0931	0.16	-4.56	-6.91	-6.89	0.0867	1.04	0.0876
110.0	0.115	0.1955	-1.44	-6.81	-7.83	-9.22	0.1775	1.20	0.1796
120.0	0.113	0.3265	-1.78	-7.61	-8.20	-9.89	0.2942	1.33	0.2982
130.0	0.112	0.4849	-1.82	-7.97	-8.37	-10.08	0.4360	1.43	0.4423
140.0	0.111	0.6708	-1.77	-8.15	-8.44	-10.10	0.6031	1.51	0.6122
150.0	0.110	0.8852	-1.69	-8.25	-8.47	-10.05	0.7962	1.57	0.8087
160.0	0.109	1.130	-1.60	-8.31	-8.49	-9.99	1.017	1.62	1.033
170.0	0.108	1.405	-1.52	-8.34	-8.49	-9.91	1.266	1.67	1.287
180.0	0.107	1.714	-1.45	-8.35	-8.48	-9.84	1.546	1.70	1.572
190.0	0.106	2.059	-1.39	-8.36	-8.47	-9.77	1.857	1.73	1.890
200.0	0.106	2.440	-1.33	-8.36	-8.46	-9.70	2.203	1.76	2.242

Formulas (3)–(6) including the b quark mass corrections have been tested by a numerical calculation in [9]. Although performed by the same authors this calculation should be considered an independent one since it was based on a completely different technique and matrix elements equivalent to those derived in the classic papers on muon decays [12] in a form adopted in [13] for charm decays. Furthermore we have observed that these formulas after an appropriate analytical continuation are equivalent to formulas in [14] describing vacuum polarization effects from heavy quarks in the W boson propagator.

Independent calculations including a nonzero b quark mass have been performed in [3] and [4]. The authors found a numerical agreement of their results with the formulas (3)–(6).

The massless limit, Eqs. (10) and (11), derived in [2] was rederived and confirmed by a number of groups [15–17]

We proceed now to the discussion of the numerical predictions for the decay rate and the quality of different approximations. As our input we use $M_W = 80.10$ GeV [2], $m_b = 4.7$ GeV, $\alpha_s(M_Z) = 0.118 \pm 0.007$ [18], and $M_Z = 91.187$ GeV [2].

Then $\alpha_s(m_t)$ is derived from the formula

$$(Q) = \frac{4\pi}{b_0 \ln Q^2/\Lambda^2} \left[1 - \frac{b_1}{b_0^2} \frac{\ln \ln Q^2/\Lambda^2}{\ln Q^2/\Lambda^2} \right], \quad (12)$$

$$b_0 = 11 - \frac{2}{3}N_f, \quad b_1 = 102 - \frac{38}{3}N_f,$$

for $N_f=5$ quark flavors. Uncertainties in the input value of $\alpha_s(M_Z)$ as well as the second order corrections $O(\alpha_s^2)$, which have not been calculated yet, lead to an error which we estimate to be of order 1%. In Table I we give our results for the widths obtained from different approximations as well as from the formula (1). Since most other authors present their results in comparison with the zeroth-order result $\Gamma_{\text{NW}}^{(0)}$ obtained in the narrow width approximation, we define

$$\delta^{(i)} = \Gamma^{(i)}/\Gamma_{\text{NW}}^{(0)} - 1 \quad (13)$$

where $i = 0, 1$ corresponds to the Born and the QCD corrected rate, respectively, and the widths in the numerators include the effects of the W propagator, cf. Eq. (1). Analogously we define $\delta_{\text{NW}}^{(1)}$ which is given by the ratio of the QCD corrected and the Born widths, both evaluated in the narrow width approximation, and $\delta_{\text{NW}}^{(1)}(0)$ for massless b quark.

III. ELECTROWEAK CORRECTIONS

The complete one-loop electroweak correction to the standard model top decay have been calculated in [3] and [4]. If the lowest-order width is parametrized by G_F and M_W , cf. Eqs. (1) and (9), the electroweak corrections are less than 2% for realistic top masses. In particular there are no sizable effects arising from Yukawa couplings [19].³ For $100 \text{ GeV} \leq m_t \leq 200 \text{ GeV}$ and Higgs boson mass $M_H \geq 100 \text{ GeV}$ the potentially large $O(m_t^2/M_W^2)$ contribution from the diagrams with Yukawa couplings are smaller than 0.2%, and hence much smaller than other, subleading in m_t terms. The dependence of the correction on M_H is weak; see [3] for details. In the following we assume $M_H = 100 \text{ GeV}$.

Strictly speaking m_t , M_Z , M_W , and M_H cannot be treated as independent parameters. The standard model and the existing data imply a relation between them. For our choice of the masses one can neglect this effect, provided m_t is not too close to the present experimental lower limit. The corresponding change of the Born width is -2.6% , -0.8% , and less than 0.3% for $m_t = 90, 100$, and ≥ 110 GeV, respectively. Therefore we ignore the above mentioned relation and treat all the masses as independent parameters. If the measured M_W and M_H turned out to be very different from the values assumed in this paper, it would be straightforward to evaluate the corresponding change of the Born width.

³We thank Andre Hoang for checking that this important result is in agreement with [3] when the latter calculation is restricted to the leading $O(m_t^2/M_W^2)$ contribution [20].

The width of the top quark including the electroweak correction can be evaluated from the formula

$$\Gamma_t = \Gamma^{(1)} [1 + \delta_{EW}] \quad , \quad (14)$$

and a simple parametrization

$$\delta_{EW}(\%) \approx 2 - 1.5\bar{y} \quad (15)$$

has been obtained by us from Table I in [3]. The results for Γ_t calculated using (14) and (15) are given in our Table I.

It should be noted that the size of the electroweak corrections is comparable to the uncertainties from as yet uncalculated $O(\alpha_s^2)$ corrections and the present uncertainty in the value of α_s . The electroweak corrections are furthermore sensitive to the details of the Higgs sector,

as exemplified by the recent calculations in the context of the two Higgs doublet model [5].

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- [1] L. Rolandi, in *Proceedings of the XXVth International Conference on High Energy Physics*, Dallas, Texas, 1992, edited by J. Sanford, AIP Conf. Proc. No. 272 (AIP, New York, 1993); Report No. CERN-PPE/92-175 (unpublished).
- [2] M. Jeżabek and J.H. Kühn, Nucl. Phys. **B314**, 1 (1989).
- [3] A. Denner and T. Sack, Nucl. Phys. **B358**, 46 (1991); **B358**, 46 (1991).
- [4] G. Eilam, R.R. Mendel, R. Migneron, and A. Soni, Phys. Rev. Lett. **66**, 3105 (1991).
- [5] B. Grzadkowski and W. Hollik, Nucl. Phys. **B384**, 101 (1992); A. Denner and A. Hoang, *ibid.* **B397**, 483 (1993).
- [6] K. Fujii, talk given at the workshop Studies of Top Quarks at Colliding-Beam Facilities, Madison, Wisconsin, 1992 (unpublished).
- [7] M. Veltman, Physica **29**, 186 (1963).
- [8] R.G. Stuart, Phys. Lett. B **262**, 113 (1991); **272**, 353 (1991); S. Willenbrock and G. Valencia, *ibid.* **259**, 373 (1991); A. Sirlin, *ibid.* **267**, 240 (1991); A. Leike, T. Riemann, and J. Rose, *ibid.* **273**, 513 (1991); H. Veltman, Report No. DESY 92-076 (unpublished).
- [9] M. Jeżabek and J.H. Kühn, Phys. Lett. B **207**, 91 (1988); see Appendix C of [10] for details.
- [10] M. Jeżabek and J.H. Kühn, Nucl. Phys. **B320**, 20 (1989).
- [11] N. Cabibbo and L. Maiani, Phys. Lett. **79B**, 109 (1978).
- [12] R. Behrends, R. Filkenstein, and A. Sirlin, Phys. Rev. **101**, 866 (1956); S. Berman, *ibid.* **112**, 267 (1958); T. Kinoshita and A. Sirlin, *ibid.* **113**, 1652 (1959).
- [13] A. Ali and E. Pietarinen, Nucl. Phys. **B154**, 519 (1979).
- [14] T.H. Chang, K.J.F. Gaemers, and W.L. van Neerven, Nucl. Phys. **B202**, 407 (1980).
- [15] A. Czarnecki, Phys. Lett. B **252**, 467 (1990).
- [16] C.S. Li, R.J. Oakes, and T.C. Yuan, Phys. Rev. D **43**, 3759 (1991).
- [17] J. Liu and Y.-P. Yao, Int. J. Mod. Phys. A **6**, 4925 (1991).
- [18] G. Altarelli, in *QCD—20 Years Later*, Proceedings of the Workshop, Aachen, Germany, 1992, edited by P.M. Zerwas and H.A. Kastrup (World Scientific, Singapore, in press); Report No. CERN-TH.6623/92 (unpublished).
- [19] B.A. Irwin, B. Margolis, and H.D. Trottier, Phys. Lett. B **256**, 533 (1991).
- [20] A. Hoang (private communication).