PHYSICAL REVIEW D VOLUME 48, NUMBER 5 1 SEPTEMBER 1993

Asymptotic behavior of the total cross section of p-p scattering and the Akeno cosmic ray data

N. N. Nikolaev
IKP (Theorie), Forschugszentrum Juelich Gmbh, 5170 Juelich, Germany
and L.D. Landau Institute for Theoretical Physis, GSP-1, 117940, ul. Kosygina 2, 117 334 Moscow, Russia

(Received 22 April 1993)

I present a new determination of the total cross section for proton-proton collisions from the recent Akeno results on the absorption of cosmic ray protons in p-air collisions. An extrapolation to SSC energies suggests $\sigma_{\rm tot}(p\text{-}p)\approx 160-170$ mb. I also comment on a possible sensitivity of the p-air cross section determinations to assumptions on the inelasticity of nuclear collisions at high energy.

PACS number(s): 96.40.Pq, 12.40.Pp, 13.85.Lg, 13.85.Tp

At present, the accelerator experiments give the total proton-proton cross section up to $\sqrt{s}=1.8$ TeV, and the cosmic ray experiments remain a unique source of information on the proton interaction cross sections at higher energies. Recently, the Akeno Collaboration has presented the results on p-air interactions up to $\sqrt{s}=24$ TeV [1]. The subject of this Rapid Communication is a reanalysis of these data in terms of the proton-proton cross section. My major conclusion, based on the comprehensive analysis [2,3] of the relationship between the proton-nucleus and the proton-proton cross sections, is that the underestimated values of $\sigma_{\rm tot}(p\text{-}p)$ were inferred in [1].

By the nature of cosmic ray experiments the quasielastic scattering of primary protons on nuclei which retains the incident hadron is unobservable. Therefore, the cross section, measured in the Akeno experiment, must be identified with the absorption cross section given by

$$\sigma_{\rm abs}(p\text{-air})=\sigma_{\rm tot}(p\text{-air})-\sigma_{\rm el}(p\text{-air})-\sigma_{\rm Qel}(p\text{-air})=\int d^2\mathbf{b}\left\{1-\left[1-\frac{1}{A}\sigma_{\rm in}(p\text{-}p)T(b)\right]^{2A}\right\}$$
$$\approx\int d^2\mathbf{b}\{1-\exp[-\sigma_{\rm in}(p\text{-}p)T(b)]\}\ . \quad (1)$$

Here $T(b)$ is the conventional optical thickness of a nucleus, folded with the profile function $\Gamma_{pp}(b)$ of the p-p scattering:

$$T(b)=\frac{2}{\sigma_{\rm tot}(p\text{-}p)}\int d^2\mathbf{c}\Gamma_{pp}(\mathbf{c})\int dz n_A(z,\mathbf{b}-\mathbf{c})\ , \quad (2)$$

$n_A(\mathbf{r})$ is the nuclear matter density for the pointlike nucleons (the usually quoted nuclear charge distributions include the charge radius of the proton), and $\Gamma_{pp}(\mathbf{b})$ is defined by

$$f_{pp}(\mathbf{q})=\frac{ik}{2\pi}\int d^2\mathbf{b}\Gamma_{pp}(b)(-i\mathbf{q}\cdot\mathbf{b})$$
$$\approx\frac{ik}{4\pi}\sigma_{\rm tot}(p\text{-}p)\exp(-\tfrac{1}{2}B_{pp}q^2)\ . \quad (3)$$

The above simple Glauber model [4] formulas must be corrected for Gribov's inelastic shadowing [5], which was done in [2]. Because of strong absorption this inelastic shadowing correction is numerically small (for more details see [2,6]).

The major observation is that by virtue of the s-channel unitarity there is a strong correlation between the p-p interaction radius and the p-p cross section [2]. This correlation is less stringent than the one dictated by the naive geometrical scaling $B_{pp}\propto\sigma_{\rm in}(p\text{-}p)$, but sufficiently strong to ensure that there emerges a universal relationship

$$\sigma_{\rm in}(p\text{-}p)=(100\ \text{mb})[\sigma_{\rm abs}(p\text{-air})/507\ \text{mb}]^{1.89}\ . \quad (4)$$

The remarkable virtue of this relationship is that it does not depend on the detailed form of the energy dependence of the p-p cross section. This relationship, derived in [2], includes corrections for Gribov's inelastic shadowing. A very conservative estimate of the theoretical systematic uncertainty of the inelastic p-p cross section determined from (1) and (4) does not exceed 5 mb. Gaisser *et al.* [3] have studied in much detail $\sigma_{\rm abs}(p\text{-air})$ as a function of B_{pp} and $\sigma_{\rm in}(p\text{-}p)$, $\sigma_{\rm tot}(p\text{-}p)$ and have found results very close to (4).

The total cross section $\sigma_{\rm tot}(p\text{-}p)$ is obtained from (4) adding the elastic cross section: $\sigma_{\rm tot}(p\text{-}p)=\sigma_{\rm in}(p\text{-}p)+\sigma_{\rm el}(p\text{-}p)$. By virtue of the same $\sigma_{\rm in}(p\text{-}p)-B_{pp}$ correlation, the ratio $R=\sigma_{\rm el}(p\text{-}p)/\sigma_{\rm in}(p\text{-}p)$ too is, to a crude approximation, a function of the predominantly absolute value of $\sigma_{\rm in}(p\text{-}p)$. In the QCD motivated models of the Pomeron [2] the ratio R rises from $R\approx 0.19$ at $\sigma_{\rm in}(p\text{-}p)=30$ mb to $R\approx 0.35$ at $\sigma_{\rm in}(p\text{-}p)=100$ mb and $R\approx 0.46$ at $\sigma_{\rm in}(p\text{-}p)=150$ mb. Such a rise of R is typical for the onset of the black disk regime. With the parameters of the QCD Pomeron which are consistent with the

CERN Super Proton Synchroton ($Sp\bar{p}S$) [7] and Fermilab[8] data, the predictions [2] for R in the vicinity of $\sigma_{in}(p\text{-}p) \sim 100$ mb follow an approximate law

$$R \approx 0.1 + 0.25[\sigma_{in}(p\text{-}p)/100 \text{ mb}] . \quad (5)$$

For instance, E710 measured $R = 0.23 \pm 0.012$ at $\sigma_{in}(p\text{-}p) = 55.5 \pm 4$ mb compared to $R = 0.24$ given by Eq. (5). The conservative estimate of the theoretical systematic error in R at $\sigma_{in}(p\text{-}p) \sim 100$ mb is $\Delta R \sim 0.05$. The overall theoretical uncertainty in the so-determined $\sigma_{tot}(p\text{-}p)$ will not exceed 5–10 mb. The results of my reanalysis of the Akeno data are shown in Table I.

The values of $\sigma_{tot}(p\text{-}p)$ presented in Table I are ~ 35 mb larger than those cited in [1]. The origin of this difference is as follows: The Glauber formula for the inelastic cross section $\sigma_{in}(p\text{-air}) = \sigma_{tot}(p\text{-air}) - \sigma_{el}(p\text{-air})$ which includes the unobservable quasielastic cross section, reads

$$\sigma_{in}(p\text{-air}) = \int d^2\mathbf{b} \left\{ 1 - \left[1 - \frac{1}{2A}\sigma_{tot}(p\text{-}p)T(b) \right]^{2A} \right\}$$
$$\approx \int d^2\mathbf{b} \{ 1 - \exp[-\sigma_{tot}(p\text{-}p)T(b)] \} . \quad (6)$$

Notice the close similarity between the $\sigma_{in}(p\text{-}p) - \sigma_{abs}(p\text{-air})$ relationship, Eq. (1), and the $\sigma_{tot}(p\text{-}p) - \sigma_{in}(p\text{-air})$ relationship, Eq. (6). (This is a particular case of the universality of nuclear cross sections discussed in [6].) Therefore, if one treats the cosmic ray cross section as $\sigma_{in}(p\text{-air})$ (which for the above discussed reasons is illegitimate), then $\sigma_{in}(p\text{-}p)$ given by Eq. (4) will erroneously be taken for $\sigma_{tot}(p\text{-}p)$. Such a determination of $\sigma_{tot}(p\text{-}p)$ will be short of the elastic scattering cross section $\sigma_{el}(p\text{-}p)$, which is precisely a discrepancy between the results of [1] and the present study. Indeed, the authors of Ref. [1] have identified their measured cross section with $\sigma_{in}(p\text{-air})$ calculated by Durand and Pi [9].

It is worthwhile to comment that the quantity measured in the Akeno experiment, as in all the air shower experiments, is the observed mean free path for the development of air showers. The relation of this quantity to the true absorption length depends on the so-called inelasticity coefficient K_{in}, the rate of transfer of energy from the projectile proton to the secondary particles. The analysis [1] assumed, in fact, the energy-independent elasticity coefficient K_{el}, a fraction of the projectile's energy carried away by the leading particles.

To a crude approximation, the energy dissipation rate is controlled by a sort of transport cross section $\sigma_{abs}(p\text{-air})[1 - K_{el}(p\text{-air})]$. According to an analysis [2], in the p-air collisions the elasticity coefficient decreases from $K_{el}(p\text{-air}) \approx 0.4$ at $\sigma_{abs}(p\text{-air}) \sim 300$ mb typical of the accelerator (Fermilab) energies, down to $K_{el}(p\text{-air}) \approx 0.31$ at $\sigma_{abs}(p\text{-air}) \sim 400$ mb and $K_{el}(p\text{-air}) \approx 0.23$ at $\sigma_{abs}(p\text{-air}) \sim 500$ mb. (Other models of the scaling violations and their implications for the air shower development are discussed by Gaisser *et al.* [10].) Therefore, assuming the exact Feynman scaling of the projectile fragmentation spectra, one will overestimate $\sigma_{abs}(p\text{-air})$. A very crude estimate of the so introduced "scaling-violation bias,"

$$\Delta\sigma_{abs} \approx -\sigma_{abs}(\text{Akeno})\frac{\Delta K_{el}(p\text{-air})}{1 - K_{el}(p\text{-air})} , \quad (7)$$

suggests that the absorption cross sections cited in [1] could have been overestimated by ~ 70–80 mb. This possible systematic bias exceeds the error bars quoted in [1]. If the Akeno results are corrected for such a bias, then the relationship (4) suggests that the proton-proton cross sections quoted in Table I must be lowered too:

$$\frac{\Delta\sigma_{in}(p\text{-}p)}{\sigma_{in}(p\text{-}p)} \approx 1.9\frac{\Delta\sigma_{abs}(p\text{-air})}{\sigma_{abs}(p\text{-air})}$$
$$\approx 1.9\frac{\Delta K_{el}(p\text{-air})}{1 - K_{el}(p\text{-air})} . \quad (8)$$

This amounts to a possible scaling-violation bias of $\Delta\sigma_{tot}(p\text{-}p) \sim -30$ mb. A more detailed analysis of the scaling-violation bias requires modeling the development of showers similar to what is being done by Gaisser *et al.* [10], which goes beyond the scope of this paper.

TABLE I. Determination of the inelastic and total cross section for p-p scattering from the Akeno data on the absorption cross section for p-air collisions.

$\log_{10}E$ (GeV)	$\sigma_{abs}(p\text{-air})$ (mb)	$\sigma_{in}(p\text{-}p)$ (mb)	$\sigma_{tot}(p\text{-}p)$ (mb)
7.17–7.41	480±33	90±12	120±15
7.41–7.65	500±38	97±14	130±18
7.65–7.89	537±33	111±13	154±17
7.89–8.13	507±61	100±22	135±29
8.13–8.37	498±64	97±24	129±30
8.37–8.61	550±72	117±29	162±38

CONCLUSIONS

A reanalysis of the Akeno data on absorption of the cosmic ray protons in the Earth's atmosphere gives a proton-proton scattering cross section ~ 30 mb larger than found in [1]. An extrapolation of the results of this reanalysis to the design energy of the Superconducting Super Collider (SSC) suggests $\sigma_{tot}(\sqrt{s} = 40 \text{ TeV}) \approx 160$–$170$ mb. The potentially large scaling-violation bias is pointed out, which suggests that the Akeno analysis [1] might have overestimated the proton-air absorption cross sections.

ACKNOWLEDGMENT

I am grateful to Tom Gaisser for the encouraging correspondence.

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