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Coherent effects, parton distributions, and baryon rich matter in central high energy heavy ion collisions

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We argue that the parton distributions measured in heavy ion collisions depend on the trigger for the centrality of the collisions as a result of coherent effects specific for the collisions of energetic composite particles. Percolation phase transitions in central heavy ion collisions are predicted and methods to form and to investigate such baryon rich matter are suggested.

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I. INTRODUCTION

There is little doubt that quantum chromodynamics (QCD) is relevant for the strong interactions between hadrons. Asymptotic freedom in perturbative QCD [1] as well as the more detailed predictions of perturbative QCD (PQCD) have been confirmed in numerous hard processes. One of the pressing problems now is to develop a theoretical framework that will allow us to use hard processes in order to investigate softer phenomena, to search for possible phase transitions in superdense hadron matter. The aim of the present paper is to show how coherent phenomena can be used for this purpose.

In order to convey the main idea, let us consider the scattering of a sufficiently energetic composite particle h from a target T at rest. It follows from the uncertainty principle that if

$$\frac{2E_h}{M_n^2 - M_h^2} \gg 2r_T, \tag{1}$$

then the time necessary for a transition between different quark-gluon configurations $|n\rangle$ with the invariant mass M_n to occur is larger than the passage time through the target (r_T is the radius of the target T, $\hbar = c = 1$). Moreover, the calculation of the minimal momentum transferred to the target shows that in the case of diffractive processes the difference between the masses of different configurations satisfying Rel. (1) may be neglected. As a result the sums over intermediate states in the inelastic eikonal formulas can be accounted for by closure. These features ensure that such configurations of the constituents are frozen during the collision and the interaction process of the projectile with the target has a coherent character. The degeneracy between different configurations is removed by the interaction with the target.

The eigenstates of the scattering matrix S form a natural basis to decompose the wave function of the energetic projectile h (Pomeranchuk and Feinberg [2], Good and Walker [3]):

$$|h\rangle = \sum_n c_n |n\rangle, \tag{2}$$

where

$$S|n\rangle = d_n |n\rangle. \tag{3}$$

The scattering eigenstates $|n\rangle$ acquire a phase after the interaction with the target and therefore

$$S|h\rangle = \sum d_n c_n |n\rangle. \tag{4}$$

Inelastic processes will occur if the phases d_n of the different states $|n\rangle$ are different. By selecting certain final states, it is possible to effectively enhance the role of some scattering states in a projectile.

It is known by now that, within the eikonal approximation, configurations of different transverse spatial size r are scattering eigenstates, characterized by different cross sections. In particular in PQCD, for $r^2 \ll \langle r^2\rangle$,

$$\sigma \approx \frac{r^2}{\langle r^2\rangle}\langle\sigma\rangle \tag{5}$$

as a result of color screening. Here $\langle\sigma\rangle$ is the measured mean value of the interaction cross section and $\pi\langle r^2\rangle$ is the average cross section of such configurations. Such a dependence of cross section on the area occupied by constituents is more general than PQCD. Any configuration of constituents aligned along hadron h momentum is characterized by a small cross section as well. For a review and earlier references on the theoretical and experimental aspects in support of Eq. (5) in perturbative and nonperturbative QCD, on the region of applicability of Eq. (5) and of the emerging physical picture, see Ref. [4].

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The number of frozen configurations $|n\rangle$ during a collision depends on the initial energy, see Rel. (1), and this number increases with increasing energy of the projectile. In the case of a nucleon projectile this physics becomes relevant for $A \approx 200$ at $E_N > 40$ GeV in the nucleus rest frame [at CERN, the BNL Relativistic Heavy Ion Collider (RHIC), and CERN Large Hadron Collier (LHC)], which follows from using $M_n = m_{N^*}$ in Rel. (1)—the first excited state of a nucleon [5,6]. The states with smaller invariant masses are not important, since the pion is a pseudo Goldstone particle of the spontaneously broken chiral symmetry in QCD.

II. PHENOMENOLOGY OF INTERACTION CROSS SECTION DISTRIBUTION

The coherence of the configurations satisfying Rel. (1), but having different interactions with the target, can be formally accounted for by introducing a distribution over the values of the cross section $P(\sigma)$ [7], instead of an average value of the cross section only. In the scattering eigenstate basis transitions between different states are absent and this makes it possible to describe physical processes in terms of $P(\sigma)$. The width of the cross section probability distribution $P(\sigma)$ has been extracted from the experimental data on diffractive hadron production in pp scattering [7] and on the inelastic shadowing correction to the total cross section of pd scattering [5]. In the several hundred GeV energy range the second relative cumulant of the cross section probability distribution so obtained is

$$\kappa_2 = \frac{\langle\sigma^2\rangle}{\langle\sigma\rangle^2} - 1 \approx 0.2\text{–}0.3. \tag{6}$$

Recent Fermilab Tevatron data on single diffraction in pp scattering correspond to a value of the second relative cumulant $\kappa_2 = 0.17$–0.25 [9], barring the uncertainties related to unmeasured diffractive production of small masses. The analysis of the diffractive hadron production off deuteron [5] leads to

$$\begin{aligned} &\frac{\langle\sigma^3\rangle}{\langle\sigma\rangle^3} - 1 \approx 3\kappa_2 \approx 0.6\text{–}0.9\ , \\ &\kappa_3 = \frac{\langle\sigma^3\rangle}{\langle\sigma\rangle^3} - 3\frac{\langle\sigma^2\rangle}{\langle\sigma\rangle^2} + 2 \approx 0, \end{aligned} \tag{7}$$

where κ_3 is the third relative cumulant of the cross section probability distribution. By definition

$$\langle\sigma^n\rangle = \frac{\int_0^\infty P(\sigma)\sigma^n d\sigma}{\int_0^\infty P(\sigma) d\sigma}. \tag{8}$$

One can show also that

$$\frac{\langle\sigma^4\rangle}{\langle\sigma\rangle^4} - 1 \geq 6\kappa_2 + \kappa_2^2 + 4\kappa_3 + \frac{\kappa_3^2}{\kappa_2} \approx 6\kappa_2 + \kappa_2^2 \approx 1.24\text{–}1.89 \tag{9}$$

and establish similar lower bounds for higher moments as well. In Ref. [5] $P(\sigma)$ has been reconstructed for nucleon and meson projectiles, using as input the lowest four moments of $P(\sigma)$, a cutoff at large values of σ and by imposing the small σ behavior for $P(\sigma) \sim \sigma^{N_q - 2}$, dictated by the quark counting rules. Here N_q is the number of valence quarks. (The zeroth moment is simply the normalization of the probability distribution, the first moment is the average total cross section, etc.) For a meson projectile Eq. (5) and $P(\sigma \approx 0)$ is rather close to the QCD calculation at small r, where such an approach is legitimate [5].

Experimental data clearly demonstrate that the σ distribution is rather wide; see Rels. (6), (7), and (9). This implies large inelastic shadowing corrections to the total cross sections of the hadron-nucleus scattering, which have been observed long ago—cf. discussion and references in Ref. [5]. We want to use these large fluctuations, already observed experimentally, to analyze the possibility of transforming colliding nuclei into excited baryon rich matter. The idea is that within the generalized eikonal approximation, which includes inelastic intermediate states, the number of inelastic collisions is determined by moments of the cross section distribution [10]. Equations (6), (7), and (9) imply that, for the events with more than one inelastic collision, the interacting nucleon has effectively a larger than average spatial size. (In a single collision one measures $\langle\sigma\rangle$, in a double inelastic collision $\langle\sigma^2\rangle$ and so forth.) The cross section fluctuations lead to an enhancement of the fluctuations of the number of inelastic collisions and hence to larger fluctuations in transverse energy E_t [8] in comparison with those computed in an independent NN-collision description, with a constant (nonfluctuating) cross section [11]. Using the magnitude of the nucleon-nucleon cross section fluctuations measured at Fermilab and the CERN Intersection Storage Rings (ISR) in single diffractive processes, it has been found [8] that this effect contributes significantly to the broadening of the E_t tail found by NA34 at CERN [12]. The above argument leads us to conclude that the conventional use of large E_t's as a trigger for centrality in heavy ion collisions, selects at the same time in the wave function of interacting nucleon quark-gluon configurations of larger than average spatial size. We shall call such a configuration a "huskyon."

III. ESTIMATES OF THE FORMATION PROBABILITY OF "HUSKYONS"

For the sake of simplicity of the presentation we shall consider only the properties of the projectile. However, a similar picture is valid for the target nuclei as well. In an independent particle model of the nucleus, the probability of a nucleon to be in a larger than average size configuration, during a single NN collision, i.e., to be a "huskyon," is given by the formula

$$p_1(\sigma_0) = \frac{\int_{\sigma_0}^\infty \sigma P(\sigma) d\sigma}{\langle\sigma\rangle}, \tag{10}$$

where σ_0 characterizes the size and/or huskiness of an interacting nucleon. In the case of a trigger for n NN-inelastic collisions the probability of the huskyon formation is

$$p_n(\sigma_0) = \frac{\int_{\sigma_0}^{\infty} P(\sigma)\sigma^n d\sigma}{\langle \sigma^n \rangle}. \tag{11}$$

Equation (11) accounts for the fact that the probability of n inelastic collisions is proportional to $\langle \sigma^n \rangle$ [10]. Examples of such triggers are $E_t \gg \langle E_t \rangle$, $E_t^n \gg \langle E_t^n \rangle$, etc. Relation (11) for $n = 2$ is equivalent to Eq. (12) of Ref. [8], obtained by using the Abramovsky, Gribov, and Kancheli (AGK) rules [13] for the calculation of the inclusive spectrum. We consider in this paper the contribution of soft processes and neglect minijet production. In this case the applicability of AGK rules [13] is practically insensitive to the way the energy of the nucleus is divided between its constituents. We use the cross section distribution extracted in Ref. [5] for the minimal value of κ_2 in Rel. (6), when the role of fluctuations is the smallest. A simple numerical calculation shows that, for $n = 1, \ldots, 5$,

$$p(= p_n) \geq 0.5 \tag{12}$$

for $\sigma_0 > \langle \sigma \rangle$, see Fig. 1. Evidently, these numbers are sensitive to lowest moments of $P(\sigma)$ but not to its precise form.

In order to get a rough idea of the consequences of the cross section fluctuations, let us consider an oversimplified model, where the probability for a nucleon to be a huskyon is p and the N nucleons are distributed uniformly. Here N is the number of nucleons at the same impact parameter (for heavy nuclei $N \approx A^{1/3} \approx 6$). The probability P_k to have at least k huskyons at a given impact parameter is given by the formulas

$$P_k = \sum_{r=k}^{N} \binom{N}{r} p^r (1-p)^{N-r}, \tag{13}$$
$$P_1 = 1 - (1-p)^N \ ,$$
$$P_2 = 1 - (1-p)^N - Np(1-p)^{N-1}, \ldots \ .$$

Unless p is extremely small, the probability to have a relatively large number of huskyons in a large nucleus is close to one. For example, if $p = 0.5$ and $N = 6$, then $P_1 = 0.98$, $P_2 = 0.89$, and $P_3 = 0.66$. Consequently, depending on the specific trigger chosen, a half or more of the total number of nucleons are huskyons in the case of large E_t as a trigger for the centrality of the collision. Obviously, this oversimplified model underestimates the number of huskyons, since it neglects the correlations between three and more particles, which are large; see Rel. (7). Under such conditions it follows that the physical object under consideration is a nucleus, where instantaneously many nucleon centers are excited, have a larger than average size, and consequently may overlap. The natural question arises whether they form a network. This is the standard problem of the percolation theory, see reviews [14]. To account for the geometry of percolation we model nuclei in the colliding nucleus rest frame as a system of spheres, which may be huskyons with probability p and normal nucleons with probability $1-p$. Results quoted in Refs. [14] show that for discrete lattices an infinite cluster (either bond or site percolation cluster) appears whenever $p > 0.12$–0.32, depending on the specific model. This condition is well satisfied in our case.

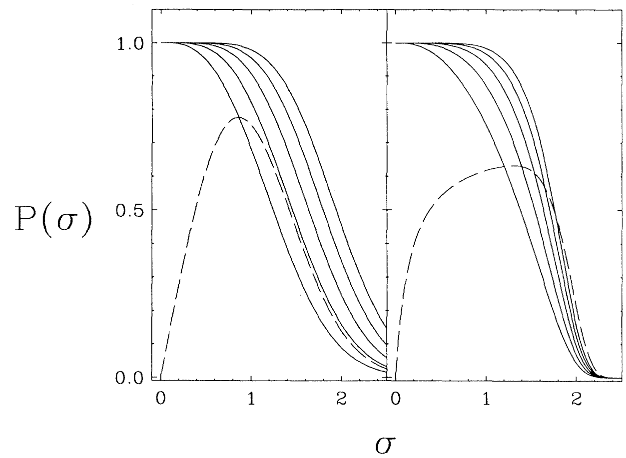

FIG. 1. The σ dependence of the probability $p = p_n(\sigma)$ (solid lines), see Eq. (11), for $n = 1, \ldots, 5$ $[p_{n+1}(\sigma) > p_n(\sigma)]$. Two parametrizations of the cross section probability distribution $P(\sigma) = N\sigma/(\sigma + a\sigma_1)\exp[-(\sigma/\sigma_1 - 1)^m/\Omega^m]$ (dashed line), see Ref. [6], with $m = 2$, $a = 1$, $\sigma_1 = 0.63$, $\Omega = 1.5$ (on the left-hand side) and $m = 10$, $a = 1$, $\sigma_1 = 0.16$, $\Omega = 11$ (on the right-hand side) have been used. Both these parametrizations are characterized by a dispersion $\kappa_2 = 0.25$, see Rel. (6). σ is in units of $\langle \sigma \rangle$.

Below the percolation threshold the average size of a cluster has a power law behavior, $N \sim (p - p_c)^{-\gamma}$, ($\gamma \approx 1.8$), where N is the average number of particles in a cluster. At the same time the size fluctuations are extremely large (the dispersion is infinite near the critical point [14]). It means that in a finite system the probability that a percolation cluster will straddle the whole volume will be close to unity near the percolation phase transition in an infinite system. In this respect the percolation phase transition in a finite system is completely similar to the "phase transitions" observed in atomic clusters, where melting and boiling are not as sharply defined as in the bulk, they occur over a range of temperatures and "appear" earlier [15]. The fact that a nucleus is a finite object does not prevent the onset of percolation, i.e., the formation of large clusters, but simply changes the long range behavior of the correlation functions. For these reasons we will keep the terminology of percolation phase transitions for the case of a finite system [17]. Our nontrivial observation is that percolation exists for relatively large values of σ_0; see Fig. 1. The bottom line is that central nucleus-nucleus collisions provide a natural method to select clusters of superdense nuclear matter in nuclei.

IV. PREDICTIONS

We have found that the relevant piece in the wave function of the colliding nuclei depends strongly on the trigger and on the energy of the collision. The question arises how to observe and to use this distinctive property. In the following we shall consider several patterns for the central nucleus-nucleus collisions only, i.e., $b \approx 0$.

A. Long range correlations in impact parameter space

The percolation characterizes the coherence of the quark-gluon orbits in different nucleons of the same nucleus, i.e., correlations between nucleons at different impact parameters. When percolation occurs, the spatial distribution of the huskyons looks like an "inverted Swiss cheese," which for the lack of a better term we shall sometimes refer to as a fractal (which seems to be the convention anyway [14]). It follows from the AGK rules [13] that single, double, etc., inclusive spectra of hadrons are proportional to the moments of huskyon distributions in nuclei. Thus near the percolation phase transition fluctuations of the distribution of the produced hadrons should reveal the fractal properties of the colliding nuclei. Modeling of this phenomenon is outside the scope of this paper.

B. Nuclear effects in parton distributions: Violation of Bjorken scaling and factorization theorem for the central heavy ion collisions

This amounts to the determination of the ratio of the structure functions of a nucleus and a nucleon defined as

$$R_i(x,Q^2,b,s) = \frac{D_i^{\rm beam}(x,Q^2,b)}{AD_i^N(x,Q^2)}. \qquad (14)$$

Here x is the usual Bjorken variable, b is the impact parameter in nucleus-nucleus collisions, s is invariant energy of collision, and i = valence quark, sea quark, or gluon. The major effects are the dependence of the parton distributions in projectile nucleus on the atomic number of the target and projectile nucleus and on energy of collision. We give here few examples of expected phenomena. Remember that factorization theorem, being proved for a hard inclusive spectrum, is not applicable for the reactions with a hadron trigger. Failure of factorization theorem for the hard single diffraction processes has been demonstrated in Ref. [16].

1. Trigger dependence of parton distributions

The depletion of the parton distributions at small x should be more pronounced than in the case of ordinary nuclei and extend over a larger region of x. This is because the huskion cross section is larger than NN cross section, which leads to an increase of shadowing, cf. Ref. [18], i.e., $R_i < 1$ for $2xm_N r_{NN} < 1$. Here r_{NN} is the mean internucleon distance in nuclei. The dependence of the structure functions of colliding nuclei on the impact parameter of the collision, i.e., the necessity to modify the factorization theorem, has been predicted in Ref. [19]. Here we discuss the dependence of the parton distributions on the energy of the collision, which reflects the change with energy of the relative importance of the different components of the wave function of the colliding nuclei. We want to draw attention to the fact that the lack of forward nucleons as trigger for centrality of the collision does not lead to the effects discussed in the paper.

2. Enhancement of valence quark and gluon distributions and suppression of antiquark distribution in nuclei with increasing energy

The application of the exact sum rules for the conservation of the baryon charge and of the total momentum,

$$\int_0^A \frac{1}{A} V_A(x_A,Q^2)dx_A - \int_0^1 V_N(x,Q^2)dx = 0, \qquad (15)$$

$$\int_0^A x_A \frac{1}{A}\left[V_A(x_A,Q^2)+G_A(x_A,Q^2)+S_A(x_A,Q^2)\right]dx_A - \int_0^1 x\left[V_N(x,Q^2)+G_N(x,Q^2)+S_N(x,Q^2)\right]dx = 0 \qquad (16)$$

(where $V_{A,N}$, $G_{A,N}$, $S_{A,N}$ represent the valence, gluon, and sea quark parton distributions in the nucleus A and in the free nucleon N, respectively, x is the usual Bjorken variable, and $x_A = AQ^2/2M_A Q_0$), leads to the conclusion, cf. Ref. [18], that in order to compensate the increased shadowing, the absolute value of the enhancement of the parton distributions R_i at larger x should also increase with energy. Thus, the qualitative prediction is that the gluon and valence quark distributions at $x \approx 2m_N r_{NN}$ should increase with energy in central heavy-ion collisions at fixed x and Q^2. This enhancement of the gluon and valence quark distributions can be searched for in any high p_t phenomena, in the bottom quark production, etc. Depending on the trigger for centrality the radius of the huskyon can be made as much as $\approx$ 30–40 % larger than the radius of a nucleon. When the number of huskyons in the nucleus is comparable with its atomic number then $R_v(x > 0.3) \le 1$, due to the larger radius of the huskyon. The estimate of this effect follows calculations of [20], which assume that the radius of a bound nucleon is larger than the radius of the free nucleon by $\approx$ 20%. Since a large huskyon radius can be achieved in heavy ion collisions, the estimates of Ref. [21] show that the expected effect should be larger than the observed European Muon Collaboration (EMC) effect. Similar effects are expected in central hadron-nucleus collisions.

The enhancement of the valence quark distribution, required by momentum sum rule, implies an enhancement of the gluon distribution, but not of the antiquark distributions in nuclei. These nuclear effects have been interpreted in Ref. [21] as being a consequence of the color screening phenomenon. If so, the ratio $R_v(x > 0.3)$ should decrease with the energy of the collision up to the energies where nucleons dissolve into quarks and gluons. This prediction is in agreement with the experimental data on deep inelastic lepton scattering off nuclei, which have put in evidence an enhancement of the valence quark distribution at $x \approx 2m_N r_{NN}$ when compared to that of a free nucleon. At the same time, no enhancement for the sea quark distribution in nuclei (which is expected if short range internucleon forces are due to meson exchanges) was seen [22,23]. Consequently, we do not expect an enhancement of the antiquark distributions in central nucleus-nucleus collisions as a result of the screening of the color fields of different nucleons. On the other hand, the enhancement of the antiquark distributions in the nucleon projectile, in central pA collisions, is natural, due to the increased meson cloud in a larger projectile. Such nuclear effects may be investigated also in the Drell-Yan lepton pair production in pA collisions with some hadron trigger in the final state [22].

3. Percolation versus quark-gluon phase

If the percolation phase transition in colliding nuclei in central heavy ion collisions corresponds to the quark-gluon phase, one may expect the occurrence of the effects discussed in connection with the conventional quark-gluon plasma (see Ref. [24] for a review and additional references): the enhancement of strange and charmed particle production for $x < A^{-1/3}$, etc. In the opinion of the present authors, the existence of the percolation phase transitions looks like an inevitable logical conclusion, in the light of the emerging current physical picture of high-energy collisions and of the observation of large cross section of diffractive processes.

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