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Dependence of the Gauss-law constraints on the regularization scheme in non-Abelian chiral gauge theory

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Mitra's regularizaton of the masslike term, which was originally discussed in an Abelian model, is used to calculate the anomaly in the commutator of the Gauss-law operators for anomalous $d=2$ non-Abelian chiral theory. The Schwinger term in the commutator of the different Gauss-law operator is shifted, because of regularization ambiguities, to the commutator of the Gauss-law operators with themselves. The Poisson brackets of the Gauss-law constraints correspond to the Kac-Moody algebra, and the Gauss-law constraints are similar to the chiral constraints. In a sense, this kind of Gauss-law constraint structure differs from what Faddeev suggested.

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Since Faddeev [1] proposed that there is an anomalous term in the commutator of the Gauss-law operators in chiral gauge theories, various methods ranging from point splitting [2], the Bjorken-Johnson-Low (BJL) limit [3–6], to others [7] have been used to calculate the Schwinger term in the commutator of the Gauss-law operators. It seems that the cohomological prediction for the Gauss-law commutator is realized up to a coboundary term. From the suggestion [8] of a connection between anomalies in space time, we know that the Schwinger term exists in the commutator of the different Gauss-law operators. And the appearance of this term drastically changes the nature of the constraints: instead of being first class they become second class.

For the chiral gauge theories in two dimensions, the bosonization method is often used to obtain the effective Lagrangian, which has the advantage that in the process of setting up the classical Hamiltonian dynamics associated with the effective Lagrangian, we can already confront and treat the consequences of the anomaly. The regularization may be specified by bosonizing the original model. The class of regularizations [9] that involve a dimensionless parameter a verify Faddeev's conjecture regarding the presence of a nontrivial two-cocycle in the gauge algebra. The general non-Abelian chiral gauge theories in two dimensions with the above regularization scheme have been studied in detail [10]. They show that there is an anomalous term in the commutator of the different Gauss-law operator indeed.

As is to be expected in a two-dimensional model, the regularization ambiguity can only affect the quadratic term in A_μ [11]. This is the reason why the now standard version of the chiral Schwinger model was given the arbitrary (parameter-dependent) masslike term. Starting with this masslike term, one can get the Gauss-law constraint structure which fits into Faddeev scenario—the commutator of the different Gauss-law operator $G^a(x)$ and $G^b(y)$ is nonzero.

Recently, Mitra [12] proposed a new regularization scheme and showed that the Gauss-law constraint $G(x)$ became second class through an anomaly in the Poisson brackets of $G(x)$ and $G(y)$ for the chiral Schwinger model with Mitra's regularization of the masslike term. From this new masslike term, the commutator of the different Gauss-law operators is zero, but these operators do not commute with themselves; i.e., the usual Schwinger term disappears and the new Schwinger term (the commutator of the Gauss-law operators with themselves) appears. The new commutator of the Gauss-law operators corresponds to a Kac-Moody algebra [13]. This kind of Gauss-law constraint structure is not in accordance with what Faddeev suggested. This is because the various Schwinger terms are different manifestations of the same anomaly [14] and, hence, regularization dependent. In other words the anomaly in the commutator of the different Gauss-law operators may be shifted, because of regularization ambiguities, to the commutator of the Gauss-law operators with themselves and vice versa.

Because the Gauss-law operators do not commute with themselves, our model has N^2-1 self-dual bosons in its spectrum [for the SU(N) gauge group]. The production mechanism of the self-dual bosons differs from other schemes [15] in which they introduce the chiral constraints to the model by hand. Also one finds that starting with the new regularization scheme the degrees of freedom are fewer than what Faddeev's mechanism involves.

Now let us consider a theory of massless fermions with only one of its chiral components coupled to an SU(N) gauge field in two space-time dimensions. The Lagrangian density is given by

$$\mathcal{L}=-\tfrac{1}{4}\,\mathrm{tr}F_{\mu\nu}F^{\mu\nu}+\bar{\psi}\left[i\partial\!\!\!/+eA\!\!\!/\left[\frac{1-\gamma_5}{2}\right]\right]\psi\ , \tag{1}$$

where [16]

$$A^{\mu}=A^{a}_{\mu}t^{a}\ , \tag{2a}$$

$$F_{\mu\nu}=\partial_{\mu}A_{\nu}-\partial_{\nu}A_{\mu}-ie[A_{\mu},A_{\nu}]\ . \tag{2b}$$

On integrating over the fermionic degrees of freedom and introducing an auxiliary field U(x) which is an SU(N) group element at each point, we get an effective bosonized action given by

$$\mathcal{L}_{\mathrm{eff}}=-\Gamma(U)-\frac{ie}{4\pi}(\eta^{\mu\nu}+\epsilon^{\mu\nu})\,\mathrm{tr}(U^{-1}\partial_{\mu}UA_{\nu})-\tfrac{1}{4}\,\mathrm{tr}F_{\mu\nu}F_{\mu\nu}+\mathcal{L}_{\mathrm{mass}}\ , \tag{3}$$

where $\Gamma(U)$ is the functional of the Wess-Zumino-Witten (WZW) nonlinear σ model [17]:

$$\Gamma(U)=-\frac{1}{8\pi}\int d^2x\ \mathrm{tr}(\partial_{\mu}U\partial^{\mu}U^{-1})-\frac{1}{4\pi}\int_0^1 dt\int d^2x\ \epsilon^{\mu\nu}\times\mathrm{tr}(U^{-1}\partial_{\mu}UU^{-1}\partial_{\nu}UU^{-1}\partial_{t}U)\ . \tag{4}$$

In the second term in (4), U has to be understood as a smooth function of t and x: $U(x,0)=1$, $U(x,1)=U(x)$.

In Eq. (3), $\mathcal{L}_{\mathrm{mass}}$ is the masslike term in (4); the usual form one takes is

$$\mathcal{L}^{(1)}_{\mathrm{mass}}=\frac{ae^2}{8\pi}\,\mathrm{tr}(A_{\mu}A^{\mu})\ . \tag{5}$$

However, in our Brief Report, we choose

$$\mathcal{L}^{(2)}_{\mathrm{mass}}=\frac{e^2}{8\pi}\,\mathrm{tr}(A_{-}A_{-}-4A_1A_1) \tag{6}$$

as our masslike term, which was first discussed by Mitra [12].

Here we should note that $\mathcal{L}_{\mathrm{eff}}$ is not Lorentz invariant. However, there are many Lorentz invariant theories where the invariance is not manifest. An example of special interest in this context is the theory of self-dual bosons [18]. The demonstration that $\mathcal{L}_{\mathrm{eff}}$ does not violate Lorentz invariance in spite of appearances to the contrary can be found in Refs. [12,19].

Before developing a canonical Hamiltonian formalism for the action (3), let us introduce some arbitrary canonical θ^i on the group manifold ($1\le i\le \dim G$); we define the vielein $V^a_i(\theta)$

$$iV^a_i(\theta)t^a=U^{-1}(\theta)\frac{\partial U(\theta)}{\partial\theta^i}\ . \tag{7}$$

In order to obtain the Hamiltonian corresponding to $\mathcal{L}_{\mathrm{eff}}$, let us define the quantities W^{ai} as

$$W^{ai}(\theta)V^b_i(\theta)=\delta^{ab}\ . \tag{8}$$

From the action $\mathcal{L}_{\mathrm{eff}}$ it is easy to calculate the canonical momenta corresponding to the coordinates θ^i, and after subtracting the contribution coming from the three-dimensional integrals, we obtain

$$\pi_i=\frac{V^a_i}{4\pi}(V^a_j\dot{\theta}^j+eA^a_{-})\ . \tag{9}$$

The nonvanishing equal-time Poisson brackets involving π_i are

$$\{\theta^i(x),\pi_j(y)\}=\delta^i_j\delta(x^1-y^1)\ , \tag{10a}$$

$$\{\pi_i(x),\pi_j(y)\}=-\frac{1}{4\pi}f^{abc}V^a_iV^b_jV^c_k\theta'^k\delta(x^1-y^1)\ . \tag{10b}$$

The nonvanishing of Eq. (10b) reflects the presence of the three-dimensional integral in Eq. (4), which is first order in time derivatives.

For the gauge field, we denote by π^a_0 and E^a the canonical conjugate momenta of A^a_0 and A^a_1:

$$\pi^a_0=0\ , \tag{11a}$$

$$E^a=F^a_{01}\ . \tag{11b}$$

As is usually done, Eq. (11a) has to be considered as a primary constraint and it holds only in a "weak" sense. The canonical Poisson brackets in this case are the familiar ones:

$$\{A^a_0(x),\pi^b_0(y)\}=\delta^{ab}\delta(x^1-y^1)\ , \tag{12a}$$

$$\{A^a_1(x),E^b(y)\}=\delta^{ab}\delta(x^1-y^1)\ . \tag{12b}$$

Let us now define the left- and right-handed currents $\rho_{\pm}(x)$ according to the Witten bosonization rules [17]

$$\rho_{+}(x)=\rho^a_{+}(x)t^a=\frac{-i}{4\pi}U^{-1}\partial_{+}U+\frac{e}{4\pi}A_{-}\ , \tag{13a}$$

$$\rho_{-}(x)=\rho^a_{-}(x)t^a=\frac{-i}{4\pi}U\partial_{-}U^{-1}-\frac{e}{4\pi}A_{-}\ . \tag{13b}$$

In terms of the π_i's, the Witten currents can be written as

$$\rho^a_{+}(x)=W^{ai}\pi_i+\frac{V^a_i}{4\pi}\theta'^i\ , \tag{14a}$$

$$\rho^a_{-}(x)=L^{ab}(-W^i\pi_i+\theta'^i)\ , \tag{14b}$$

where

$$L^{ab}(\theta)=\mathrm{tr}(t^aUt^bU^{-1})\ . \tag{15}$$

Then the Hamiltonian density $\mathcal{H}$ can be put in the form

$$\mathcal{H}=\tfrac{1}{2}E^aE^a-A_0^a(D_1E)^a+\pi(\rho_+^a\rho_+^a+\rho_-^a\rho_-^a)$$
$$-e\rho_+^aA_-^a+\frac{e^2}{2\pi}A_1^aA_1^a+V^a\pi_0^a\ . \tag{16}$$

In Eq. (16) we have added a term $V^a\pi_0^a$ in order to take into account the primary constraint given by (11a). V^a is a Lagrange multiplier, which is undetermined at this stage.

Using Eq. (10) it is not difficult to check that the equal-time Poisson brackets of the currents $\rho_\pm^a$ with themselves correspond to a Kac-Moody algebra:

$$\{\rho_\pm^a(x),\rho_\pm^b(y)\}=f^{abc}\rho_\pm^c\delta(x^1-y^1)\pm\frac{\delta^{ab}}{2\pi}\delta'(x^1-y^1)\ , \tag{17a}$$

$$\{\rho_+^a(x),\rho_-^b(y)\}=0\ . \tag{17b}$$

The Hamiltonian density $\mathcal{H}$ given in (16), together with the Poisson brackets appearing in (12) and (17), define our Hamiltonian system.

Let us now proceed with the constraint analysis. Our primary constraints are

$$\pi_0^a(x)=0\ . \tag{18}$$

The consistency of these constraints under time evolution requires that

$$G^a(x)=\{\pi_0^a(x),H\}=(D_1E)^a(x)+e\rho_+^a(x)=0\ ; \tag{19}$$

which are the generalizations of the usual Gauss-law constraints. As

$$\{G^a(x),G^b(y)\}=ef^{abc}G^c(x)\delta(x^1-y^1)+\frac{e^2}{2\pi}\delta^{ab}\delta(x^1-y^1)\ , \tag{20}$$

which corresponds to the Kac-Moody algebra, i.e.,

$$\{G^a(x),G^b(y)\}=ef^{abc}G^c(x)\delta(x^1-y^1)\quad\text{for } a\neq b\ , \tag{21a}$$

$$\{G^a(x),G^a(y)\}=\frac{e^2}{2\pi}\delta'(x^1-y^1)\quad\text{for each } a\ . \tag{21b}$$

In Eq. (21b), the repeated index a does not indicate the summation over a; it only means the index a is equal to b.

Equation (21) shows that the commutator of the different Gauss-law operators vanishes, but it does not commute with itself, which is similar to the so-called chiral constraint. The corresponding Gauss-law constraints are the second-class ones, and no further constraints are needed. It can be checked that

$$\{\pi_0^a(x),G^b(y)\}=0\ , \tag{22}$$

which means that the primary constraints $\pi_0^a(x)$ are first class. So in our model there are N^2-1 first-class constraints (π_0^a) and N^2-1 second-class constraints (G^a), where $G^a(x)$ are similar to the chiral constraints. However, in Ref. [15] the authors imposed chiral constraints on the model by hand, and in that case there are $2(N^2-1)$ primary constraints π_0^a and ρ_-^a (chiral constraints). Then the induced Gauss-law operators do not satisfy the Kac-Moody algebra.

Now we have completed the analysis of the Gauss-law constraint structure. The commutator of the Gauss-law operators corresponds to the Kac-Moody algebra. The Schwinger term in the commutator of the different Gauss-law operators may be shifted to the commutator of the Gauss-law operators with themselves. This is quite reminiscent of the well-known shift between covariant and consistent anomalies.

[1] L. D. Faddeev, Phys. Lett. **145B**, 81 (1984).

[2] L. D. Faddeev and S. L. Shatashvili, Phys. Lett. **167B**, 225 (1986); S. Hosono and K. Seo, Phys. Rev. D **38**, 1296 (1988).

[3] S. G. Jo, Phys. Rev. D **35**, 3179 (1987); Nucl. Phys. **B259**, 616 (1985); Phys. Lett. **163B**, 353 (1985).

[4] M. Kobayashi, K. Seo, and A. Sugamoto, Nucl. Phys. **B273**, 607 (1986).

[5] H. J. Rothe and K. D. Rothe, Phys. Rev. D **40**, 545 (1989); F. S. Otto, H. J. Rothe, and A. Rechnagel, *ibid.* **42**, 1203 (1990); F. S. Otto, *ibid.* **43**, 548 (1991).

[6] J. G. Zhou, C. Y. Xiao, and Y. Y. Liu, Phys. Rev. D **45**, 705 (1992).

[7] A. J. Niemi and G. W. Semenoff, Phys. Rev. Lett. **56**, 1019 (1986); H. Sonoda, Nucl. Phys. **B266**, 410 (1986); J. G. Zhou, Q. H. Dai, and Y. Y. Liu, Phys. Rev. D **43**, 613 (1991).

[8] T. Fujiwara, Phys. Lett. **152B**, 103 (1985); J. Mickelesson, Commun. Math. Phys. **97**, 361 (1985).

[9] R. Jackiw and R. Rajaraman, Phys. Rev. Lett. **54**, 1219 (1985); R. Rajaraman, Phys. Lett. **154B**, 305 (1985).

[10] R. Rajaraman, Phys. Lett. **162B**, 148 (1985); E. Abdalla and K. D. Rothe, Phys. Rev. D **36**, 3190 (1987); N. K. Falck and G. Kramer, Phys. Lett. B **193**, 257 (1987); M. V. Manias, F. A. Schaposnik, and M. Trobo, *ibid.* **195**, 209 (1987); A. V. Ramallo, Z. Phys. C **42**, 125 (1989).

[11] K. Harada, I. Kubota, and I. Tsutsui, Phys. Lett. B **173**, 77 (1986); R. Banerjee, Phys. Rev. Lett. **56**, 1889 (1986).

[12] P. Mitra, Phys. Lett. B **284**, 23 (1992).

[13] For a review and detailed reference see P. Goddard and D. Olive, Int. J. Mod. Phys. A **1**, 303 (1986).

[14] B. Zumino, Nucl. Phys. **B253**, 477 (1985); R. Banerjee, Phys. Rev. Lett. **61**, 514 (1988).

[15] K. Harada, Phys. Rev. Lett. **64**, 139 (1990); Y. Oz, Phys. Lett. B **245**, 497 (1990).

[16] Our conventions are

$$\eta_{00}=-\eta_{11}=1,\quad \epsilon_{01}=-\epsilon_{10}=1\ ,$$
$$\gamma^0=\sigma_1,\ \gamma^1=-i\sigma_2,\ \gamma_5=\gamma^0\gamma^1=\sigma_3,\ \epsilon_{\mu\nu}\gamma^\nu=\gamma_\mu\gamma_5\ ,$$
$$[t^a,t^b]=if^{abc}t^c,\quad \mathrm{tr}(t^at^b)=\delta^{ab}\ ,$$
$$A_\pm=A_0\pm A_1,\quad \partial_\pm=\partial_0\pm\partial_1\ .$$

[17] E. Witten, Commun. Math. Phys. **92**, 455 (1984).

[18] R. Floreanini and R. Jackiw, Phys. Rev. Lett. **59**, 1873 (1987).

[19] P. Mitra and R. Rajaraman, Phys. Rev. D **37**, 448 (1988).