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Exact effective action and spacetime geometry in gauged WZW models

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We present an effective quantum action for the gauged WZW model G_{-k}/H_{-k}. It is conjectured that it is valid to all orders of the central extension $(-k)$ on the basis that it reproduces the exact spacetime geometry of the zero modes that was previously derived in the algebraic Hamiltonian formalism. In addition to the metric and dilaton, the new results that follow from this approach include the exact axion field and the solution of the geodesics in the exact geometry. It is found that the axion field is generally nonzero at higher orders of $1/k$ even if it vanishes at large k. We work out the details in two specific coset models: one non-Abelian, i.e., SO(2,2)/SO(2,1), and one Abelian, i.e., $SL(2,\mathbb{R})\otimes SO(1,1)^{d-2}/SO(1,1)$. The simplest case $SL(2,\mathbb{R})/\mathbb{R}$ corresponds to a limit.

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I. INTRODUCTION

A gauged Wess-Zumino-Witten (WZW) model can be rewritten in the form of a nonlinear σ model by choosing a unitary gauge that eliminates some of the degrees of freedom from the group element and then integrating out the nonpropagating gauge fields [1,2]. The remaining degrees of freedom are identified with the string coordinates $X^\mu(\tau,\sigma)$. The resulting action exhibits a gravitational metric $G_{\mu\nu}(X)$ and an antisymmetric tensor $B_{\mu\nu}(X)$ at the classical level. At the one-loop level, there is also a dilation $\Phi(X)$. These fields govern the spacetime geometry of the manifold on which the string propagates. Conformal invariance at the one-loop level demands that they satisfy coupled Einstein equations. Because of exact conformal properties of the gauged WZW model, these equations are automatically satisfied.

For a restricted list of noncompact gauged WZW models, there is only one time coordinate [1,3,4], thus making them suitable for a string-theory interpretation in curved spacetime. The list may be extended to supersymmetry heterotic models [5–7]. Then these models can be viewed as generating automatically a solution of these rather unyielding Einstein equations. One need only do some straightforward algebra based on group theory to extract the explicit forms of $G_{\mu\nu},B_{\mu\nu},\Phi$. Following the lead of [2], in which the $SL(2\mathbb{R})_{-k}/\mathbb{R}$ case at $k=\frac{9}{4}$ [1] was interpreted as a string propagating in the background geometry of a black hole in two dimensions, several groups have worked out the geometry for all possible cases up to dimension 4 [5,8–10]. The resulting new geometries are generally nonisotropic and have singularities that are more intricate than a black hole, and may have physical interpretations in the early string Universe. The global aspects of these higher geometries have been understood [11,12,7]. They all have very interesting duality properties that correspond to interchanges of patches of the global geometry. This duality may be viewed as inversions in group space [11] and are related to asymmetric left-right gauging that involves a twist on the right relative to the left of the group element [8].

Since these are singular geometries, it is clearly desirable to go beyond the one-loop expansion of the effective σ model and consider the effect of the exact conformal invariance that underlies the gauged WZW model. It is the purpose of the present paper to accomplish this by considering the full quantum effective action. Of course, the full quantum action is of interest in its own right since the range of its applications goes far beyond the exact geometry of the model. However, at this point, rather than a complete derivation, we are able to present a conjecture on the form of the full quantum effective action. We will justify its form by deriving the exact geometry and comparing to our previous exact results obtained with algebraic Hamiltonian techniques. Therefore let us first briefly review the status of conformally exact results.

In recent papers [12–14] we showed how to improve on the perturbative Lagrangian results by using algebraic Hamiltonian techniques to compute globally valid and conformally exact geometrical quantities such as the metric and dilaton (and, in principle, other fields) in gauged WZW models. We have applied the method to bosonic, heterotic, and type-II supersymmetric four-dimensional (4D) string models that use the noncompact cosets. The main idea is as follows. It is part of the folklore of string theory that $L_0+\bar{L}_0$ is the Laplacian and that when applied to the tachyon T it takes the form

$$(L_0+\bar{L}_0)T=\frac{-1}{e^{\Phi}\sqrt{-G}}\partial_\mu(e^{\Phi}\sqrt{-G}\,G^{\mu\nu}\partial_\nu T)\ . \quad (1.1)$$

This equation follows from the general form of the low-energy effective action of string theory which concentrates on the low-lying spectrum. Equation (1.1) was used in [15] where the $SL(2,\mathbb{R})_{-k}/\mathbb{R}$ geometry to all orders in $1/k$ was "conjectured" to arise from it. Indeed, this simplest case has been checked to work up to four loops for the bosonic string and up to five loops for the type-II superstring [16]. In [13] we developed the general methods to use (1.1) to extract the *global* and conformally *exact* geometry for all G/H models, including the heterotic superstring case. This was based on the following proof of (1.1), which was implicit but was not stated explicitly in [13]: Evidently, $L_0+\bar{L}_0$, as constructed from currents in a G/H theory, is exact to all orders in $1/k$. The tachyon is annihilated by all $n\geq 1$ currents J_n^G, so that only the zero-mode currents J_0^G are relevant, as they appear in $L_0+\bar{L}_0$. We further made the reasonable assumption that the tachyon wave function depends only on the zero modes of the group parameters. Therefore we only need to know how to construct the zero-mode currents from the zero modes of the group parameters as differential operators. Then, after using the crucial observation that the tachyon T is constructed from certain gauge-invariant combinations of group parameters and then applying the chain rule as described in [13], $L_0+\bar{L}_0$ indeed takes the general form of the Laplacian in (1.1) with a nontrivial dilation and metric. Since this Laplacian is exact to all orders in $1/k$, the resulting metric and dilaton must be identified with the exact ones to all orders in $1/k$.

The Hamiltonian approach has effectively concentrated on the zero modes. Therefore, in comparing the old results to the new exact quantum action of the present paper, we must take care that the exact geometry is in agreement first and foremost for the zero modes. We shall see that the geometry for the higher modes may be nonlocal on the world sheet. Our approach here will apply to Abelian cosets such as

$$\mathrm{SL}(2,\mathbb{R})/\mathbb{R}\ ,$$

$$\mathrm{SL}(2,\mathbb{R})\otimes \mathrm{SO}(1,1)^{d-1}/\mathrm{SO}(1,1)\ ,$$

or

$$\mathrm{SL}(2,\mathbb{R})_{-k'}\otimes \mathrm{SU}(2)_k/(\mathbb{R}\otimes\tilde{\mathbb{R}})\ ,$$

as well as non-Abelian ones such as

$$\mathrm{SO}(2,2)/\mathrm{SO}(2,1)\sim \mathrm{SL}(2,\mathbb{R})\otimes \mathrm{SL}(2,\mathbb{R})/\mathrm{SL}(2,\mathbb{R})$$

or

$$\mathrm{SO}(3,2)/\mathrm{SO}(3,1)\ .$$

For related results for the Abelian coset $\mathrm{SL}(2,\mathbb{R})/\mathbb{R}$, see also a paper by Tseytlin [17] with whom the present investigations were initiated [18].

II. EFFECTIVE QUANTUM ACTION

The effective quantum action for any field theory is derived by introducing sources and then applying a Legendre transform [19]. The effective action, which is then used as a classical field theory, incorporates all the higher-loop effects. Based on a perturbative analysis in [20,21], it has been argued [17] that for the ungauged WZW model G_{-k} this procedure gives

$$\begin{aligned} S_{\mathrm{WZW}}^{\mathrm{eff}}&=(-k+g)I_0(g)\ ,\\ I_0(g)&=\frac{1}{8\pi}\int_M \mathrm{Tr}(\partial_+g^{-1}\partial_-g)\\ &\quad+\frac{1}{24\pi}\int_B \mathrm{Tr}(g^{-1}dg)^3\ . \end{aligned} \tag{2.1}$$

Therefore the full quantum effective action differs from the classical one only by the overall renormalization that replaces $(-k)$ by $(-k+g)$, where g is the Coxeter number for the group G, not to be confused with the group element $g(\sigma^+,\sigma^-)$ (we have also assumed a conformally critical theory with the Virasoro central charge at $c=26$ that fixes the value of k and we have neglected possible field renormalizations [27] for the group element g since they give rise to nonlocal terms in the σ model). Instead of relying on the perturbative approach in [20,21,17], we can justify the result (2.1) by the following argument on the geometry: Before the quantum effects are taken into account, the classical σ-model geometry of the WZW model is given by the group manifold metric and the antisymmetric tensor (the axion), both *multiplied by* $(-k)$. To derive the exact geometry by the algebraic Hamiltonian approach, one must use the quantum exact stress tensor to construct $L_0+\bar{L}_0$ as described in Sec. I. The conformally exact quantum stress tensor follows from the classical one by a well-known renormalization that replaces $(-k)$ by $(-k+g)$. It follows from this that the exact geometry in the Hamiltonian approach is the same as the classical geometry except for the aforementioned renormalization. To agree with this quantum result, the exact effective action must be the same as the classical one except for the proportionality constant $(-k+g)$ as given in (2.1). Furthermore, $g(\sigma^+,\sigma^-)$ is now treated as a classical field.

We now extend these arguments to the gauged WZW (GWZW) model for G_{-k}/H_{-k}, which is defined by the classical action [22,23]

$$\begin{aligned} S_{\mathrm{GWZW}}&=-kI_0(g)-kI_1(g,A_+,A_-)\ ,\\ I_1(g,A_+,A_-)&=\frac{1}{4\pi}\int_M \mathrm{Tr}(A_-\partial_+gg^{-1}-A_+g^{-1}\partial_-g\\ &\quad+A_-gA_+g^{-1}-A_+A_-)\ . \end{aligned} \tag{2.2}$$

Here g is a group element in G and $A_\pm$ is valued in the Lie algebra for the subgroup H. This action is invariant under the local gauge transformations that belong to the subgroup H:[1]

$$g\to\Lambda^{-1}g\Lambda,\quad A_\pm\to\Lambda^{-1}(A_\pm-\partial_\pm)\Lambda\ . \tag{2.3}$$

It is useful to make a change of variables to group elements $h_\pm\in H$, $A_+=\partial_+h_+h_+^{-1}$, $A_-=\partial_-h_-h_-^{-1}$. After picking up a determinant and an anomaly from the measure, the path integral is rewritten with a new form for the action [24,23]

$$S_{\mathrm{GWZW}}=-kI_0(h_-^{-1}gh_+)+(k-2h)I_0(h_-^{-1}h_+)\ , \tag{2.4}$$

[1]The more general left-right-asymmetric gauging of [8] may also be discussed in a straightforward fashion (for an application, see [12]).

which is manifestly gauge invariant under $h_\pm \to \Lambda^{-1} h_\pm$. The new path-integral measure is the Haar group measure $D_g Dh_+ Dh_-$. We want to take advantage of the similarity of this action to the classical WZW action: The first term is appropriate for G with central extension $(-k)$, and the second term is appropriate for H with central extension $(k-2h)$. Defining the new fields $g'=h_-^{-1}gh_+$, $h'=h_-^{-1}h_+$, $h''=h_-$ and taking advantage of the properties of the Haar measure, we can rewrite the measure and action in decoupled form $Dg'Dh'Dh''$ and $S=-kI_0(g')+(k-2h)I_0(h')$. This decoupled form emphasizes the close connection to the WZW path integral and gives us a clue as to how to guess the effective quantum action.

However, g',h' are not really decoupled, since we must consider sources coupled to the *original fields*. Indeed, to derive the quantum effective action one must introduce source terms and perform a Legendre transform. Since these coupled g',h',h'' integrations are not easy to perform, we will guess [18] the answer based on the remarks above and then try to justify it. By analogy to (2.1) we suggest that the quantum effective action is given by simply shifting $(-k)$ to $(-k+g)$ and $(k-2h)$ to $(k-2h)+h=k-h$ (again we neglect possible field renormalization):

$$S^{\text{eff}}_{\text{GWZW}}=(-k+g)I_0(h_-^{-1}gh_+)-(-k+h)I_0(h_-^{-1}h_+) \ . \tag{2.5}$$

This may now be rewritten back in terms of *classical* fields g, A_+, A_- by using the definitions given before. We obtain

$$S^{\text{eff}}_{\text{GWZW}}=(-k+g)\left[I_0(g)+I_1(g,A_+,A_-)+\frac{g-h}{-k+g}I_2(A_+,A_-)\right] , \tag{2.6}$$

$$I_2(A_+,A_-)=I_3(A_+)+I_3(A_-)+\frac{1}{4\pi}\int_M d^2\sigma\,\text{Tr}(A_+A_-) ,$$

where we have defined $I_3(A_+)\equiv I_0(h_+)$, $I_3(A_-)\equiv I_0(h_-^{-1})$ and used the Polyakov-Wiegman formula [24] to rewrite $I_0(h_-^{-1}h_+)\equiv I_2(A_+,A_-)$ in the form above. Note that $I_2(A_+,A_-)$ is gauge invariant. Our proposed effective action differs from the purely classical action (2.2) by the overall renormalization $(-k+g)$ and by the additional term proportional to $(g-h)$. In the large-k limit (which is equivalent to small $\hbar$), the effective quantum action reduces to the classical action, as it should.

This is not yet the end of the story, because what we are really interested in is the effective action for the σ model after the gauge fields are integrated out (and a unitary gauge fixed for g). In other words, sources are not introduced for the original $A_\pm$, but only for g. The effect of this is that the path integral over the above $A_\pm$ (or $h_\pm$) still needs to be performed. At the outset, with the classical action, the path integral over $A_\pm$ was purely Gaussian, and therefore it could be performed by simply substituting the classical solutions for $A_\pm=A_\pm(g)$ back into the action. This integration also introduces an anomaly which can be computed exactly as a one-loop effect. The anomaly gives the dilation piece to be added to the effective action

$$S_{\text{dil}}\sim\int_M d^2\sigma\sqrt{\gamma}R^{(2)}(\gamma)\Phi(g) , \tag{2.7}$$

where γ_{ab}, γ, and $R^{(2)}$ are the metric, its determinant, and curvature on the world sheet for any genus, respectively. In order to obtain the exact dilaton, we need to perform the $A_\pm$ integrations with the effective action, not the classical one. However, in (2.6) the parts $I_3(A_\pm)$ are nonlocal in the $A_\pm$ (although they are local in $h_\pm$). The reason is that

$$I_3(A_+)=I_0(h_+)\sim\int\text{Tr}(A_+\partial_-h_+h_+^{-1})+\cdots$$

and we cannot write $\partial_-h_+h_+^{-1}$ as a local function of A_+. Furthermore, in the non-Abelian case, $I_3(A_\pm)$ have additional nonlinear terms. So, if we believe that the quantum effective action is indeed (2.6), then the effective σ-model action we are seeking seems to be generally nonlocal even in the Abelian case (see also [17]). We will therefore concentrate on just the zero modes. As shown below, we have managed to obtain exactly the zero-mode sector of the σ model and proved that the geometry does indeed reproduce correctly the exact geometry derived before in the Hamiltonian formalism [13,14]. This is our justification for (2.6).

III. ZERO-MODE SECTOR

To restrict ourselves to the zero-mode sector, we do dimensional reduction by taking all the fields as functions of only τ (i.e., world line rather than world sheet). This extracts the low-energy point-particle content of the string. This technique proved to be very useful in the analysis of the GWZW model at the classical limit [11], and we now use it for the conformally exact action. The derivatives $\partial_\pm$ get replaced by ∂_τ, and $A_\pm$ get replaced by $a_\pm=\partial_\tau h_\pm h_\pm^{-1}$. Then all nonlocal and nonlinear terms drop out and we obtain the effective action in the zero-mode sector:

$$S_{\text{eff}}=\frac{-k+g}{4\pi}\int d\tau\,\text{Tr}(\tfrac{1}{2}\partial_\tau g^{-1}\partial_\tau g+a_-\partial_\tau g g^{-1}-a_+g^{-1}\partial_\tau g+a_-ga_+g^{-1}-a_+a_-)-\frac{g-h}{8\pi}\int d\tau\,\text{Tr}(a_+-a_-)^2 \ . \tag{3.1}$$

This action is gauge invariant for τ-dependent gauge transformations $\Lambda(\tau)$. Most notably, the path integral over $a_\pm$ is now Gaussian, and this permits the elimination of $a_\pm$ through the classical equations of motion,

$$\begin{aligned}(D_+gg^{-1})_H&=\frac{g-h}{k-g}(a_+-a_-) ,\\ (g^{-1}D_-g)_H&=\frac{g-h}{k-g}(a_+-a_-) ,\end{aligned} \tag{3.2}$$

where we have defined the covariant derivatives $D_\pm$ on the world line $D_\pm g = \partial_\tau g - [a_\pm, g]$ and the subscript H indicates a projection to the Lie algebra of the subgroup H. The system of equations (3.2) is linear and algebraic in $a_\pm$, and therefore it can be easily solved. To do that and for further convenience, it is useful to introduce a set of matrices $\{t_A\}$ in the Lie algebra of G which obey $\mathrm{Tr}(t_A t_B) = \eta_{AB}$, where the Killing metric η_{AB} is diagonal and normalized to have ± 1 eigenvalues. The subset of matrices belonging to the Lie algebra of the subgroup H will be denoted by $\{t_a\}$ with a lowercase subscripts or superscripts. Then we define the quantities

$$L^H = (g^{-1}\partial_\tau g)_H, \quad L^A_\mu \partial_\tau X^\mu = \mathrm{Tr}(g^{-1}\partial_\tau g t^A) ,$$
$$R^H = (-\partial_\tau g g^{-1})_H, \quad R^A_\mu \partial_\tau X^\mu = -\mathrm{Tr}(\partial_\tau g g^{-1} t^A) , \tag{3.3}$$
$$M_{ab} = \mathrm{Tr}(t_a g t_b g^{-1} - t_a t_b) ,$$

where X^μ, $\mu = 0, 1, \ldots, d-1$, are of the $d = \dim(G/H)$ parameters in g that are left over after going to a unitary gauge for g. Then the solution of (3.2) for $a_\pm$ is

$$a_+ = [M^T M - \lambda(M + M^T)]^{-1}[M^T R^H - \lambda(L^H + R^H)] , \tag{3.4}$$
$$a_- = [M M^T - \lambda(M + M^T)]^{-1}[M L^H - \lambda(L^H + R^H)] ,$$

where $\lambda = (g-h)/(k-g)$. Substitution of these expressions back into (3.1) gives

$$S^{\mathrm{eff}}_{\mathrm{point}} = \frac{k-g}{\pi}\int d\tau\, G_{\mu\nu}\partial_\tau X^\mu \partial_\tau X^\nu , \tag{3.5}$$

where the metric $G_{\mu\nu}$ is defined as

$$G_{\mu\nu} = g_{\mu\nu} + \tfrac{1}{8}\{[M^T M - \lambda(M+M^T)]^{-1}(M^T - \lambda I)\}_{ab} L^a_{\{\mu} R^b_{\nu\}} - \tfrac{1}{8}\lambda[M^T M - \lambda(M+M^T)]^{-1}_{ab} L^a_\mu L^b_\nu$$
$$- \tfrac{1}{8}\lambda[M M^T - \lambda(M+M^T)]^{-1}_{ab} R^a_\mu R^b_\nu , \tag{3.6}$$

with $g_{\mu\nu}$ being the part of the metric due to the kinetic first term in $I_0(g)$;

$$g_{\mu\nu} = \tfrac{1}{8} L^A_\mu L^B_\nu \eta_{AB} = \tfrac{1}{8} R^A_\mu R^B_\nu \eta_{AB} , \tag{3.7}$$

and where the curly brackets denote symmetrization with respect to the appropriate indices.

We will illustrate applications of the above result for several Abelian and non-Abelian cosets. The simplest case is $\mathrm{SL}(2,\mathbb{R})/\mathbb{R}$, but since this can be presented as a limit of more complicated cases, we will give the results for it after discussing others. This provides a check of our methods.

Let us specialize to the three-dimensional non-Abelian coset SO(2,2)/SO(2,1) whose exact metric and dilaton was found in [13] with the Hamiltonian approach. We will find the metric in a patch of the manifold corresponding, in the notation of [11,13] to $b = \cosh 2r$, $u = \sin^2\theta(\cosh 2t - 1)$, $v = \cosh 2t + 1$, where $\{b, u, v\}$ are the global coordinates which cover the entire manifold. The set of three matrices $\{t_a\}$ in the subgroup $H = \mathrm{SO}(2,1)$ is

$$t_{01} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} ,$$
$$t_{02} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} , \tag{3.8}$$
$$t_{12} = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} .$$

The columns of the matrices $L_\mu{}^a$, $R_\mu{}^a$ may be given as vectors $L_\mu, R_\mu, \mu = t, \theta, r$:

$$L_t = \sqrt{2}\begin{bmatrix} 2c_r - 2s_\theta^2(c_r - 1) \\ 2s_\theta c_\theta(c_r - 1) \\ 0 \end{bmatrix} , \quad R_t = \sqrt{2}\begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix} ,$$

$$L_\theta = \sqrt{2}\begin{bmatrix} 0 \\ 0 \\ 1 - c_r \end{bmatrix} , \quad R_\theta = \sqrt{2}\begin{bmatrix} 0 \\ s_t(1 - c_r) \\ c_t(1 - c_r) \end{bmatrix} , \tag{3.9}$$

$$L_r = 0, \quad R_r = 0 ,$$

and similarly the matrix M_{ab} is

$$(M_{ab}) = \begin{bmatrix} c_\theta^2(c_r - 1) & s_\theta c_\theta(c_r - 1) & 0 \\ c_t s_\theta c_\theta(c_r - 1) & c_t(c_\theta^2 + s_\theta^2 c_r) - 1 & s_t c_r \\ -s_t s_\theta c_\theta(c_r - 1) & -s_t(c_\theta^2 + s_\theta^2 c_r) & 1 - c_t c_r \end{bmatrix} , \tag{3.10}$$

where $c_r = \cosh 2r$, $c_\theta = \cos\theta$, $c_t = \cosh 2t$ and $s_r = \sinh 2r$, $s_\theta = \sin\theta$, $s_t = \sinh 2t$. The nonzero components of the matrix $g_{\mu\nu}$ are $g_{tt} = g_{rr} = 1$, $g_{\theta\theta} = (c_r - 1)/2$.

Using (3.6)–(3.10), the nonzero components of the metric (3.6) take the form

$$G_{rr}=1,\quad G_{tt}=\beta\left[\tanh^2 r\coth^2 t\tan^2\theta-\coth^2 r\frac{1}{\cos^2\theta}-\frac{1}{k-1}\frac{1}{\cos^2\theta\sinh^2 t}\right],$$
$$G_{\theta\theta}=\beta\left[\tanh^2 r-\frac{1}{k-1}\frac{1}{\cos^2\theta}\right],\quad G_{t\theta}=\beta\tanh^2 r\coth t\tan\theta\ , \tag{3.11}$$

where the function $\beta(r,t,\theta)$ is defined as

$$\beta^{-1}=1-\frac{1}{k-1}\left[\frac{\coth^2 r}{\cos^2\theta}-\tanh^2 r\left[\frac{1}{\sinh^2 t}+\coth^2 t\tan^2\theta\right]\right]-\frac{1}{(k-1)^2}\frac{1}{\cos^2\theta\sinh^2 t}\ . \tag{3.12}$$

It is not hard to check that the expression for the metric (3.11) is the same as the one we found in [13] with the Hamiltonian approach.[2]

IV. EXACT AXION

To obtain the axion $B_{\mu\nu}$, we need to retain the $\partial_\pm$ on the world sheet and then read off the coefficient of

$$\tfrac{1}{2}(\partial_-X^\mu\partial_+X^\nu-\partial_-X^\nu\partial_+X^\mu)B_{\mu\nu}(X)\ .$$

As already explained above, we cannot do this fully because of the nonlocal terms and non-Abelian nonlinearities, but we can still obtain the axion as follows. We formally replace the R^H, L^H in the expressions for $a_\pm$ and elsewhere by $R^H_\pm, L^H_\pm$, where $R^H_\pm=(-\partial_\pm g g^{-1})_H$ and $L^H_\pm=(g^{-1}\partial_\pm g)_H$. We justify this step by the conformal transformation properties for left and right movers. We then substitute these forms of $A_\pm$ back into the action (2.6) and extract the desired axion from the quadratic part (which is local and a partner of the metric). The expression we find for the axion $B_{\mu\nu}(X)$ is

$$B_{\mu\nu}=b_{\mu\nu}+\tfrac{1}{8}\{[M^TM-\lambda(M+M^T)]^{-1}\times(M^T-\lambda I)\}_{ab}L^a_{[\mu}R^b_{\nu]}\ , \tag{4.1}$$

where $b_{\mu\nu}$ is the part of the axion due to the Wess-Zumino term in $I_0(g)$ and the brackets denote antisymmetrization with respect to the appropriate indices.

In the particular case of the SO(2,2)/SO(2,1) coset model, we have found [8] that for the semiclassical geometry ($k\to\infty$) the axion field vanishes. However, when k is finite we obtain a nonvanishing result, which is given by the expression

$$B_{t\theta}=\frac{\beta}{2(k-1)}\tanh^2 r\coth t\tan\theta\ , \tag{4.2}$$

with the rest of the components being zero. In terms of the global coordinates $\{b,u,v\}$, the corresponding expression is

$$B_{vu}=\frac{\beta}{8(k-1)}\frac{b-1}{b+1}\frac{1}{(v-2)(v-u-2)}\ . \tag{4.3}$$

In Sec. VII we will obtain the exact axion for the three-dimensional black string model discussed in the semiclassical limit $k\to\infty$ in (second reference in [9]) and for any k in [14].

V. EXACT DILATON

To obtain the exact dilaton we must compute the anomaly in the integration over $A_\pm$. However, as was the case with the metric and axion, the local part of the dilaton can be obtained by going to the point-particle limit. The effective action (3.1) contains a quadratic part in the gauge fields, which can be rewritten

$$\frac{-k+g}{4\pi}\int d\tau\,\mathrm{Tr}\left[a_-(M-\lambda I)a_++\frac{\lambda}{2}(a_-^2+a_+^2)\right]. \tag{5.1}$$

Integrating out the gauge fields $a_\pm$ gives a determinant that produces the exact dilaton by identifying, determinant$=e^\Phi$, that is,

$$\Phi(X)=\ln[\det(M)(\det\{I-\lambda[M^{-1}+(M^T)^{-1}]\})^{1/2}]+\text{const} \tag{5.2}$$

As an example, for the non-Abelian coset SO(2,2)/SO(2,1) this gives

$$\Phi=\ln\left[\frac{\sinh^2 2r\sinh^2 t\cos^2\theta}{\sqrt{\beta}}\right]+\text{const}\ , \tag{5.3}$$

or, in terms of the global coordinates $\{b,u,v\}$,

$$\Phi=\ln\left[\frac{(b^2-1)(v-u-2)}{\sqrt{\beta}}\right]+\text{const}\ , \tag{5.4}$$

which is exactly the expression found in [13] with the Hamiltonian approach.

We can use the general expressions for the exact metric (3.6) and dilaton (5.2) to check a theorem which we suggested before [13]. We noted some time ago [8] that the combination $e^\Phi\sqrt{-G}$ that appears in the Laplacian (1.1) is actually independent of k. We had first conjectured this by noting that, in the large-k limit, we could write this quantity as the product of the Haar measure for g times the Faddeev-Popov determinant for fixing a unitary

[2]To compare, one should change variables from (b,u,v) to (t,θ,r) according to the prescription above.

gauge for any G/H gauged WZW model[3]:

$$e^{\Phi}\sqrt{-G} = (\text{Haar}) \times (\text{Faddeev-Popov}) . \tag{5.5}$$

Both $G_{\mu\nu}$ and Φ receive $1/k$ corrections. But by noting that the right-hand side is purely group theoretical, we first conjectured that the combination $e^{\Phi}\sqrt{-G}$ must remain k independent. In our later work for several non-Abelian cases [13], we verified that this conjecture is indeed true. Therefore we stated the following theorem:

$$e^{\Phi}\sqrt{-G}\,(\text{any } k) = e^{\Phi}\sqrt{-G}\,(\text{at } k=\infty) . \tag{5.6}$$

We can reinforce this result by making additional observations. First, the path integral reasoning that allowed us to observe (5.5) is equally valid when the effective action (2.6) is used in place of the classical action (2.2). Since the right-hand side of (5.5) is purely group theoretical, (5.5) should be valid both for the exact and classical $G_{\mu\nu}$ and Φ. Since we have already computed the exact metric and dilaton, one is now in principle in a position to check the relation (5.6) in general. However, the algebra required to compute $\sqrt{-G}$ seems difficult. Instead, the result for all cases relevant to strings in four-dimensional curved spacetime has already been computed explicitly in our previous papers, and indeed for Abelian and non-Abelian cases the theorem (5.6) is true.

To include the effects of the dilaton, we must add one more piece to the effective σ-model action

$$S^{\text{eff}}_{\text{total}} = S^{\text{eff}}_{\sigma} + S^{\text{eff}}_{\text{dil}} , \tag{5.7}$$

where $S^{\text{eff}}_{\text{dil}}$ has the same form as (2.7) but with the exact dilaton replacing the perturbative one. Here we have discussed mainly the zero-mode part of the total effective string action. The effective action for the higher modes that follows from (2.5) and (2.6) is generally nonlocal.

VI. GEODESICS IN THE EXACT GEOMETRY

In [11] the string (or particle) coordinates were defined as certain gauge-invariant combinations of the group parameters in g. In a specific unitary gauge, these invariants are related to the gauge-fixed form of g that defines the string coordinates. Using this formalism, a group-theoretical method for obtaining the solution to the geodesic equation was found and used to obtain the geodesics in the classical geometry. It was shown that the solution to the geodesic equation, which generally is a complicated nonlinear differential equation for the string coordinates and hard to solve directly, could be obtained by first solving the equations of motion of the original variables $g(\tau), a_{\pm}(\tau)$ (which is easy) and then forming the gauge-invariant combinations *from the solutions* for the group parameters in $g(\tau)$. We now apply the same method to solve the geodesic equations in the exact geometry. So we seek a solution to the classical equations of motion given by (3.2) and

$$D_{-}(D_{+}gg^{-1}) = \partial_{\tau}(a_{-} - a_{+}) + [a_{-}, a_{+}] , \tag{6.1}$$

which follows from varying g and where $D_{\pm}$ have the same meaning as in (3.2). The method for solving these equations is identical to that used in [11], and the solution as a function of the proper time τ is

$$\begin{aligned} g(\tau) &= \exp\left[\frac{k-g}{k-h}\alpha\tau\right] g_0 \exp[(P-\alpha)\tau] , \\ [g_0(P-\alpha)g_0^{-1}]_H &+ \alpha = 0 , \end{aligned} \tag{6.2}$$

where α and P are constant matrices in the Lie algebra of H and G/H, respectively, and g_0 is a constant group element. These matrices, which are constrained by the second equation in (6.2), define the initial conditions for any geodesic at $\tau=0$. The line element evaluated at this general solution becomes

$$\left[\frac{ds}{d\tau}\right]^2 = \frac{k-g}{8\pi}\text{Tr}\left[P^2 + \frac{g-h}{k-g}\alpha^2\right] . \tag{6.3}$$

The sign of this quantity determines whether the geodesic is timelike, spacelike, or lightlike, and it can be chosen *a priori* as an initial condition. The large-k analysis of (6.2) was given in [11]. With the new k dependence and using the same methods as [11], we have checked in a few specific cases that the geodesic equations for the exact metric are indeed solved with this group-theoretical technique.

VII. AXIAL GAUGING AND THE SL(2,R)⊗SO(1,1)$^{d-2}$/SO(1,1) MODELS

So far, we have concentrated on the vector gauging of the WZW models. For the axial gauging, the subgroup H should be Abelian with zero Coxeter number. The action is given by (2.2), but with $I_1(g, A_+, A_-)$ replaced by

$$\begin{aligned} I_1^{\text{axial}}(g, A_+, A_-) = \frac{1}{4\pi}\int_M \text{Tr}(& A_-\partial_+ gg^{-1} + A_+ g^{-1}\partial_- g \\ & - A_- g A_+ g^{-1} - A_+ A_-) . \end{aligned} \tag{7.1}$$

Then, if $A_{\pm} = -\partial_{\pm}h_{\pm}h_{\pm}^{-1}$, the analogue of (2.6) is

$$S^{\text{eff,axial}}_{\text{GWZW}} = (-k+g)\left[I_0(g) + I_1^{\text{axial}}(g, A_+, A_-) + \frac{g}{-k+g}I_0(h_-^{-1}h_+)\right] . \tag{7.2}$$

Let us specialize to the SL(2,R)⊗SO(1,1)$^{d-2}$/SO(1,1)

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[3]For a related statement for SL(2,R)/R, see also [25], and for clarifications, see [17]. In our previous work [8,5], we erroneously stated that the path integral for the GWZW model requires an extra gauge-invariant factor $F(g)$ in the measure. Our error was due to the omission of an anomaly factor. The correct measure at the outset is the Haar measure for g times the naive measure for the gauge fields $A_{\pm}$ and $F=1$. This correction does not alter our theorem. We thank E. Kiritsis and A. A. Tseytlin for comments on this point.

coset models. For $d=3$ the semiclassical aspects of the model were worked out in the second reference in [9], for $d=4$ in the fifth reference in [9], and for general d in [4]. The conformally exact geometry was found in [14] with the Hamiltonian approach. It is convenient to parametrize the group element of $G=\mathrm{SL},(2,\mathbb{R})\otimes \mathrm{SO}(1,1)^{d-2}$ as

$$g=\begin{bmatrix} g_0 & 0 & \cdots & 0 \\ 0 & g_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & g_{d-2} \end{bmatrix}, \qquad (7.3)$$

where

$$g_0=\begin{bmatrix} a & u \\ -v & b \end{bmatrix}, \quad ab+uv=1\ , \qquad (7.4)$$

and

$$g_i=\begin{bmatrix} \cosh 2r_i & \sinh 2r_i \\ \sinh 2r_i & \cosh 2r_i \end{bmatrix}, \quad i=1,2,\ldots,d-2\ . \qquad (7.5)$$

The infinitesimal generators for $\mathrm{SL}(2,\mathbb{R})$ are

$$j_0=\frac{q_0}{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad j_+=q_0\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad j_-=q_0\begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}, \qquad (7.6)$$

and those for the SO(1,1)'s are

$$j_i=q_i\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad i=1,2,\ldots,d-2\ . \qquad (7.7)$$

The coefficients q_i parametrize the embedding of $H=\mathrm{SO}(1,1)$ into the factored SO(1,1)'s in G and are normalized to $\sum_{i=0}^{d-2}q_i^2=1$. The subgroup elements $h_\pm$ are parametrized in terms of two variables $\phi_\pm$ as

$$h_\pm=\exp\left[-\frac{1}{q_0}J_{\mathrm{U}(1)}\phi_\pm\right], \qquad (7.8)$$

where

$$J_{\mathrm{U}(1)}=\begin{bmatrix} j_0 & 0 & \cdots & 0 \\ 0 & j_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & j_{d-2} \end{bmatrix}. \qquad (7.9)$$

If we define two new variables $\phi=\phi_- - \phi_+$, $\tilde{\phi}=\phi_- + \phi_+$, then in the gauge $b=\pm a$ the action (7.2) takes the form

$$\begin{aligned} S_{\mathrm{GWZW}}^{\mathrm{eff,axial}}=\frac{k'}{16\pi}\int \frac{1}{uv-1}&[\partial_+(uv)\partial_-(uv)-2(uv-1)(\partial_+u\partial_-v+\partial_+v\partial_-u)]+4\frac{k}{k'}\sum_{i=1}^{d-2}\kappa_i\partial_+r_i\partial_-r_i \\ &+\left[u\partial_+v-v\partial_+u-2\frac{k}{k'}\sum_{i=1}^{d-2}\kappa_i\eta_i\partial_+r_i\right](\partial_-\phi+\partial_-\tilde{\phi}) \\ &+\left[u\partial_-v-v\partial_-u+2\frac{k}{k'}\sum_{i=1}^{d-2}\kappa_i\eta_i\partial_-r_i\right](\partial_+\phi-\partial_+\tilde{\phi})+\left[uv-1-\frac{k}{k'}\rho^2-\frac{2}{k'}\right]\partial_+\phi\partial_-\phi \\ &+\left[1-uv+\frac{k}{k'}\rho^2\right](\partial_+\tilde{\phi}\partial_-\tilde{\phi}+\partial_-\phi\partial_+\tilde{\phi}-\partial_+\phi\partial_-\tilde{\phi})\ , \end{aligned} \qquad (7.10)$$

where $k'=k-2$ is the renormalized value for the central extension k and $\eta_i\equiv q_i/q_0$, $\kappa_i\equiv k_i/k$, $\rho^2\equiv\sum_{i=1}^{d-2}\eta_i^2\kappa_i$.

To extract the effective string model, we now need to integrate out ϕ and $\tilde{\phi}$, which is equivalent to integrating out $A_\pm$. As discussed before, this gives nonlocal contributions. Therefore we may again concentrate on the zero modes by dimensional reduction. Furthermore, as discussed in Sec. IV, we may restore formally $\partial_\tau\to\partial_\pm$ in order to compute the axion. This procedure extracts the local part of the effective action and preserves gauge invariance with respect to τ-dependent gauge transformations $\Lambda(\tau)$. In fact, the local part of the effective action is an ambiguous notion and the principle of τ-dependent gauge invariance resolves this ambiguity.[4] The upshot of these steps boils down to keeping the local part of the solutions of the classical equations for the gauge fields ϕ and $\tilde{\phi}$:

[4]Our gauge-invariant results differ in general from the local part discussed in [17], which is not gauge invariant with respect to $\Lambda(\tau)$. Our form is required to produce the correct geometry that agrees with the algebraic results. However, for the special case $\mathrm{SL}(2,\mathbb{R})/\mathbb{R}$ the results for the metric and dilaton agree accidentally with [17].

$$\partial_\pm\phi|_{\rm local}=\frac{v\partial_\pm u-u\partial_\pm v}{uv-1-(k/k')\rho^2-2/k'}\ ,$$
$$\partial_\pm\tilde{\phi}|_{\rm local}=-2\frac{k}{k'}\frac{\sum_{i=1}^{d-2}\kappa_i\eta_i\partial_\pm r_i}{uv-1-(k/k')\rho^2}\ . \tag{7.11}$$

Substitution of the above expressions into the action (7.10) gives the following expression for the local part of the effective action:

$$S_{\rm eff}^{\rm local}=\frac{k'}{4\pi}\int\frac{1}{k'/k(uv-1)-\rho^2-2/k}\left[-\frac{\rho^2+2/k}{4(uv-1)}\partial_+(uv)\partial_-(uv)+\frac{1+\rho^2}{2}(\partial_+u\partial_-v+\partial_-u\partial_+v)\right]$$
$$+\frac{k}{k'}\sum_{i,j=1}^{d-2}\kappa_i\left[\delta_{ij}+\frac{\eta_i\eta_j\kappa_j}{k'/k(uv-1)-\rho^2}\right]\partial_+r_i\partial_-r_j$$
$$+\frac{1/2}{k'/k(1-uv)+\rho^2}\sum_{i=1}^{d-2}[(u\partial_+v-v\partial_+u)\kappa_i\eta_i\partial_-r_i-(u\partial_-v-v\partial_-u)\kappa_i\eta_i\partial_+r_i]\ . \tag{7.12}$$

The first two lines in the above expressions define a metric which is precisely that found in [14] with the Hamiltonian approach. The third line defines an antisymmetric tensor (axion). As in Ref. [14], it is useful to diagonalize the metric. Since the procedure is exactly the same, we are not going to repeat it here. The answer is that only a three-dimensional part of the metric is nontrivial, and the rest corresponds to flat directions. The three-dimensional nontrivial part of the metric, which describes a black string, has the form [14]

$$ds_{3\rm D}^2=-\left[1-\frac{r_+}{r}\right]dt^2+\left[1-\frac{r_--r_q}{r-r_q}\right]dx^2$$
$$+\frac{k'}{8r^2}\left[1-\frac{r_+}{r}\right]^{-1}\left[1-\frac{r_-}{r}\right]^{-1}dr^2\ , \tag{7.13}$$

where [14] $r_+=\sqrt{2/k'}(\rho^2+1)C$, $r_-=\sqrt{2/k'}\times(\rho^2+2/k)C$, and $r_q=2/k\sqrt{2/k'}C$ (for C, see below the expression for the dilaton). For the axion and its field strength, we obtain a new result: $B_{tr}=B_{xr}=0$, and

$$B_{tx}=\left[\frac{r_--r_q}{r_+}\right]^{1/2}\frac{r-r_+}{r-r_q}\ ,$$
$$H_{rtx}=\partial_rB_{tx}=\left[\frac{r_--r_q}{r_+}\right]^{1/2}\frac{r_+-r_q}{(r-r_q)^2}\ . \tag{7.14}$$

To obtain the dilaton, one has to integrate out ϕ and $\tilde{\phi}$ in (7.10). Then one gets for the conformally exact dilaton the expression which was found in [14]:

$$Ce^{\Phi}=(1-uv)\left\{\left[1+\rho^2+(\rho^2+2/k)\frac{uv}{1-uv}\right]\right.$$
$$\left.\times\left[1+\rho^2-2/k+\rho^2\frac{uv}{1-uv}\right]\right\}^{1/2}\ , \tag{7.15}$$

where C is an arbitrary constant. In the variables which diagonalize the metric, the dilaton takes the form [14]

$$\Phi=\tfrac{1}{2}\ln[r(r-r_q)]+\tfrac{1}{2}\ln k'\ . \tag{7.16}$$

Therefore an additional piece $S_{\rm dil}^{\rm eff}(\Phi)$ must be added to the action in (7.12). The expressions for the metric, the dilaton, the axion, and its field strength tend to their semiclassical values (see the second reference in [9]) in the $k\to\infty$ limit, because then $r_q\to 0$.

It would be interesting to check that the expressions we found for the metric, the axion, and dilaton in this simple Abelian model indeed satisfy the perturbative equations for conformal invariance beyond the one-loop approximation. For large k the backgrounds of the (2D black hole)$\otimes\mathbb{R}$ and the 3D black string are related by a duality transformation as it was shown in [26]. Knowing the exact backgrounds (any k) for both geometries may shed some light into the form of the duality transformation beyond leading order in $\alpha'\sim 1/k$.

VIII. SL(2,R)/R MODEL

Since the simplest case $SL(2,\mathbb{R})/\mathbb{R}$ is just a limit of the previous case, we will briefly derive in this section all the well-known results. In order to specialize the action (7.12) to the case of the $SL(2,\mathbb{R})_{-k}/\mathbb{R}$ model, one should take $k_i=0$. It follows that $\kappa_i=\rho^2=0$, and the action (7.12) and dilaton (7.15) take the form

$$S_{\rm local}^{\rm eff}=\frac{k}{8\pi}\int\frac{1}{uv-1-2/k'}\left[-\frac{1/k}{uv-1}\partial_+(uv)\partial_-(uv)\right.$$
$$\left.+(\partial_+u\partial_-v+\partial_-u\partial_+v)\right]$$
$$+S_{2\rm D}^{\rm dil}(\Phi) \tag{8.1}$$

and

$$C'e^{\Phi}=(1-uv)\left[1-\frac{2}{k}\frac{uv}{uv-1}\right]^{1/2}\ , \tag{8.2}$$

where C' is a constant related to C in (7.15). In the region where $uv>1$, we change variables from $(u,v)\to(t,r)$ as

$$u=\cosh r e^{t}, \quad v=\cosh r e^{-t} . \tag{8.3}$$

Then the action (8.1) and the dilaton (8.2) can be written as

$$\begin{aligned} S_{2D}^{\text{local}} &= \frac{k'}{4\pi}\int \partial_+ r\partial_- r - f(r)\partial_+ t\partial_- t + S_{2D}^{\text{dil}}(\Phi) , \\ \Phi &= \ln[\sinh 2r/f(r)] + \text{const} \end{aligned} \tag{8.4}$$

where $f(r)=1/(\tanh^2 r-2/k)$, thus reproducing the exact expressions for the metric and dilaton of the two-dimensional (2D) black hole as they were computed in [15,13]. One could also use the effective action appropriate for vectorial gauging (2.6) to obtain all of the results in this section.

IX. CONCLUSION

We have suggested the form of the effective quantum action for the general Abelian or non-Abelian GWZW model and verified that it works, at least in the zero-mode sector. Furthermore, we have obtained new general results for the conformally exact axion field and geodesics. The zero-mode sector determines the point-particle behavior of the underlying string theory and is the only part relevant for the low-energy physics. Therefore, although our methods have yielded incomplete results for the full string theory, they are adequate to extract the most relevant physical information on the curved space-time geometry. Based on the agreement with the algebraic Hamiltonian approach in the zero-mode sector, we conjecture that, before the integration over $A_{\pm}$, the forms (2.5), (2.6), and (7.2) may be trusted for all the higher modes as well.

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