Acceleration through the Dirac-Pauli vacuum and effects of an external field

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The Dirac equation for a charged spin-$\frac{1}{2}$ particle with an anomalous magnetic moment μ' in a background magnetic field is solved in Rindler coordinates (uniformly accelerated frame). A method based on the existence of a spin operator is presented, which permits us to solve the Dirac-Pauli equation in curvilinear coordinates. This is applied to the Rindler coordinates, and the spectra of conserved quantities such as the energy, particle, and spin densities are calculated. The ratio of the energy and particle densities is not given by a Fermi-Dirac distribution, except in the limit $\mu' \to 0$. Furthermore, the spectrum of the energy density takes a complicated form that cannot be simply interpreted as thermal even in the zero mass limit. Finally, it is shown that, at this level of approximation, a possible effect on the spin polarization cannot arise from acceleration effects on the electron.

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I. INTRODUCTION

It has been shown by Pauli [1] that the Dirac equation in the presence of an external field can represent a particle having an arbitrary magnetic moment μ' if one adds a term $\frac{1}{2}\mu' F_{\mu\nu}\gamma^\mu\gamma^\nu$, that is,

$$[\gamma^\mu(\partial_\mu - ieA_\mu) + im + \tfrac{1}{2}\mu' F_{\mu\nu}\gamma^\mu\gamma^\nu]\psi = 0 \quad (1)$$

with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ , \quad (2)$$

whereupon the particle behaves as if it had an "anomalous" magnetic moment μ' in addition to its normal moment $e/2m$. Equation (1) is conventionally called the Dirac-Pauli (DP) equation. In the case of an electron, this anomalous term incorporates radiative corrections to the electrodynamic vertex in an effective way.

Only a few exact solutions of Eq. (1) are known to date. Most authors [2–4] investigated the solutions of this equation in a constant and homogeneous magnetic field, and studied how the anomalous moment affects the electron spin precession. Exact solutions in Minkowski coordinates of the DP equation for a constant and homogeneous magnetic field were first obtained in Ref. [5].

The aim of this paper is to study the DP field in a uniformly accelerated frame, including a background magnetic field which interacts with the particle. To this end, we find an exact solution in Rindler coordinates. We also present a method for solving the Dirac or DP equation using the spin operator integral of the motion.

The study of spin fields in a uniformly accelerated frame was first carried by Candelas and Deutsch [6], who calculated the vacuum expectation value of the energy-momentum tensor for the simplest case of a massless free field and found that the energy density spectrum has a Planckian form multiplied by a factor which depends on the spin. Later on, the massive Dirac field in Rindler coordinates with a uniform magnetic field was studied by Jáuregui, Torres, and Hacyan [7], who calculated the energy density with the Bogoliubov coefficients relating Rindler and Minkowski modes. The ratio of the energy and particle densities is given by a Fermi-Dirac distribution, but the spectrum of these quantities takes a complicated form that cannot be simply interpreted as a thermal spectrum [8]. In this work we extend the previous results to the case of a particle with an anomalous magnetic moment using the DP equation which includes the effects of this additional magnetic moment interaction as effective radiative correction.

This paper is organized as follows. Section II briefly presents the main equations and definitions of the energy momentum and spin tensors appropriate to the problem under consideration. In Sec. III, we give the exact solutions of the DP equation with a constant and homogeneous magnetic field, both in Minkowski and Rindler coordinates. Finally, in Sec. IV, the Bogoliubov coefficients are calculated, and we give explicit expressions for the particle, energy, and spin densities calculated in the Minkowski vacuum state as detected by an uniformly accelerated observer; we also consider the massless case with $\mu' \neq 0$.

II. THE GENERAL RELATIVISTIC DP EQUATION, BASIC TENSORS, AND CONSERVED QUANTITIES

In an arbitrary coordinate system one substitutes the derivative with minimal coupling:

$$(\partial_\mu - ieA_\mu)\psi \to D_\mu\psi \equiv (\nabla_\mu - ieA_\mu)\psi = (\partial_\mu + \Gamma_\mu - ieA_\mu)\psi \ , \quad (3)$$

where Γ_μ are the affine connections. The covariant γ^μ matrices generate the Clifford algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \ , \quad (4)$$

where $g^{\mu\nu}$ is the metric tensor.

Using the DP equation one can easily verify the fact that the current $\mathcal{J}^\mu = e\bar{\psi}\gamma^\mu\psi$ obeys the conservation equation

$$\nabla_\mu \mathcal{J}^\mu = 0 \tag{5}$$

and hence, the scalar product can be defined by the integral over a spacelike surface

$$(\psi_1,\psi_2) = \int_\Sigma \sqrt{-g}\,\bar{\psi}_1 \gamma^\mu \psi_2 d\sigma_\mu \ . \tag{6}$$

By varying the action with respect to the metric $g^{\mu\nu}$, one gets the energy-momentum tensor

$$T_{\mu\nu} = -\frac{i}{2}\bar{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi + \mu'\bar{\psi}\sigma_{\alpha(\mu}\psi F^{\alpha}_{\ \nu)} \tag{7}$$

and its divergence takes the form

$$\nabla_\mu T^{\mu\nu} = F^\nu_{\ \alpha} J^\alpha \ , \tag{8}$$

where

$$J^\alpha = e\bar{\psi}\gamma^\alpha\psi + \mu'\nabla_\mu(\bar{\psi}\sigma^{\alpha\mu}\psi) \tag{9}$$

is the DP current which clearly satisfies the continuity equation (5). The current J^α can be found easily by varying the action with respect to the vector potential A_μ, as has been shown by Pauli [1].

To appreciate the physical significance of the additional term in the current, split J^α into two parts:

$$J^\alpha = J_1^\alpha + J_2^\alpha \ , \tag{10}$$

where

$$J_1^\alpha = \left[\frac{e}{2m} + \mu'\right]\nabla_\nu(\bar{\psi}\sigma^{\alpha\nu}\psi) \tag{11}$$

and

$$J_2^\alpha = \frac{ie}{2m}\bar{\psi}\overleftrightarrow{D}^\alpha\psi + \frac{\mu' e}{4m}F^{\mu\nu}\bar{\psi}\{\gamma^\alpha,\sigma_{\mu\nu}\}\psi \ . \tag{12}$$

Looking closely to Eq. (11) we note that the additional term in the current, Eq. (9), is a radiative correction associated with the intrinsic magnetization (magnetic dipole density [9]) of the electron due to the coupling of the external field with the anomalous magnetic moment.

For the DP equation, an important role is assigned to the spin-operator-valued integrals of the motion. It is convenient to define the spin tensor as

$$S_\lambda^{\alpha\beta} = \bar{\psi}\gamma_\lambda S^{\alpha\beta}\psi + \text{H.c.} \ , \tag{13}$$

where $S^{\alpha\beta}$ is the spin operator

$$S^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta}\gamma_5\gamma_\alpha D_\beta \tag{14}$$

with

$$\gamma_5 = -\frac{1}{4!}\sqrt{-g}\,\epsilon_{\mu\nu\alpha\beta}\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta \ . \tag{15}$$

Integrals of motion have been found in Refs. [10,11] for a few external fields in Minkowski coordinates using this spin operator. At this point, a remark must be made with respect to the tensor spin operator studied in Refs. [7,12–15]:

$$S'^{\mu\nu} = i(\gamma^\mu D^\nu - \gamma^\nu D^\mu) - m\sigma^{\mu\nu} \ . \tag{16}$$

If ψ is a solution of the Dirac equation ($\mu'=0$), then

$$S'^{\mu\nu}\psi = S^{\mu\nu}\psi \tag{17}$$

and it makes no difference which of the two operators is used. However, Eq. (17) does not hold when an interaction such as the Pauli term is added to the Dirac equation.

It is well known that a divergence free four-vector yields a conserved quantity which is its time component integrated over all space. The current (9), for instance, is divergence free and gives the particle number as a conserved quantity. Thus, the problem is reduced to defining the appropriate four-vector for quantities of interest such as the energy density and the spin, which are conserved only if the spacetime admits a Killing vector associated to time invariance or rotational symmetries.

Thus, given a timelike Killing vector t_α one can construct the four-vector $T^{\alpha\beta}t_\beta$ which is divergence free provided that the "electric" four-vector $t_\alpha F^{\alpha\beta}$ is zero, and the total energy

$$e = \int T_0^0 dV \tag{18}$$

turns out to be time independent.

If dealing with the projection of the total spin in the direction of a given axis through which the system exhibits a rotational symmetry, we may consider the spin four-vector S^α as proposed in [7]:

$$S^\alpha = \tfrac{1}{2}S^{\alpha\beta\gamma}\nabla_\beta k_\gamma \ , \tag{19}$$

where k_γ is the Killing vector associated to the rotational symmetry. Then, in flat space with a constant and homogeneous magnetic field along the z axis, where the Killing vector is $k^\alpha = (0,-y,x,0)$ and its properties

$$\nabla_\beta k_\alpha = \delta^1_{[\beta}\delta^2_{\alpha]} \ , \quad k^\alpha\partial_\alpha = x\partial_y - y\partial_x \ , \tag{20}$$

we obtain the basic conservation equation

$$\nabla_\gamma S^\gamma = 0 \ . \tag{21}$$

Therefore, the integral over a spacelike surface

$$S_z = \int S^{012} dV \tag{22}$$

is conserved and is the projection of the total spin along the symmetry axis. In the particular cases of some external fields, the component S'^{12} of the operator (16) commutes with the DP Hamiltonian (the case of a homogeneous and constant magnetic field is an example, as we shall see in the following). Explicitly,

$$\begin{aligned} H &= H_0 - i\mu' B\gamma^0\gamma^1\gamma^2 \ , \\ S'^{12} &= i(\gamma^1 D^2 - \gamma^2 D^1) - im\gamma^1\gamma^2 \ , \end{aligned} \tag{23}$$

where H_0 is the Dirac Hamiltonian, and hence

$$[H,S'^{12}] = -i\mu' B[\gamma^0\gamma^1\gamma^2, S'^{12}] = 0 \ ,$$

where use has been made of the fact that $[H_0,S'^{12}]=0$ [16]. It is an easy task to verify that the system

$$S'^{12}\psi = \pm k\psi$$

has a solution of form (26) using that

$$(S'^{12})^2 = (D^1)^2 + (D^2)^2 + eB\sigma_3 + m^2$$

is diagonal. However, when one subjects ψ to be an eigenfunction of the DP Hamiltonian

$$H\psi = E\psi ,$$

the trivial solution $C_i=0$ is obtained for the spin coefficients. This is because the spin-polarization operator should have a term proportional to μ' accounting for the anomalous magnetic moment of the particle, which, as can be seen, does not emerge from $S'^{\mu\nu}$. This problem does not exist when one uses the spin operator (14).

III. THE DP EQUATION IN MINKOWSKI AND RINDLER COORDINATES (EXACT SOLUTIONS)

A. Minkowski coordinates

In the case of the magnetic field ($\mathbf{B}=B\hat{\mathbf{e}}_z$) parallel to the z direction, the vector potential is

$$A_\mu=(0,0,xB,0) . \tag{24}$$

The DP equation is a set of first-order coupled equations

$$i\partial_t\psi=H\psi , \quad H=\boldsymbol{\alpha}\cdot\mathbf{p}-exB\alpha_2+\beta m+\mu' B\beta\sigma_3 , \tag{25}$$

where H is the Hamiltonian. To solve them, one proposes an eigenfunction of the form [17]

$$\psi=e^{-iEt}\frac{e^{ik_2y}}{\sqrt{2\pi}}\begin{bmatrix} C_1\varphi_n[\sqrt{eB}\,(x-k_2/eB)] \\ C_2\varphi_{n-1}[\sqrt{eB}\,(x-k_2/eB)] \\ C_3\varphi_n[\sqrt{eB}\,(x-k_2/eB)] \\ C_4\varphi_{n-1}[\sqrt{eB}\,(x-k_2/eB)] \end{bmatrix} , \tag{26}$$

where $\varphi_n(\rho)$ are related to the Hermite polynomials H_n:

$$\varphi_n(\rho)=2^{-n/2}(n!)^{-1/2}\pi^{-1/4}e^{-\rho^2/2}H_n(\rho) , \tag{27}$$

where $\rho=\sqrt{eB}\,(x-k_2/eB)$ and $n=0,1,2,\ldots$ are the principal quantum numbers. The coefficients in Eq. (26) satisfy the normalization condition

$$\sum_i |C_i|^2=1 . \tag{28}$$

A complete determination of the wave function is achieved introducing the operator

$$S^{12}=m\sigma_3+\mu' B+i\beta\alpha_1(i\partial_y+exB)+\beta\alpha_2\partial_x , \tag{29}$$

which is the projection of the total spin on the direction of the magnetic field. Note that the difference between the spin operators S^{12} and S'^{12} is the presence of the anomalous term $\mu' B$; according to the discussion following Eq. (17), the spinor (26) should be eigenfunction of this operator, and therefore

$$S^{12}\psi=\theta\sqrt{E^2-k_3^2}\,\psi\equiv\theta k\psi , \quad \theta=\pm 1 . \tag{30}$$

One could have used this equation as a starting point to find ψ, since the square of the operator S^{12} is diagonal and we get a system of second-order decoupled equations from where it follows that the general solution has necessarily the form (26).

The systems (25) and (30) can be solved together to yield

$$C_1=\frac{1}{2\sqrt{2}}\left[\frac{k-\theta\mu' B+\theta m}{k-\theta\mu' B}\right]^{1/2}\left[\sqrt{1+\tanh\phi}+\theta\sqrt{1-\tanh\phi}\right] , \tag{31a}$$

$$C_2=\frac{1}{2\sqrt{2}}\left[\frac{k-\theta\mu' B-\theta m}{k-\theta\mu' B}\right]^{1/2}\left[\sqrt{1+\tanh\phi}-\theta\sqrt{1-\tanh\phi}\right] , \tag{31b}$$

$$C_3=\frac{1}{2\sqrt{2}}\left[\frac{k-\theta\mu' B+\theta m}{k-\theta\mu' B}\right]^{1/2}\left[\sqrt{1+\tanh\phi}-\theta\sqrt{1-\tanh\phi}\right] , \tag{31c}$$

$$C_4=\frac{1}{2\sqrt{2}}\left[\frac{k-\theta\mu' B-\theta m}{k-\theta\mu' B}\right]^{1/2}\left[\sqrt{1+\tanh\phi}+\theta\sqrt{1-\tanh\phi}\right] , \tag{31d}$$

where we have used the usual representation of the Dirac matrices

$$\gamma^0=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} , \quad \gamma^i=\begin{bmatrix}0 & \sigma^i\\ -\sigma^i & 0\end{bmatrix} , \tag{32}$$

in which the σ^i, $i=1,2,3$, are Pauli spin matrices, and

$$E=\pm k\cosh\phi , \quad k_3=k\sinh\phi . \tag{33}$$

B. Rindler coordinates

Owing to the dynamics of the uniformly accelerated observer, the Minkowski space is divided into four sectors: right (I), left (II), future (F), and past (P) with respect to the origin $z=t=0$, where z is the axis of acceleration. One defines the Rindler coordinates (η,x,y,ξ) in sectors I and II according to (regions F and P will not be considered here)

$$t=\xi\sinh\eta , \quad z=\xi\cosh\eta , \tag{34a}$$

$$\eta=\operatorname{arctanh}(t/z) , \quad \xi=\operatorname{sgn}(z)\sqrt{z^2-t^2} . \tag{34b}$$

The line element in the new coordinates is

$$ds^2=\xi^2d\eta^2-dx^2-dy^2-d\xi^2 , \tag{35}$$

and curves of constant ξ correspond to world lines of observers undergoing uniform acceleration of ξ^{-1}.

Let us consider a relativistic spin-$\frac{1}{2}$ particle with charge e possessing an anomalous magnetic moment in the presence of a constant and homogeneous magnetic field in the direction of acceleration ($\hat{\mathbf{e}}_z$). The DP equation in Rindler coordinates has the form

$$\left[\frac{i}{\xi}\gamma^0(\partial_\eta-\tfrac{1}{2}\gamma_0\gamma_3)+i\gamma^3\partial_\xi+i\gamma^1\partial_x + i\gamma^2(\partial_y-iexB)-i\mu' B\gamma^1\gamma^2\right]\Phi=m\Phi . \quad (36)$$

Hereafter, we will concentrate on region I and the positive-energy solutions. The Dirac matrices have the form

$$\gamma^\mu=e^\mu_n\gamma^n , \quad (37)$$

where γ^n are the standard Dirac matrices, and e^μ_n is a tetrad such that $e^\mu_0=(\xi^{-1},\mathbf{0})$; $e^\mu_i=-\delta^\mu_i$ for $i=1,2,3$. The affine connection in I is given by

$$\Gamma_\mu=(-\tfrac{1}{2}\gamma_0\gamma_3,\mathbf{0}) . \quad (38)$$

Under these conditions, we can separate the functional dependence of the wave function in the form

$$\Phi(x,y,\xi,\eta)=N^{(R)}e^{-i\epsilon\eta}\frac{e^{ik_2 y}}{\sqrt{2\pi}}\mathbf{M}_k(\xi)\chi_k(x) , \quad (39)$$

where k is a separation constant ($k^2=E^2-k_3^2$ in Minkowski coordinates) and ϵ is the "energy" in the Rindler coordinate system, $\mathbf{M}$ are 4×4 matrices and χ are bispinors. To determine each functional dependence of the exact solution, it is convenient to introduce the operator of projection of the total spin on the direction of the magnetic field; i.e., on the z axis

$$S^{12}_R=\gamma_5\gamma^0\partial_\xi+\frac{1}{\xi}\gamma_5\gamma^3(\partial_\eta-\tfrac{1}{2}\gamma_0\gamma_3) , \quad (40)$$

this quantity commutes with the operator in the large parentheses on the left-hand side of (36) and, therefore, the wave function is an eigenfunction of the operator (40):

$$S^{12}_R\Phi=\theta k\Phi , \quad (41)$$

where $\theta=\pm1$ corresponds to the spin polarization relative to the direction of the magnetic field: $\theta=+1$ along the field and $\theta=-1$ against the field. The substitution of Φ in Eq. (41) gives

$$\left[\gamma_5\gamma^0\partial_\xi-\frac{1}{\xi}\gamma_5\gamma^3(i\epsilon-\tfrac{1}{2}\gamma^0\gamma^3)\right]\mathbf{M}_k(\xi)\chi_k(x) =\theta k\mathbf{M}_k(\xi)\chi_k(x) , \quad (42)$$

and applying again the spin operator we get

$$\left[\partial^2_\xi+\frac{1}{\xi}\partial_\xi+\frac{\epsilon^2}{\xi^2}-\frac{1}{4\xi^2}+\frac{i\epsilon}{\xi^2}\gamma^0\gamma^3\right]\mathbf{M}_k(\xi)\chi_k(x) =k^2\mathbf{M}_k(\xi)\chi_k(x) . \quad (43)$$

To separate this equation, we propose the following ansatz for the matrix $\mathbf{M}$:

$$\mathbf{M}_k(\xi)=K_+\mathbf{P}_++K_-\mathbf{P}_- , \quad (44)$$

where $\mathbf{P}_\pm$ are projection operators defined as

$$\mathbf{P}_\pm=\tfrac{1}{2}(1\pm\gamma^0\gamma^3) , \quad (45)$$

and with the property

$$\gamma^0\gamma^3\mathbf{P}_\pm=\pm\mathbf{P}_\pm , \quad (46)$$

so that

$$\left[\partial^2_\xi+\frac{1}{\xi}\partial_\xi+\frac{\epsilon^2}{\xi^2}-\frac{1}{4\xi^2}\pm\frac{i\epsilon}{\xi^2}\right]K_\pm\mathbf{P}_\pm\chi_k(x) =k^2K_\pm\mathbf{P}_\pm\chi_k(x) . \quad (47)$$

This equation is satisfied only if

$$\left[\partial^2_\xi+\frac{1}{\xi}\partial_\xi+\frac{\epsilon^2}{\xi^2}-\frac{1}{4\xi^2}\pm\frac{i\epsilon}{\xi^2}\right]K_\pm=k^2K_\pm , \quad (48)$$

from which it follows that

$$K_\pm(\xi)=K_{\pm1/2-i\epsilon}(k\xi) , \quad (49)$$

where $K_\nu(x)$ are the modified Bessel functions of the third kind, which have a regular asymptotic behavior ($\xi\to\infty$).

Now, we come to the problem of finding the bispinors χ_k. Using the DP equation we find

$$S^{12}_R\Phi=[\gamma^2\partial_x+\gamma^1(\partial_y-iexB)+im\gamma^1\gamma^2+\mu' B]\Phi , \quad (50)$$

from which it follows that

$$[\gamma^2\partial_x+i\gamma^1(k_2-exB)+im\gamma^1\gamma^2+\mu' B]\mathbf{P}_\pm\chi_k(x) =\theta k\mathbf{P}_\pm\chi_k(x) , \quad (51)$$

and using the usual representation of the Dirac matrices (32), together with the fact that the square of the spin-polarization operator is diagonal,

$$(S^{12})^2=m^2+2\theta\mu' Bk-(\mu' B)^2+eB\sigma_3 +(k_2-exB)^2-\partial^2_x , \quad (52)$$

and hence that we are led to a second-order differential equation for each component of the spinor Φ, we get the bispinors

$$\chi_k(x)=\begin{bmatrix} C_1\varphi_n[\sqrt{eB}(x-k_2/eB)] \\ C_2\varphi_{n-1}[\sqrt{eB}(x-k_2/eB)] \\ C_3\varphi_n[\sqrt{eB}(x-k_2/eB)] \\ C_4\varphi_{n-1}[\sqrt{eB}(x-k_2/eB)] \end{bmatrix} , \quad (53)$$

$$k=\sqrt{2eBn+m^2}+\theta\mu' B , \quad (54)$$

where $\varphi_n(\rho)$ are connected with the Hermite polynomials (27), with $n=0,1,2,\ldots$ the principal quantum number. The spin coefficients C_i are interrelated by the normalization condition (28); solving the DP equation and (51) simultaneously, we obtain the coefficients in the form

$$C_1=\frac{1}{2\sqrt{2}}\left[\frac{k-\theta\mu' B+m\theta}{k-\theta\mu' B}\right]^{1/2}(1-i\theta)\ , \tag{55a}$$

$$C_2=\frac{1}{\sqrt{2}\sqrt{2}}\left[\frac{k-\theta\mu' B-m\theta}{k-\theta\mu' B}\right]^{1/2}(-1+i\theta)\ , \tag{55b}$$

$$C_3=C_1^*,\quad C_4=C_2^*\ . \tag{55c}$$

It only remains to determine the normalization constant $N^{(R)}$ in (39). To this end we use the condition

$$\int dx\, dy\, d\xi \Phi^\dagger_{\epsilon kn}\Phi_{\epsilon' k' n'}=\delta(\epsilon-\epsilon')\delta_{kk'}\delta_{nn'}\ , \tag{56}$$

and finally arrive at the result

$$N^{(R)}=\left[\frac{2}{\pi^2}\cosh(\pi\epsilon)\right]^{1/2}\ . \tag{57}$$

IV. BOGOLIUBOV COEFFICIENTS: ENERGY, PARTICLE, AND SPIN DENSITIES

Having solved the relativistic wave equation, both in Rindler and Minkowski coordinates, one can quantize these fields in a straightforward way. Φ and ψ are now regarded as operator fields (Φ, Rindler field; ψ, Minkowski field) and the usual quantum field theory can be obtained if the DP operator is expanded in terms of the Minkowski solutions

$$\psi=\sum_w [a_w\psi_w^{(+)}+b_w^\dagger\psi_w^{(-)}]\ , \tag{58}$$

where $\psi_w^{(\pm)}$ are the positive (+) and negative (−) energy solutions. The operators a_w, b_w obey the usual anticommutation relations

$$\{a_w, a_{w'}^\dagger\}=\delta_{ww'} \tag{59}$$

and the Minkowski vacuum $|0\rangle_M$ is defined by the condition

$$a_w|0\rangle_M=b_w|0\rangle_M=0\ , \tag{60}$$

the label w in these expressions denotes all the quantum or continuum numbers (k_3,k_2,θ,E). Similarly, we can expand Φ in terms of the Rindler modes

$$\Phi=\sum_W\left[\sum_\Omega({}^\Omega\tilde{a}_W\Phi^{(+)}_{W\Omega}+{}^\Omega\tilde{b}_W\Phi^{(-)}_{W\Omega})\right]\ , \tag{61}$$

taken on both regions I and II ($\Omega=\mathrm{I},\mathrm{II}$).

We have taken into account both positive- and negative-energy solutions to construct a wave function in region $B=\mathrm{I}\cup\mathrm{II}$, and defined

$$\Phi_\Omega=\begin{cases}\delta_{\mathrm{I}\Omega}\Phi_{\mathrm{I}} & \text{in I}\ ,\\ \delta_{\mathrm{II}\Omega}\Phi_{\mathrm{II}} & \text{in II}\ ,\end{cases} \tag{62}$$

and the parameter W in Eq. (61) represents the quantum or continuum numbers associated with the accelerated observer (ϵ,k,θ,k_2), such as w above.

Finally, the Rindler vacuum $|0\rangle_R$ is defined by

$${}^\Omega\tilde{a}_W|0\rangle_R={}^\Omega\tilde{b}_W|0\rangle_R=0\ . \tag{63}$$

We can now expand the Minkowski modes, after a Lorentz boost $\Lambda=\exp(-\frac{1}{2}\gamma^0\gamma^3\eta)$ to the instantaneous inertial frame defined by η=const, in terms of the Rindler modes; that is,

$$\Lambda\psi^{(+)}=\sum_W\left[\sum_\Omega(\alpha_\Omega^*\Phi_\Omega^{(+)}-\beta_\Omega\Phi_\Omega^{(-)})\right]\ , \tag{64}$$

where α and β are the Bogoliubov coefficients, which, in general, are functions of the quantum numbers $[\alpha=\alpha(W,w)]$. The integral (6) over the hypersurface η=const can be used as an internal product, and it follows that the Bogoliubov coefficients are given by

$$\alpha_\Omega^*=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}dx\, dy\, d\xi\,\Phi_\Omega^{(+)\dagger}\Lambda\psi^{(+)} \tag{65}$$

and

$$\beta_\Omega=-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}dx\, dy\, d\xi\,\Phi_\Omega^{(-)\dagger}\Lambda\psi^{(+)}\ . \tag{66}$$

By a direct substitution of the Minkowski and Rindler solutions, together with the transformation equations (33) and (34), one gets

$$\alpha_{\mathrm{I}}^*=\frac{1+i}{2}\frac{e^{\pi\epsilon/2}e^{i\phi\epsilon}}{[2\pi k\cosh(\pi\epsilon)\cosh\phi]^{1/2}}\delta_{nn'}\delta(k_2-k_2')\delta_{\theta\theta'}\ , \tag{67}$$

and

$$\beta_{\mathrm{I}}=\frac{1-i}{2}\frac{e^{-\pi\epsilon/2}e^{-i\phi\epsilon}}{[2\pi k\cosh(\pi\epsilon)\cosh\phi]^{1/2}}\delta_{nn'}\delta(k_2-k_2')\delta_{\theta\theta'}\ , \tag{68}$$

while

$$\alpha_{\mathrm{II}}=\alpha_{\mathrm{I}}^*\ ,\quad \beta_{\mathrm{II}}=\beta_{\mathrm{I}}^*\ . \tag{69}$$

It must be noted here that the number of particles, which is given directly by $|\beta_{\mathrm{I}}|^2$ [18], has a Planckian spectrum.

In order to calculate other field variables we first apply the Lorentz transformation Λ to Eq. (58) and get

$$\Phi'\equiv\Lambda\psi=\sum_W\left[\frac{1}{2\sqrt{\cosh(\pi\epsilon)}}\left[\sum_\Omega a^1e^{\pi\epsilon/2}\Phi^{(+)}_{\Omega,n,k_2}+a^2e^{-\pi\epsilon/2}\Phi^{(-)}_{\Omega,n,-k_2}+b^{1\dagger}e^{\pi\epsilon/2}\Phi^{(-)}_{\Omega,n,k_2}+b^{2\dagger}e^{-\pi\epsilon/2}\Phi^{(+)}_{\Omega,n,-k_2}+(\epsilon\leftrightarrow-\epsilon)\right]\right]\ . \tag{70}$$

The operators a^1 and a^2 are related to the Minkowski annihilation operators through

$$a^1=\int_{-\infty}^{\infty}\frac{dk_3}{[2\pi k\cosh\phi]^{1/2}}\frac{1-i}{2}e^{-i\phi\epsilon}a_k\ , \tag{71}$$

$$a^2=\int_{-\infty}^{\infty}\frac{dk_3}{[2\pi k\cosh\phi]^{1/2}}\frac{1+i}{2}e^{i\phi\epsilon}a_k\ , \tag{72}$$

with analogous formulas for b^1 and b^2.

The expectation value of any operator $\mathbf{E}$ can be evaluated in terms of the Fourier transforms of the two-point Wightman functions [19]:

$${}_M\langle 0|\overline{\Phi}'\mathbf{E}\Phi'|0\rangle_M=\frac{2}{\pi}\int_0^{\infty}d\omega\, e^{i\omega\sigma}[G_+^{(E)}(\tau,\omega)+G_-^{(E)}(\tau,\omega)]\ , \tag{73}$$

where

$$G_\pm^{(E)}(\tau,\omega)=\int_{-\infty}^{\infty}d\sigma\, e^{i\omega\sigma}G^{(E)}(\tau\mp\sigma/2,\tau\pm\sigma/2) \tag{74}$$

are the Fourier transform (with respect to the proper time) of the Wightman functions:

$$G^{(E)}(x,y)=\tfrac{1}{4}{}_M\langle 0|\overline{\Phi}(y)\mathbf{E}\Phi'(x)|0\rangle_M\ . \tag{75}$$

In particular, the energy density corresponds for $\mathbf{E}=-i\gamma^0\overleftrightarrow{\nabla}_0$ to its expectation value. By a direct substitution of (70) one gets

$$e=\frac{4B}{\pi^2 a}\int_0^{\infty}d\omega\sum_{n,\theta}\omega\sinh(\pi\omega/a)[\sqrt{2enB+m^2}+\theta\mu' B]\left|K_{1/2+i(\omega/a)}\left[\frac{1}{a}(\sqrt{2enB+m^2}+\theta\mu' B)\right]\right|^2 \tag{76}$$

when evaluated along the trajectory of a uniformily accelerated observer ($\xi=1/a$). Similarly, the particle and spin densities can be obtained setting $\mathbf{E}=\gamma^0+(\mu'\sigma^{0\mu}\nabla_\mu-\text{H.c.})$ and $\mathbf{E}=\frac{1}{2}\gamma^0 S^{12}+\text{H.c.}$, respectively, in Eq. (73), from where it follows that

$$n=(1-2m\mu')\frac{2B}{\pi^2}\int_0^{\infty}\sum_{n,\theta}k\cosh\left[\frac{\pi\omega}{a}\right]|K_{1/2+i\omega/a}(k/a)|^2 d\omega$$
$$+\mu'^2B^2\frac{2m}{\pi^2}\int_0^{\infty}\sum_{n,\theta}\left[\frac{k\theta}{k-\theta\mu' B}\right]\cosh\left[\frac{\pi\omega}{a}\right]|K_{1/2+i\omega/a}(k/a)|^2 d\omega\ , \tag{77}$$

$$S_z=\frac{Bm}{4\pi^2}\int_0^{\infty}\sum_{n,\theta}\left[\frac{k\theta}{k-\theta\mu' B}\right]\sinh\left[\frac{\pi\omega}{a}\right]\text{Im}[K_{1/2+i\omega/a}(k/a)]^2 d\omega\ , \tag{78}$$

$$n_1=\left[\frac{1}{2m}+\mu'\right]\left[-\frac{4Bm}{\pi^2}\int_0^{\infty}\sum_{n,\theta}k\cosh\left[\frac{\pi\omega}{a}\right]|K_{1/2+i\omega/a}(k/a)|^2 d\omega\right.$$
$$\left.+\mu' B^2\frac{2m}{\pi^2}\int_0^{\infty}\sum_{n,\theta}\left[\frac{k\theta}{k-\theta\mu' B}\right]\cosh\left[\frac{\pi\omega}{a}\right]|K_{1/2+i\omega/a}(k/a)|^2 d\omega\right]\ , \tag{79}$$

$$n_2=n-n_1\ . \tag{80}$$

n_1 and n_2 are the space-integrated time components of the currents J_1,J_2, and are conserved quantities which corresponds to the expectation values of the operators $(1/2m+\mu')(\sigma^{0\nu}\nabla_\nu-\text{H.c.})$ and $(i/2m)\overleftrightarrow{\nabla}_0-(\mu' B/m)\gamma^0\sigma_3$, respectively. Notice that in the expression for S_z contrary to the case of the Dirac field ($\mu'=0$) [7] we have a contribution from every state with $n\geq 0$ due to the fact that the presence of the anomalous magnetic moment has broken the degeneracy of the Landau levels, Eq. (54). Equations (77) and (79) for the particle density and magnetic dipole density can be written in the somewhat suggestive form

$$(1-2m\mu')\frac{de}{d\omega}=\left[-\omega+\frac{2\omega}{e^{2\pi\omega/a}+1}\right]\frac{dn}{d\omega}-\mu'^2B^2\frac{2m}{\pi^2}\sum_{n,\theta}\left[\frac{k\theta}{k-\theta\mu' B}\right]\cosh\left[\frac{\pi\omega}{a}\right]|K_{1/2+i\omega/a}(k/a)|^2\ , \tag{81}$$

$$2(1+m\mu')\frac{de}{d\omega}=-\left[-\omega+\frac{2\omega}{e^{2\pi\omega/a}+1}\right]\frac{dn_1}{d\omega}+(1+m\mu')\mu' B^2\frac{2}{\pi^2}\sum_{n,\theta}\left[\frac{k\theta}{k-\theta\mu' B}\right]\cosh\left[\frac{\pi\omega}{a}\right]|K_{1/2+i\omega/a}(k/a)|^2\ . \tag{82}$$

Thus, the relation between the energy and the particle densities is no longer given by a Fermi-Dirac distribution, as in the case $\mu'=0$, unless the mass m is zero. Furthermore, any simple relation between the last term in (81) and the energy density does not seem to exist. In conclusion, the energy spectrum of the DP sea in the accelerated frame does not exhibit a Fermi-Dirac distribution.

In the case of a weak field and $\mu' \ll 1$, we can ignore the second term in (81) and write

$$\frac{de}{d\omega} \approx (1+2m\mu') \left[-\omega + \frac{2\omega}{e^{2\pi\omega/a}+1} \right] \frac{dn}{d\omega} , \qquad (83)$$

from which we note an "anomalous" contribution to the Dirac sea. Indeed, this expression is valid if our particles are electrons, but for the proton roughly 60% of the observed magnetic moment is "anomalous" and the second term in (79) may give an important contribution. Notice also from Eq. (76) for the energy density that the zero mass limit ($\mu' \neq 0$) does not bring any further simplification, due to the fact that the $\theta\mu' B$ term appears as a mass contribution.

V. DISCUSSIONS

We have solved the DP equation in Rindler coordinates, including a background constant magnetic field in the direction of acceleration, and applied the usual canonical quantization to the DP field in this simple system. It turned out that the coefficients connecting Rindler and instantaneous Minkowski spinors, Φ and $\Lambda\psi$, do not mix spin components (67)–(69). Therefore, there are no effects on the spin polarization due to acceleration at this level [20]. We also derived an explicit expression for the energy and particle densities of the Minkowski vacuum as seen by a Rindler observer.

A formal argument based on the time periodicity of the Feynman propagator was given by Gibbons and Perry [21], who show that the Green's function in Rindler coordinates is formally equivalent to a thermal Green's function [22]. As a consequence, one gets a Planckian spectrum for the particle number. This is true even with the inclusion of an anomalous magnetic moment, as it can be seen directly from the Bogoliubov coefficient in Eq. (68), and is due to the fact that the coordinate transformation from Rindler coordinates to Minkowski coordinates is periodic in imaginary Rindler time.

However, this does not imply that the energy-momentum tensor is that of a thermal distribution. This is hardly a surprise, since even in the massless free case the energy spectrum has a $(\omega^2+a^2s^2)$ term, which is related to the density of modes, and does not allow the spectrum to be Planckian [6,23,24]. This term is even more complicated in the massive case.

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