CP-violating electric and weak dipole moments of the τ lepton from threshold to 500 GeV

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A number of CP-odd momentum correlations are investigated for τ pair production in e^+e^- collisions covering the energy range from threshold to 500 GeV. These correlations are in particular sensitive to the electric and weak dipole form factor of the τ lepton. Taking into account the main decay modes of the τ, the correlations are calculated in terms of these form factors. Sensitivity estimates, which apply to present and future e^+e^- facilities, are given.

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I. INTRODUCTION

In a series of articles [1–5] we proposed a number of CP-odd correlations which can be used to search for CP-violating effects in e^+e^- collisions. Detailed investigations were made at the Z resonance, in particular for $Z\to\tau^+\tau^-$. Specifically, when applied to the latter decay, these correlations, which involve easily measurable momenta of the decay products of the τ leptons, provide direct information about the weak dipole form factor $d_\tau^Z(s=m_Z^2)$ of the τ lepton, which parametrizes possible CP-violating couplings in the on-shell $Z\tau^+\tau^-$ amplitude. A nonzero expectation value of any CP-odd observable would be evidence for interactions beyond the standard model since the effect of the Kobayashi-Maskawa mechanism of CP violation in the reaction $e^+e^-\to\tau^+\tau^-$ is practically negligible. Sizable dipole form factors of heavy fermions are possible in extensions of the standard model, in particular in models with an extended Higgs sector and in leptoquark models (see, e.g., Refs. [6,7]).

Following the proposal of Ref. [5], OPAL and ALEPH gave the first upper limits on the weak dipole form factor d_τ^Z resulting from direct CP tests [8,9].

In this paper we extend our investigations of [2,5] by considering the reaction $e^+e^-\to\tau^+\tau^-$ in the continuum, covering the energy range of the CERN e^+e^- colliders LEP 1 and LEP 2, and of a number of proposed e^+e^- facilities, such as τ-charm and B factories and a linear collider with a c.m. energy up to 500 GeV. At energies below the Z pole CP-odd correlations are also sensitive to the electric dipole form factor $d_\tau^\gamma(s)$ of the τ lepton.[1] (At zero momentum transfer this form factor is the familiar *electric dipole moment* of the τ lepton.) In the continuum above the Z resonance it will be possible to determine the electric and the weak dipole form factors simultaneously by using an appropriate set of CP-odd observables. The correlations which we consider involve momenta of the observable τ decay products. In addition to the decays $\tau^-\to\nu_\tau\bar\nu_l l^-$ $(l=e,\mu)$, $\tau^-\to\nu_\tau\pi^-$, and $\tau^-\to\nu_\tau\pi^0\pi^-$ we also take into account here the three pion decay modes $\tau^-\to\nu_\tau\pi^0\pi^0\pi^-$ and $\tau^-\to\nu_\tau\pi^-\pi^-\pi^+$. Our proposal does not require the measurement or the reconstruction of the $\tau^\pm$ directions off light. However, the knowledge of the τ momenta would increase the sensitivity substantially. For related proposals see Refs. [10–13] and references contained in Ref. [5].

This paper is organized as follows. In Sec. II we remark briefly on the basic ideas of CP tests in e^+e^- experiments. We then derive the final-state density matrix for the reaction $e^+e^-\to\tau^+\tau^-$, taking into account CP-violating terms induced by nonzero form factors $d_\tau^{\gamma,Z}$. After a discussion of the above-mentioned τ decay channels (Sec. IV) we consider in Sec. V the $\tau^+\tau^-$ angular distribution in terms of measurable particles emerging from τ decays. From this we derive a basic set of CP-odd observables and some useful relations among their expectation values with respect to different final states. In Sec. VI the mean values and fluctuations of these observables and the resulting ideal statistical errors for $d_\tau^{\gamma,Z}$ are given for an energy range from threshold to 500 GeV. In the Appendix we collect in detail the numerical results.

II. CP TESTS

CP tests in the reaction $e^+e^-\to\tau^+\tau^-$ require an analysis of the final-state spin correlations. A list of observables which involve the spins and the momenta of the τ leptons was given in Ref. [5]. These observables can be directly used if the momenta of the τ leptons are measured and the polarization is analyzed through their decays. Here we restrict ourselves to the case that only the momenta of the τ decay products are observed. Specifically we consider the reactions

$$e^+(\mathbf{p})+e^-(-\mathbf{p})\to\tau^++\tau^-\to a(\mathbf{q}_-)+\bar b(\mathbf{q}_+)+X\ , \quad (2.1)$$

in the laboratory system, where a ($\bar b$) can be identified as a particle originating from τ^- (τ^+) decay. An identification of the residual particles X is not required but would considerably improve the analysis. The differential cross section for the process (2.1) is given by

$$d\sigma_{a\bar b}(\mathbf{p};\mathbf{q}_-,\mathbf{q}_+)=\frac{1}{2s}\frac{d^3q_-}{(2\pi)^3 2E_-}\frac{d^3q_+}{(2\pi)^3 2E_+}\times\mathcal{R}_{a\bar b}(\mathbf{p};\mathbf{q}_-,\mathbf{q}_+)\ , \quad (2.2)$$

where we neglect the electron mass, $s=(p_{e^+}+p_{e^-})^2$, and

[1] In Ref. [5] we used the notation d_τ ($\tilde d_\tau$) for the electric (weak) dipole form factor.

$$\mathcal{R}_{a\bar{b}}(\mathbf{p};\mathbf{q}_-,\mathbf{q}_+)=\sum_{\text{spins}}{}' \int d\Gamma_X (2\pi)^4\delta^{(4)}(p_{e^+}+p_{e^-}-q_+-q_--q_X)|\langle a(\mathbf{q}_-)\bar{b}(\mathbf{q}_+)X|\mathcal{T}|e^+(\mathbf{p})e^-(-\mathbf{p})\rangle|^2 . \quad (2.3)$$

Now we recall the conditions on $\mathcal{R}$ which can be derived by assuming invariance of the basic interactions under the *CP, T,* and *CPT* transformations. From *CP* invariance we get the condition

$$\mathcal{R}_{a\bar{b}}(\mathbf{p};\mathbf{q}_-,\mathbf{q}_+)=\mathcal{R}_{b\bar{a}}(\mathbf{p};-\mathbf{q}_+,-\mathbf{q}_-) . \quad (2.4)$$

If we neglect the absorptive parts $\mathrm{Im}\mathcal{T}=(\mathcal{T}-\mathcal{T}^\dagger)/2i$ of the amplitude, invariance under T and the *CPT* transformation implies

$$\mathcal{R}_{a\bar{b}}(\mathbf{p};\mathbf{q}_-,\mathbf{q}_+)=\mathcal{R}_{a\bar{b}}(-\mathbf{p};-\mathbf{q}_-,-\mathbf{q}_+) \quad (2.5)$$

and

$$\mathcal{R}_{a\bar{b}}(\mathbf{p};\mathbf{q}_-,\mathbf{q}_+)=\mathcal{R}_{b\bar{a}}(-\mathbf{p};\mathbf{q}_+,\mathbf{q}_-) , \quad (2.6)$$

respectively. In general, the relations (2.5) and (2.6) can be violated by *T*- and *CPT-conserving* interactions if the anti-Hermitian part $\mathrm{Im}\mathcal{T}$ of the T matrix does not vanish due to initial- or final-state interactions or the propagators of resonances. However, the relation (2.4) is always respected by *CP*-conserving interactions.

Next we consider a *CP*-odd observable

$$\mathcal{O}(\mathbf{q}_+,\mathbf{q}_-)=-\mathcal{O}(-\mathbf{q}_-,-\mathbf{q}_+)$$

which is a function of the momenta $\mathbf{q}_\pm$ defined in (2.1). From Eq. (2.4) we conclude that the expectation value

$$[\langle\mathcal{O}\rangle]_{ab}\equiv\frac{[\langle\mathcal{O}\rangle]_{a\bar{b}}+[\langle\mathcal{O}\rangle]_{b\bar{a}}}{2} =\frac{1}{2}\left[\frac{\int d\sigma_{a\bar{b}}\mathcal{O}}{\int d\sigma_{a\bar{b}}}+\frac{\int d\sigma_{b\bar{a}}\mathcal{O}}{\int d\sigma_{b\bar{a}}}\right] \quad (2.7)$$

projects[2] onto a possible *CP*-odd part of $\mathcal{R}$. The *CP*-odd and *CPT*-even part of $\mathcal{R}$ is analyzed by observables which are *CP* odd and *T* odd (i.e., change sign with respect to reflection of the momenta). With *CP*-odd and *T*-even observables one can analyze possible *CP*-violating absorptive parts. An example for such observables are functions of the energies $E_\pm$ which are odd under the exchange $E_+\leftrightarrow E_-$. However, these observables turn out to be rather insensitive compared to momentum correlations.

We mention also the possibility to study unitarity corrections with *CPT*- and *T*-odd tensor correlations. Such observables receive contributions from absorptive parts generated by *CP*-conserving interactions [14–16].

III. SPIN CORRELATIONS OF THE $\tau^+\tau^-$ SYSTEM

In this section we give explicit formulas for the spin-density matrix of the τ leptons produced in the reaction

$$e^+(\mathbf{p})+e^-(-\mathbf{p})\to\gamma,\ Z\to\tau^+(\mathbf{k})+\tau^-(-\mathbf{k}) \quad (3.1)$$

in the c.m. system, assuming unpolarized e^+ and e^- beams and neglecting the electron mass. The spin-density matrix (up to a normalization factor) is given by

$$\chi_{\alpha\alpha'\beta\beta'}=\frac{1}{4}\sum_{B_1,B_2=\gamma,Z}\chi_{\alpha\alpha'\beta\beta'}(B_1,B_2) , \quad (3.2)$$

with

$$\chi_{\alpha\alpha'\beta\beta'}(B_1,B_2)=\frac{1}{2}\sum_{\gamma,\delta}\mathcal{T}(e^+_\gamma e^-_\delta\to B_1\to\tau^+_\alpha\tau^-_\beta)\times\mathcal{T}^*(e^+_\gamma e^-_\delta\to B_2\to\tau^+_{\alpha'}\tau^-_{\beta'})+(B_1\leftrightarrow B_2) . \quad (3.3)$$

Greek letters denote the spin indices of the initial and the final state. If the electron mass is neglected then up to and including interactions of mass dimension $d=6$ the most general ansatz describing possible *CP*-violating effects in the amplitude for (3.1) can be parametrized by the electric and weak dipole form factors $d_\tau^{\gamma,Z}(s)$ of the τ lepton, described by the effective Lagrangian [3]

$$\mathcal{L}_{\text{eff}}^{CP}=-(i/2)\bar{\tau}\sigma^{\mu\nu}\gamma_5\tau[d_\tau^\gamma(s)(\partial_\mu A_\nu-\partial_\nu A_\mu)+d_\tau^Z(s)(\partial_\mu Z_\nu-\partial_\nu Z_\mu)] , \quad (3.4)$$

where A and Z are the photon and the Z boson fields. In Fig. 1 the diagrams corresponding to these effective *CP*-violating interactions together with the standard model graphs at tree level are shown.

Now we expand the spin-density matrix of the τ leptons in powers of the dipole form factors

$$\chi=\chi_{\text{SM}}+\chi_{CP}+\chi_{d^2} , \quad (3.5)$$

where χ_{SM} is the standard model part, χ_{CP} results from the interference of *CP*-conserving and *CP*-violating cou-

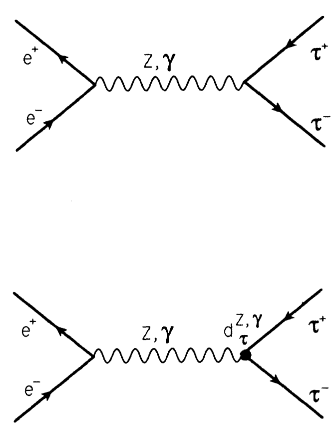

FIG. 1. The diagrams contributing to the reaction $e^+e^-\to\tau^+\tau^-$.

[2]We always assume cuts in phase space to be C and P blind and to satisfy the conditions specified in Ref. [14] if statements concerning T and CPT are made.

plings, and χ_{d^2} is the CP-even contribution bilinear in the dipole form factors. We shall expand χ in terms of the matrices

$$\begin{aligned} \mathbb{1} &\equiv (\mathbb{1}\otimes\mathbb{1})_{\alpha\alpha',\beta\beta'} = \delta_{\alpha\alpha'}\delta_{\beta\beta'} , \\ \boldsymbol{\sigma}_+ &\equiv (\boldsymbol{\sigma}\otimes\mathbb{1})_{\alpha\alpha',\beta\beta'} = \boldsymbol{\sigma}_{\alpha\alpha'}\delta_{\beta\beta'} , \\ \boldsymbol{\sigma}_- &\equiv (\mathbb{1}\otimes\boldsymbol{\sigma})_{\alpha\alpha',\beta\beta'} = \delta_{\alpha\alpha'}\boldsymbol{\sigma}_{\beta\beta'} , \end{aligned} \tag{3.6}$$

where $\frac{1}{2}\boldsymbol{\sigma}_\pm$ is the spin operator of $\tau^\pm$. A useful abbreviation for the couplings of the electron and the τ lepton to the photon and to the Z boson is

$$\begin{aligned} &V_f^\gamma = Q_f e, \quad A_f^\gamma = 0 , \\ &V_f^Z = \frac{(T_f)_3 - 2Q_f \sin^2\theta_W}{2\sin\theta_W\cos\theta_W} e, \quad A_f^Z = \frac{(T_f)_3}{2\sin\theta_W\cos\theta_W} e , \end{aligned} \tag{3.7}$$

with Q_f the electric charge and T_f the weak isospin of the fermion $f = e, \tau$. Using these conventions, the standard model part of the spin-density matrix can be written in tree approximation as

$$\begin{aligned} \chi_{\rm SM}(B_1,B_2) = {} & \frac{s(V_e^{B_1}V_e^{B_2} + A_e^{B_1}A_e^{B_2})}{(s-m_{B_1}^2 + im_{B_1}\Gamma_{B_1})(s-m_{B_2}^2 - im_{B_2}\Gamma_{B_2})} \\ & \times \big(V_\tau^{B_1}V_\tau^{B_2}\{ k_0^2 + m_\tau^2 + |\mathbf{k}|^2(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})^2 - \boldsymbol{\sigma}_+\cdot\boldsymbol{\sigma}_-|\mathbf{k}|^2[1-(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})^2] \\ & \qquad + 2(\hat{\mathbf{k}}\cdot\boldsymbol{\sigma}_+)(\hat{\mathbf{k}}\cdot\boldsymbol{\sigma}_-)[|\mathbf{k}|^2 + (k_0-m_\tau)^2(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})^2] + 2k_0^2(\hat{\mathbf{p}}\cdot\boldsymbol{\sigma}_+)(\hat{\mathbf{p}}\cdot\boldsymbol{\sigma}_-) \\ & \qquad - 2k_0(k_0-m_\tau)\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}[(\hat{\mathbf{k}}\cdot\boldsymbol{\sigma}_+)(\hat{\mathbf{p}}\cdot\boldsymbol{\sigma}_-) + (\hat{\mathbf{k}}\cdot\boldsymbol{\sigma}_-)(\hat{\mathbf{p}}\cdot\boldsymbol{\sigma}_+)]\} \\ & + A_\tau^{B_1}A_\tau^{B_2}|\mathbf{k}|^2\{ 1 + (\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})^2 + \boldsymbol{\sigma}_+\cdot\boldsymbol{\sigma}_-[1-(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})^2] - 2(\hat{\mathbf{p}}\cdot\boldsymbol{\sigma}_+)(\hat{\mathbf{p}}\cdot\boldsymbol{\sigma}_-) \\ & \qquad + 2\hat{\mathbf{k}}\cdot\hat{\mathbf{p}}[(\hat{\mathbf{k}}\cdot\boldsymbol{\sigma}_+)(\hat{\mathbf{p}}\cdot\boldsymbol{\sigma}_-) + (\hat{\mathbf{k}}\cdot\boldsymbol{\sigma}_-)(\hat{\mathbf{p}}\cdot\boldsymbol{\sigma}_+)]\} \\ & + (A_\tau^{B_1}V_\tau^{B_2} + A_\tau^{B_2}V_\tau^{B_1})|\mathbf{k}|\{\hat{\mathbf{k}}\cdot(\boldsymbol{\sigma}_+ + \boldsymbol{\sigma}_-)[k_0 + (k_0-m_\tau)(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})^2] + m_\tau(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})\hat{\mathbf{p}}\cdot(\boldsymbol{\sigma}_+ + \boldsymbol{\sigma}_-)\} \\ & + c(B_1,B_2)V_\tau^{B_1}V_\tau^{B_2}2k_0\{m_\tau\hat{\mathbf{p}}\cdot(\boldsymbol{\sigma}_+ + \boldsymbol{\sigma}_-) + (k_0-m_\tau)(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})\hat{\mathbf{k}}\cdot(\boldsymbol{\sigma}_+ + \boldsymbol{\sigma}_-)\} \\ & + c(B_1,B_2)A_\tau^{B_1}A_\tau^{B_2}2|\mathbf{k}|^2(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})\hat{\mathbf{k}}\cdot(\boldsymbol{\sigma}_+ + \boldsymbol{\sigma}_-) \\ & + c(B_1,B_2)(A_\tau^{B_1}V_\tau^{B_2} + A_\tau^{B_2}V_\tau^{B_1})|\mathbf{k}| \\ & \quad \times\{m_\tau[(\hat{\mathbf{k}}\cdot\boldsymbol{\sigma}_+)(\hat{\mathbf{p}}\cdot\boldsymbol{\sigma}_-) + (\hat{\mathbf{k}}\cdot\boldsymbol{\sigma}_-)(\hat{\mathbf{p}}\cdot\boldsymbol{\sigma}_+)] + 2k_0\hat{\mathbf{k}}\cdot\hat{\mathbf{p}} + 2(k_0-m_\tau)(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})(\hat{\mathbf{k}}\cdot\boldsymbol{\sigma}_+)(\hat{\mathbf{k}}\cdot\boldsymbol{\sigma}_-)\}\big) . \end{aligned} \tag{3.8}$$

Here m_γ and Γ_γ are understood to be zero. The terms proportional to

$$c(B_1,B_2) = \frac{V_e^{B_1}A_e^{B_2} + V_e^{B_2}A_e^{B_1}}{V_e^{B_1}V_e^{B_2} + A_e^{B_1}A_e^{B_2}} \tag{3.9}$$

result from the parity-violating interference of the electron vector and axial-vector couplings. The CP-violating spin correlation term reads

$$\begin{aligned} \chi_{CP}(B_1,B_2) = {} & \frac{s(V_e^{B_1}V_e^{B_2} + A_e^{B_1}A_e^{B_2})2k_0|\mathbf{k}|}{(s-m_{B_1}^2 + im_{B_1}\Gamma_{B_1})(s-m_{B_2}^2 - im_{B_2}\Gamma_{B_2})} \\ & \times \big((V_\tau^{B_2}\mathrm{Re}d_\tau^{B_1} + V_\tau^{B_1}\mathrm{Re}d_\tau^{B_2})\{ -[m_\tau + (k_0-m_\tau)(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})^2](\boldsymbol{\sigma}_+\times\boldsymbol{\sigma}_-)\cdot\hat{\mathbf{k}} + k_0(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})(\boldsymbol{\sigma}_+\times\boldsymbol{\sigma}_-)\cdot\hat{\mathbf{p}} \\ & \qquad - c(B_1,B_2)k_0[\hat{\mathbf{k}}\times(\boldsymbol{\sigma}_+ - \boldsymbol{\sigma}_-)]\cdot\hat{\mathbf{p}}\} \\ & + (A_\tau^{B_2}\mathrm{Re}d_\tau^{B_1} + A_\tau^{B_1}\mathrm{Re}d_\tau^{B_2})|\mathbf{k}|\{ -(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})[\hat{\mathbf{k}}\times(\boldsymbol{\sigma}_+ - \boldsymbol{\sigma}_-)]\cdot\hat{\mathbf{p}} + c(B_1,B_2)(\hat{\mathbf{k}}\cdot\boldsymbol{\sigma}_+)(\hat{\mathbf{k}}\times\boldsymbol{\sigma}_-)\cdot\hat{\mathbf{p}} \\ & \qquad - c(B_1,B_2)(\hat{\mathbf{k}}\cdot\boldsymbol{\sigma}_-)(\hat{\mathbf{k}}\times\boldsymbol{\sigma}_+)\cdot\hat{\mathbf{p}}\} \\ & - (V_\tau^{B_2}\mathrm{Im}d_\tau^{B_1} + V_\tau^{B_1}\mathrm{Im}d_\tau^{B_2})\{ -[m_\tau + (k_0-m_\tau)(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})^2](\boldsymbol{\sigma}_+ - \boldsymbol{\sigma}_-)\cdot\hat{\mathbf{k}} + k_0(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})(\boldsymbol{\sigma}_+ - \boldsymbol{\sigma}_-)\cdot\hat{\mathbf{p}} \\ & \qquad - c(B_1,B_2)k_0[(\hat{\mathbf{k}}\cdot\boldsymbol{\sigma}_+)(\hat{\mathbf{p}}\cdot\boldsymbol{\sigma}_-) - (\hat{\mathbf{k}}\cdot\boldsymbol{\sigma}_-)(\hat{\mathbf{p}}\cdot\boldsymbol{\sigma}_+)]\} \\ & - (A_\tau^{B_2}\mathrm{Im}d_\tau^{B_1} + A_\tau^{B_1}\mathrm{Im}d_\tau^{B_2})|\mathbf{k}|\{ -(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})[(\hat{\mathbf{k}}\cdot\boldsymbol{\sigma}_+)(\hat{\mathbf{p}}\cdot\boldsymbol{\sigma}_-) - (\hat{\mathbf{k}}\cdot\boldsymbol{\sigma}_-)(\hat{\mathbf{p}}\cdot\boldsymbol{\sigma}_+)] \\ & \qquad + c(B_1,B_2)[\hat{\mathbf{p}}\cdot(\boldsymbol{\sigma}_+ - \boldsymbol{\sigma}_-) - (\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})\hat{\mathbf{k}}\cdot(\boldsymbol{\sigma}_+ - \boldsymbol{\sigma}_-)]\}\big) . \end{aligned} \tag{3.10}$$

Finally, we give the term which is bilinear in the dipole form factors:

$$\chi_{d^2}(B_1,B_2)=\frac{s(V_e^{B_1}V_e^{B_2}+A_e^{B_1}A_e^{B_2})k_0^2|\mathbf{k}|^2}{(s-m_{B_1}^2+im_{B_1}\Gamma_{B_1})(s-m_{B_2}^2-im_{B_2}\Gamma_{B_2})}4(\mathrm{Re}d_\tau^{B_1}\mathrm{Re}d_\tau^{B_2}+\mathrm{Im}d_\tau^{B_1}\mathrm{Im}d_\tau^{B_2})[1-(\hat{\mathbf{k}}\cdot\hat{\mathbf{p}})^2](1-\boldsymbol{\sigma}_+\cdot\boldsymbol{\sigma}_-) . \tag{3.11}$$

The corresponding expressions in Ref. [5] which are valid only at the Z pole can be recovered by setting $B_1=B_2=Z$.

It may easily be verified that the CP-conserving part of χ is symmetric in $\boldsymbol{\sigma}_+$ and $\boldsymbol{\sigma}_-$, while the CP-odd contribution, i.e., χ_{CP}, is antisymmetric. We allow the dipole form factors to have absorptive parts $\mathrm{Im}d_\tau^{\gamma,Z}$, giving rise to CPT-odd transforming terms of the spin correlation. Absorptive parts due to CP-conserving interactions are neglected. In particular, in the continuum the finite width of the Z boson is a radiative correction of the order α and is neglected there for consistency.

IV. DECAY MODES OF THE τ LEPTON

In order to calculate the differential cross section for the reaction (2.1) we need the decay distribution of a polarized τ lepton. The decay matrix is defined by

$$\mathcal{D}_{\alpha'\alpha}[\tau\to a(\mathbf{q})+X]=\Gamma^{-1}(\tau\to a+X)\frac{1}{2m_\tau}\int d\Gamma_X(2\pi)^4\delta^{(4)}(k_\tau-q-q_X)\langle a(\mathbf{q})X|\mathcal{T}|\tau_\alpha\rangle\langle a(\mathbf{q})X|\mathcal{T}|\tau_{\alpha'}\rangle^* , \tag{4.1}$$

where we consider the τ lepton in its rest system and α,α' are spin indices. We integrate over the momenta of all final-state particles X except for the particle a whose momentum enters into the CP-odd observables which will be introduced in the following section. The decay matrix satisfies the normalization condition

$$\int\frac{d^3q}{(2\pi)^3 2q^0}\mathcal{D}_{\alpha\alpha'}=\delta_{\alpha\alpha'}\langle n_a\rangle , \tag{4.2}$$

with $\langle n_a\rangle$ denoting the average multiplicity of the particle a. The decay matrix which describes the charge-conjugated process can be obtained from CPT invariance if the absorptive parts of the amplitude $\langle aX|\mathcal{T}|\tau\rangle$ are negligible. Given the production matrix defined in Eq. (3.2) the differential cross section (2.1) reads

$$\begin{aligned} d\sigma_{a\bar{b}}=&\frac{\sqrt{1-4m_\tau^2/s}}{16\pi s}\frac{d\Omega_\tau}{4\pi}\chi_{\alpha\alpha'\beta\beta'}\frac{d^3q_-}{(2\pi)^3 2q_-^0}\\ &\times\mathcal{D}_{\beta'\beta}[\tau^-\to a(\mathbf{q}_-)+X]\frac{d^3q_+}{(2\pi)^3 2q_+^0}\\ &\times\mathcal{D}_{\alpha'\alpha}[\tau^+\to\bar{b}(\mathbf{q}_+)+X'] . \end{aligned} \tag{4.3}$$

After the energy dependence of the decay matrix is integrated out, $\mathcal{D}$ reduces in the τ rest system (indicating quantities referring to it by an asterisk) to

$$\int\frac{q_0^* dq_0^*}{(2\pi)^2}\mathcal{D}[\tau\to\underbrace{a(\mathbf{q}^*)+X}_{A}]=\mathbb{1}+\alpha_A\hat{\mathbf{q}}^*\cdot\boldsymbol{\sigma} , \tag{4.4}$$

where α_A ($|\alpha_A|\leq 1$) is a measure of the spin-analyzing power of the decay channel A. It is well known that in the decay $\tau^-\to\nu_\tau\pi^-$ the parameter α takes its maximal value $\alpha_{\nu\pi}=1$. Other useful channels are $\tau^-\to\nu_\tau\bar{\nu}_l l^-$ with $l=e,\mu$ and $\tau^-\to\nu_\tau\pi^0\pi^-$, where the charged pion is observed. The decay distributions of these reactions were collected in Refs. [5,17]. In this paper we investigate the reaction $\tau^-\to\nu_\tau\rho^-$, where the momentum of the ρ meson must be recombined from both final-state pions. This has a much better spin-analyzer quality than the charged pion from the ρ decay. In addition, we consider $\tau^-\to\nu_\tau 2\pi^0\pi^-$ and $\tau^-\to\nu_\tau 2\pi^-\pi^+$, where the momenta of the π^- and the π^+, respectively, are taken as τ spin analyzers. The amplitude for $\tau\to\nu 2\pi^0\pi^-$ is assumed to be of the form

$$\begin{aligned}\langle\nu\pi^-(q)\pi^0(q_1)\pi^0(q_2)|\mathcal{T}|\tau^-\rangle=&\frac{G_F}{\sqrt{2}}\bar{u}_\nu\gamma^\mu(1-\gamma_5)u_\tau F_{a_1}(Q^2)\frac{-i2\sqrt{2}}{3f_\pi}\cos\theta_C\left[g_{\mu\nu}-\frac{Q_\mu Q_\nu}{Q^2}\right]\\ &\times\{(q-q_1)^\nu F_\rho[(q+q_1)^2]+1\leftrightarrow 2\} ,\end{aligned} \tag{4.5}$$

where $Q=q+q_1+q_2$, G_F is the Fermi constant, θ_C is the Cabibbo angle, and f_π is the pion decay constant. The normalization of (4.5) for $Q^2\to 0$ is determined by chiral invariance [18]. Using isospin symmetry (4.5) also gives the amplitude for $\tau^-\to\nu\pi^-\pi^-\pi^+$. The three pion final state is dominated by the a_1 resonance, parametrized by the Breit-Wigner propagator

$$F_{a_1}(Q^2)=\frac{m_{a_1}^2}{m_{a_1}^2-Q^2-im_{a_1}\Gamma_{a_1}} , \tag{4.6}$$

and its subsequent decay to $\rho\pi$. We implement the ρ resonance as a P-wave propagator

$$F_\rho(u)=\frac{m_\rho^2}{m_\rho^2-u-i\sqrt{u}\,\Gamma(u)}\ , \tag{4.7}$$

with

$$\Gamma(u)=\Gamma_\rho\frac{m_\rho^2}{(m_\rho^2-4m_\pi^2)^{3/2}}\frac{(u-4m_\pi^2)^{3/2}}{u}\ . \tag{4.8}$$

A more detailed discussion about the incorporation of resonances was given in Ref. [19].

We used the decay distributions of the τ lepton together with the tree level production matrix to write a Monte Carlo program. The spectra simulated with our program, putting the dipole form factors to zero, are in good agreement with results from the Monte Carlo code [20]. In Table I we collect the parameters α, defined in Eq. (4.4), for the decay channels which are considered here.

TABLE I. The parameter α for various decay channels of the τ lepton.

$\tau^-\to$	α
$\nu_\tau\pi^-$	1
$\nu_\tau\pi^0\pi^-$	-0.062
$\nu_\tau\rho^-$	0.46
$\nu_\tau\pi^0\pi^0\pi^-$	-0.18
$\nu_\tau\bar{\nu}_l l^-$	$-\frac{1}{3}$

V. *CP*-ODD OBSERVABLES

In this section we give first a qualitative discussion of *CP*-odd observables and their analyzing power for dipole form factors. For this we derive an expression for the differential cross section

$$\frac{d\sigma[e^+e^-\to a(\mathbf{q}_-)\bar{b}(\mathbf{q}_+)X]}{(d\Omega_-/4\pi)(d\Omega_+/4\pi)} \tag{5.1}$$

in the c.m. system. For obtaining (5.1) we use the angular decay distributions of the τ leptons according to (4.4), where the momenta are transformed to the laboratory frame and the masses of the observed particles are neglected. Taking account of the symmetry properties of the integrand with respect to C and P transformations and spatial rotations, the *CP*-odd part of the angular distribution (5.1) is found to be

$$\begin{aligned}
&\frac{d\sigma_{CP}[e^+(\mathbf{p})e^-(-\mathbf{p})\to a(\mathbf{q}_-)\bar{b}(\mathbf{q}_+)X]}{(d\Omega_-/4\pi)(d\Omega_+/4\pi)}\\
&\propto\sum_{B_1,B_2=\gamma,Z}\frac{V_e^{B_1}V_e^{B_2}+A_e^{B_1}A_e^{B_2}}{(s-m_{B_1}^2+im_{B_1}\Gamma_{B_1})(s-m_{B_2}^2-im_{B_2}\Gamma_{B_2})}\\
&\quad\times\{(V_\tau^{B_2}\mathrm{Re}d_\tau^{B_1}+V_\tau^{B_1}\mathrm{Re}d_\tau^{B_2})[F_1\alpha_A\alpha_B\hat{\mathbf{p}}\cdot(\hat{\mathbf{q}}_+\times\hat{\mathbf{q}}_-)\hat{\mathbf{p}}\cdot(\hat{\mathbf{q}}_+-\hat{\mathbf{q}}_-)+c(B_1,B_2)F_2(\alpha_A+\alpha_B)\hat{\mathbf{p}}\cdot(\hat{\mathbf{q}}_+\times\hat{\mathbf{q}}_-)]\\
&\qquad+(A_\tau^{B_2}\mathrm{Re}d_\tau^{B_1}+A_\tau^{B_1}\mathrm{Re}d_\tau^{B_2})[F_3(\alpha_A+\alpha_B)\hat{\mathbf{p}}\cdot(\hat{\mathbf{q}}_+\times\hat{\mathbf{q}}_-)\hat{\mathbf{p}}\cdot(\hat{\mathbf{q}}_+-\hat{\mathbf{q}}_-)\\
&\qquad\qquad+c(B_1,B_2)F_4\alpha_A\alpha_B\hat{\mathbf{p}}\cdot(\hat{\mathbf{q}}_+\times\hat{\mathbf{q}}_-)]\\
&\qquad+(V_\tau^{B_2}\mathrm{Im}d_\tau^{B_1}+V_\tau^{B_1}\mathrm{Im}d_\tau^{B_2})[F_5(\alpha_A+\alpha_B)\hat{\mathbf{p}}\cdot(\hat{\mathbf{q}}_++\hat{\mathbf{q}}_-)\hat{\mathbf{p}}\cdot(\hat{\mathbf{q}}_+-\hat{\mathbf{q}}_-)\\
&\qquad\qquad+c(B_1,B_2)F_6\alpha_A\alpha_B\hat{\mathbf{p}}\cdot(\hat{\mathbf{q}}_++\hat{\mathbf{q}}_-)]\\
&\qquad+(A_\tau^{B_2}\mathrm{Im}d_\tau^{B_1}+A_\tau^{B_1}\mathrm{Im}d_\tau^{B_2})[F_7\alpha_A\alpha_B\hat{\mathbf{p}}\cdot(\hat{\mathbf{q}}_++\hat{\mathbf{q}}_-)\hat{\mathbf{p}}\cdot(\hat{\mathbf{q}}_+-\hat{\mathbf{q}}_-)\\
&\qquad\qquad\times c(B_1,B_2)F_8(\alpha_A+\alpha_B)\hat{\mathbf{p}}\cdot(\hat{\mathbf{q}}_++\hat{\mathbf{q}}_-)]\}\ ,
\end{aligned} \tag{5.2}$$

where $\alpha_{A,B}$ are defined in (4.4) and $\hat{\mathbf{p}},\hat{\mathbf{q}}_\pm$ denote unit momenta. The scalar functions $F_{1\ldots8}(\hat{\mathbf{q}}_+\cdot\hat{\mathbf{q}}_-)$ can be calculated numerically. The *CPT*-even part of the differential cross section (5.2) involves

$$\begin{aligned}
\hat{T}^{ij}&=(\hat{\mathbf{q}}_+-\hat{\mathbf{q}}_-)^i\frac{(\hat{\mathbf{q}}_+\times\hat{\mathbf{q}}_-)^j}{|\hat{\mathbf{q}}_+\times\hat{\mathbf{q}}_-|}+(i\leftrightarrow j)\ ,\\
\hat{A}_1&=\hat{\mathbf{p}}\cdot\frac{\hat{\mathbf{q}}_+\times\hat{\mathbf{q}}_-}{|\hat{\mathbf{q}}_+\times\hat{\mathbf{q}}_-|}\ ,
\end{aligned} \tag{5.3}$$

whereas the *CPT*-odd term which contains the absorptive parts of the dipole form factors involves

$$\begin{aligned}
\hat{Q}^{ij}&=(\hat{\mathbf{q}}_++\hat{\mathbf{q}}_-)^i(\hat{\mathbf{q}}_+-\hat{\mathbf{q}}_-)^j+(i\leftrightarrow j)\ ,\\
\hat{A}_2&=\hat{\mathbf{p}}\cdot(\hat{\mathbf{q}}_++\hat{\mathbf{q}}_-)\ .
\end{aligned} \tag{5.4}$$

Here $i,j=1\cdots3$ are Cartesian vector indices. The expressions (5.3) and (5.4) can be taken as the basic *CP*-odd observables for the semi-inclusive reactions (2.1) since they project onto the corresponding terms of the differential cross section (5.2). We also considered the dimensionful analogues of these observables:

$$\begin{aligned}
T^{ij}&=(\mathbf{q}_+-\mathbf{q}_-)^i(\mathbf{q}_+\times\mathbf{q}_-)^j+(i\leftrightarrow j)\ ,\\
A_1&=\hat{\mathbf{p}}\cdot(\mathbf{q}_+\times\mathbf{q}_-)\ ,\\
Q^{ij}&=(\mathbf{q}_++\mathbf{q}_-)^i(\mathbf{q}_+-\mathbf{q}_-)^j-\frac{\delta^{ij}}{3}(\mathbf{q}_+^2-\mathbf{q}_-^2)+(i\leftrightarrow j)\ ,\\
A_2&=\hat{\mathbf{p}}\cdot(\mathbf{q}_++\mathbf{q}_-)\ .
\end{aligned} \tag{5.5}$$

However, except for T^{ij}, these turn out to be less sensitive than their analogues (5.3) and (5.4).

TABLE II. 1 s.d. accuracies obtainable in measuring the dipole form factors at c.m. energies between 3.67 and 500 GeV.

$\sqrt{s}$ (GeV)	$\tau^+\tau^-$ events	$\Delta\,\mathrm{Re}d_\tau^\gamma$ (e cm)	$\Delta\,\mathrm{Im}d_\tau^\gamma$ (e cm)	$\Delta\,\mathrm{Re}d_\tau^Z$ (e cm)	$\Delta\,\mathrm{Im}d_\tau^Z$ (e cm)
3.67	2.4×10^7	2.4×10^{-16}	1.4×10^{-16}		
4.25	3.5×10^7	4.1×10^{-17}	1.9×10^{-17}		
10.58	5×10^7	0.9×10^{-18}	2.7×10^{-18}		
91.16	3.3×10^5			2.0×10^{-18}	3.4×10^{-17}
180	5000	1.5×10^{-16}	7.0×10^{-17}	1.1×10^{-17}	5.8×10^{-16}
500	5000	4.6×10^{-17}	2.4×10^{-17}	4.5×10^{-18}	2.9×10^{-16}

From (5.2) we derive some useful relations which connect the expectation values of dimensionless observables with respect to different τ decay modes. At energies far below the Z pole, axial-vector couplings can be neglected which also implies $c(B_1,B_2)\approx 0$ [cf. (3.9)]. Then only the tensor observables have nonzero mean values. For the channel $\tau^+\tau^-\to AB$ they are related to the corresponding averages for the pion-pion channel by

$$[\langle \hat{T}^{ij}\rangle]_{AB}\approx\alpha_A\alpha_B[\langle \hat{T}^{ij}\rangle]_{\nu\pi,\nu\pi}\ ,$$
$$[\langle \hat{Q}^{ij}\rangle]_{AB}\approx\frac{\alpha_A+\alpha_B}{2}[\langle \hat{Q}^{ij}\rangle]_{\nu\pi,\nu\pi}\quad(\sqrt{s}\ll m_Z)\ . \tag{5.6}$$

Since $|\alpha|\le 1$, the expectation values are maximal in the pion-pion case. Note that in unfavorable cases the mean value of $\hat{Q}^{ij}$ can become small due to a cancellation between α_A and α_B. At the Z peak, where the axial-vector couplings of the τ lepton and of the electron are dominant, we find, on the other hand,

$$[\langle \hat{T}^{ij}\rangle]_{AB}\approx\frac{\alpha_A+\alpha_B}{2}[\langle \hat{T}^{ij}\rangle]_{\nu\pi,\nu\pi}\ ,$$
$$[\langle \hat{Q}^{ij}\rangle]_{AB}\approx\alpha_A\alpha_B[\langle \hat{Q}^{ij}\rangle]_{\nu\pi,\nu\pi}\quad(\sqrt{s}=m_Z)\ . \tag{5.7}$$

The relations (5.6) and (5.7) provide information about the sensitivity of an observable in a certain combination of τ decay modes, since its fluctuations are approximately constant for all channels.

VI. NUMERICAL RESULTS

The expectation values of the dimensionless tensor observables introduced in the preceding section are, to leading order in the standard model couplings and in the (nonstandard) dipole form factors of the τ lepton, given by

$$\langle \hat{T}^{ij}\rangle=\frac{\sqrt{s}}{e}[c_\gamma(s)\mathrm{Re}d_\tau^\gamma(s)+c_Z(s)\mathrm{Re}d_\tau^Z(s)]s^{ij}\ ,$$
$$\langle \hat{Q}^{ij}\rangle=\frac{\sqrt{s}}{e}[c'_\gamma(s)\mathrm{Im}d_\tau^\gamma(s)+c'_Z(s)\mathrm{Im}d_\tau^Z(s)]s^{ij}\ , \tag{6.1}$$

where s^{ij} is the tensor polarization of the intermediate γ/Z state [5]. If the beam direction is identified with the three-axis, this matrix can be written as diag$(-\frac{1}{6},-\frac{1}{6},\frac{1}{3})$. Since the diagonal elements of tensor observables are not independent, we consider only the 3,3 component, which has the largest expectation value. For the mean values of vectorlike observables we write

$$\langle \hat{A}_1\rangle=\frac{\sqrt{s}}{e}[r_\gamma(s)\mathrm{Re}d_\tau^\gamma(s)+r_Z(s)\mathrm{Re}d_\tau^Z(s)]\ ,$$
$$\langle \hat{A}_2\rangle=\frac{\sqrt{s}}{e}[r'_\gamma(s)\mathrm{Im}d_\tau^\gamma(s)+r'_Z(s)\mathrm{Im}d_\tau^Z(s)]\ . \tag{6.2}$$

Similar equations hold for the dimensionful observables defined in (5.5). The coefficients c,c',r,r' have the same mass dimension as the corresponding observables.

TABLE III. Observables $T^{ij},\hat{Q}^{ij}$ at $\sqrt{s}=3.67$ GeV (2.4×10^7 $\tau^+\tau^-$ events).

$\tau^-\to$	$\tau^+\to$	c_γ (10^{-3} GeV^3)	$\sqrt{\langle T_{33}^2\rangle}$ (GeV^3)	$\Delta\,\mathrm{Re}d_\tau^\gamma$ (10^{-16} e cm)	c'_γ (10^{-3})	$\sqrt{\langle \hat{Q}_{33}^2\rangle}$	$\Delta\,\mathrm{Im}d_\tau^\gamma$ (10^{-16} e cm)
$\nu\pi^-$	$\bar{\nu}\pi^+$	36.7	0.545	4.44	−59	0.836	4.2
$\nu\pi^0\pi^-$	$\bar{\nu}\pi^0\pi^+$	1.5	0.171	16.8	7	0.844	18
$\nu\rho^-$	$\bar{\nu}\rho^+$	5.8	0.319	7.92	−38	0.837	3.2
$\nu 2\pi^0\pi^-$	$\bar{\nu}2\pi^0\pi^+$	0	0.072	(*)	11	0.847	20
$\nu\pi^-$	$\bar{\nu}\pi^0\pi^+$	7.5	0.307	6.00	−26	0.845	4.8
$\nu\pi^-$	$\bar{\nu}\rho^+$	15.1	0.412	4.02	−48	0.840	2.6
$\nu\pi^-$	$\bar{\nu}2\pi^0\pi^+$	0.8	0.203	48.8	−23	0.848	7.2
$\nu\pi^0\pi^-$	$\bar{\nu}2\pi^0\pi^+$	0.15	0.113	106	8	0.844	14
$\nu\rho^-$	$\bar{\nu}2\pi^0\pi^+$	0.27	0.155	77.5	−14	0.847	8.2
$\nu\bar{\nu}l^-$	$\bar{\nu}\nu l^+$	2.9	0.227	7.21	19	0.843	4.1
$\nu\bar{\nu}l^-$	$\bar{\nu}\pi^+$	−10.4	0.320	3.61	−20	0.846	5.0
$\nu\bar{\nu}l^-$	$\bar{\nu}\pi^0\pi^+$	−2.1	0.185	7.12	13	0.844	5.3
$\nu\bar{\nu}l^-$	$\bar{\nu}\rho^+$	−4.3	0.258	4.96	−11	0.845	6.3
$\nu\bar{\nu}l^-$	$\bar{\nu}2\pi^0\pi^+$	−0.23	1.266	60.1	15	0.846	6.1

TABLE IV. Observables $T^{ij},\hat{Q}^{ij}$ at $\sqrt{s}=4.25$ GeV (3.5×10^7 $\tau^+\tau^-$ events).

$\tau^- \to$	$\tau^+ \to$	c_γ (10^{-3} GeV^3)	$\sqrt{\langle T_{33}^2\rangle}$ (GeV^3)	$\Delta\,\mathrm{Re}d_\tau^\gamma$ (10^{-16} e cm)	c'_γ	$\sqrt{\langle \hat{Q}_{33}^2\rangle}$	$\Delta\,\mathrm{Im}d_\tau^\gamma$ (10^{-16} e cm)
$\nu\pi^-$	$\bar{\nu}\pi^+$	256	1.050	0.88	-0.318	0.803	0.54
$\nu\pi^0\pi^-$	$\bar{\nu}\pi^0\pi^+$	11.3	0.335	3.06	0.030	0.836	2.9
$\nu\rho^-$	$\bar{\nu}\rho^+$	41.7	0.794	1.97	-0.190	0.813	0.44
$\nu 2\pi^0\pi^-$	$\bar{\nu}2\pi^0\pi^+$	0	0.140	(*)	0.060	0.841	2.6
$\nu\pi^-$	$\bar{\nu}\pi^0\pi^+$	54	0.600	1.16	-0.135	0.838	0.65
$\nu\pi^-$	$\bar{\nu}\rho^+$	105	0.909	0.91	-0.253	0.814	0.34
$\nu\pi^-$	$\bar{\nu}2\pi^0\pi^+$	5.9	0.393	9.35	-0.127	0.846	0.93
$\nu\pi^0\pi^-$	$\bar{\nu}2\pi^0\pi^+$	1.2	0.217	17.1	0.048	0.839	1.7
$\nu\rho^-$	$\bar{\nu}2\pi^0\pi^+$	2.2	0.347	15.4	-0.065	0.838	1.3
$\nu\bar{\nu}l^-$	$\bar{\nu}\nu l^+$	20.9	0.442	1.40	0.102	0.829	0.54
$\nu\bar{\nu}l^-$	$\bar{\nu}\pi^+$	-73	0.636	0.74	-0.097	0.847	0.74
$\nu\bar{\nu}l^-$	$\bar{\nu}\pi^0\pi^+$	-15.4	0.364	1.39	0.067	0.832	0.73
$\nu\bar{\nu}l^-$	$\bar{\nu}\rho^+$	-30.0	0.581	1.14	-0.044	0.840	1.1
$\nu\bar{\nu}l^-$	$\bar{\nu}2\pi^0\pi^+$	-1.7	0.246	11.0	0.080	0.839	0.84

TABLE V. Observables $T^{ij},\hat{Q}^{ij}$ at $\sqrt{s}=10.58$ GeV (5×10^7 $\tau^+\tau^-$ events).

$\tau^- \to$	$\tau^+ \to$	c_γ (GeV^3)	$\sqrt{\langle T_{33}^2\rangle}$ (GeV^3)	$\Delta\,\mathrm{Re}d_\tau^\gamma$ (10^{-17} e cm)	c'_γ	$\sqrt{\langle \hat{Q}_{33}^2\rangle}$	$\Delta\,\mathrm{Im}d_\tau^\gamma$ (10^{-17} e cm)
$\nu\pi^-$	$\bar{\nu}\pi^+$	4.43	11.39	1.85	-0.603	0.587	0.70
$\nu\pi^0\pi^-$	$\bar{\nu}\pi^0\pi^+$	0.194	3.64	6.53	0	0.582	(*)
$\nu\rho^-$	$\bar{\nu}\rho^+$	0.72	10.13	4.88	-0.175	0.345	0.69
$\nu 2\pi^0\pi^-$	$\bar{\nu}2\pi^0\pi^+$	0	1.48	(*)	0.113	0.643	3.49
$\nu\pi^-$	$\bar{\nu}\pi^0\pi^+$	0.93	6.46	2.46	-0.298	0.608	0.72
$\nu\pi^-$	$\bar{\nu}\rho^+$	1.84	10.77	2.07	-0.388	0.521	0.48
$\nu\pi^-$	$\bar{\nu}2\pi^0\pi^+$	0.096	4.17	20	-0.244	0.648	1.25
$\nu\pi^0\pi^-$	$\bar{\nu}2\pi^0\pi^+$	0.019	2.31	39	0.052	0.618	3.86
$\nu\rho^-$	$\bar{\nu}2\pi^0\pi^+$	0.040	4.12	33	-0.031	0.543	5.73
$\nu\bar{\nu}l^-$	$\bar{\nu}\nu l^+$	0.359	4.68	2.9	0.197	0.636	0.72
$\nu\bar{\nu}l^-$	$\bar{\nu}\pi^+$	-1.26	6.69	1.50	-0.201	0.653	0.92
$\nu\bar{\nu}l^-$	$\bar{\nu}\pi^0\pi^+$	-0.265	3.87	2.88	0.101	0.614	1.20
$\nu\bar{\nu}l^-$	$\bar{\nu}\rho^+$	-0.517	6.81	2.59	0.011	0.542	9.38
$\nu\bar{\nu}l^-$	$\bar{\nu}2\pi^0\pi^+$	-0.026	2.59	84	0.157	0.642	1.07

TABLE VI. Observables $T^{ij},\hat{T}^{ij}$ at $\sqrt{s}=M_Z$ (10^7 Z events).

$\tau^- \to$	$\tau^+ \to$	c_Z (GeV^3)	$\sqrt{\langle T_{33}^2\rangle}$ (GeV^3)	$\Delta\,\mathrm{Re}d_\tau^Z$ (10^{-18} e cm)	c_Z	$\sqrt{\langle \hat{T}_{33}^2\rangle}$	$\Delta\,\mathrm{Re}d_\tau^Z$ (10^{-18} e cm)
$\nu\pi^-$	$\bar{\nu}\pi^+$	-1390	789	5.83	-1.82	1.030	5.81
$\nu\pi^0\pi^-$	$\bar{\nu}\pi^0\pi^+$	-147	268	9.08	0.04	1.069	145
$\nu\rho^-$	$\bar{\nu}\rho^+$	-694	791	5.68	-0.912	1.062	5.80
$\nu 2\pi^0\pi^-$	$\bar{\nu}2\pi^0\pi^+$	-11.7	116	87.0	0.351	1.075	26.9
$\nu\pi^-$	$\bar{\nu}\pi^0\pi^+$	-495	465	4.75	-0.99	1.071	5.46
$\nu\pi^-$	$\bar{\nu}\rho^+$	-1162	805	3.50	-1.54	1.053	3.46
$\nu\pi^-$	$\bar{\nu}2\pi^0\pi^+$	-242	315	8.73	-0.77	1.081	9.4
$\nu\pi^0\pi^-$	$\bar{\nu}2\pi^0\pi^+$	-56	175	14.6	0.20	1.070	25.4
$\nu\rho^-$	$\bar{\nu}2\pi^0\pi^+$	-110	323	13.7	-0.088	1.077	56.8
$\nu\bar{\nu}l^-$	$\bar{\nu}\nu l^+$	250	377	4.79	0.618	1.062	5.46
$\nu\bar{\nu}l^-$	$\bar{\nu}\pi^+$	-240	552	9.29	-0.62	1.078	7.05
$\nu\bar{\nu}l^-$	$\bar{\nu}\pi^0\pi^+$	6	318	140	0.34	1.063	8.71
$\nu\bar{\nu}l^-$	$\bar{\nu}\rho^+$	64	565	25.0	0.093	1.074	32.5
$\nu\bar{\nu}l^-$	$\bar{\nu}2\pi^0\pi^+$	55	211	14.3	0.483	1.072	8.28

TABLE VII. Observable $\hat{Q}^{ij}$ at $\sqrt{s}=M_Z$ (10^7 Z events).

$\tau^-\to$	$\tau^+\to$	c'_Z (10^{-3})	$\sqrt{\langle \hat{Q}_{33}^2 \rangle}$	$\Delta\,\mathrm{Im}d_\tau^Z$ (10^{-16} e cm)
$\nu\pi^-$	$\bar\nu\pi^+$	-11.4	0.114	1.0
$\nu\pi^0\pi^-$	$\bar\nu\pi^0\pi^+$	0	0.086	(*)
$\nu\rho^-$	$\bar\nu\rho^+$	-2.87	0.041	0.70
$\nu 2\pi^0\pi^-$	$\bar\nu 2\pi^0\pi^+$	2.4	0.141	5.2
$\nu\pi^-$	$\bar\nu\pi^0\pi^+$	-5.9	0.100	0.86
$\nu\pi^-$	$\bar\nu\rho^+$	-7.5	0.085	0.57
$\nu\pi^-$	$\bar\nu 2\pi^0\pi^+$	-4.6	0.127	1.9
$\nu\pi^0\pi^-$	$\bar\nu 2\pi^0\pi^+$	0	0.118	(*)
$\nu\rho^-$	$\bar\nu 2\pi^0\pi^+$	0	0.103	(*)
$\nu\bar\nu l^-$	$\bar\nu\nu l^+$	4.2	0.137	1.0
$\nu\bar\nu l^-$	$\bar\nu\pi^+$	-4.0	0.125	1.3
$\nu\bar\nu l^-$	$\bar\nu\pi^0\pi^+$	1.6	0.115	2.0
$\nu\bar\nu l^-$	$\bar\nu\rho^+$	0.7	0.103	4.1
$\nu\bar\nu l^-$	$\bar\nu 2\pi^0\pi^+$	3.5	0.141	1.6

By a simultaneous measurement of $\hat{T}$ and $\hat{A}_1$ one can disentangle the real parts of the electric and the weak dipole form factors at a fixed c.m. energy. For a given number of events N, the 1 s.d. (standard deviation) statistical sensitivities read

$$\Delta\mathrm{Re}d_\tau^\gamma=\frac{e}{\sqrt{s}}\frac{1}{\sqrt{N}}\frac{\sqrt{\langle(3r_Z\hat{T}_{33}-c_Z\hat{A}_1)^2\rangle}}{|c_\gamma r_Z-c_Z r_\gamma|}\,,$$
$$\Delta\mathrm{Re}d_\tau^Z=\frac{e}{\sqrt{s}}\frac{1}{\sqrt{N}}\frac{\sqrt{\langle(3r_\gamma\hat{T}_{33}-c_\gamma\hat{A}_1)^2\rangle}}{|c_Z r_\gamma-c_\gamma r_Z|}\,, \tag{6.3}$$

and analogously for the absorptive parts of the dipole form factors. The 1 s.d. statistical significance can be regarded as the best possible experimental upper limit which is achieved if the mean value vanishes. Of course, Eqs. (6.3) simplify significantly at the Z resonance and for energies far below, where either the weak or the electric dipole form factor dominates the expectation values (6.1). In the continuum above the Z pole, these form factors cannot be determined independently: their correlation is given by the matrix

$$\rho=\begin{bmatrix}1 & \rho_{\gamma Z}\\ \rho_{Z\gamma} & 1\end{bmatrix}, \tag{6.4}$$

with the off-diagonal elements

$$\rho_{\gamma Z}=\rho_{Z\gamma}=\frac{\langle(3r_Z\hat{T}_{33}-c_Z\hat{A}_1)(c_\gamma\hat{A}_1-3r_\gamma\hat{T}_{33})\rangle}{[\langle(3r_Z\hat{T}_{33}-c_Z\hat{A}_1)^2\rangle\langle(c_\gamma\hat{A}_1-3r_\gamma\hat{T}_{33})^2\rangle]^{1/2}}\,. \tag{6.5}$$

For the dimensionless observables $\hat{T},\hat{Q},\hat{A}_{1,2}$ and their dimensionful analogues $T,Q,A_{1,2}$ the coefficients c,c',r,r' and the CP-even expectation values occurring in (6.3) were calculated numerically. In general, the dimensionless quantities prove to be more sensitive, except for T which provides better results than $\hat{T}$ at energies up to the Z pole. Also CPT-odd observables which are functions only of energies, e.g., $\mathrm{sgn}(E_+-E_-)$, are less sensitive than the angular correlations $\hat{Q}$ and $\hat{A}_2$. In the Appendix we collect the numerical results for a number of c.m. energies from threshold to 500 GeV, together with the statistical significance with which the dipole form factors $d_\tau^{\gamma,Z}$ can be determined.

At a future τ-charm facility (see, e.g., Ref. [21]) one will investigate the reaction $e^+e^-\to\tau^+\tau^-$ away from hadronic resonances to avoid the background as much as possible. For our analysis we choose the c.m. energies 3.67 GeV, where the $\tau^+\tau^-$ cross section is largest between the threshold and the $\psi(3.69)$, and 4.25 GeV, which is between the $\psi(4.16)$ and the $\psi(4.41)$ resonances. At these c.m. energies one expects 2.4×10^7 and 3.5×10^7 $\tau^+\tau^-$ events per year, respectively [21]. A B factory, as proposed in Ref. [22], will provide about 5×10^7 $\tau^+\tau^-$ events at the $\Upsilon(4S)$ resonance. [Note that for an asymmetrically designed B factory the observables (5.3)–(5.5) must be transformed to the e^+e^- c.m. system.] At the Z peak at LEP 1 a production of 10^7 Z bosons is assumed. For $\sqrt{s}=180$ and 500 GeV the $\tau^+\tau^-$ event rates/year expected at LEP 2 and at a future linear collider [23] were taken.

TABLE VIII. Observables $\hat{T}^{ij}$, $\hat{A}_1$ at $\sqrt{s}=180$ GeV.

$\tau^-\to$	$\tau^+\to$	c_γ	c_Z	$\sqrt{\langle\hat{T}_{33}^2\rangle}$	r_γ	r_Z	$\sqrt{\langle\hat{A}_1^2\rangle}$	$\langle\hat{T}_{33}\hat{A}_1\rangle$
$\nu\pi^-$	$\bar\nu\pi^+$	0.187	-0.311	1.094	0.0291	-0.297	0.567	0.309
$\nu\pi^0\pi^-$	$\bar\nu\pi^0\pi^+$	0	0.01	1.069	0.0005	0.005	0.549	0.310
$\nu\rho^-$	$\bar\nu\rho^+$	0.013	-0.165	1.078	-0.0009	-0.147	0.553	0.311
$\nu 2\pi^0\pi^-$	$\bar\nu 2\pi^0\pi^+$	0.014	0.066	1.077	0.0044	0.056	0.550	0.317
$\nu\pi^-$	$\bar\nu\pi^0\pi^+$	-0.026	-0.171	1.069	-0.0090	-0.159	0.547	0.309
$\nu\pi^-$	$\bar\nu\rho^+$	0.047	-0.263	1.082	0.0032	-0.249	0.557	0.310
$\nu\pi^-$	$\bar\nu 2\pi^0\pi^+$	-0.056	-0.144	1.068	-0.0141	-0.123	0.545	0.312
$\nu\pi^0\pi^-$	$\bar\nu 2\pi^0\pi^+$	0.006	0.036	1.073	0.0014	0.032	0.548	0.312
$\nu\rho^-$	$\bar\nu 2\pi^0\pi^+$	-0.016	-0.018	1.067	-0.0033	-0.014	0.548	0.311
$\nu\bar\nu l^-$	$\bar\nu\nu l^+$	0.036	0.118	1.074	0.0096	0.101	0.548	0.309
$\nu\bar\nu l^-$	$\bar\nu\pi^+$	-0.087	-0.117	1.062	-0.0197	-0.100	0.541	0.310
$\nu\bar\nu l^-$	$\bar\nu\pi^0\pi^+$	0.006	0.064	1.070	0.0030	0.056	0.549	0.311
$\nu\bar\nu l^-$	$\bar\nu\rho^+$	-0.024	0.016	1.068	-0.0042	0.016	0.544	0.310
$\nu\bar\nu l^-$	$\bar\nu 2\pi^0\pi^+$	0.023	0.095	1.077	0.0065	0.080	0.550	0.312

TABLE IX. Observables $\hat{Q}^{ij}$, $\hat{A}_2$ at $\sqrt{s}=180$ GeV.

$\tau^- \to$	$\tau^+ \to$	c'_γ (10^{-3})	c'_Z (10^{-3})	$\sqrt{\langle \hat{Q}_{33}^2 \rangle}$ (10^{-3})	r'_γ (10^{-3})	r'_Z (10^{-3})	$\sqrt{\langle \hat{A}_2^2 \rangle}$ (10^{-3})	$\langle \hat{Q}_{33} \hat{A}_2 \rangle$ (10^{-3})
$\nu\pi^-$	$\bar{\nu}\pi^+$	−36.7	−3.2	55	−7.20	−0.90	27.6	0.84
$\nu\pi^0\pi^-$	$\bar{\nu}\pi^0\pi^+$	−1.3	0	43	−0.30	−0.01	22.1	0.511
$\nu\rho^-$	$\bar{\nu}\rho^+$	−9.0	−0.85	20	−1.79	−0.23	10.2	0.112
$\nu 2\pi^0\pi^-$	$\bar{\nu}2\pi^0\pi^+$	7.4	0.81	79	1.44	0.20	42	1.7
$\nu\pi^-$	$\bar{\nu}\pi^0\pi^+$	−19.1	−1.9	50	−3.75	−0.47	25.6	0.68
$\nu\pi^-$	$\bar{\nu}\rho^+$	−22.9	−2.1	42	−4.23	−0.58	21.2	0.485
$\nu\pi^-$	$\bar{\nu}2\pi^0\pi^+$	−14.9	−1.4	68	−2.92	−0.35	35.5	1.1
$\nu\pi^0\pi^-$	$\bar{\nu}2\pi^0\pi^+$	3.0	0.1	62	0.63	0.01	33.1	1.0
$\nu\rho^-$	$\bar{\nu}2\pi^0\pi^+$	−0.9	−0.06	57	−0.15	−0.05	33	0.8
$\nu\bar{\nu}l^-$	$\bar{\nu}\nu l^+$	12.7	1.2	78	2.6	0.33	40	1.5
$\nu\bar{\nu}l^-$	$\bar{\nu}\pi^+$	−12.1	−1.2	68	−2.38	−0.28	36	1.27
$\nu\bar{\nu}l^-$	$\bar{\nu}\pi^0\pi^+$	5.8	0.5	63	1.14	0.17	31.7	0.97
$\nu\bar{\nu}l^-$	$\bar{\nu}\rho^+$	1.9	0.16	57	0.39	0.04	29	0.88
$\nu\bar{\nu}l^-$	$\bar{\nu}2\pi^0\pi^+$	10.1	0.91	77	1.93	0.25	39	1.6

TABLE X. Observables $\hat{T}^{ij}$, $\hat{A}_1$ at $\sqrt{s}=500$ GeV.

$\tau^- \to$	$\tau^+ \to$	c_γ	c_Z	$\sqrt{\langle \hat{T}_{33}^2 \rangle}$	r_γ	r_Z	$\sqrt{\langle \hat{A}_1^2 \rangle}$	$\langle \hat{T}_{33} \hat{A}_1 \rangle$
$\nu\pi^-$	$\bar{\nu}\pi^+$	0.208	−0.206	1.099	0.0233	−0.248	0.570	0.261
$\nu\pi^0\pi^-$	$\bar{\nu}\pi^0\pi^+$	0	0.005	1.070	0.0002	0.004	0.548	0.256
$\nu\rho^-$	$\bar{\nu}\rho^+$	0.019	−0.106	1.079	−0.0010	−0.123	0.554	0.258
$\nu 2\pi^0\pi^-$	$\bar{\nu}2\pi^0\pi^+$	0.014	0.041	1.080	0.0035	0.047	0.553	0.260
$\nu\pi^-$	$\bar{\nu}\pi^0\pi^+$	−0.017	−0.118	1.071	−0.0065	−0.132	0.548	0.257
$\nu\pi^-$	$\bar{\nu}\rho^+$	0.057	−0.181	1.084	0.0024	−0.208	0.558	0.257
$\nu\pi^-$	$\bar{\nu}2\pi^0\pi^+$	−0.059	−0.098	1.068	−0.0112	−0.102	0.545	0.260
$\nu\pi^0\pi^-$	$\bar{\nu}2\pi^0\pi^+$	0.003	0.025	1.075	0.0015	0.029	0.551	0.261
$\nu\rho^-$	$\bar{\nu}2\pi^0\pi^+$	−0.017	−0.013	1.070	−0.0029	−0.012	0.547	0.259
$\nu\bar{\nu}l^-$	$\bar{\nu}\nu l^+$	0.037	0.072	1.073	0.0080	0.084	0.549	0.257
$\nu\bar{\nu}l^-$	$\bar{\nu}\pi^+$	−0.090	−0.078	1.061	−0.0161	−0.083	0.541	0.258
$\nu\bar{\nu}l^-$	$\bar{\nu}\pi^0\pi^+$	0.006	0.041	1.067	0.0022	0.047	0.547	0.258
$\nu\bar{\nu}l^-$	$\bar{\nu}\rho^+$	−0.026	0.010	1.064	−0.0034	0.012	0.544	0.258
$\nu\bar{\nu}l^-$	$\bar{\nu}2\pi^0\pi^+$	0.023	0.056	1.074	0.0053	0.066	0.553	0.261

TABLE XI. Observables $\hat{Q}^{ij}$, $\hat{A}_2$ at $\sqrt{s}=500$ GeV.

$\tau^- \to$	$\tau^+ \to$	c'_γ (10^{-3})	c'_Z (10^{-3})	$\sqrt{\langle \hat{Q}_{33}^2 \rangle}$ (10^{-3})	r'_γ (10^{-3})	r'_Z (10^{-3})	$\sqrt{\langle \hat{A}_2^2 \rangle}$ (10^{-3})	$\langle \hat{Q}_{33} \hat{A}_2 \rangle$ (10^{-3})
$\nu\pi^-$	$\bar{\nu}\pi^+$	−14.3	−0.93	19.5	−2.18	−0.21	9.78	0.092
$\nu\pi^0\pi^-$	$\bar{\nu}\pi^0\pi^+$	−0.51	−0.01	15.5	−0.08	0	8.00	0.055
$\nu\rho^-$	$\bar{\nu}\rho^+$	−3.51	−0.24	7.21	−0.54	−0.053	3.66	0.12
$\nu 2\pi^0\pi^-$	$\bar{\nu}2\pi^0\pi^+$	2.82	0.21	31.0	0.43	0.042	15.3	0.18
$\nu\pi^-$	$\bar{\nu}\pi^0\pi^+$	−7.38	−0.57	17.8	−1.12	−0.12	9.15	0.074
$\nu\pi^-$	$\bar{\nu}\rho^+$	−8.96	−0.60	14.8	−1.36	−0.132	7.53	0.052
$\nu\pi^-$	$\bar{\nu}2\pi^0\pi^+$	−5.8	−0.41	26.1	−0.87	−0.08	12.1	0.14
$\nu\pi^0\pi^-$	$\bar{\nu}2\pi^0\pi^+$	1.1	0.05	22.3	0.19	0.01	11.1	0.13
$\nu\rho^-$	$\bar{\nu}2\pi^0\pi^+$	−0.35	−0.02	21	−0.04	0	10.6	0.10
$\nu\bar{\nu}l^-$	$\bar{\nu}\nu l^+$	5.1	0.35	29	0.74	0.069	17	0.17
$\nu\bar{\nu}l^-$	$\bar{\nu}\pi^+$	−4.7	−0.31	25.6	−0.71	−0.07	11.9	0.13
$\nu\bar{\nu}l^-$	$\bar{\nu}\pi^0\pi^+$	2.3	0.13	22.0	0.33	0.03	12.4	0.12
$\nu\bar{\nu}l^-$	$\bar{\nu}\rho^+$	0.73	0.04	22	0.12	0.016	10.2	0.09
$\nu\bar{\nu}l^-$	$\bar{\nu}2\pi^0\pi^+$	3.9	0.27	31	0.60	0.06	14.9	0.14

TABLE XII. 1 s.d. accuracies obtainable in measuring the dipole form factors assuming 5000 $\tau^+\tau^-$ events at c.m. energies 180 and 500 GeV, respectively.

		$\sqrt{s}=180$ GeV				$\sqrt{s}=500$ GeV			
$\tau^-\to$	$\tau^+\to$	$\Delta\,\mathrm{Re}d_\tau^\gamma$ (10^{-17} e cm)	$\Delta\,\mathrm{Re}d_\tau^Z$ (10^{-17} e cm)	$\Delta\,\mathrm{Im}d_\tau^\gamma$ (10^{-17} e cm)	$\Delta\,\mathrm{Im}d_\tau^Z$ (10^{-17} e cm)	$\Delta\,\mathrm{Re}d_\tau^\gamma$ (10^{-17} e cm)	$\Delta\,\mathrm{Re}d_\tau^Z$ (10^{-17} e cm)	$\Delta\,\mathrm{Im}d_\tau^\gamma$ (10^{-17} e cm)	$\Delta\,\mathrm{Im}d_\tau^\gamma$ (10^{-17} e cm)
$\nu\pi^-$	$\bar{\nu}\pi^+$	27.3	3.07	17.2	145	8.44	1.21	5.64	68.1
$\nu\pi^0\pi^-$	$\bar{\nu}\pi^0\pi^+$	1020	144	110	2190	1030	66.4	20.2	321
$\nu\rho^-$	$\bar{\nu}\rho^+$	144	2.99	14.9	124	37.4	1.23	4.66	55.2
$\nu 2\pi^0\pi^-$	$\bar{\nu}2\pi^0\pi^+$	409	29.8	162	1240	121	8.89	65.7	759
$\nu\pi^-$	$\bar{\nu}\pi^0\pi^+$	123	6.41	22.7	191	67.9	3.20	6.05	65.3
$\nu\pi^-$	$\bar{\nu}\rho^+$	47.4	1.45	11.6	97.1	13.9	0.644	3.83	46.1
$\nu\pi^-$	$\bar{\nu}2\pi^0\pi^+$	67.6	7.32	52.6	473	20.9	2.43	19.9	243
$\nu\pi^0\pi^-$	$\bar{\nu}2\pi^0\pi^+$	460	18.7	65.5	2040	405	19.7	167	3260
$\nu\rho^-$	$\bar{\nu}2\pi^0\pi^+$	156	36.9	139	566	49.6	13.8	62.4	800
$\nu\bar{\nu}l^-$	$\bar{\nu}\nu l^+$	51.0	4.58	27.6	234	16.2	1.56	12.1	154
$\nu\bar{\nu}l^-$	$\bar{\nu}\pi^+$	25.1	4.72	40.6	362	7.96	1.68	10.1	119
$\nu\bar{\nu}l^-$	$\bar{\nu}\pi^0\pi^+$	504	26.3	24.2	174	112	4.89	11.8	170
$\nu\bar{\nu}l^-$	$\bar{\nu}\rho^+$	59.0	16.4	185	1930	18.2	6.50	19.8	173
$\nu\bar{\nu}l^-$	$\bar{\nu}2\pi^0\pi^+$	98.1	7.54	31.6	260	30.8	2.48	15.0	176

The resulting sensitivity estimates for the dipole form factors given in Table II were obtained from an exclusive analysis of the τ decay channels listed in Table I. From an experimental point of view it might be useful to consider inclusive classes of τ decay channels (e.g., one prong semileptonic decays) instead of the exclusive measurements as proposed above, for instance, in order to reduce the systematic errors. In that case the inclusive mean value of an observable $\mathcal{O}$ is given by

$$[\langle\mathcal{O}\rangle]_{\mathrm{incl}}=\frac{\sum[\langle\mathcal{O}\rangle]_{AB}B_AB_B}{\sum B_AB_B}\ , \tag{6.6}$$

where the sum runs over the τ decay modes in the considered event class and B_A, B_B are the corresponding branching ratios. Some care has to be taken of the rare channels whose decay distributions have not been calculated. One may assume that on average their spin-analyzer quality is poor, i.e., that the parameter α defined in (4.4) vanishes for these decay modes. Under these assumptions one can calculate the inclusive expectation value using (5.6) and (5.7). It must, however, be emphasized that this procedure can lower the statistical significance considerably.

To summarize, we presented a set of *CP*-odd correlations which can be used to measure the electric and the weak dipole form factor of the τ lepton in e^+e^- experiments from the threshold up to several hundreds of GeV. The accuracies reachable at the Z pole with LEP 1 and in the continuum at future e^+e^- facilities are of the same order of magnitude as the present upper limit [24] for the electric dipole moment of the muon.

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APPENDIX

In Tables III–XII we collect the numerically calculated expectation values, the correlations, and the coefficients c, c', r, r' as defined in (6.1) and (6.2) for the observables given in (5.3)–(5.5). The 1 s.d. ideal statistical errors, based on the number of events given in Table II, are determined using (6.3). Note that the nondiagonal entries, e.g., $\tau^+\tau^-\to\bar{\nu}\pi^+\nu\rho^-$, include already the charge-conjugated combination. In very rare cases (marked by an asterisk) the mean value of an observable becomes zero within the numerical uncertainty and the sensitivity is omitted.

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