Lowest-order graviton interactions with a charged fermion and a photon

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(Received 9 September 1992)

Graviton interactions with a charged fermion and a photon are studied in the context of linearized gravity coupled to QED. It is found that, apart from an overall kinematical factor, each graviton interaction process has essentially the same transition-amplitude structure as the process involving a photon in place of the graviton. While the angular dependence of cross sections in both cases is different due to the overall factor, the polarization effects of the graviton are identical to those of the corresponding photon, except that the graviton Stokes parameters enter instead of the photon Stokes parameters. The consequences of the electromagnetic and gravitational low-energy theorems and some possible extensions of our results are discussed.

PACS number(s): 12.25.+e, 04.60.+n, 13.88.+e

I. INTRODUCTION

Newton's law of gravity is long ranged, and this suggests the existence of a gravity force mediated by a massless particle called a graviton. In view of the fact that even-spin particle exchanges are fundamentally attractive, observed universal gravitational attraction selects the graviton spin to be an even integer. Until now, all observations have supported a spin-2 graviton picture [1–3].

At present, there does not exist a complete theory of quantum gravity. The main problem is that Einstein gravity is nonrenormalizable [4–6] because the gravitational constant G has dimensions of inverse mass square. In this respect the theory of gravitation is more like other nonrenormalizable effective theories such as the Fermi theory of weak interaction. However, Weinberg [7] showed that it is quite impossible to construct a Lorentz-invariant quantum theory of particles of mass zero and helicity ± 2 without introducing some sort of gauge invariance into the theory. It is well known that the classical theory of gravitational radiation in the linearized version of general relativity has gauge invariance, which is related to the general covariance of the full theory. To gain further insight concerning the massless spin-2 graviton, we consider here physical processes in the context of the linearized gravity coupled to QED.

Several graviton interaction processes have been studied previously. The gravitational Compton scattering ($ge \to ge$) and the graviton photoproduction ($\gamma e \to ge$) were considered in the lowest-order Born approximation in Refs. [8, 9]. The cross sections of two annihilation processes $e\bar{e} \to g\gamma$ and $e\bar{e} \to gg$ were calculated some time ago in Ref. [10]. Also, recently, first-order cross sections for the processes such as $ge \to \gamma e$, bremsstrahlung, and $e\bar{e}$-pair production by a graviton in the Coulomb field were calculated by Saif [11]. But all these calculations do not agree with the results of Voronov [12]. According to the above statement of Weinberg, it is so crucial to maintain general covariance in the theory that one should introduce gravitational gauge invariance on determining the interaction Lagrangian. Following that point of view, we utilize here the same interaction Lagrangian as in Ref. [12].

In the present work it is found that the gravitational gauge invariance forces a graviton interaction with a charged fermion and a photon to have its transition amplitude factorized into an energy-momentum-dependent part and a spin- or polarization-dependent part. On the other hand, similar factorizations [13–16] exist in the transition amplitudes of photon interactions with charged particles because of the $U(1)_{EM}$ gauge invariance of the theory. With two similar factorization properties in mind, we compare a graviton process with the process involving a photon instead of the graviton and find that, apart from a kinematical factor, the two processes have the same transition-amplitude structure and exhibit identical polarization effects. The similarity between a graviton interaction process and the corresponding process where the graviton is replaced by a photon has been considered by Good [17], and Łukaszuk and Szymanowski [18].

Generally, the theory of gravitation is complicated by the nonlinearity of Einstein's equations. But the complicated generally covariant and $U(1)_{EM}$ gauge-invariant Lagrangian $\mathcal{L}$ [12, 19] is greatly simplified with the following constraints.

(i) Only terms up to the first order in $f(=\sqrt{8\pi G})$ are considered. Here G is the gravitational constant.

(ii) Gravitation self-interaction is not included; that is, a graviton takes part in any reaction just as an external particle.

(iii) The gauge with which the graviton wave function $h^{\mu\nu}$ satisfies

$$\partial_\mu h^{\mu\nu} = 0 = h^\mu{}_\mu, \quad h^{\mu\nu} = h^{\nu\mu}, \tag{1.1}$$

is taken. This gauge is called the harmonic or de Donder gauge.

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Here the graviton wave function $h_{\mu\nu}$ represents a deviation of the metric tensor $g_{\mu\nu}$ from the flat tensor $\eta_{\mu\nu}$:

$$g_{\mu\nu} = \eta_{\mu\nu} + 2fh_{\mu\nu}, \tag{1.2}$$

where the flat tensor $\eta_{\mu\nu}$ is the Minkowski tensor with signature $(+1, -1, -1, -1)$. With the above constraints, the linearized Lagrangian $\mathcal{L}$ is given by

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_F + \mathcal{L}_A + \mathcal{L}_{FA} + \mathcal{L}_{GF} + \mathcal{L}_{GA} + \mathcal{L}_{GFA}, \tag{1.3}$$

with

$$\mathcal{L}_G = -\frac{1}{4}h_{\mu\nu}\Box h^{\mu\nu}, \tag{1.4}$$

$$\mathcal{L}_F = \frac{i}{2}(\bar\psi\gamma^\mu\partial_\mu\psi - \partial_\mu\bar\psi\gamma^\mu\psi) - m\bar\psi\psi, \tag{1.5}$$

$$\mathcal{L}_A = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \tag{1.6}$$

$$\mathcal{L}_{FA} = -e\bar\psi\gamma^\mu\psi A_\mu, \tag{1.7}$$

$$\mathcal{L}_{GF} = \frac{i}{4}fh_{\mu\nu}(\bar\psi\gamma^\mu\partial^\nu\psi - \partial^\mu\bar\psi\gamma^\nu\psi), \tag{1.8}$$

$$\mathcal{L}_{GA} = \tfrac{1}{2}fh_{\mu\nu}(F^\mu{}_\alpha F^{\nu\alpha} - \tfrac{1}{4}g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}), \tag{1.9}$$

$$\mathcal{L}_{GFA} = -\frac{1}{2}ef\bar\psi\gamma^\mu\psi A^\nu h_{\mu\nu}, \tag{1.10}$$

where γ^μ are Dirac matrices, ψ is a spin-$\frac{1}{2}$ fermion field, A^μ is the photon field, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Given the explicit form of the interaction Lagrangian, one can construct graviton wave functions, propagators, and Feynman rules.

The paper is organized as follows. We prove in Sec. II that, when an external graviton interacts with a charged fermion and a photon, the process has essentially the same transition-amplitude structure as the process involving a photon in place of the graviton. Only the overall kinematical factor differs, and this is explicitly determined. Even the polarization effects of the graviton with helicity ± 2 are identical to those of the corresponding photon. In Sec. III, we explain this aspect explicitly with the help of a density matrix formalism. In Sec. IV, we discuss possible generalizations of our results by employing electromagnetic and gravitational low-energy theorems. Finally, in Sec V, some discussions and concluding remarks are presented.

II. FACTORIZATION

The Feynman diagrams for processes under investigation can be generically represented as in Fig. 1. There are a contact interaction term and a photon-graviton coupling term in which the graviton can couple with a photon without mass but with energy. Even without Fig. 1(d), the transition amplitude is $U(1)_{EM}$ gauge invariant, but this diagram should be included in order to ensure gravitational gauge invariance. However, as we will show later, one does not have to do any tedious calculation to get cross sections. One can use only the well-known QED results with a photon instead of the graviton. These QED processes have no contact terms for fermion-photon interactions and no photon self-interacting terms due to the Abelian property of the gauge group $U(1)_{EM}$.

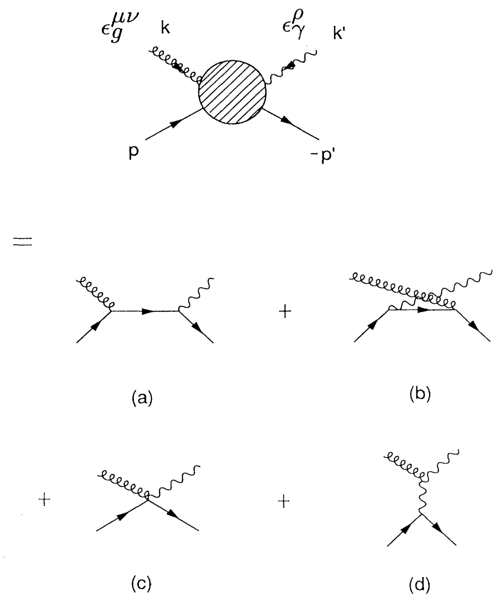

FIG. 1. Diagrams for graviton interactions with a charged fermion and a photon.

Changing particle momentum directions and taking complex conjugates of fermion spinors, the photon wave vector, and the graviton wave tensor, one can investigate essentially four kinds of two-to-two reactions: $ge \to \gamma e$, $g\gamma \to e\bar e$, $\gamma e \to ge$, and $e\bar e \to g\gamma$.

In the standard model, any four-particle lowest-order transition amplitude, where one particle is an on-shell photon, has been well known to be always factorizable [13–16] into one factor which contains the dependence on the charge or other internal-symmetry indices, and another which contains the dependence on the spin or polarization indices.

Let us describe briefly how to get such a factorization [13]. In general, we can decompose the transition amplitude into three components:

$$\mathcal{M} = \sum_{i=1}^{Z} \frac{A_i B_i}{C_i}, \tag{2.1}$$

where charge factors (A_i), polarization-dependent terms (B_i), and propagator denominators (C_i) satisfy

$$\sum_{i=1}^{Z} A_i = \sum_{i=1}^{Z} B_i = \sum_{i=1}^{Z} C_i = 0, \tag{2.2}$$

from charge conservation, energy-momentum conservation

$$\sum_{i=1}^{Z} p_i = -k, \tag{2.3}$$

and $k^2 = 0$ and $k \cdot \epsilon_\gamma = 0$, the massless and transverse properties of the on-shell photon. Then the transition amplitude $\mathcal{M}$ for $Z = 3$ can be written in a factorized form:

$$\mathcal{M} = \frac{A_1B_1}{C_1} + \frac{A_2B_2}{C_2} + \frac{A_3B_3}{C_3},$$
$$= -\frac{C_1C_2}{C_3}\left(\frac{A_1}{C_1} - \frac{A_2}{C_2}\right)\left(\frac{B_1}{C_1} - \frac{B_2}{C_2}\right), \tag{2.4}$$

or in equivalent forms where the indices (1,2,3) are permuted. In any case, Eq. (2.4) definitely exhibits the factorization of the transition amplitude into the charge-dependent part and the polarization-dependent part.

However, one can also get a factorized transition amplitude $\mathcal{M}_g$ for graviton interactions with a charged fermion and a photon. In this case, we do not have any charge conservation law. But there is one even stronger conservation law that a graviton and any elementary particle interact with the universal coupling constant, $f(=\sqrt{8\pi G})$. Of course, the structure of interaction vertices is different depending on the particle spin.

To derive the factorization, let us at first write the explicit form of the transition amplitude $\mathcal{M}_g$ for the process in Fig. 1. Here we note that the graviton spin-2 wave tensor $\epsilon_g^{\mu\nu}(2\lambda)$ with helicity 2λ ($\lambda = \pm 1$) can be taken to be a multiplication [20] of two massless spin-1 wave vectors $\epsilon_g^{\mu}(\lambda)$ and $\epsilon_g^{\nu}(\lambda)$ as

$$\epsilon_g^{\mu\nu}(2\lambda) = \epsilon_g^{\mu}(\lambda)\epsilon_g^{\nu}(\lambda), \tag{2.5}$$

with the properties

$$k \cdot \epsilon_g(\lambda) = 0, \quad \epsilon_g(\lambda) \cdot \epsilon_g(\lambda') = -\delta_{\lambda,-\lambda'}, \tag{2.6}$$

and thus the wave tensor $\epsilon_g^{\mu\nu}(2\lambda)$ satisfies

$$k_\mu \epsilon_g^{\mu\nu}(2\lambda) = \epsilon_g^{\mu\nu}(2\lambda)k_\nu = 0, \quad \epsilon_{g\,\mu}^{\mu}(2\lambda) = 0, \tag{2.7}$$

where k is the graviton four-momentum. It is assumed that all particles are incoming to the bulb vertex in Fig. 1, $p(p')$ are the four-momenta of two fermions, and k' is the photon four-momentum. The transition amplitude $\mathcal{M}_g$ is composed of four parts:

$$\mathcal{M}_g = \mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c + \mathcal{M}_d, \tag{2.8}$$

with their explicit forms

$$\mathcal{M}_a = -\frac{i}{2}ef\frac{1}{q^2 - m^2}\bar{w}'(-p', s')\left[\not\epsilon_\gamma(\not q + m)\not\epsilon_g\, \epsilon_g\cdot(p+q)\right]w(p,s), \tag{2.9}$$

$$\mathcal{M}_b = -\frac{i}{2}ef\frac{1}{q'^2 - m^2}\bar{w}'(-p', s')\left[(q'-p')\cdot\epsilon_g\ \not\epsilon_g(\not q' + m)\ \not\epsilon_\gamma\right]w(p,s), \tag{2.10}$$

$$\mathcal{M}_c = ief\bar{w}'(-p', s')\left[\not\epsilon_g \epsilon_g\cdot\epsilon_\gamma\right]w(p,s), \tag{2.11}$$

$$\mathcal{M}_d = ief\frac{1}{l^2}\bar{w}'(-p', s')\left\{-l^2\ \not\epsilon_g\epsilon_g\cdot\epsilon_\gamma + 2k'\cdot\epsilon_g\left[\not\epsilon_g\, k\cdot\epsilon_\gamma - \epsilon_g\cdot k'\ \not\epsilon_\gamma + \epsilon_g\cdot\epsilon_\gamma\ \not k'\right]\right\}w(p,s), \tag{2.12}$$

where the momentum transfers q, q', and l are given in terms of particle momenta p, p', k, and k' as

$$q = p + k = -p' - k', \quad q' = p + k' = -p' - k, \quad l = k + k' = -p - p', \tag{2.13}$$

and the spinors w' and w are replaced by the familiar u, v spinors as the particle is a fermion or an antifermion, respectively. Note that the contact term $\mathcal{M}_c$ and the graviton-photon coupling term $\mathcal{M}_d$ have the same momentum-independent component $[\not\epsilon_g\epsilon_g\cdot\epsilon_\gamma]$ between two spinors w' and w, and their sum vanishes because gravitational gauge invariance is imposed in the theory. Combining $\mathcal{M}_c$ and $\mathcal{M}_d$, we get the following expression:

$$\mathcal{M}'_c = ief\frac{2}{l^2}\bar{w}'(p', s')k'\cdot\epsilon_g\left[\not\epsilon_g\, k\cdot\epsilon_\gamma - \epsilon_g\cdot k'\ \not\epsilon_\gamma + \epsilon_g\cdot\epsilon_\gamma\ \not k'\right]w(p,s). \tag{2.14}$$

It can be directly verified that the transition amplitude $\mathcal{M}_g$ is invariant under the gauge transformations

$$\begin{aligned}&\epsilon_g^\mu \to \epsilon_g^\mu + \Lambda k^\mu \ \text{ for the graviton,}\\ &\epsilon_\gamma^\mu \to \epsilon_\gamma^\mu + \Lambda' k'^\mu \ \text{ for the photon,}\end{aligned} \tag{2.15}$$

where Λ and Λ' are any scalar functions.

Now let us determine what A_i, B_i, and C_i ($i = 1,2,3$) are. The A_i terms which denote the strength of graviton interaction with matter lines are

$$A_1 = f(p+q)\cdot\epsilon_g, \quad A_2 = f(p'-q')\cdot\epsilon_g, \quad A_3 = 2fk'\cdot\epsilon_g. \tag{2.16}$$

It is noted that all couplings have the same gravitational coupling f and their strength is proportional to the sum of four-momenta of two matter lines. Naturally, these terms will be replaced by charges in a (non-)Abelian gauge theory. The B_i terms are polarization-dependent parts, which are given as

$$\begin{aligned}B_1 &= -\frac{i}{2}e\bar{w}'(-p', s')\left[\not\epsilon_\gamma(\not q + m)\not\epsilon_g\right]w(p,s),\\ B_2 &= \frac{i}{2}e\bar{w}'(-p', s')\left[\not\epsilon_g(\not q' + m)\not\epsilon_\gamma\right]w(p,s),\\ B_3 &= ie\bar{w}'(-p', s')\left[k\cdot\epsilon_\gamma\ \not\epsilon_g - k'\cdot\epsilon_g\ \not\epsilon_\gamma + \epsilon_g\cdot\epsilon_\gamma\ \not k'\right]w(p,s).\end{aligned} \tag{2.17}$$

The C_i terms are propagator denominators:

$$C_1 = q^2 - m^2, \quad C_2 = q'^2 - m^2, \quad C_3 = l^2. \tag{2.18}$$

One can easily check that the conditions (2.2) are satisfied owing to the energy-momentum conservation

$$p + p' + k + k' = 0, \tag{2.19}$$

and two Dirac equations

$$\begin{aligned} \not{p}' w'(-p', s') &= -m\, w'(-p', s'), \\ \not{p} w(p, s) &= m\, w(p, s). \end{aligned} \tag{2.20}$$

Then we obtain the factorized form of the transition amplitude

$$\mathcal{M}_g = ief \frac{k\cdot p\, k\cdot p'}{k\cdot k'} \left[\frac{p^\mu}{k\cdot p} - \frac{p'^\mu}{k\cdot p'}\right] \epsilon_{g\mu} \bar{w}'(-p', s') \left[\not{\epsilon}_\gamma \frac{1}{\not{q} - m} \not{\epsilon}_g + \not{\epsilon}_g \frac{1}{\not{q}' - m} \not{\epsilon}_\gamma\right] w(p, s). \tag{2.21}$$

This is the final factorized expression. Two factors in Eq. (2.21),

$$\left[\frac{p^\mu}{k\cdot p} - \frac{p'^\mu}{k\cdot p'}\right] \tag{2.22}$$

and

$$\bar{w}'(-p', s') \left[\not{\epsilon}_\gamma \frac{1}{\not{q} - m} \not{\epsilon}_g + \not{\epsilon}_g \frac{1}{\not{q}' - m} \not{\epsilon}_\gamma\right] w(p, s), \tag{2.23}$$

appear quite often in QED interaction processes. The latter is nothing but the transition amplitude for three two-to-two QED reactions: the Compton scattering [21] $\gamma e \to \gamma e$, the pair creation $\gamma\gamma \to e\bar{e}$, and the pair annihilation $e\bar{e} \to \gamma\gamma$. On the other hand, with two vectors p and p', we can always construct two vectors [22, 23]

$$n_1^\mu = N\left[\frac{p^\mu}{k\cdot p} - \frac{p'^\mu}{k\cdot p'}\right], \quad n_2^\mu = N\, \frac{\varepsilon^{\mu\nu\alpha\beta} k_\nu p_\alpha p'_\beta}{k\cdot p\, k\cdot p'}, \tag{2.24}$$

which satisfy the conditions

$$k\cdot n_i = 0, \quad n_i \cdot n_j = -\delta_{ij}, \tag{2.25}$$

with the normalization factor N defined as

$$N = \left[\frac{2k\cdot k'}{k\cdot p\, k\cdot p'} - m^2 \left(\frac{1}{k\cdot p} + \frac{1}{k\cdot p'}\right)^2\right]^{-1/2}. \tag{2.26}$$

They enable us to determine the spin-1 wave vector $\epsilon_g(\lambda)$:

$$\epsilon_g^\mu(\lambda) = \frac{1}{\sqrt{2}}(n_1 + i\lambda n_2), \tag{2.27}$$

where $\lambda = \pm 1$ is for the (right-) left-handed polarization, respectively. What should be noted is that the scalar product $n_1 \cdot \epsilon_g(\lambda)$ of this wave vector and the four-vector n_1 is in this case independent of the helicity value λ. Certainly one can take another set of (n_1', n_2') as a basis satisfying Eq. (2.25), but they are different from the set (n_1, n_2) simply by a phase. Therefore, there will be no change in the cross section.

Finally we come to the result

$$\mathcal{M}_g = \frac{f}{e}\sqrt{F}\,(n_1\cdot\epsilon_g)\,\mathcal{M}_\gamma, \tag{2.28}$$

where the relation (2.2) is used and the overall kinematical factor F is

$$F = 2\,\frac{k\cdot p\, k\cdot p'}{k\cdot k'} - m^2, \tag{2.29}$$

and the transition amplitude $\mathcal{M}_\gamma$ is

$$ie^2 \bar{w}'(-p', s') \left[\not{\epsilon}_\gamma \frac{1}{\not{q} - m} \not{\epsilon}_g + \not{\epsilon}_g \frac{1}{\not{q}' - m} \not{\epsilon}_\gamma\right] w(p, s). \tag{2.30}$$

In the center-of-mass frame, this overall kinematical factor takes the forms

$$F = \begin{cases} \frac{1}{4}(s - 4m^2)\sin^2\theta, & \theta = \angle(g, e) \quad \text{for } g\gamma \to e\bar{e} \text{ and } e\bar{e} \to g\gamma, \\ s\cot^2\frac{\theta}{2}, & \theta = \angle(g, \gamma) \quad \text{for } ge \to \gamma e \text{ and } \gamma e \to ge. \end{cases} \tag{2.31}$$

The result leads to a number of comments.

(i) The ratio f/e of the gravitational coupling constant f to the electromagnetic coupling constant e is an extremely small value $\approx 1.3 \times 10^{-18} m_P^{-1}$, where m_P is the proton mass. Therefore, it will be very difficult to detect or produce any measurable amount of gravitons in a terrestrial experiment.

(ii) Because $n_1 \cdot \epsilon_g$ can be made independent of the helicity value λ of the graviton which will be discussed in detail later, the graviton process has essentially the same transition-amplitude structure as the QED process involving a photon in place of the graviton, except the coupling constant and the kinematical overall factor F.

(iii) Owing to the kinematical factor F in Eq. (2.31), the $g\gamma \to e\bar{e}$ and $e\bar{e} \to g\gamma$ cross sections show different angular distributions from those of $\gamma\gamma \to e\bar{e}$ and $e\bar{e} \to \gamma\gamma$, respectively. The former are more transversely peaked and vanish in the forward and backward directions, while the latter are forwardly peaked. Also, the $ge \to \gamma e$ and $\gamma e \to ge$ cross sections are more forwardly peaked than the cross section of the Compton scattering $\gamma e \to \gamma e$ and vanish in the backward direction. On the other hand, our calculation of differential cross sections agrees with the result of Voronov [12].

(iv) The overall kinematical factor F increases almost linearly with the square of the center-of-mass energy s, while the total cross sections σ_γ of the photon-replaced QED processes decrease as s increases [24]:

$$\sigma_\gamma \to \frac{\pi\alpha^2}{m^2}\frac{2}{\sqrt{s}}\ln\sqrt{s}, \quad \text{as} \quad s\to\infty. \tag{2.32}$$

Therefore, the graviton cross sections might violate unitarity at some very high energies. This indicates the linearized graviton Lagrangian $\mathcal{L}$ in Eq. (1.2) is nonrenormalizable.

III. POLARIZATION

Now let us show in detail that the polarization effects of the graviton are identical to those of the corresponding photon. This aspect can be formally shown by using a density matrix formalism. Physically measurable quantities are not transition amplitudes but their absolute squares, e.g., cross sections. Before obtaining the absolute squares of transition amplitudes, first let us rewrite the transition amplitudes $\mathcal{M}_\gamma$ as

$$\mathcal{M}_\gamma = \epsilon_{g\nu}\, M_\gamma{}^\nu. \tag{3.1}$$

Formally, the square of the graviton transition amplitude $|\mathcal{M}_g|^2$ can be written as

$$|\mathcal{M}_g|^2(\lambda\lambda') = \left(\frac{f}{e}\right)^2 F\, P_g{}^{\mu\nu}(\lambda,\lambda')\, M_{\gamma_\mu} M^*_{\gamma_\nu}, \tag{3.2}$$

where the graviton polarization dictating tensor operator $P_g{}^{\mu\nu}(\lambda,\lambda')$ is defined as

$$P_g{}^{\mu\nu}(\lambda,\lambda') = n_1\cdot\epsilon_g(\lambda)\, n_1\cdot\epsilon_g^*(\lambda')\, \epsilon_g^\mu(\lambda)\epsilon_g^{*\nu}(\lambda'). \tag{3.3}$$

It is well known that the polarization of a photon (i.e., a massless spin-1 particle) beam is completely described [21, 22, 25] in terms of the so-called Stokes parameters $\xi^\gamma{}_i$ $(i=1,2,3)$. In the helicity basis, ξ_2^γ is the degree of circular polarization and the others are degrees of linear polarization. Because a graviton beam has only two helicity values, one can introduce its Stokes parameters ξ_i^g $(i=1,2,3)$ [26]. As in the case of the photon beam, ξ_2^g is the degree of graviton circular polarization and the others are degrees of graviton linear polarization. On the whole, the photon or graviton polarization density matrix $\rho_{\gamma,g}$ is given in the helicity basis by

$$\rho_{\gamma,g} = \frac{1}{2}\begin{pmatrix} 1+\xi_2^{\gamma,g} & -\xi_3^{\gamma,g}+i\xi_1^{\gamma,g} \\ -\xi_3^{\gamma,g}-i\xi_1^{\gamma,g} & 1-\xi_2^{\gamma,g}\end{pmatrix}. \tag{3.4}$$

For a photon beam, we should in general replace the photon projection operator $\epsilon_\gamma^\mu(\lambda)\epsilon_\gamma^{*\nu}(\lambda')$ with its photon covariant density matrix $\rho_\gamma{}^{\mu\nu}$:

$$\begin{aligned}\rho_\gamma{}^{\mu\nu} = \frac{1}{2}\,&[(n_1^\mu n_1^\nu + n_2^\mu n_2^\nu) - (n_1^\mu n_2^\nu + n_2^\mu n_1^\nu)\xi_1^\gamma \\ &+ i(n_2^\mu n_1^\nu - n_1^\mu n_2^\nu)\xi_2^\gamma + (n_2^\mu n_2^\nu - n_1^\nu n_1^\mu)\xi_3^\gamma]\,.\end{aligned} \tag{3.5}$$

In the graviton case, the covariant density matrix $\rho_g{}^{\mu\nu\,:\,\alpha\beta}$, which should be substituted for the graviton projection operator $\epsilon_g^{\mu\nu}(2\lambda)\epsilon_g^{*\alpha\beta}(2\lambda')$, is a little complicated, but can be written in terms of the graviton Stokes parameters ξ_i^g $(i=1,2,3)$ as

$$\begin{aligned}\rho_g{}^{\mu\nu:\alpha\beta} = \frac{1}{4}\Big\{ & (n_1^\mu n_1^\alpha + n_2^\mu n_2^\alpha)(n_1^\nu n_1^\beta + n_2^\nu n_2^\beta) - (n_2^\mu n_1^\alpha - n_1^\mu n_2^\alpha)(n_2^\nu n_1^\beta - n_1^\nu n_2^\beta) \\ & - \Big[(n_2^\mu n_2^\alpha - n_1^\mu n_1^\alpha)(n_1^\nu n_2^\beta + n_2^\nu n_1^\beta) - (n_1^\mu n_2^\alpha + n_2^\mu n_1^\alpha)(n_2^\nu n_2^\beta - n_1^\nu n_1^\beta)\Big]\xi_1^g \\ & + i\Big[(n_1^\mu n_1^\alpha + n_2^\mu n_2^\alpha)(n_2^\nu n_1^\beta - n_1^\nu n_2^\beta) + (n_2^\mu n_1^\alpha - n_1^\mu n_2^\alpha)(n_1^\nu n_1^\beta + n_2^\nu n_2^\beta)\Big]\xi_2^g \\ & - \Big[(n_2^\mu n_2^\alpha - n_1^\mu n_1^\alpha)(n_2^\nu n_2^\beta - n_1^\nu n_1^\beta) - (n_1^\mu n_2^\alpha + n_2^\mu n_1^\alpha)(n_1^\nu n_2^\beta + n_2^\nu n_1^\beta)\Big]\xi_3^g\Big\}.\end{aligned} \tag{3.6}$$

In the same way, the tensor operator $P_g{}^{\mu\nu}$ is to be replaced by the tensor operator $\rho_g{}^{\mu\nu}$ obtained by folding the graviton covariant density matrix $\rho_g{}^{\mu\alpha\,:\,\nu\beta}$ with $n_{1\alpha}n_{1\beta}$. Note that the operator $\rho_g{}^{\mu\nu}$ takes exactly the same form as the covariant density matrix $\rho_\gamma{}^{\mu\nu}$ in Eq. (3.5) where the photon Stokes parameters ξ_i^γ take the place of the graviton Stokes parameters ξ_i^g, respectively. In a concrete way, one obtains

$$\rho_g{}^{\mu\nu} = \rho_\gamma{}^{\mu\nu}(\xi_i^\gamma \to \xi_i^g) \quad (i=1,2,3). \tag{3.7}$$

The conclusion is that, when a graviton interacts with a charged fermion and a photon, the polarization effects of the graviton are identical to those of its photon partner if the graviton Stokes parameters are used instead of the photon Stokes parameters.

IV. LOW-ENERGY THEOREMS

The fact that the graviton behaves like a photon in the graviton interaction with a charged fermion and a photon looks quite general in the sense that only the gravitational gauge invariance is required in the theory.

Now let us consider the case in which the graviton four-momentum is near zero in order to get some hints on how general our results are. What can be employed are the so-called low-energy theorems—electromagnetic and gravitational [7, 12, 27, 28].

Let $\mathcal{M}_{\beta\alpha}$ be the transition amplitude for some reaction $\alpha \to \beta$, the states α and β consisting of various charged and uncharged particles, possibly including gravitons and photons. The same reaction can also occur with absorption (or emission) [29] of a very soft extra photon or graviton of four-momentum k^μ.

These absorption (or emission) matrix elements have poles at $k^\mu = 0$. The poles arise because the virtual particle connecting the photon or graviton vertex with the rest of the diagram gives a vanishing denominator:

$$\frac{1}{(p_n - k)^2 - m_n^2} = \frac{-1}{2k\cdot p_n} \quad : \text{ outgoing particle } n,$$
$$\frac{1}{(p_n + k)^2 - m_n^2} = \frac{1}{2k\cdot p_n} \quad : \text{ incoming particle } n. \tag{4.1}$$

For the absolute value of k^μ sufficiently small, these poles will completely dominate the absorption matrix element. The singular factors (4.1) will be multiplied by a factor $-i(2\pi)^{-4}$ associated with the extra internal line, a factor arising from the interaction vertices

$$\frac{2ieQ_n(p_n\cdot\epsilon_g)(2\pi)^4}{(2\pi)^{3/2}(2\omega)^{1/2}} \tag{4.2}$$

or

$$\frac{2if(p_n\cdot\epsilon_g)^2(2\pi)^4}{(2\pi)^{3/2}(2\omega)^{1/2}}, \tag{4.3}$$

where ω is the photon or graviton energy and Q_n is the electric charge of the particle n, and a factor $S_{\beta\alpha}$ for the rest of the diagram. Therefore, the transition amplitude for soft photon or graviton absorption is written in the limit $\omega \to 0$ as

$$M^\gamma_{\beta\alpha} \to (2\pi)^{-3/2}(2\omega)^{-1/2} e\left[\sum_n \eta_n Q_n \frac{p_n^\mu}{k\cdot p_n}\right]\epsilon_{g\mu} S_{\beta\alpha}, \tag{4.4}$$

or

$$M^g_{\beta\alpha} \to (2\pi)^{-3/2}(2\omega)^{-1/2} f\left[\sum_n \eta_n \frac{p_n^\mu p_n^\nu}{k\cdot p_n}\right]\epsilon_{g\mu}\epsilon_{g\nu} S_{\beta\alpha}, \tag{4.5}$$

with the sign η_n being $+1$ or -1 according to whether particle n is incoming or outgoing. What is noted is that, for the soft graviton or photon, we have exactly the same structure $S_{\beta\alpha}$. For convenience, let us consider all particles incoming for which $\eta_n = 1$. Comparing two asymptotic expressions, we note that the energy-momentum vector p_n^μ is to the strength of graviton interaction what the electric charge Q_n is to that of photon interaction. The properties of the polarization-dependent $S_{\beta\alpha}$ are determined by the participating matter fields.

First of all, let us investigate what constraints [19, 29] gravitational gauge invariance and $\mathrm{U}(1)_{\mathrm{EM}}$ invariance impose on the transition amplitude. Naturally, both of the transition amplitudes should satisfy such invariance properties not only to all orders in k, but also to leading order. Requiring them, we get the relations

$$\sum_n Q_n = 0, \quad \sum_n p_n^\mu = 0. \tag{4.6}$$

The former relation corresponds to electric charge conservation and the latter is nothing but the energy-momentum conservation for the $k = 0$ process. It is certain that the latter is not valid unless the graviton coupling is universal. This is the content of the equivalence principle.

Now let us consider the case in which the number of external matter lines Z is three. Charge conservation, energy-momentum conservation, and the massless $k^2 = 0$ and transverse $k\cdot\epsilon_g = 0$ properties of the on-shell graviton give us an interesting identity:

$$\sum_{n=1}^{3}\frac{[p_n\cdot\epsilon_g]^2}{k\cdot p_n} = \left[\frac{k\cdot p_i\, p_j\cdot\epsilon_g - k\cdot p_j\, p_i\cdot\epsilon_g}{k\cdot p_i\, Q_j - k\cdot p_j\, Q_i}\right]\sum_{n=1}^{3} Q_n\frac{p_n\cdot\epsilon_g}{k\cdot p_n}, \tag{4.7}$$

where $i \neq j$ $(i,j = 1,2,3)$. In the low-energy limit, we get

$$\mathcal{M}^g_{\beta\alpha} = \left(\frac{f}{e}\right)\left[\frac{k\cdot p_i\, p_j\cdot\epsilon_g - k\cdot p_j\, p_i\cdot\epsilon_g}{k\cdot p_i\, Q_j - k\cdot p_j\, Q_i}\right]\mathcal{M}^\gamma_{\beta\alpha}. \tag{4.8}$$

In particular, when $Q_1 = -Q_2$ and the third particle is an external photon, the relation (4.8) takes the same form as Eq. (2.21).

V. DISCUSSION AND CONCLUSION

The relation (4.8) derived from the electromagnetic and gravitational low-energy theorems puts forward the following question: Is the relation (4.8) true for any linearized gravity coupled to a (non-)Abelian gauge theory? While the detailed analysis of the aspect will be reported elsewhere, some speculations are offered below. The crucial point for Eq. (4.8) is that all properties are closely related with the fact that the gravitational coupling is universal. In the present work, we have shown our conjecture to be correct explicitly for the case when a graviton interacts with a charged fermion and a photon. On the other hand, even for a photon with a finite energy, note that the transition amplitude $\mathcal{M}^\gamma_{\beta\alpha}$ for a photon interaction process involving three external matter lines is always factorized [13–16] as

$$\mathcal{M}^\gamma_{\beta\alpha} = \left[\frac{Q_i}{k\cdot p_i} - \frac{Q_j}{k\cdot p_j}\right]\tilde{\mathcal{M}}^\gamma_{\beta\alpha}, \tag{5.1}$$

where $\tilde{\mathcal{M}}^\gamma_{\beta\alpha}$ is an even simpler matrix element. The factorization property (5.1) and the relation (4.8) derived from the electromagnetic and gravitational low-energy theorems strongly suggest that our results can be generalized.

To conclude, the transition amplitudes for the graviton interactions with a charged fermion and a photon have essentially the same transition-amplitude structure as those involving a photon instead of the graviton, apart from a simple overall kinematical factor. And the polarization effects involving the graviton are identical to those for the corresponding photon if the graviton Stokes parameters are used in place of the photon Stokes parameters. Finally, the electromagnetic and gravitational low-energy theorems strongly suggest that our results can be generalized to a broad class of graviton interactions with matter.

ACKNOWLEDGMENTS

The work was supported in part by the Korea Science and Engineering Foundation through the SRC program and in part by the Korean Ministry of Education. The authors would like to thank Professor C. Lee for very helpful discussions.

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