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Black hole radiation in the presence of a short distance cutoff

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A derivation of the Hawking effect is given which avoids reference to field modes above some cutoff frequency $\omega_c \gg M^{-1}$ in the free-fall frame of the black hole. To avoid reference to arbitrarily high frequencies, it is necessary to impose a boundary condition on the quantum field in a timelike region near the horizon, rather than on a (spacelike) Cauchy surface either outside the horizon or at early times before the horizon forms. Because of the nature of the horizon as an infinite redshift surface, the correct boundary condition at late times outside the horizon cannot be deduced, within the confines of a theory that applies only below the cutoff, from initial conditions prior to the formation of the hole. A boundary condition is formulated which leads to the Hawking effect in a cutoff theory. It is argued that it is possible the boundary condition is *not* satisfied, so that the spectrum of black hole radiation may be significantly different from that predicted by Hawking, even without the back reaction near the horizon becoming of order unity relative to the curvature.

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I. INTRODUCTION

The Hawking radiation from a black hole of mass M is most copious at a wavelength of order M.[1] In this sense it is a long-distance effect, whose scale is set by the mass of the hole. Thus it is odd that all derivations of the Hawking effect refer in some manner to arbitrarily short distances. For instance, consider Hawking's original derivation [1]: the annihilation operator for an outgoing quantum field mode at late times is expressed, via the free-field equations, in terms of annihilation and creation operators for ingoing modes at early times, before the matter has collapsed to form the hole. The thermal character of the state at late times is then deduced from the boundary condition specifying that the initial state is the vacuum (or vacuum plus some excitations of finite total energy).

The fishy thing about this derivation is that the frequency of the ingoing modes diverges as the time of the corresponding outgoing modes goes to infinity. This is because all of the outgoing modes, for all eternity, originate as incoming modes that arrive at the hole *before* the formation of the event horizon. An infinite number of oscillations of the incoming modes must thus be packed into a finite time interval, so their frequency must diverge.

Other derivations of the Hawking effect also make reference to arbitrarily short distances. A recent derivation by Fredenhagen and Haag [2] is based on the form of the singularity in the two-point function $\langle\phi(x)\phi(y)\rangle$ as x approaches y just outside the horizon. Similarly, arguments based on the properties of the correlation functions on the Euclidean continuation of the black hole metric [3] assume that the correlation functions have the requisite analytic behavior, which involves the form of the short-distance singularities. Finally, arguments (for conformal fields in two dimensions) based on conservation of the stress-energy tensor [4, 5] assume the value of the trace anomaly, which is the result of regulating a short-distance divergence of the theory.

Since the scale of the process is set by the mass of the hole, it would seem that it should be possible to avoid the role of ultrahigh frequencies much higher than M^{-1} in deriving its existence. In a previous paper [6] this issue was discussed in detail, and two arguments were offered to support this point of view, one involving the response of accelerated particle detectors and one involving conservation of the stress-energy tensor. These arguments were not conclusive but they did make it plausible that the Hawking effect would occur even if there were a Planck frequency cutoff in the frame of free-fall observers that fall from rest far from the hole.

It now seems a mistake to focus on a Planck frequency cutoff, since the same arguments would support the existence of Hawking radiation as long as the high frequency cutoff ω_c is much larger than M^{-1}. In the present paper it will be shown how Hawking's original analysis can be modified to avoid reference to ultrahigh frequencies. This will require the use of an alternate boundary condition, which states roughly that observers falling freely into the black hole (starting from rest far away) see no particles at frequencies much higher than M^{-1} but less than some cutoff ω_c. That this condition implies the existence of black hole radiation was implicit in Hawking's original paper [1], and was later stressed by Unruh [7]. One contribution of the present paper is to demonstrate in detail how the derivation can be structured so as to entirely avoid invoking the behavior of ultrahigh frequency modes. This analysis involves several sticky technicalities, which we have attempted to address as thoroughly

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[1]We use units with $G = c = \hbar = 1$.

as possible.

This alternate boundary condition is not an *initial* condition, since it is imposed for all times. Moreover, for the reason explained above, it cannot be derived from the early time vacuum inital condition. It is in the nature of the horizon as an infinite redshift surface that the state of the outgoing field modes at low frequencies descends from presently unknown physics at very high frequency (in the free-fall frame). Thus the validity of the boundary condition cannot be proved within a theory that is only valid below some high frequency cutoff. The possibility of justifying the boundary condition on energetic grounds will be addressed in Sec. VI. Our conclusion will be that it is quite possible the boundary condition is *not* satisfied, so that the spectrum of black hole radiation may be significantly different from that predicted by Hawking, even without the back reaction near the horizon becoming of order unity relative to the curvature. Violations of the boundary condition leading to a large back reaction also seem possible; however, in such a situation the quasistatic, semiclassical framework of our calculations is unjustified.

The rest of the paper is organized as follows. In Sec. II Hawking's original derivation is reviewed. In Sec. III the role of ultrahigh frequencies in this derivation is discussed and in Sec. IV our alternate boundary condition is formulated and discussed in detail. It is shown in Sec. V that this boundary condition implies the existence of the usual Hawking radiation. In Sec. VI the physical basis of the boundary condition is discussed, and the implications of a violation of the boundary condition are studied. Section VII contains some concluding remarks, and the Appendixes contain technical material needed in the rest of the paper.

II. HAWKING'S REASONING

In this section Hawking's original derivation [1] of black hole radiation from a nonrotating, uncharged black hole will be reviewed. We use Wald's formulation [9] in terms of individual wave packets, rather than Bogoliubov transformations between orthonormal bases, because selection of a complete basis is distracting and unnecessary for our our purposes.

Consider an outgoing positive frequency wave packet P at late times far from the black hole, centered on frequency $\bar{\omega}$ and retarded time $\bar{u}$. (The retarded time coordinate is defined in Appendix A.) Suppose P is normalized in the Klein-Gordon norm, so the annihilation operator for this wave packet is given by

$$a(P) = \langle P, \Phi \rangle, \tag{2.1}$$

where the angular brackets denote the Klein-Gordon (KG) inner product. (See Appendix B for the definition of the KG inner product and Appendix C for a discussion of this characterization of annihilation and creation operators.) We are interested in the state of the quantum field "mode" corresponding to this wave packet. This is partly[2] characterized by the expectation value of the number operator:

$$\langle N(P) \rangle = \langle \Psi | a^\dagger(P) a(P) | \Psi \rangle. \tag{2.2}$$

Using the field equation $\nabla^2 \Phi = 0$, this number operator can be expressed in terms of operators whose expectation values are fixed by initial conditions or other assumptions on the properties of the state $|\Psi\rangle$.

Propagating the wave packet P backward in time, it breaks up into a "reflected piece" R that scatters off the curvature outside the matter and out to past null infinity $\mathcal{I}^-$, and a "transmitted" piece T that propagates back through the collapsing matter and then out to $\mathcal{I}^-$. (See Fig. 1.) The original wave packet P can be expressed as the sum of these two solutions as

$$P = R + T, \tag{2.3}$$

and the annihilation operator for P (2.1) can thus be decomposed as

$$a(P) = a(R) + a(T). \tag{2.4}$$

Since both the wave packets and the field operator satisfy the wave equation, the KG inner products in (2.4) are conserved, and can therefore be evaluated on any Cauchy hypersurface.

Because of time translation invariance in the part of the spacetime exterior to the matter, the reflected packet R consists of the same frequencies with respect to the Schwarzschild time coordinate at $\mathcal{I}^-$ as the packet P at

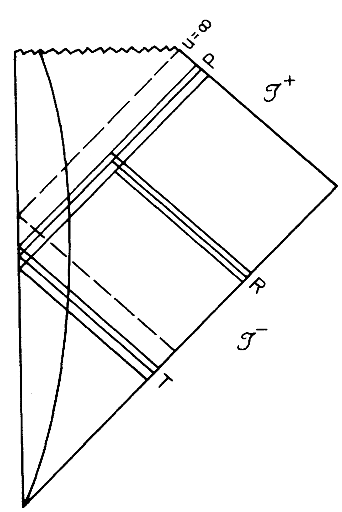

FIG. 1. Conformal diagram depicting wave fronts of the wave packet $P = R + T$ propagating in the spacetime of a spherically symmetric collapsing body.

[2]For simplicity we focus on the expectation value of the number operator. In fact, the form of the annihilation operator $a(P)$ discussed below implies also the true thermal nature of the state. (See, for example, [9–11].)

future null infinity $\mathcal{I}^+$. Thus the operator $a(R) = \langle R, \Phi \rangle$ is an annihilation operator for an incoming wave-packet centered on frequency $\bar{\omega}$. Assuming that this mode of the quantum field started out in its ground state, we have

$$a(R)|\Psi\rangle = 0, \tag{2.5}$$

so that

$$\langle N(P) \rangle = \langle \Psi | a^\dagger(T) a(T) | \Psi \rangle. \tag{2.6}$$

At $\mathcal{I}^-$ the transmitted packet T is composed of both positive and negative frequency components with respect to the asymptotic Schwarzschild time,

$$T = T^{(+)} + T^{(-)}, \tag{2.7}$$

and we have the expansion

$$a(T) = a(T^{(+)}) - a^\dagger(T^{(-)*}). \tag{2.8}$$

Thus $a(T)$ is a combination of annihilation and creation operators for incoming wave packets at $\mathcal{I}^-$. Assuming that both the positive frequency part and the complex conjugate of the negative frequency part of the packet T started out in their ground states, we have

$$a(T^{(+)})|\Psi\rangle = 0, \quad a(T^{(-)*})|\Psi\rangle = 0. \tag{2.9}$$

Thus, using (2.6), (2.8), (2.9), and the commutation relation between annihilation and creation operators (C3) and (C4) we have

$$\langle N(P) \rangle = -\langle T^{(-)}, T^{(-)} \rangle. \tag{2.10}$$

For a wave packet with a spread of frequencies $\Delta\omega \ll \kappa$, the Klein-Gordon norm of the negative frequency packet $T^{(-)}$ can be evaluated in terms of the norm of T as described in Appendix D, and one finds, using (D5),

$$-\langle T^{(-)}, T^{(-)} \rangle = \langle T, T \rangle [\exp(2\pi\bar{\omega}/\kappa) - 1]^{-1}. \tag{2.11}$$

This is just what the emission would be from a body at temperature $\kappa/2\pi = 1/8\pi M$, for a mode of energy $\bar{\omega}$ with absorption coefficient $\langle T, T \rangle$.

III. ULTRAHIGH FREQUENCIES

The difficulty with this analysis is that at past null infinity, the incoming packet T consists of extremely high frequency components, whose frequency (with respect to the asymptotic rest frame of the hole) grows as $\sim \exp(\bar{u}/4M)\,\omega$ as the retarded time $\bar{u}$ of the outgoing wave packet goes to infinity. This exceeds Planck frequency ω_P for $\bar{u} > 4M \ln(\omega_P/\omega)$, that is, after only several light crossing times for the hole. That the frequency diverges in some such manner is immediately evident from inspection of Fig. 1. An infinite amount of time at infinity corresponds to the interval between any finite u and the horizon at $u = \infty$. The correspondingly infinite number of field oscillations must all be packed into the finite range of advanced times between some v and v_0, the advanced time of formation of the horizon.

It is unsatisfactory from a physical point of view to base the prediction of black hole evaporation on an assumption that involves the behavior of arbitrarily high frequency modes. We are ignorant of what physics might look like at those high frequencies or corresponding short distances. In order to be confident of the prediction of Hawking radiation, one should formulate a derivation that avoids this ignorance while invoking only known physics—or at least only more reasonable extrapolations of known physics.

It is not the unknown physics of high energy *interactions* that we are concerned about here. Although we are dealing with incoming wave packets with arbitrarily high frequency relative to the frame of the collapsing matter that forms the black hole, there is no interaction between these wave packets and the collapsing matter. The reason is that these incoming wave-packet modes are in their *ground state*, so there is nothing for the collapsing matter to interact with.

What we are concerned about is the need to assume that the physics is Lorentz invariant under arbitrarily large boosts. Assuming Lorentz invariance, one can of course argue that although the frequency of the transmitted wave packet T grows as $\exp(\bar{u}/4M)$ with respect to the asymptotic rest frame of the black hole, there is always a local Lorentz frame in which the frequency appears as low as one wishes. The velocity of this frame relative to the black hole approaches the speed of light as $\bar{u} \to \infty$, with a boost factor $\gamma = (1 - v^2)^{-1/2} = \exp(\bar{u}/4M)$.

We have no observations that confirm Lorentz invariance at the level of such arbitrarily high velocity boosts [12–14]. Probably the highest boost factors at which Lorentz invariance might be checked anytime soon arise in cosmic-ray proton collisions. We are basically at rest with respect to the cosmic microwave background (CMB) radiation. Assuming Lorentz invariance, one predicts that for proton energies greater than about 10^{20} eV (relative to the CMB frame), the head-on collision of a proton with a CMB photon can produce a pion. This process would leave its mark on the cosmic-ray proton spectrum. If this mark is eventually observed, it will lend support to the assumption of Lorentz invariance that went into the calculation.[3] The boost factor here relating the CMB frame to the center-of-mass frame of the collision is a "modest" $\gamma \sim 10^{12}$.

In the black hole situation, after a retarded time interval $\Delta u \sim 4M \ln 10^{12} \simeq 10^2 M$, the boost factor required to transform an incoming wave packet to low frequency would have increased by more than 10^{12}. Thus the above derivation of a steady flux of Hawking radiation depends on the assumption of Lorentz invariance arbitrarily far beyond its observationally verified domain of validity.

IV. CUTOFF BOUNDARY CONDITION

To avoid the need to make assumptions regarding arbitrarily high frequency behavior we will have to give up

[3] According to Sokolsky [15] it should be possible to confirm this prediction in the coming decade.

the attempt to derive the properties of the state of the quantum field at late times from the initial condition that it is the vacuum state before the hole forms. Instead, we will formulate a different "boundary" condition on the state that will still imply the existence of Hawking radiation.

The alternate boundary condition is expressed in terms of the particle states defined by free-fall observers near the horizon that have fallen in from rest far away from the hole. For frequencies much higher than M^{-1}, these particle states are well defined by field modes with positive frequency with respect to the proper time of the free-fall observers. Our boundary condition will be that outgoing, high frequency field modes are in their ground states. How high is "high"? Roughly, to predict Hawking radiation to an accuracy $\eta \ll 1$, it will suffice to assume that the outgoing modes of free-fall frequency $\sim \eta^{-2}M^{-1}$ are in their ground state. The statement of the boundary condition just given is appropriate for a *massless, free* field. We defer to Sec. IV F a brief discussion of the modifications required for a treatment of massive and/or interacting fields.

To derive this alternate boundary condition from the condition that the *initial* state is vacuum requires appeal to arbitrarily high frequency modes, for the reason discussed earlier. Thus we make no attempt here to *derive* this alternate boundary condition, but rather take it as given. The question of physical plausibility of the condition will be taken up in Sec. VI.

A. Precise formulation of the boundary condition

Actually imposing the alternate boundary condition in terms of the proper time of the family of free-fall observers is somewhat complicated. Instead, we shall employ the affine parameter along radial ingoing null geodesics as the relevant "time" variable. This turns out to amount to the same thing near the horizon, as will now be explained.

First note that the usual radial coordinate r is an affine parameter along the radial null rays (see Appendix A). To find the rate of change of r with respect to the proper time τ along the free-fall geodesic, note that the quantity $p_v = g_{v\mu}dx^\mu/d\tau = (1-\frac{2M}{r})dv/d\tau - dr/d\tau$ is conserved, since the metric is independent of v in Eddington-Finkelstein (EF) coordinates (A1d). If the geodesic starts from rest at ∞, one has at infinity $dv/d\tau = 1$ and $dr/d\tau = 0$, so $p_v = 1$. It follows then that at the horizon $r = 2M$, one has $dr/d\tau = -1$. That is, r is changing at the same rate as the proper time.

An outgoing solution f to the wave equation near the horizon is nearly independent of v in EF coordinates, since the lines of constant r are nearly null there. Along the free-fall world line near the horizon, we therefore have $df/d\tau \cong (\partial f/\partial r)dr/d\tau \cong -(\partial f/\partial r)$. Thus, for outgoing modes near the horizon, the frequency with respect to r on a constant v surface is effectively the negative of the frequency with respect to the free-fall observers.

The particle states of our boundary condition will correspond to wave packets f composed of field modes on a constant v null hypersurface Σ of the form

$$f_{\omega lm}(r,\theta,\phi) = r^{-1}\exp(i\omega r)Y_{lm}(\theta,\phi)\ . \tag{4.1}$$

In an effort to avoid confusion I will call these *positive* r-frequency modes, because they have positive frequency with respect to the proper time of the free-fall observers. We can regard the operator $a(f) = \langle f,\Phi\rangle$ as (proportional to) an annihilation operator for a one-particle state provided that the Klein-Gordon (KG) norm of f is positive. (This is discussed in Appendix C.)

To evaluate the Klein-Gordon inner product (B1) on Σ, we use the metric components in EF coordinates (A1d) and the surface element (B2) to find $\sqrt{-g}g^{\mu\nu}d\Sigma_\nu = -\delta_r{}^\mu\, r^2 \sin\theta\, dr\, d\theta\, d\phi$. Thus the KG inner product takes the form

$$\langle f,g\rangle = -\frac{i}{2}\int d\Omega \int_0^\infty dr\, r^2\, (f^*\partial_r g - g\partial_r f^*). \tag{4.2}$$

This shows that the modes $f_{\omega lm}$ (4.1) indeed have positive norm for $\omega > 0$, as do localized wave packets constructed by superposing them.[4]

Our alternate boundary condition can thus be implemented as follows. We choose to calculate the expectation value of the number operator corresponding to wave packets P with the property that on *some* constant v surface, $v = v_c$, their transmitted piece T has only components with an r frequency $\omega^{(r)}$ much higher than M^{-1} but less than some cutoff frequency ω_c,

$$\omega_c > \omega^{(r)} \gg M^{-1}. \tag{4.3}$$

(If the frequency at infinity ω is much greater than M^{-1}, we also require $\omega^{(r)} \gg \omega$.) Then, instead of propagating the transmitted piece T of the wave packet P all the way back through the collapsing matter and out to past null infinity, we stop when it reaches $v = v_c$. There we decompose it into its positive and negative r-frequency parts and impose the boundary condition that the positive r-frequency part (and the complex conjugate of the

[4]It is tempting to try to define a full Hilbert space of one-particle states on a constant v surface using the positive r-frequency modes. However, the fact that the r integral runs only over the interval $[0,\infty)$ leads to a problem with this definition. Positive frequency modes of the form $f_{\omega lm}$ and $f_{\omega' lm}$ (4.1) are *not* orthogonal for $\omega \neq \omega'$, and linear combinations of positive frequency modes can have negative norm. This is not a problem if one restricts attention to wave packets that have negligible support near $r = 0$, since for them it makes no difference whether the r integration is over $[0,\infty)$ or $(-\infty,\infty)$. (One cannot take wave packets of compact support since that would be inconsistent with their being composed of purely positive frequencies.) In any case, we will refer to only one wave packet at a time, with no need to consider the full Hilbert space of one-particle states.

negative frequency part) are in their ground states.[5] To carry out this program, it must first be established that there *exist* positive u-frequency wave packets with the property than on some surface $v = v_c$, their r-frequency components satisfy (4.3). This will be accomplished in Sec. IV C below.

B. Self-consistency of the boundary condition

Note that for a wave packet centered on frequency $\bar{\omega}$ and retarded time $\bar{u}$, the surface $v = v_c$ must necessarily move to the future as $\bar{u}$ grows with $\bar{\omega}$ fixed, in order to avoid the occurrence of r-frequency components above the cutoff frequency. Thus our boundary condition is not being imposed on a single Cauchy surface, so it is not an "initial" condition. This raises the question whether our boundary condition is consistent with the field dynamics.

For simplicity, let us think of the boundary condition as being imposed on a surface of fixed radius, $r = r_{\rm BC}$, just outside the horizon.[6] This surface is timelike, so the site of the part of the boundary condition imposed at advanced time v includes, within its past, sites of parts of the condition imposed at earlier advanced times. Is the condition imposed at v consistent with the earlier ones?

The boundary condition refers to the state of outgoing modes with r frequency $\omega^{(r)}$ in the range $\omega_c > \omega^{(r)} \gg M^{-1}$. The modes of frequency ω_c come from two sources: modes that propagate out from yet closer to the horizon with yet higher frequencies, and modes that have scattered off the geometry. The state of the former modes can be freely specified, since they are above the cutoff until they reach advanced time v and hence no condition at all is imposed on them until then. Thus there is enough freedom to consistently assign the state of the outgoing modes at ω_c. But one may still ask if the gound state boundary condition is the *appropriate* one, in view of the contributions from the modes that have backscattered. For instance, some Hawking radiation can scatter back toward the hole and then scatter again out from the hole, apparently leading to some nonzero occupation number in an outgoing mode that the boundary condition assigns to its ground state. The scattering amplitude for these modes in this region of the spacetime is very small however, so such processes should affect the state only very little.

Now let us consider the modes with frequency *less* than the cutoff. The state of these modes cannot really be independently specified, since they can be traced back (primarily) to modes yet closer to the horizon with frequency ω_c, on which a (ground state) boundary condition has already been imposed. Thus the state of the modes with frequency $\omega^{(r)} < \omega_c$ must be *calculated*, not assigned. In fact, it follows from the argument in Sec. V that no modes are excited while they are propagating close to the horizon; it is not until they climb away significantly (on the scale of M) that the presence of Hawking radiation becomes apparent in the free-fall frame.

Thus it appears not inconsistent to impose our ground state boundary condition, at least to the order of precision of our calculations. Note that we can really only check self-consistency of the calculation: As shown in the next two subsections, the unavoidable spread of the wave packets makes it necessary to impose a boundary condition on a wide range of frequencies from the beginning. Then all we can do is verify that this boundary condition is self-consistent.

C. Existence of the required wave packets

Let $p_{\omega lm}$ denote the solution to the massless scalar wave equation in Schwarzschild spacetime that is purely outgoing at future null infinity (and is therefore outgoing at the horizon as well), and is of the form

$$p_{\omega lm} = (2\pi\omega)^{-1/2} \exp(-i\omega t) r^{-1} f_{\omega l}(r) Y_{lm}(\theta, \phi), \qquad (4.4)$$

with

$$f_{\omega l}(r) = \begin{cases} e^{i\omega r^*} + A_{\omega l} e^{-i\omega r^*} & \text{as } r^* \to +\infty, \\ B_{\omega l} e^{i\omega r^*} & \text{as } r^* \to -\infty, \end{cases} \qquad (4.5)$$

where r^* is the tortoise coordinate defined in Eq. (A2a). These modes are normalized according to $\langle p_{\omega lm}, p_{\omega' l' m'} \rangle = \delta(\omega - \omega') \delta_{ll'} \delta_{mm'}$. Using these modes, we seek to construct wave packets that satisfy the condition (4.3) restricting the r-frequency components on a constant v surface, $v = v_c$.

The wave packets we will employ are of the form

$$P_{\bar{\omega}\bar{u}lm} = \mathcal{N} \int_{\bar{\omega}}^{\bar{\omega}+\Delta\omega} d\omega\, B_{\omega l}^{-1} \exp(i\omega\bar{u})\, p_{\omega lm}. \qquad (4.6)$$

$P_{\bar{\omega}\bar{u}lm}$ is a unit norm, positive t-frequency wave packet centered on frequency $\bar{\omega} + \frac{\Delta\omega}{2}$. $\mathcal{N}$ is a normalization factor, and the factor $B_{\omega l}^{-1}$ (inverse of the transmission amplitude) is included in the integrand so that we will have control over the spread of the part of the packet near the horizon. The wave packet $P_{\bar{\omega}\bar{u}lm}$ is defined by its (purely outgoing) behavior at $\mathcal{I}^+$ and the fact that it vanishes on the horizon. Alternatively, propagating it backward in time from $\mathcal{I}^+$ as in Sec. II, one sees that it is generated by data on a Cauchy hypersurface formed by a constant v surface $v = v_c$ together with the part of $\mathcal{I}^-$ that lies to the future of v_c. The wave packet generated by the data at $v = v_c$ alone will be called the "transmitted packet" $T_{\bar{\omega}\bar{u}lm}$, and that generated by the data at $\mathcal{I}^-$ will be called the "reflected packet" $R_{\bar{\omega}\bar{u}lm}$. Thus we have $P_{\bar{\omega}\bar{u}lm} = T_{\bar{\omega}\bar{u}lm} + R_{\bar{\omega}\bar{u}lm}$.

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[5]Although the wave packet P is completely *outside* the horizon, its positive and negative r-frequency parts have support both inside and outside the horizon. [See Eqs. (5.5a) and (D3).]

[6]Actually, the boundary condition refers to the region *inside* the horizon as well, since the positive and negative frequency parts have support inside the horizon. It is therefore more accurate to think of the boundary condition as being imposed on a *pair* of surfaces of constant r, one just outside the horizon and one just inside. Since the one inside is *spacelike*, no question of consistency arises for that part of the boundary condition.

For each $\bar{\omega}$ and for $\bar{u}$ sufficiently long after the collapse that formed the black hole, one can always choose v_c sufficiently early in the past so that $T_{\bar{\omega}\bar{u}lm}$ is concentrated near the horizon. In this case, the asymptotic form $f_{\omega l} \cong B_{\omega l}\exp(i\omega r^*)$ can be accurately substituted in the integrand (4.6) and one obtains

$$T_{\bar{\omega}\bar{u}lm} = \mathcal{N}(2\pi)^{-1/2} r^{-1} Y_{lm}(\theta,\phi) \times \int_{\bar{\omega}}^{\bar{\omega}+\Delta\omega} d\omega\, \omega^{-1/2} \exp[i\omega(\bar{u}-u)]. \quad (4.7)$$

This transmitted wave packet is localized in retarded time u, centered roughly on $\bar{u}$, with a spread $\Delta u \simeq 8\pi/\Delta\omega$. More precisely, the spread of $T_{\bar{\omega}\bar{u}lm}$ in u is of course infinite, but the packet is well localized in the following sense.[7] After carrying out the angular integrations the KG norm (4.2) of the packet (4.7) calculated at $v = v_c$ reduces to a numerical factor times an integral over x of the quantity $(\sin x/x)^2$, where $x = \Delta\omega(u-\bar{u})/2$. One can show that $\int_0^y (\sin x/x)^2\, dx = (\pi/2)[1-(1/\pi y) + O(y^{-2})]$. Thus, defining η as the fraction of the full norm omitted in a range Δu, one has $\eta \simeq 1/\pi y = 4/\pi\Delta\omega\,\Delta u$, or

$$\eta \simeq 1/\Delta\omega\,\Delta u\,. \quad (4.8)$$

In Hawking's paper [1], wave packets of the form (4.6) (without the factor of $B_{\omega l}^{-1}$) were also employed, however $\Delta\omega$ was chosen very small compared with the surface gravity $\kappa = 1/4M$, so that the wave packets relevant to the black hole radiation would be very peaked in frequency, thus simplifying the analysis. From our point of view, the difficulty with this is that such a packet cannot be squeezed close enough to the horizon without containing r frequencies above the cutoff ω_c. In fact, one must take $\Delta\omega \gtrsim \kappa$, and to maximize the precision of our derivation one should take $\Delta\omega \sim \sqrt{\omega_c\kappa}$, as will now be shown.

D. Precision of the derivation

The precision of the derivation we will give is limited by the fact that the wave packets will not be infinitely squeezed up against the horizon. The resulting "error" is of order $C_{\max} \equiv (1-2M/r_{\max})$, where $r_{\max}$ is the largest value of r to occur in the wave packet.[8] Of course, strictly speaking, $r_{\max} = \infty$, but a fraction $(1-\eta)$ if the wave packet is contained within a smaller range of r values, given by $\Delta u \simeq 1/\eta\Delta\omega$. Thus to minimize the errors we should minimize the combined error due to the fraction η of the wave packet beyond $r_{\max}$, and due to $C_{\max}$ not vanishing. To carry out this minimization calculation, we must express $C_{\max}$ as a function of η and $\Delta\omega$, and minimize the error function

$$E^2(\eta,\Delta\omega) \equiv \eta^2 + C_{\max}^2(\eta,\Delta\omega)\,. \quad (4.9)$$

The relation between u and r at constant v is given [cf. (A2a) and (A2b)] by $\partial u/\partial r|_v = -2(1-\frac{2M}{r})^{-1} = -2C^{-1}$, where $C = (1-\frac{2M}{r})$. It is this factor that converts between u frequency and r frequency at fixed v, $\omega^{(r)} = -2C^{-1}\omega$. We assume that on the constant v surface, the wave packet is squeezed very near to the horizon, since that is in any case required in order to deduce the existence of Hawking radiation from our boundary condition. Then we have (with $\kappa = 1/4M$)

$$C_{\max}/C_{\min} \simeq \exp(\kappa\Delta u) \sim \exp(\kappa/\eta\Delta\omega)\,. \quad (4.10)$$

Now assuming the highest r frequency present in the wave packet is the cutoff frequency, we have $\omega_c = \omega_{\max}^{(r)} = C_{\min}^{-1}\omega_{\max}$, so that $C_{\min} = \omega_{\max}/\omega_c$. Together with (4.10) this yields

$$C_{\max} \sim \exp(\kappa/\eta\Delta\omega)\,(\bar{\omega}+\Delta\omega)/\omega_c\,, \quad (4.11)$$

where we have returned to the notation $\bar{\omega} \equiv \omega_{\min}$. For the purposes of minimizing the error, we will consider $\bar{\omega}$ as fixed, since this is really determined by which frequencies we want to learn about.

Already (4.11) shows us that it is not acceptable to choose $\Delta\omega \ll \kappa$ as Hawking did. For instance, suppose that $\bar{\omega} \sim \kappa$), so the frequencies most copious in the Hawking radiation will be included, and suppose that $\Delta\omega = 0.01\kappa$ and $\eta = 0.01$. Then we have $C_{\max} = \exp(10\,000)\ \kappa/\omega_c$, which will be smaller than unity only if κ/ω_c is much smaller than we want to assume.

To minimize the error (4.9) we use (4.11) and set $\partial E/\partial\eta = 0$ and $\partial E/\partial\Delta\omega = 0$. Up to factors of order 1, this yields, at the minimum,

$$\eta \sim (\Delta\omega)^3/\kappa\omega_c^2 \qquad \text{and} \qquad C_{\max} \sim (\Delta\omega)^5/\kappa^2\omega_c^3\,, \quad (4.12)$$

where $\Delta\omega$ satisfies

$$\bar{\omega} + \Delta\omega \simeq (\Delta\omega)^5/\kappa^2\omega_c^2\,. \quad (4.13)$$

As long as $\bar{\omega} \ll \sqrt{\kappa\omega_c}$, the solution is given by

$$\Delta\omega \sim \sqrt{\kappa\omega_c}, \qquad \eta \sim \sqrt{\kappa/\omega_c}, \qquad C_{\max} \sim \sqrt{\kappa/\omega_c}. \quad (4.14)$$

Note that for such a "minimum error" wave packet with $\omega_{\max}^{(r)} = \omega_c$, we have $\omega_{\min}^{(r)} = C_{\max}^{-1}\bar{\omega} \sim (\bar{\omega}^2\omega_c/\kappa^3)^{1/2}\,\kappa$, which will satisfy the condition $\omega^{(r)} \gg \kappa$ as long as $\bar{\omega} \gg (\kappa/\omega_c)^{1/2}\,\kappa$.

We conclude that one can work with wave packets with

[7]The wave packet $P_{\bar{\omega}\bar{u}lm}$ at $\mathcal{I}^+$ does not have the same width in u as does $T_{\bar{\omega}\bar{u}lm}$ at v_c. The wave packet is somewhat dispersed, since the different frequency components have unequal transmission amplitudes. We included the factor $B_{\omega l}^{-1}$ in the definition (4.6) of $P_{\bar{\omega}\bar{u}lm}$ so that our packet would be well localized at v_c; it will not bother us that $P_{\bar{\omega}\bar{u}lm}$ is not as well localized at $\mathcal{I}^+$.

[8]Actually, since only a fraction of the wave packet is located at $r \sim r_{\max}$, with the rest at smaller values of r, the error is somewhat smaller. To keep the crude analysis that follows from getting too complicated, we will simply make the conservative error estimate using the largest value of r.

r frequencies in the required range, with a built-in imprecision of the calculation[9] limited to an error of order $\sqrt{\kappa/\omega_c}$.

E. Horizon fluctuations

Another point that should be checked is how close to the horizon is our boundary condition being imposed? If this is within the expected range of quantum fluctuations of the horizon itself, then we will not have succeeded in formulating a derivation free of short-distance uncertainties. To estimate the radius $r_{\rm BC}$ at which the boundary condition is being imposed, note that for a mode of frequency M^{-1} coming from a hole of mass M, we have $\omega^{(r)} \sim \omega_c$ when $(1-\frac{2M}{r})^{-1}M^{-1} \sim \omega_c$, or $r_{\rm BC} \sim 2M + l_c$, where $l_c = \omega_c^{-1}$. The scale of quantum fluctuations of the horizon δr can be estimated by using the Beckenstein-Hawking entropy $S = \frac{1}{4}A/l_P^2$ and setting $\delta S \sim 1$, which is characteristic of thermal fluctuations about equilibrium.[10] Assuming the horizon should be treated as $N \equiv A/l_P^2$ independent fluctuating area elements, each of area a and radius r, we have $\delta A \sim \sqrt{N}\delta a \sim l_P \delta r$, so $\delta A \sim l_P^2$ gives $\delta r \sim l_P$. Thus for a Planck scale cutoff, we are perhaps not justified in ignoring the quantum fluctuations of the horizon in our derivation. The simple way out is to take the cutoff length much longer than the Planck length. This is fine until we come to discussing the physical justification for the boundary condition, or violations of it. It should be kept in mind that if the modes are followed all the way back to where they are squeezed up within one Planck length of the horizon, several grains of salt should be added to the whole analysis.

F. Massive or interacting fields

In order to apply the arguments of Sec. V, it is necessary that the propagation be governed by the massless wave equation for a sufficiently long interval of advanced time v. Thus for a free field of mass m one must impose the boundary condition on wave packets satisfying $\omega^{(r)} \gg m$, in addition to the condition $\omega^{(r)} \gg M^{-1}$ already discussed in Sec. IV A. Then one finds that particles corresponding to these wave packets are created near the black hole just as are massless ones, and they then propagate away from the hole as massive particles. As long as the mass is much less than the cutoff frequency, $m \ll \omega_c$, there is no obstruction to extending our argument to cover the case of massive particles.

It is generally believed that the Hawking effect occurs for interacting fields as well as for free fields, although this has never been demonstrated explicitly. For the purposes of determining what would be emitted by a real black hole, some researchers [18] have assumed that the process can be divided into two stages, much as for the massive free field just discussed. In the first stage, which takes place very near the horizon, the dynamics of the field is governed by the asymptotically free regime. In QCD, for example, free quarks and gluons are assumed to be radiated with a thermal spectrum. In the second stage, as the particles climb away from the horizon, the self-interactions of the field become important, and the free particle states hadronize into jets.

A direct demonstration of the validity of this picture has never been given, although there are various arguments that support it. Gibbons and Perry [3] argued that the periodicity of the Euclidean section of Schwarzschild spacetime implies the thermal character of Hawking radiation for interacting fields. This argument applies only to the thermal equilibrium state on the eternal black hole spacetime. Moreover, it rests heavily on the assumption that a state that is regular on the horizon must arise by analytic continuation from a state that is regular on the (periodically identified) Euclidean section. While this condition seems natural in some sense, it has not been demonstrated to be necessary.

Another argument advanced in favor of thermality is that of Unruh and Weiss [19], who demonstrated that the *Minkowski* vacuum of an interacting field theory is a thermal state when viewed by a uniformly accelerating family of observers. More precisely, correlation functions in the Rindler wedge are given by the thermal density matrix relative to the Hamiltonian that generates translations along the boost Killing field. This is a purely kinematical result. It is, in a sense, a local version of the Euclidean section argument that avoids the need for assumptions about regularity of the analytically continued correlation functions on the Euclidean section. To turn it into a derivation of Hawking radiation for interacting fields, one presumably must assume the field is in a state that "looks like" the Minkowski vacuum very near the horizon, use the Unruh-Weiss result to describe it from the point of view of the static observers as a thermal state, and then propagate this thermal state out away from the hole. The result will depend on the interactions and on what state is incoming from infinity, since this would interact with the outgoing Hawking radiation.

For weakly coupled fields one can study this process using perturbation theory. Massless $\lambda\phi^4$ theory in a two-dimensional black hole spacetime was studied by Leahy and Unruh [20], who showed that for an ingoing thermal state at the Hawking temperature, the interaction preserves the thermal nature of the outgoing state. For an ingoing vacuum state however, the outgoing state is *not* thermal.

It does not appear to be entirely straightforward to ex-

[9]It may be that the derivation can be improved, reducing the imprecision. The wave-packet analysis employed here seems a rather clumsy approach to the problem. The problem can also be formulated using the approach of Fredenhagen and Haag [2], which focuses on the behavior of the two-point function. That approach may turn out to be more suitable for maximizing the precision of the derivation.

[10]This gives the same scale as the one obtained by York using the uncertainty principle and the spectrum of quasinormal modes [16], or using the Euclidean partition function approach [17].

tend the arguments of our paper to the case of interacting fields, since we use the linearity of the field equation to express the annihilation operator corresponding to a wave packet at one time in terms of annihilation and creation operators associated with wave packets at another time. In order to extend our argument, one can presumably use the fact that for the first part of the process, as the excitations are created, only the propagation of the field "near the light cone" is relevant. That is, one can presumably show that only small spacetime intervals are involved, and thus use the fact that the correlation functions behave like free-field ones in this region, due to asymptotic freedom. This picture of the process was outlined by Fredenhagen and Haag in the discussion section of [2], but to my knowledge it has never been worked out in any detail.

V. HAWKING RADIATION IN THE PRESENCE OF A CUTOFF

Having formulated in the previous section a boundary condition on the quantum state near the horizon that refers only to modes below the cutoff, it is now our task to determine the properties of the state far from the hole.

A. Evaluating the occupation numbers

Suppose now that $P_{\bar{\omega}\bar{u}lm} = R_{\bar{\omega}\bar{u}lm} + T_{\bar{\omega}\bar{u}lm}$ is an outgoing wave packet of the form (4.6), and propagate $T_{\bar{\omega}\bar{u}lm}$ back to a constant v surface $v = v_c$ on which its r-frequency components satisfy $\omega_c > \omega^{(r)} \gg M^{-1}$. (More precisely, it will be composed of both positive and negative r-frequency modes with frequencies in this range.)

Now we would like to evaluate the expectation value of the number operator $N(P_{\bar{\omega}\bar{u}lm})$, subject to our "boundary conditions" on the quantum state. These are that (1) the reflected piece $R_{\bar{\omega}\bar{u}lm}$ is in its ground state at $\mathcal{I}^-$ and (2) the positive r-frequency part and the complex conjugate of the negative r-frequency part of the transmitted piece $T_{\bar{\omega}\bar{u}lm}$ are in their ground states on the surface $v = v_c$ on which the r-frequency components satisfy $\omega_c > \omega^{(r)} \gg M^{-1}$.

Subject to these boundary conditions, the evaluation of $\langle N \rangle$ goes through as in Sec. II and we find

$$\langle N(P_{\bar{\omega}\bar{u}lm})\rangle = -\langle T^{(-,r)}_{\bar{\omega}\bar{u}lm}, T^{(-,r)}_{\bar{\omega}\bar{u}lm}\rangle, \tag{5.1}$$

where $T^{(-,r)}_{\bar{\omega}\bar{u}lm}$ denotes the negative r-frequency part of the wave packet $T_{\bar{\omega}\bar{u}lm}$, evaluated on the surface $v = v_c$.

Now the KG norm in (5.1) cannot have the form of (2.11) because, as explained in Secs. IV C and IV D, the spread of frequencies $\Delta\omega$ in the packet must be taken to be at least of order κ (or even much larger in order to maximize the precision). In order to exploit the simple formula (D3b) that is applicable to the negative r-frequency part of a wave packet of the form (4.6) with $\Delta\omega \ll \kappa$, we break up the packet $P_{\bar{\omega}\bar{u}lm}$ into a large number of pieces, defining

$$P_{\bar{\omega}\bar{u}lm} = \sum_{j=0}^{N-1} p_j, \tag{5.2}$$

$$p_j = \mathcal{N} \int_{\bar{\omega}+j\Delta\omega/N}^{\bar{\omega}+(j+1)\Delta\omega/N} d\omega\; B^{-1}_{\omega l}\, \exp(i\omega\bar{u})\, p_{\omega lm}, \tag{5.3}$$

and the corresponding transmitted packets t_j. The $\{p_j\}$ (and the $\{t_j\}$) are an orthogonal (but non-normalized) set of wave packets, of the type used in Hawking's original derivation when N is chosen large enough so that $\Delta\omega/N \ll \kappa$. [Note that for such large N, $B^{-1}_{\omega l}$ does not vary much over the range of integration in (5.3) and can be pulled out of the integral.]

Each packet t_j has a width of order $\Delta u \sim N/\eta\kappa$, and therefore contains r-frequency components in the ratio $\omega^{(r)}_{\rm max}/\omega^{(r)}_{\rm min} \sim e^{N/\eta}$ [see Eq. (4.10)]. Nevertheless, the full wave packet $T_{\bar{\omega}\bar{u}lm}$ contains only r frequencies in the range (4.3); the other r-frequency components in the t_j's must cancel in the sum (5.2), since the sum gives a much more localized wave packet (which suffers much less differential redshift). It is important to stress that although we work with the packets t_j as a technique to evaluate the right-hand side (RHS) of (5.1) we do not attribute any direct physical significance or quantum state to them.

Since extracting the negative frequency part is a linear operation we have

$$\langle T^{(-,r)}_{\bar{\omega}\bar{u}lm}, T^{(-,r)}_{\bar{\omega}\bar{u}lm}\rangle = \sum_{j,k}\langle t^{(-,r)}_j, t^{(-,r)}_k\rangle. \tag{5.4}$$

To evaluate the KG inner products $\langle t^{(-,r)}_j, t^{(-,r)}_k\rangle$ we would like to make use of the expression (D3b) for $t^{(-,r)}_j$ as a linear combination of t_j and the "time reflected" packet $\tilde{t}_j$. That is, we would like to use the formula

$$t^{(-,r)}_j = c_-(e^{-\pi\omega_j/\kappa} t_j + \tilde{t}_j), \tag{5.5a}$$

where

$$c_- = e^{-\pi\omega_j/\kappa}(e^{-2\pi\omega_j/\kappa} - 1)^{-1} \tag{5.5b}$$

and

$$\omega_j = \bar{\omega} + j\Delta\omega/N. \tag{5.5c}$$

Now this expression for $t^{(-,r)}_j$ was derived in Appendix D assuming that the wave packet t_j is squeezed close to the horizon. However, although $T_{\bar{\omega}\bar{u}lm}$ is squeezed close to the horizon, the individual wave packets t_j may not be, since their width Δu is much larger than that of $T_{\bar{\omega}\bar{u}lm}$.

Fortunately this is not a problem, for the following reason. Since the KG norm is conserved, we can choose to evaluate (5.1) on an earlier surface $v < v_c$, on which not only $T_{\bar{\omega}\bar{u}lm}$ but also all the t_j are squeezed close to the horizon. Moreover, the negative r-frequency part of $T_{\bar{\omega}\bar{u}lm}$ at $v = v_c$ evolves to the negative r-frequency part at $v < v_c$. This is because $T_{\bar{\omega}\bar{u}lm}$ is a function of r only through u in this region. Since $u = v - 2r - 4M\ln(\frac{r}{2M} - 1)$, a shift in v is equivalent to a scaling of r near the horizon (where the logarithm is dominant) by a linear transformation $r \to ar + b$, which leaves the negative r-frequency part unchanged. This means we can evaluate the RHS of (5.1) at a surface upon which the t_j *are* sufficiently sqeezed to justify use of the formula (5.5a).

The cross terms in the sum (5.4) vanish, since $\langle t_j, t_k\rangle =$

$\langle \widetilde{t}_j, \widetilde{t}_k \rangle = \langle t_j, \widetilde{t}_k \rangle = 0$ for $j \neq k$. The diagonal terms are given by the result (D5), so we have finally

$$\langle N(P_{\bar{\omega}\bar{u}lm}) \rangle = \sum_j \langle t_j, t_j \rangle [\exp(2\pi\omega_j/\kappa) - 1]^{-1}. \quad (5.6)$$

This is just what the expected occupation number would be for a wave-packet mode of the form (4.6) [equivalently (5.2)] emitted from a body at temperature $\kappa/2\pi$ with absorption coefficients $\langle t_j, t_j \rangle$ for the component wave packets p_j.

VI. PHYSICS OF THE BOUNDARY CONDITION

In this section we take up the question of whether there is any way to argue that the boundary condition is in fact satisfied. Recall that because of the gravitational redshift there is no way, within a cutoff theory, to *derive* the quantum state of the high frequency outgoing modes just outside the horizon. The natural expectation would be that they will be in their "free-fall" ground state, because from their point of view, there is nothing special about the horizon and they are merely propagating along just as they would in flat spacetime. The problem with this line of reasoning is that it ignores the very question we are trying to address: does the fact that these modes have been redshifted down from physics above any cutoff scale leave an imprint on their quantum state?

A. Is this a one-scale problem?

Together with the presence of the horizon, the absence of any scale other than the size of the black hole is really the essence of the Hawking effect. One can almost deduce the Hawking result from the assumption that the boundary condition introduces no length scale other than the Schwarzschild radius into the problem. In our form, the boundary condition states that field modes near the horizon with r frequencies satisfying $\omega_c > \omega^{(r)} \gg M^{-1}$ are in their ground state. (Since we impose this boundary condition for all times, no condition need be imposed on modes with $\omega^{(r)} > \omega_c$.) This ground state is a pure state, however the state of every mode outside the horizon is correlated to that of another mode inside the horizon. When only the field outside is accessible, there is missing correlation information. An observer far from the hole can never determine the state of the modes inside the horizon, so the relative phases of the states of all those outgoing modes at infinity that emerged from the region of the horizon are completely unknown. The state thus cannot be a pure state, but is rather one in which the missing information must be *maximized* in some sense. A maximum entropy state is a thermal one, so the state of the outgoing modes should appear thermal (modulo absorption coefficients) far from the hole. Since the cutoff ω_c plays no quantitative role in the problem as formulated, the only scale is M, so the temperature must be proportional to $1/M$. Calculation shows it to be $T_H = 1/8\pi M$.

In the formulation where the boundary condition is imposed in the asymptotic past, the insensitivity of the black hole radiation to the details of the inital state before the hole forms follows from the nature of the horizon as an infinite redshift surface: the more time passes, the higher the frequency of the relevant ingoing modes. In the limit of infinite time, all that matters is the fact that the infinitely high frequency modes are assumed to be initially in their ground state.

But what if one does not *assume* that physics is invariant under infinite blueshifting of scale? If there is new physics at some short-distance scale, whether it be the Planck scale or something longer, then the gravitational redshift may lead to a *communication* from short- to long-distance scales outside the horizon. That is, *the redshift effect leads to a breakdown of the usual separation of scales.*

Thus it seems perfectly possible that the quantum state of the outgoing field modes near the horizon might *not* be the ground state. The precise state of these modes could reflect details of physics at much shorter distances. For instance, there may be amplitudes for the excited states that could only be calculated from a knowledge of the short-distance theory. If this is the case, then the spectrum of black hole radiation may be quite different from that deduced by Hawking.[11] For example, if one of these modes were to emerge at the cutoff in an excited state, then the emission in that mode would be a combination of the spontaneous Hawking radiation, the stimulated emission, and the original excitation.[12] Thus the flux of energy at infinity would be greater than the Hawking flux.

B. Constraints on the stress-energy tensor

In this subsection we will analyze the implications for the stress-energy tensor of a violation of the ground state boundary condition near the horizon. The goal is to determine what restrictions energy considerations may place on the form of the quantum state of the outgoing modes near the horizon. If the components of $\langle T_{\mu\nu} \rangle$ in the free-fall frame become too large, then neglect of the back reaction is unjustified. I see no reason in principle why this may not happen in actuality. It may be that, in fact, the problem of quantum fields interacting with gravity in a black hole spacetime defies treatment which neglects the back reaction or which treats it as a small perturbation that produces only slow evaporation of the black hole mass. However, if this is the case, then the (static) method of analysis used in this paper is inapplicable.

[11] This has nothing to do with the fact that for interacting fields, the spectrum of black hole radiation will reflect the dressing and decay of the interacting particle states. Rather, we are referring to a difference in the state of the high frequency modes, before the interactions have had their effect.

[12] Stimulated emission by black holes is analyzed in Ref. [21].

Under what conditions can the back reaction be treated as a small perturbation? From the semiclassical Einstein equation $G_{\mu\nu} = 8\pi l_P^2 \langle T_{\mu\nu}\rangle$, we infer that the back reaction will be small provided the stress tensor components in the free-fall frame near the horizon are small compared with l_P^{-2} times the typical curvature components there, i.e.,

$$\langle T_{\mu\nu}\rangle \ll 1/l_P^2 M^2. \tag{6.1}$$

In the Unruh or Hartle-Hawking states, one has $\langle T_{\mu\nu}\rangle = O(M^{-4})$ in the free-fall frame near the horizon; hence, in that state the back reaction is very small indeed as long as the hole is much larger than Planck size. In fact, one must increase the stress tensor by a factor of order $(M/M_P)^2$ before the back reaction becomes more than a small perturbation. This leaves a lot of leeway in the form of the state near the horizon, and demonstrates that, even within the approximation that treats the back reaction as a small perturbation, there is no particular reason why the ground state boundary condition at the horizon should hold.

This leeway in the state at the horizon does not necessarily mean that the black hole flux would differ significantly from the Hawking flux however. The reason is that the energy carried by outgoing modes near the horizon is vastly redshifted by the time they make it out far from the hole. In order to make a significant difference in the flux at infinity, an excited outgoing mode near the horizon must have a very high energy with respect to the free-fall frame.

To obtain a very crude estimate of the energy density associated with such an excited mode, consider a wave packet that far from the hole is centered on a frequency ω with a width $\Delta\omega \sim \omega$ and a spread in retarded time $\Delta u \sim \omega^{-1}$. Suppose this mode is occupied in a one-particle state near the horizon at some r. As discussed in Sec. IV C, its energy relative to the free-fall frame will be roughly $(1-\frac{2M}{r})^{-1}\,\omega$, and the proper volume of the thin spherical shell containing it will be roughly $(1-\frac{2M}{r})\,\omega^{-1}M^2$ (since it has a thickness $\Delta u \sim \omega^{-1}$ at infinity). Thus the energy density will be roughly $(1-\frac{2M}{r})^{-2}\,\omega^2 M^{-2}$.[13] If this mode is followed all the way back to the horizon, the energy density diverges, and the neglect of the back reaction is totally unjustified. If on the other hand the mode is followed only back to the value of r for which the r frequency is equal to the cutoff ω_c, then one has $(1-\frac{2M}{r})^{-1} \sim \omega_c/\omega$, and the energy density is $1/l_c^2 M^2$. Note that this result is independent of ω, even though the extra power emitted $\omega/\Delta u \sim \omega^2$ is not.

Now if the cutoff represents not just an arbitrary scale beyond which we are pleading ignorance, but is rather a physical scale at which the nature of propagation might fundamentally change, then it might make sense to halt the backward-in-time propagation when the r frequency reaches ω_c. Let us entertain this possibility.

Suppose then that the energy density near the horizon due to the presence of an extra particle in the black hole radiation is given by $1/l_c^2 M^2$ as suggested by the above computation. More extra particles would just multiply this by the number of particles, irrespective of their frequency.[14] (Note however that in order not to overcount the degrees of freedom the independent modes should be spaced in frequency by the spread adopted above, $\Delta\omega \sim \omega$.) Similarly, for each particle *missing* from the Hawking flux, one expects a *negative* contribution to the energy density of the same magnitude.

Now if the back reaction is a large effect, then our analysis on the static black hole background is actually not correct. We see no reasoning by which this scenario can be ruled out, but we can say nothing more about it. If on the other hand the back reaction is small, then at least one of the following must be true: (1) The outgoing modes have only small amplitudes to be not in their ground state; (2) there is near-perfect cancellation between the energy densities due to "overoccupied" and "underoccupied" modes; (3) the cutoff length l_c is much longer than the Planck length.

It is not even entirely clear that (1) is consistent with a small back reaction, since it only implies a small *expectation value* for the energy density, but still allows *fluctuations* of order $1/l^2M^2$. It would seem to require a full quantum theory of gravity to determine whether or not the back reaction could really be neglected in such circumstances. While (2) cannot be ruled out, it seems somewhat implausible, since there is no apparent reason for such cancellation to occur. Also (3) does not seem very likely, since there is currently no evidence of any fundamental length scale other than the Planck length. Nevertheless, let us just accept these as the logical possibilities that they are. Is there any further difficulty with such a scenario of deviation from the Hawking spectrum maintaining small back reaction?

If the *spectrum* of radiation is different, but the *luminosity* is the same as the Hawking luminosity, then there must be cancellations of positive and negative energy contributions, as mentioned in item (2) above. Although this scenario does not seem likely, there seems to be no way to rule it out. The possibility that the net luminosity differs from the Hawking luminosity appears to be somewhat constrained however by general properties of the stress energy tensor if the back reaction is to remain small.

As first shown in the 1970s [4, 5], given some relatively "theory-independent" constraints on the behavior of the stress-energy tensor one can derive a formula for the net radiation flux far from a (quasi)static black hole. These constraints are $\langle T_{\mu\nu}\rangle^{;\nu} = 0$, $\langle T_{\mu\nu}\rangle$ is nonsingular on

[13] As discussed in Sec. IV C, the finite width of the wave packet leads to a differential redshift across the packet, so this simple analysis is too crude to produce reliable numerical coefficients.

[14] The estimated energy density breaks down however if the frequency is too low, because $\Delta u \sim \omega^{-1}$ will become so broad that the differential redshift across the wave packet totally invalidates the assignment of a particular r frequency to the packet near the horizon.

and outside the horizon (in regular coordinates), $\langle T_{\mu\nu}\rangle$ is static and spherically symmetric (in four dimensions), and no radiation is incoming from infinity at late times. If all these properties hold then it can be shown [5] that the luminosity L of the black hole is given in two spacetime dimensions by

$$L = \frac{1}{2} M \int_{2M}^{\infty} dr\, r^{-2} \langle T^{\alpha}_{\alpha}\rangle \qquad (D=2) \tag{6.2}$$

and in four dimensions by

$$L = 2\pi\, M \int_{2M}^{\infty} dr\, \langle T^{\alpha}_{\alpha}\rangle + 4\pi \int_{2M}^{\infty} dr\, (r-3M)\, \langle T^{\theta}_{\theta}\rangle \qquad (D=4). \tag{6.3}$$

Let us consider first the two-dimensional case. Then the luminosity is determined entirely by the trace of the stress-energy tensor. If we consider a conformally coupled massless scalar field, the trace is determined in a state-independent manner by the trace anomaly to be $\langle T^{\alpha}_{\alpha}\rangle = R/24\pi$ where R is the Ricci scalar. Putting this in (6.2) yields the Hawking flux $L_H = 1/768\pi M^2$.

Any deviation from the Hawking flux for a conformally coupled field in two dimensions thus implies that at least one of the properties of the stress tensor assumed above must fail to hold. It seems that the most questionable assumption is that of the value of the trace. But what would be the physical basis for a deviation from the usual trace anomaly formula?

It was argued in [6] that the presence of a high frequency cutoff ω_c is only likely to affect the value of the trace by terms of order $O(R/\omega_c^2)$. This argument was based on the assumption that the origin of the quantum violation of conformal invariance can be located entirely in the regulated functional measure in the manner of Fujikawa [22]. If correct this implies that the corrections to the trace (and to the Hawking flux) are very small indeed for holes much larger than the cutoff length. However, if there is fundamentally new physics at the cutoff scale, then the violation of conformal invariance will not be due simply to the noninvariance of the regulated functional measure. This opens up the possibility of a more significant deviation from the usual trace anomaly. Nevertheless, the fact that the usual trace anomaly is state independent (assuming the state has the usual short-distance form down to some cutoff much smaller than the radius of curvature of the spacetime) suggests strongly that no significant deviation from the usual trace would occur. Thus, at least in this two-dimensional model, it is hard to see how the flux could differ from the Hawking flux and still have a small back reaction.

In the four-dimensional case (6.3) the situation is perhaps different. For a conformally coupled field the trace is still determined by the trace anomaly, and is given by

$$\langle T^{\alpha}_{\alpha}\rangle = \beta C^2/48 = \beta M^2/r^6, \tag{6.4}$$

where $60\pi^2\beta = 1,\ \frac{7}{4},\ 33/60\pi^2$ for fields of spin 0, 1/2, and 1, respectively, and C^2 is the square of the Weyl tensor. Now however, the trace does not suffice to determine the luminosity. One free function of r remains undetermined. The reason is that, unlike in two dimensions where all metrics are conformally flat, the Schwarzschild spacetime is *not* conformally flat, so even a conformally coupled field scatters in a nontrivial way. Both the spin of the field and the detailed radial dependence of the metric affect the radial dependence of $\langle T_{\mu\nu}\rangle$ and the net flux at infinity. Numerical computations [23] show, for example, that for a massless, minimally coupled scalar field in the Unruh vacuum in Schwarzschild spacetime, the contribution of the second integral to (6.3) is relatively small, and the Hawking luminosity is of order $L_H \sim (4800\pi M^2)^{-1} \sim 10^{-4} M^{-2}$.

A deviation from the Hawking luminosity could be produced, as in the two-dimensional case, by a deviation from the usual trace anomaly, however the same arguments as given in that case make this seem unlikely. But in four dimensions there is another possibility: Any change in the tangential stress $\langle T^{\theta}_{\theta}\rangle$ will entail a change in the luminosity L, without violating the above assumptions on the behavior of $\langle T_{\mu\nu}\rangle$. Can this be exploited to allow for a deviation from the Hawking luminosity? While it is not clear why there should be any fundamental difference between the two- and four-dimensional cases with regard to the possibility of deviating from the Hawking radiation, let us just take the result (6.3) and see what can be done with it.

Note first that the r dependence of $\langle T^{\theta}_{\theta}\rangle$ has a lot to do with the scattering behavior of fields propagating in the Schwarzschild geometry. Thus at most, we should think of the possibility of freely modifying $\langle T^{\theta}_{\theta}\rangle$ at one point, letting the behavior everywhere else be determined by the scattering off the background geometry. A change in the luminosity of order δL could be produced, consistent with (6.3), in two qualitatively different ways: (i) a change $\delta T^{\theta}_{\theta} = O(\delta L/M^{-2})$ over a range $\delta r \sim M$ or (ii) a very large change $\delta T^{\theta}_{\theta}$ over a very small range of r near the horizon. The second way seems inconsistent with the scattering behavior of the field, since the effective potential that governs the scattering is well behaved near the horizon. The first way requires only a relative change $\delta T^{\theta}_{\theta}/T^{\theta}_{\theta}$ of order unity to change the luminosity by order unity. Thus there seems to be no obstacle to the physics at the cutoff scale leading to a deviation from the Hawking luminosity, even if the back reaction is to remain small.

VII. CONCLUSION

What has been accomplished in this paper? We have succeeded in formulating a derivation of the Hawking effect (for massless free fields) that avoids reference to field modes above some cutoff frequency in the frame of the free-fall observers that are asymptotically at rest. To stay below the cutoff it is necessary to impose a boundary condition on the field near the horizon *for all times*. The boundary condition states roughly that the outgoing high frequency field modes are in their "ground state" as viewed by free-fall observers. This boundary condition is not derivable from the initial state within the cutoff theory.

The precision of our derivation is controlled by the ratio of the cutoff length to the Schwarzschild radius of the black hole, and is limited by $\sqrt{l_c/M}$. For a black hole large compared with the cutoff length, the largest source of imprecision is the unavoidable spread of the wave packets employed, and the associated large differential in the redshift suffered across the packet when it is near the horizon.

The boundary condition we impose may or may not be physically the correct one. If it fails to hold, then there will be a deviation from the Hawking spectrum. It seems this could occur either with or without a large back reaction. If the back reaction is to remain small, then either the deviation must be small, or there must be cancellation between positive and negative energy contributions, or there must be a physical cutoff much longer than the Planck length. In four dimensions, the generally expected behavior of the stress tensor cannot be used to definitively rule out any of these scenarios.

Even a very small deviation from the thermal nature of the Hawking radiation would seem to entail a breakdown in the generalized second law of thermodynamics [24–26]. Thus one has reason to suspect that the physics at the cutoff scale somehow conspires to produce precisely the "thermal" state. However, that is not to say that the ordinary effects of quantum field propagation in the black hole background should not leave their mark on the radiation. The scattering of wave packets by the geometry is one well-known aspect of this mark, but it is conceivable that the redshifting of the physics at the cutoff is another one. If that is the case, then the thermodynamic behavior of physics in a black hole spacetime may turn out to be much more subtle than was previously thought.

Given a candidate theory with a short-distance cutoff, it will certainly be interesting to study its behavior in a black hole spacetime, in which the redshift effect acts as a microscope to reveal consequences of short-distance physics at larger scales.

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APPENDIX A: BLACK HOLE LINE ELEMENT

The static, spherically symmetric black hole line element in Schwarzschild, tortoise, double-null, and ingoing Eddington-Finkelstein coordinates takes the form

$$ds^2 = (1 - \tfrac{2M}{r})dt^2 - (1 - \tfrac{2M}{r})^{-1}dr^2 - r^2 d\Omega^2 \quad \text{(A1a)}$$

$$= (1 - \tfrac{2M}{r})(dt^2 - dr^{*2}) - r^2 d\Omega^2 \quad \text{(A1b)}$$

$$= (1 - \tfrac{2M}{r})du\, dv - r^2 d\Omega^2 \quad \text{(A1c)}$$

$$= (1 - \tfrac{2M}{r})dv^2 - 2\, dv\, dr - r^2 d\Omega^2, \quad \text{(A1d)}$$

with

$$r^* = r + 2M \ln(\tfrac{r}{2M} - 1), \quad \text{(A2a)}$$

$$u = t - r^*, \quad v = t + r^*. \quad \text{(A2b)}$$

The coordinates u and v are called the *retarded* and *advanced* time coordinates, respectively.

If $x^\mu(\lambda)$ is an affinely parametrized geodesic, then it is a stationary point of the integral $\int g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu d\lambda$, where the dot $\cdot = d/d\lambda$. To see that r is an affine parameter along ingoing radial null geodesics, it is convenient to use the Eddington-Finkelstein coordinates, so that $\dot{v} = \dot{\theta} = \dot{\phi} = 0$. Upon varying $v(\lambda)$, one immediately finds $\ddot{r} = 0$, so $r = a\lambda + b$ for some constants a and b.

APPENDIX B: KLEIN-GORDON INNER PRODUCT

The Klein-Gordon inner product $\langle f, g\rangle$ between two initial data sets f and g on a Cauchy surface Σ is defined by

$$\langle f, g\rangle = \int j^\mu \, d\Sigma_\mu, \quad \text{(B1a)}$$

$$j^\mu = \tfrac{i}{2}\sqrt{-g} g^{\mu\nu}(f^* \partial_\nu g - g\partial_\nu f^*). \quad \text{(B1b)}$$

The surface element $d\Sigma_\mu$ is given by

$$d\Sigma_\mu = \tfrac{1}{6}\epsilon_{\mu ijk}\, d\sigma^i d\sigma^j d\sigma^k, \quad \text{(B2)}$$

where σ^i $(i = 1, 2, 3)$ are coordinates on the surface Σ. For solutions of the KG equation of compact support, (B1a) is independent of the Cauchy surface on which the integral is evaluated, since the current vector density j^μ is divergence free, $\partial_\mu j^\mu = 0$. We shall have occasion to evaluate the KG inner product on a surface that is null, which can be thought of as a limiting case of Cauchy surfaces.

APPENDIX C: QUANTUM FIELD THEORY

The field operator Φ for a real, free scalar field is a Hermitian operator that satisfies the wave equation $\nabla^2\Phi = 0$. We define an annihilation operator corresponding to an initial data set f on a surface Σ by

$$a(f) = \langle f, \Phi\rangle_\Sigma. \quad \text{(C1)}$$

If the data f is extended to a solution of the wave equation then we can evaluate the KG product in (C1) on whichever surface we wish. The Hermitian adjoint of $a(f)$ is called the creation operator for f and it is given by

$$a^\dagger(f) = -\langle f^*, \Phi\rangle_\Sigma. \quad \text{(C2)}$$

The commutation relations between these operators follow from the canonical commutation relations satisfied by the field operator. The latter are equivalent to

$$[a(f), a^\dagger(g)] = \langle f, g\rangle, \quad \text{(C3)}$$

provided this holds for all choices of f and g. Now it is clear that only if f has positive, unit KG norm are the appellations "annihilation" and "creation" appropriate for these operators. From (C3) and the definition of the KG inner product it follows identically that we also have the commutation relations

$$[a(f), a(g)] = -\langle f, g^*\rangle, \quad [a^\dagger(f), a^\dagger(g)] = -\langle f^*, g\rangle. \quad \text{(C4)}$$

A Hilbert space of "one-particle states" can be defined by choosing a decomposition of the space S of complex initial data sets (or solutions to the wave equation) into a direct sum of the form $S = S_p \oplus S_p{}^*$, where all the data sets in S_p have positive KG norm and the space S_p is orthogonal to its conjugate $S_p{}^*$. Then all of the annihilation operators for elements of S_p commute with each other, as do the creation operators. A "vacuum" state $|\Psi\rangle$ corresponding to S_p is defined by the condition $a(f)|\Psi\rangle = 0$ for all f in S_p, and a Fock space of multiparticle states is built up by repeated application of the creation operators to $|\Psi\rangle$.

[Instead of thinking of the Hilbert space as the Fock space corresponding to some decomposition $S_p \oplus S_p{}^*$ as above, it is perhaps conceptually preferable to take the point of view of the algebraic approach to quantum field theory [27], according to which a "state" is simply a positive linear functional ρ on the $\star$ algebra of field operators. Thus, for example, to express the idea that a given field mode f is in its ground state, one says that the state ρ satisfies $\rho(\mathcal{O}a(f)) = 0$ for all operators $\mathcal{O}$. This language is preferable if, as is often the case for quantum fields in curved space, one wishes to simultaneously consider a state as an element of two completely differently constructed (for example, "in" and "out") Fock spaces. In the algebraic approach, no mysterious "identification" of the two Fock spaces is required. Another advantage is that whereas the statement that the field operator is "Hermitian" is meaningless until the Hilbert space on which it acts has been specified, the statement that Φ goes into itself under the abstract $\star$ operation is always well defined. The algebraic approach is clearly preferable in contexts (e.g., [27, 28]) in which one wishes to obtain results valid for a class of quantum states that is as wide as possible.]

APPENDIX D: NEGATIVE FREQUENCY PART OF THE TRANSMITTED WAVE PACKET

Consider the transmitted part $t_{\bar\omega\bar u}$ of a wave packet $p_{\bar\omega\bar u}$ propagating in the Schwarzschild black hole spacetime, narrowly peaked in u frequency about $\bar\omega$ (at large r) and about some late retarded time $\bar u$. For the original Hawking argument one needs to determine the KG norm of the negative frequency part of $t_{\bar\omega\bar u}$ at $\mathcal{I}^-$ in terms of the norm of $t_{\bar\omega\bar u}$ itself. For our argument in this paper, it is the negative r-frequency part on a constant v surface that is of interest. It was Hawking's original argument that these two are related, using the geometrical optics approximation to propagate the very high frequency modes in question back out to $\mathcal{I}^-$.

Consider a collection of null surfaces, wave fronts for such a mode. In Fig. 2 one surface is shown that is outgoing at retarded time u and ingoing at advanced time $v(u)$. The key fact is that for late retarded times, the value of the affine parameter $r(u, v_c)$ where this wave front intersects the surface $v = v_c$ is linearly related to the advanced time $v(u)$.[15] Therefore the negative r-frequency part of $t_{\bar\omega\bar u}$ at $v = v_c$ propagates back to the negative v-frequency part at $\mathcal{I}^-$. Thus the corresponding KG norms are identical, so in both cases we can carry out the calculation at $v = v_c$.

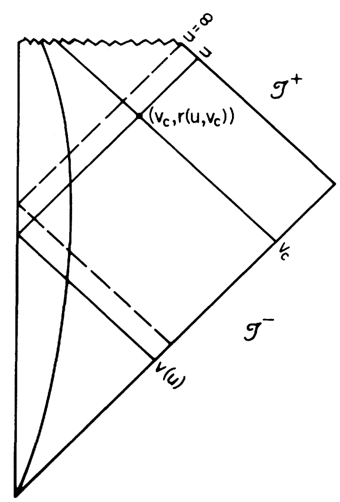

FIG. 2. Conformal diagram depicting the propagation of a wave front. The point $(v_c, r(u, v_c))$ is connected by a radial null geodesic to a point on $\mathcal{I}^-$ at advanced time $v(u)$. The affine parameter r along the line $v = v_c$ is linearly related to $v(u)$ for late retarded times u.

Now there is an observation [8, 9] that makes the extraction of the negative frequency part simple: let U be defined by $\kappa u = -\ln(-\kappa U)$, and consider the functions q and $\widetilde{q}$, defined by

$$q(U) = \begin{cases} e^{-i\omega u} & \text{for } U < 0, \\ 0 & \text{for } U > 0, \end{cases} \quad \text{(D1)}$$

and

$$\widetilde{q}(U) = q(-U). \quad \text{(D2)}$$

That is, $\widetilde{q}$ is just the function q reflected over the line $U = 0$ ($u = \infty$). Then one can easily show that the

[15] Hawking argued that this is because as one goes from (u, v_c) back along the wave front and out to $\mathcal{I}^-$, the "vector" that connects the wave front to the horizon (and earlier to the null ray that becomes the generator of the horizon) is parallel transported into itself. This is not actually correct, since the connecting vector satisfies not the parallel transport equation but the geodesic deviation equation. Nevertheless, one still obtains a finite linear scaling of the connecting vector, which is all that is required for the argument [29].

functions

$$q^{(+)} = c_+(q + e^{-\pi\omega/\kappa}\widetilde{q}) \quad \text{(D3a)}$$

and

$$q^{(-)} = c_-(e^{-\pi\omega/\kappa}q + \widetilde{q}) \quad \text{(D3b)}$$

are pure positive and negative U-frequency packets, respectively. One can solve for the normalization factors c_+ and c_- by setting $q = q^{(+)} + q^{(-)}$. This yields

$$c_- = -e^{-\pi\omega/\kappa}\, c_+, \quad \text{(D4a)}$$

$$c_+ = (1 - e^{-2\pi\omega/\kappa})^{-1}. \quad \text{(D4b)}$$

Finally, the KG norm of $q^{(-)}$ is calculated from (D3b) and (D4) using $\langle\widetilde{q},\widetilde{q}\rangle = -\langle q,q\rangle$ and $\langle q,\widetilde{q}\rangle = 0$, yielding

$$\langle q^{(-)}, q^{(-)}\rangle = -\langle q,q\rangle(e^{2\pi\omega/\kappa} - 1)^{-1}. \quad \text{(D5)}$$

The preceding calculation is directly applicable to the wave packet $t_{\bar{\omega}\bar{u}}$, squeezed near the horizon on the surface $v = v_c$. The relation between r and u along $v = v_c$ is given by (A2), $u = v_c - 2r^* = v_c - 2r - 4M\ln(\frac{r}{2M} - 1)$. For our wave packet near the horizon, the spread in r is very small compared with $2M$, so the wave packet only has support where one has $\kappa u \simeq -\ln(\frac{r}{2M} - 1) +$ const. Thus U and r are linearly related (via $-\kappa U = \frac{r}{2M} - 1$), so the negative r-frequency part $q^{(-,r)}$ is equal to the negative U-frequency part $q^{(-)}$ (D3b) with $\omega = \bar{\omega}$, provided the packet is sufficiently peaked in frequency about $\bar{\omega}$ ($\Delta\omega \ll \kappa$) so that the expressions (D3) for the positive and negative frequency parts still hold. This is the result used in the text.

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