Nonsingular Lagrangians for two-dimensional black holes

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We introduce a large class of modifications of the standard Lagrangian for two-dimensional dilaton gravity, whose general solutions are nonsingular black holes. A subclass of these Lagrangians have extremal solutions which are nonsingular analogues of the extremal Reissner-Nordström spacetime. It is possible that quantum deformations of these extremal solutions are the end point of Hawking evaporation when the models are coupled to matter, and that the resulting evolution may be studied entirely within the framework of the semiclassical approximation. Numerical work to verify this conjecture is in progress. We point out however that the nonextremal solutions always contain Cauchy horizons, and may be sensitive to small perturbations.

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I. INTRODUCTION

Recently, there has been significant progress in unraveling the mystery which enshrouds the end point of Hawking evaporation of black holes [1]. In particular, we now believe that the simple arguments that appeared to rule out stable remnants as a plausible end point for black hole evaporation are wrong. The context in which these ideas have been developed was that of extremal magnetically charged black holes in the version of gravity (dilaton gravity) which appears in the low-energy limit of string theory [2–5]. It has long been argued that extremal charge black holes might be the natural final state for a black hole that manages to retain its charge in the process of Hawking evaporation. In the case of dilaton gravity, the geometry of the extremal magnetic black hole (shown in Fig. 1) is completely static, horizon-free, and has no singularities at finite points of space. It has the form of an infinite funnel or horn, attached to an asymptotically flat space. The only singularity of the solution is the divergence of the effective coupling an infinite distance down the horn.

Although an external observer with sufficiently coarse resolving power will see such a black hole as a pointlike object, it is not an elementary particle. In a previous publication [3], we have named such objects horned particles or cornucopions. The infinite volume of the horn is a repository for an infinite number of states of quantum fields propagating on the background geometry. It has been argued that states which differ only by excitations localized far down the horn, will be essentially degenerate in Arnowitt-Deser-Misner (ADM) energy. The extremal hole is thus a candidate for the kind of infinitely degenerate remnant which might resolve the information loss paradox of Hawking evaporation. Its large size also allows it to evade the apparent phenomenological problems of infinite production cross sections, and infinite contributions to virtual loops, that usually plague the idea of black hole relics [6].

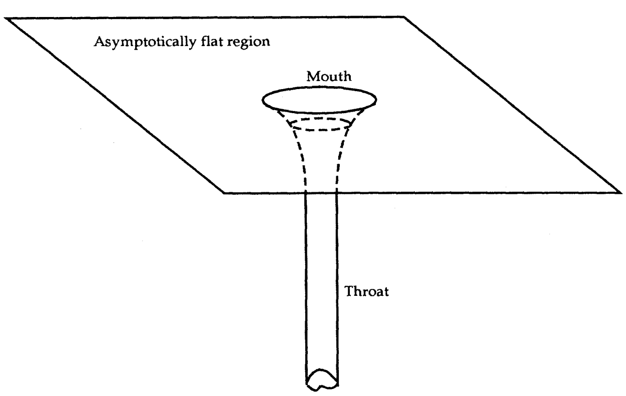

FIG. 1. The spatial geometry of an extremal dilatonic magnetic black hole. The cross sections of the throat are two-spheres.

It is important to emphasize that the conceptual picture that has been built up for magnetically charged black holes is, in general terms, equally applicable to neutral holes. In the charged case, we can find a completely classical picture of the remnant,[1] but one can conjecture that a similar picture (with a bit of quantum fuzz near the region of the classical singularity) might be applicable to neutral holes as well. The idea of geometries in which the Schwarzchild singularity is replaced by an expanding internal universe [7,8] is an old one. Linde has argued that black holes with internal universes appear naturally in the chaotic inflation scenario, and Farhi and Guth proposed creating one in the laboratory [7]. While such a scenario is not consistent with the classical Einstein field equations when matter satisfies the dominant energy condition, it could arise due to quantum effects [8,9]. In many of these discussions, the universe on the other side of the black hole throat is taken to be a de Sitter space. Strominger [10] has argued that nonsingular black hole geometries with an internal de Sitter structure might appear in the solutions of a particular class of semiclassical

[1]This is somewhat illusory. The weak-coupling approximation breaks down deep in the hole.

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equations describing two-dimensional black hole evaporation.

The spacetimes envisaged in these scenarios differ quantitatively but not qualitatively from the cornucopion. The geometry beyond the mouth of the latter can be viewed [5] as the limit of an anisotropically and inhomogeneously expanding four-dimensional universe with the shape of an expanding cigar cross a two-sphere of constant radius. In the spacetimes of the previous paragraph, the internal geometry far from the throat is isotropic and uniformly expanding. The common element in all these scenarios is that black hole formation results in the production of a new asymptotic region of space, separated from the old one by a neck of small size. While such a spacetime can be foliated by spacelike surfaces, and the evolution of quantum fields between these surfaces is unitary, the S matrix of the initial asymptotic region is not. An observer in this region loses information into what are for her the "internal states" of a pointlike remnant. She can construct a sensible quantum mechanics of these remnants without examining their internal structure, as long as she does not study processes in which they are created or annihilated. The latter require her to understand the internal structure of the remnant [6].

While the general picture provided by these model spacetimes is satisfying, a crucial part of the puzzle remains unsolved. To date, no one has demonstrated the collapse of a nonextremal black hole to one of the hypothetical remnants into which it is supposed to evolve. Clearly, such a demonstration will require us to understand physics at a level beyond the classical Einstein theory of gravity, but it is not clear what extension of Einstein's theory is necessary. The essence of the problem is the black hole singularity, and this is widely viewed as a problem having to do with short-distance physics. String theory is the most plausible candidate for a short-distance extension of Einstein's theory, and there have been repeated suggestions that the existence of a fundamental length in classical string theory might eliminate black hole singularities.[2] On the other hand, it has often been argued that the high curvature region near the singularity of a collapsing black hole will be a region where quantum fluctuations are very important. One might hope, for example, that quantum fluctuations in some scalar field near the singularity create a locally large value of the cosmological constant, setting off chaotic inflation inside the black hole, and producing one of the de Sitter remnants described above. While these ideas are alluring, they do not lend themselves to systematic mathematical investigation.

The main advantage of studying extremal dilaton black holes, rather than neutral ones, is that one can plausibly argue [12] that the collapse and evaporation of near extremal dilaton black holes can be entirely described by an effective two-dimensional field theory. In this effective theory, the black hole singularity can (by an appropriate, and string theoretically natural, choice for the conformal frame of the metric) be attributed entirely to the blow up of the coupling constant. One is led to hope that a fully quantum-mechanical solution of the low-energy effective field theory may be all that is necessary for an understanding of the singularity.[3] All extant attempts to solve this problem have relied on the quantum fluctuations of a set of massless degrees of freedom, and treated the metric and dilaton as classical mean fields. This approach has led to the first explicit description of Hawking radiation with back reaction included, but the semiclassical metric becomes singular (now at a finite value of the coupling), and the mean-field expansion breaks down [1].

There are two kinds of corrections to the mean-field description which might become important in the strong-coupling regime. The first are quantum fluctuations of the metric and dilaton, and within the Callan-Giddings-Harvey-Strominger (CGHS) model, these are the only corrections to mean-field theory. It is obviously a problem of great conceptual interest to learn how to treat these fluctuations correctly, but it is also a very difficult problem. If we view the CGHS model as the low-energy effective theory of a more complete Lagrangian, then there are other quantum corrections that might be equally important. These corrections, first discussed in [3], come from integrating out quantum fluctuations of the heavy fields in the full Lagrangian. In effective field theory language, they correspond to relevant and marginal operators in the low-energy theory which have different scaling behavior than the CGHS Lagrangian when the dilaton is shifted by a constant. An l loop contribution will scale like $e^{2l\phi}$. If we treat the graviton and dilaton fields classically, there are an infinite number of relevant and marginal operators that can be added to the Lagrangian, the most general renormalizable Lagrangian having the form [13]

$$\mathcal{L}=\sqrt{-g}\,[D(\phi)R+G(\phi)(\nabla\phi)^2+H(\phi)]\ , \tag{1.1}$$

with

$$D(\phi)\rightarrow\frac{G(\phi)}{4}\rightarrow H(\phi)\rightarrow e^{-2\phi}\ , \tag{1.2}$$

as $\phi\rightarrow-\infty$. In conformal gauge, with conformal factor $e^{2\rho}$, this is a nonlinear model with a two-dimensional Minkowski signature target space. The ρ direction is lightlike [14]. The renormalization group equations for such a model are hyperbolic, and their solutions are determined by the initial data on a surface of constant ρ given by the three functions in (1.1).

It is a formidable task to find the functions D, G, and H that are obtained by integrating out heavy degrees of freedom in a realistic model. In this paper, we will sim-

[2]Witten [11] has argued that his exact classical black hole solution of two-dimensional string theory shows that this conjecture is false. We feel that too little is known about the actual properties of this solution to justify this conclusion.

[3]This is the case in $(1+1)$-dimensional string theory. Although we still lack a proper nonperturbative definition of this theory, it is clear that the singularity of the classical solution with vanishing tachyon condensate is eliminated by quantum mechanics.

ply explore the possibilities, with a view to answering the following question: is it possible for the quantum corrections to the effective action to smooth out the strong-coupling singularity of the CGHS Lagrangian? If this is the case, we might be able to solve the black hole evaporation problem without dealing with the intricate interpretational problems of the quantum theory of gravity. The mean-field approach pioneered by CGHS would be sufficient. Of course, finding a particular Lagrangian which avoids singularities is not very satisfactory. In view of our ignorance of the form of the realistic corrections to the effective Lagrangian, it would be well to find that the presence or absence of singularities was a generic property of large classes of Lagrangians of the form (1.1).

We will show in Sec. II that a very large subclass of the Lagrangians of (1.1) have solutions which are asymptotic to the CGHS black holes for weak coupling, but are completely nonsingular spacetimes which are geodesically complete as the coupling goes to infinity. Thus, for these Lagrangians, the infinite coupling singularity is located at an infinite geodesic distance for any value of the black hole mass. The rather generic occurrence of nonsingular black holes with internal de Sitter asymptotics is interesting, but by itself does not prove that a semiclassical description of Hawking evaporation is possible. We must face another important issue. In four-dimensional Einstein gravity and the CGHS model, the nonsingular "vacuum" solution is the boundary between solutions which have naked singularities and those whose singularity is hidden from the asymptotic region by a horizon. The positive energy theorem holds only for solutions which do not have naked singularities. For many of our nonsingular Lagrangians, there is no qualitative difference between solutions with positive and negative ADM mass. Thus, one might expect that once our models are coupled to dynamical matter fields, so that black holes can radiate, the radiation will go on forever, down to infinitely negative energy.[4] Precisely such a disaster occurs in exactly soluble semiclassical models of black hole evaporation which exploit Strominger's [10] mechanism for avoiding singularities [15].

It turns out that one can obtain nonsingular Lagrangians which have a dichotomy between "positive" mass and "negative" mass solutions. We put the terms positive and negative in quotes because we measure the ADM mass of all of these solutions relative to a certain extremal solution of the equations, rather than to the linear dilaton vacuum of the CGHS model. We believe that this is the correct procedure. The linear dilaton vacuum is not a solution of any of the modified Lagrangians that we have studied. The proper reference point for ADM mass is the stable solution of the equations which we expect to be the end point of Hawking evaporation. We will explain the detailed geometries of these solutions in a more leisurely manner in Sec. III.

In Sec. IV we briefly discuss the semiclassical equations which arise by coupling the models of III to massless CGHS f fields in the leading order of a certain large N expansion. This discussion is preliminary, as we have not yet solved the equations. We conclude by outlining the further steps which must be taken to carry out the analysis which we have begun in this paper.

II. NONSINGULAR LAGRANGIANS AND NEGATIVE ENERGY

In [13] we wrote down the most general renormalizable Lagrangian for the two-dimensional metric and a single scalar field. For a general choice of field variables, it takes the form (1.1). We will require that the coupling functions satisfy the CGHS boundary condition (1.2). The metric in these Lagrangians is, by definition, the stringy metric, which is distinguished by its simple coupling to propagating strings.[5] For purposes of solving the equations of motion, it is convenient to perform a Brans-Dicke transformation to eliminate the G term in the Lagrangian. This can be done in a nonsingular way whenever $dD/d\phi \neq 0$ over the entire range of ϕ. For the CGHS Lagrangian, this criterion is satisfied for any finite value of the string coupling $e^{2\phi}$ but fails when the coupling goes to infinity. As we will see in a moment, it is no accident that this is also the locus of singularities of the classical solutions of this Lagrangian.

The appropriate Brans-Dicke transformation is

$$g_{\mu\nu}^{\mathrm{CGHS}} = e^{2S(\phi)} g_{\mu\nu} , \tag{2.1}$$

where

$$4\frac{dS}{d\phi}\frac{dD}{d\phi} = -G(\phi) . \tag{2.2}$$

If $W \equiv He^{2S}$, then the Lagrangian of the transformed fields is

$$\sqrt{-g}\,(DR + W) . \tag{2.3}$$

It is amusing to note that the field equation for ϕ which follows from this Lagrangian involves no derivatives. Thus, this Lagrangian is equivalent to a higher derivative Lagrangian involving only the gravitational field

$$\mathcal{L}_{\mathrm{HD}} = \sqrt{-g}\,\mathcal{F}(R) . \tag{2.4}$$

Note, however, that the function $\mathcal{F}$ results from inverting the functional relation between ϕ and R given by the field equations, so that it might be multivalued.

To find the general solution of the field equations, we recall our general result [13] that any such solution has a Killing vector, and that the dilaton is constant along the Killing flows. If t is a coordinate along the homogeneous direction, and σ a coordinate orthogonal to it, we can choose σ so that the metric is conformally flat, with conformal factor $e^{2\beta(\sigma)}$, and the dilaton is a function only of

[4]The importance of this potential problem was brought to our attention by A. Strominger.

[5]There is, in fact, an ambiguity in what we mean by the stringy metric arising from renormalization scheme ambiguities in world sheet σ models. This problem arises in higher orders in the string tension expansion and should not effect the low-energy considerations of this paper.

σ. In this coordinate system, the constraint equations reduce to a single equation (dots refer to σ derivatives),

$$\ddot{D}=2\dot{\hat{\rho}}\dot{D} \ , \tag{2.5}$$

while the variational equation for the conformal factor is

$$\ddot{D}=e^{2\hat{\rho}}W \ . \tag{2.6}$$

The first equation is solved by

$$\dot{D}=2\beta e^{2\hat{\rho}} \ , \tag{2.7}$$

where β is a constant. Plugging this back into the second equation we get

$$h(\phi)\equiv\frac{d\phi}{d\sigma}(\phi)=\frac{1}{2\beta D'}\int d\phi\, W(\phi)D'(\phi) \ . \tag{2.8}$$

Here, primes denote derivatives with respect to ϕ. Thus, the solution of ϕ in terms of σ is reduced to quadratures. Note, from (2.8), that generically $\dot{\phi}=0$ at a number of values of σ equal to the number of zeros of $W(\phi)D'(\phi)$, plus one.

The two qualitative features of these solutions that we would like to discuss at this point are the behavior of the solutions at the horizon, and the behavior in the strong-coupling region. In the coordinates in which we are working, an apparent horizon is a point where (ρ is the Liouville field in the CGHS frame) $e^{2\rho}=0$ and $\dot{\phi}=0$. The fact that these two conditions coincide for finite values of ϕ follows from the equation

$$e^{2\rho}=e^{2\hat{\rho}+2\phi}=\frac{D'(\phi)\dot{\phi}e^{2\phi}}{2\beta} \ , \tag{2.9}$$

and our fundamental assumption that the derivative of D vanishes nowhere. When h has a linear zero, the behavior of the solution is exactly the same as that of the standard dilaton gravity black hole. The contours of constant dilaton field change from being spacelike to timelike or vice versa, but the dilaton is monotonic across the horizon. It is also, of course, monotonic in regions where $h\neq 0$. Thus, the sort of "bounce" solutions that appeared in the static quantum equations of [16–18] are not obtained in any of the models we are studying.

To find singularities, we express the curvature of the CGHS metric in terms of our solutions:

$$R_{\rm CGHS}=8e^{-2\rho}\partial_+\partial_-\rho=-4\beta\frac{e^{-2\phi}}{hD'}\ddot{\rho}=-\frac{2\beta e^{-2\phi}}{D'}\left[\frac{h'D''}{D'}+2h'+h''+\frac{D'''h}{D'}-\frac{(D'')^2h}{(D')^2}\right] . \tag{2.10}$$

This is finite at any finite value of ϕ as long as D and W are smooth, and D' does not vanish. In the weak-coupling region $\phi\to-\infty$, R goes to zero because of our requirement (1.2) that the Lagrangian approach that of CGHS. Let us assume that in the strong-coupling region, $D\sim e^{n\phi}$ and $W\sim e^{m\phi}$. Then $h\sim e^{m\phi}$, and barring accidental cancellations, $R_{\rm CGHS}\sim e^{(m-n-2)\phi}$, so that, if $n\geq m-2$, it remains finite when the coupling goes to infinity.

We can also examine the geodesic distance to the infinite coupling region:

$$\int e^{\rho}d\sigma=\int\sqrt{D'h}\ e^{\phi}d\sigma$$
$$=\int\left[\frac{D'}{h}\right]^{1/2}e^{\phi}d\phi\sim\exp\left[\frac{n-m+2}{2}\phi\right] . \tag{2.11}$$

This is infinite whenever $n\geq m-2$. Note, by the way, that the distance to any apparent horizon from a point where ϕ takes on a general finite value is generically finite. Only if $h(\phi)$ has a double zero will it be infinite. This observation will be important in the sequel.

We have thus exhibited a large class of models whose general black hole solution is no more singular than the linear dilaton vacuum of the CGHS action. Indeed, they are less singular. Although the "string coupling" $e^{2\phi}$ becomes infinitely strong asymptotically in our solutions, this no longer signals a region of large quantum fluctuation. Quantum fluctuations in the graviton and dilaton fields are controlled by the value of $1/D'(\phi)$, which is bounded even in the "strong-coupling" region.

To conclude this section, we record a simple set of Lagrangians which generate nonsingular black holes with asymptotically de Sitter interiors. We take $D=e^{-2\phi}-(\gamma^2/n)e^{2n\phi}$ and $W=-2\lambda^2D'(\phi)e^{2\phi}$. This gives (with $\beta=\lambda$, obtained by choosing asymptotically linear dilaton coordinates)

$$h(\phi)=\frac{2\lambda}{1+\gamma^2e^{2(n+1)\phi}}\left[-\frac{1}{2}+\frac{M}{2\lambda}e^{2\phi}+\frac{\gamma^2}{n}e^{2(n+1)\phi}+\frac{\gamma^4}{4n+2}e^{4(n+1)\phi}\right] . \tag{2.12}$$

Since this has only a single real zero, the metric has one horizon. The metric and dilaton are given by

$$\sigma(\phi)=\frac{1}{2\lambda}\int d\phi\left[\frac{1+\gamma^2e^{2(n+1)\phi}}{-\frac{1}{2}+\frac{M}{2\lambda}e^{2\phi}+\frac{\gamma^2}{n}e^{2(n+1)\phi}+\frac{\gamma^4}{4n+2}e^{4(n+1)\phi}}\right] . \tag{2.13}$$

Asymptotically, and in the vicinity of the horizon, we find

$$\sigma(\phi)=\begin{cases} -\dfrac{\phi}{\lambda}-\dfrac{M}{2\lambda^2}e^{2\phi}+\cdots\ , & \phi\to-\infty\ , \\ \dfrac{\alpha}{2\lambda}\ln(\phi-\phi_0)\ , & \phi\to\phi_0\ , \\ -\dfrac{2n+1}{2\lambda\gamma^2(n+1)}e^{-2(n+1)\phi}\ , & \phi\to\infty\ . \end{cases} \tag{2.14}$$

Inverting to obtain the metric and dilaton,

$$\phi(\sigma)=\begin{cases} -\lambda\sigma-\dfrac{M}{2\lambda}e^{-2\lambda\sigma}+\cdots\ , & \phi\to-\infty\ , \\ \phi_0-e^{2(\lambda/\alpha)\sigma}+\cdots\ , & \phi\to\phi_0\ , \\ -\dfrac{1}{2(n+1)}\ln(\sigma_\infty-\sigma)\ , & \phi\to\infty\ , \end{cases} \tag{2.15}$$

$$e^{2\rho}=\begin{cases} 1-\dfrac{M}{\lambda}e^{-2\lambda\sigma}+\cdots\ , & \phi\to-\infty\ , \\ e^{(2\lambda/\alpha)\sigma}\ , & \phi\to\phi_0\ , \\ \dfrac{\text{const}}{(\sigma-\sigma_\infty)^2}\ , & \phi\to\infty\ . \end{cases} \tag{2.16}$$

They exhibit the advertized asymptotic de Sitter behavior in the strong-coupling region.

III. MODELS WITH SEMICLASSICALLY STABLE SOLUTIONS?

Although we have exhibited a general class of effective Lagrangians with nonsingular black hole solutions, we do not think that the results of the previous section constitute a demonstration that we can construct sensible semiclassical models of black hole evaporation. In a sense, we have done our work too well. The models that we have constructed have nonsingular solutions for all values of the ADM mass, *including negative ones.* One suspects that when the black holes of these models are coupled to matter, they will suffer the fate of the models of [15]. Hawking evaporation will proceed forever, leaving behind nonsingular black holes of ever larger negative ADM mass.

To see why we anticipate this disaster, consider the class of models, which are nonsingular according to the criteria of the previous section, and in which $W(\phi)$ has no zeros. $h(\phi)$ then has a single zero, which is a finite geodesic distance from points with finite σ coordinates. This is true for all finite positive and negative values of the ADM mass. The asymptotically timelike Killing vector of the solution is null at this horizon, and becomes spacelike on the other side of it. It is easily seen in the semiclassical approximation that matter propagating in any of these geometries will Hawking radiate. This makes it seem implausible that the equations with back reaction included will have solutions which become asymptotically static. There is no apparent reason for the radiation to turn off.

While this is not a proof that these models suffer the fate of those studied in [15], we were sufficiently convinced of this to search for models in which the horizon moved off to infinity at some finite value of ADM mass. Such models can be constructed by choosing $W(\phi)$ to change sign once in the physical range of ϕ. For the generic solution, $h(\phi)$ then has two zeros, and there is a particular value of the ADM mass for which these zeros coincide. If we impose restrictions on the strong-coupling behavior of D and W which guarantee the absence of singularities, the generic geometry has two half infinite static regions, connected through a pair of horizons, to a homogeneous expanding universe. This is not the complete geometry, however, since there are geodesics which enter into the expanding region and never come back. The second horizon is a Cauchy horizon for the region connected to weak coupling. If we follow the usual procedure of analytically continuing through this Cauchy horizon, we obtain the full Penrose diagram shown in Fig. 2. It has the periodic structure familiar from the Kerr and Reissner-Nordström geometries, but is everywhere nonsingular. Parts of this spacetime can be foliated by spacelike surfaces, but it contains Cauchy horizons. This may mean that most of the structure is unstable to small perturbations.

For the extremal value of the ADM mass, the two horizons coincide, and the extended spacetime now has zero Hawking temperature. The causal structure is identical to that of the $M^2=Q^2$ extremal Reissner-Nordström solution, except that the timelike singularity has been eliminated. To see this explicitly, let us investigate the behavior of the metric near a general zero of h:

$$h(\phi)=-\alpha^2(\phi_0-\phi)^k\ . \tag{3.1}$$

Integrating to get $\sigma(\phi)$, we obtain

$$\sigma=\sigma_0-\frac{(\phi_0-\phi)^{-(k-1)}}{\alpha^2(k-1)}\ ,\quad k\neq1\ ,$$
$$=\sigma_0+\frac{\ln(\phi_0-\phi)}{\alpha^2}\ ,\quad k=1\ . \tag{3.2}$$

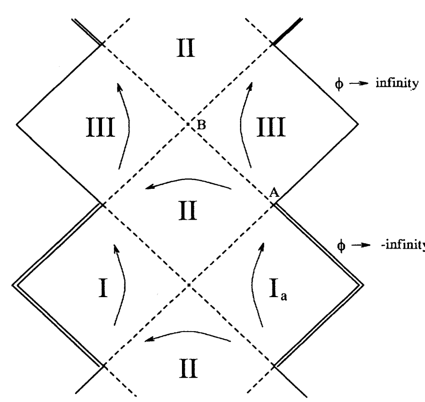

FIG. 2. Extended Penrose diagram of the classical metric for the case of two zeros in h. Regions I and III are static, but region II, is time dependent. The arrows indicate the direction of the Killing vector used to obtain these solutions. The line AB indicates a Cauchy horizon for region I_a. The double line indicates the region that is asymptotically linear dilaton vacuum.

Using (2.7), we can write, for the metric ($k \neq 1$),

$$e^{2\rho}=\frac{hD'e^{2\phi}}{2\lambda}\approx(\sigma-\sigma_0)^{-k/(k-1)}, \quad \sigma\to\sigma_0,\ k<1$$

$$\approx\sigma^{-k/(k-1)}, \quad \sigma\to\infty,\ k>1 . \qquad (3.3)$$

We can see then that, for $k\geq 2$, e^{ρ} cannot be integrated through $\phi=\phi_0$ and so the distance to $\phi=\phi_0$ is infinite, while for $k<2$ the distance to this point is finite. [For $k=1$, $h(\phi)=e^{\alpha^2\sigma}$, $e^{2\rho}\approx e^{\alpha^2\sigma}$, and $\phi\to\phi_0$ as $\sigma\to-\infty$.] The curvature is given by $e^{-2\rho}\partial_\sigma^2\rho$, and using the above expressions is easily seen to be infinite for $1<k<2$ and $k<1$, finite for $k=2$, and zero for $k=1$. Note, in particular, that $k=1$ near $\phi=\phi_0$ corresponds to the behavior at the horizon of the standard dilaton-gravity black hole.

If our effective action really comes from integrating out massive fields, then we expect it to be an analytic function of the dilaton.[6] Thus, $k=1,2$ are the only sensible nonsingular choices. In fact, we expect simple zeros ($k=1$) of h to be generic. If h has a single such zero, then we have the sort of model analyzed in the previous section, which has the potentially disastrous problem of runaway Hawking radiation. If W has one or more simple zeros, then h has multiple zeros. By tuning the one parameter at our disposal, the ADM mass, we expect to find a unique value of ADM mass at which the two zeros nearest to the weak-coupling region coincide. The existence of special solutions, for which the horizons coincide (as in the extremal Reissner-Nordström black hole) is thus generic, as long as $W(\phi)$ has one or more simple zeros. We can see, then, that for $k\geq 2$, e^{ρ} cannot be integrated through $\phi=\phi_0$ and so the distance to $\phi=\phi_0$ is infinite, while for $k<2$ the distance to this point is finite [For $k=1$, $h(\phi)=e^{\alpha^2\sigma}$, $e^{2\rho}\approx e^{\alpha^2\sigma}$, and $\phi\to\phi_0$ as $\sigma\to-\infty$.] The curvature is given by $e^{-2\rho}\partial_\sigma^2\rho$, and using the above expressions is easily seen to be infinite for $1<k<2$ and $k<1$, finite for $k=2$, and zero for $k=1$. *The extremal geometry is thus a nonsingular spacetime with zero Hawking temperature.* Quantum fields placed in such a gravitational field will not give off Hawking radiation.

This observation by itself does not guarantee that the extremal geometry will furnish a satisfactory semiclassical end point for Hawking evaporation. Unlike the linear dilaton solution of the classical CGHS model, the extremal solution has nonzero curvature and will not be a solution of the one-loop-corrected mean-field equations which describe Hawking evaporation. We must enquire whether there is a static solution to these equations with properties similar to the extremal geometry, and whether perturbations of this solution by infalling matter eventually relax back to it. We will set up the equations which must be solved in the next section. We have not yet solved them.

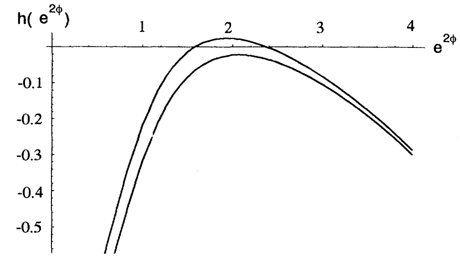

FIG. 3. A plot of $h(e^{2\phi})$ showing the crossover from two to no zeros. The conjectured extremal zero-temperature solution lies at the crossover point.

Before concluding this section, we will work out the details of a particular form of the Lagrangian which has solutions of the type we have discussed. We take

$$D=e^{-2\phi}-\tfrac{1}{2}\gamma^2e^{4\phi} \qquad (3.4)$$

and

$$W=4\lambda^2-\mu e^{4\phi} . \qquad (3.5)$$

Then

$$h=\frac{2\lambda}{1+\gamma^2e^{6\phi}}\left[-\frac{1}{2}+\frac{M}{2\lambda}e^{2\phi}-\frac{\mu}{8\lambda^2}e^{4\phi}+\frac{\gamma^2}{4}e^{6\phi}-\frac{\mu\gamma^2}{32\lambda^2}e^{10\phi}\right], \qquad (3.6)$$

where M is the ADM mass. It is easy to see that the number of zeros of h jumps from 0 to 2 at a critical value of M (see Fig. 3 for an illustration for $\lambda=\gamma=\mu=1$). This is the extremal geometry of this model. The explicit asymptotic formulas for the metric and dilaton at $\phi\to\pm\infty$ [when $h(\phi)$ has a zero the behavior at the zero will be typically of type $k=1$ discussed above, and in the extremal case, $k=2$] are

$$\sigma(\phi)=\frac{1}{2\lambda}\int d\phi\frac{1+\gamma^2e^{6\phi}}{-\frac{1}{2}+\frac{M}{2\lambda}e^{2\phi}-\frac{\mu}{8\lambda^2}e^{4\phi}+\frac{\gamma^2}{8}e^{6\phi}-\frac{\mu\gamma^2}{64\lambda^2}e^{10\phi}} , \qquad (3.7)$$

[6] Unless some heavy degrees of freedom became massless at a particular value of ϕ. The phenomenon would complicate our analysis, and we assume that it does not occur.

giving

$$\sigma(\phi)=\begin{cases}-\frac{\phi}{\lambda}-\frac{M}{2\lambda}e^{2\phi}+\cdots\ , & \phi\to-\infty\ ,\\ \sigma_\infty+\frac{8\lambda}{\mu}e^{-4\phi}\ , & \phi\to\infty\ ,\end{cases} \tag{3.8}$$

and inverting this expression to obtain

$$\phi(\sigma)=\begin{cases}-\lambda\sigma-\frac{M}{2\lambda}e^{-2\lambda\sigma}+\cdots\ , & \phi\to-\infty\ ,\\ -\frac{1}{4}\ln(\sigma-\sigma_\infty)\ , & \phi\to\infty\end{cases} \tag{3.9}$$

and

$$e^{2\rho}=\begin{cases}1-\frac{M}{\lambda}e^{-2\lambda\sigma}+\cdots\ , & \phi\to-\infty\ ,\\ \frac{\text{const}}{(\sigma-\sigma_\infty)^{5/2}}\ , & \phi\to\infty\ .\end{cases} \tag{3.10}$$

This exhibits the features that as $\phi\to\infty$, the curvature goes to zero and the distance to $\phi=\infty$ is infinite.

IV. MEAN-FIELD EQUATIONS

At this point it would be natural to study the classical collapse of matter coupled to our nonsingular versions of dilaton gravity. However, we have not been able to exactly integrate the equations of motion for even the simplest choice of matter fields, massless **f** waves. That being so, we decided to directly tackle the more ambitious problem of incorporating our nonsingular black hole solutions into a framework that allows us to follow the process of Hawking evaporation to its end point. It is difficult to do this in a way which can be justified as a systematic approximation to the dynamics of a four-dimensional dilaton black hole (as we have tried to imagine justifying the modified Lagrangians which we have studied up to this point). The only available technique for studying Hawking evaporation is based on the mean-field equations of CGHS, which may be viewed as the leading term in an expansion in the inverse number of massless matter fields in the two-dimensional effective Lagrangian. The massless CGHS **f** fields will certainly be present in string theory, and for large magnetic charge, there will be many of them. However, in the conventional large N approximation, the string coupling is of order $e^{2\phi}\sim 1/N$, and the modifications that we have made to the CGHS Lagrangian are formally of higher order in $1/N$. Furthermore, we would expect terms of the form $P(\phi)(\nabla\mathbf{f})^2$ to be just as important as those we have included, and we do not know how to evaluate the **f** functional integral in the presence of such terms. Finally, there are low-energy quantum corrections coming from the interaction of the **f** fields with the two-dimensional remnant of the electromagnetic field [19], which are also of the same nominal order in $1/N$ as our modifications of CGHS.

To make progress, we abandon a bit more of our pretense of realism, and view the Lagrangian

$$\mathcal{L}=\sqrt{-g}\left[\frac{N}{\kappa}(DR+W)+(\nabla\mathbf{f})^2\right] \tag{4.1}$$

as a "two-dimensional model field theory". We also abandon the identification of $e^{2\phi}$ with a coupling constant. Indeed, in the context of the effective Lagrangian it is $D'(\phi)$ rather than $e^{2\phi}$ which controls the magnitude of quantum fluctuations of the graviton and dilaton. We can now perform a systematic large N expansion of the theory presented in Eq. (4.1). We use the CGHS metric to regularize the **f** determinant and obtain the mean-field equations (in the conformal gauge)

$$0=D''\partial_+\phi\partial_-\phi+D'\partial_+\partial_-\phi+\tfrac{1}{4}e^{2(\rho-\phi)}W+\kappa\partial_+\partial_-\rho\ , \tag{4.2}$$

$$0=D''\partial_+\phi\partial_-\phi+2D'\partial_+\partial_-\phi-D'\partial_+\partial_-\rho-\frac{W'}{8}e^{2(\rho-\phi)}+\frac{1}{4}We^{2(\rho-\phi)}\ , \tag{4.3}$$

$$T_{\pm\pm}=0=(D''+2D')\partial_\pm\phi\partial_\pm\phi+D'\partial_\pm^2\phi-2D'\partial_\pm\phi\partial_\pm\rho+(\partial_\pm\mathbf{f})^2-\kappa(\partial_\pm\rho\partial_\pm\rho-\partial_\pm^2\rho+t_+)\ , \tag{4.4}$$

$$\partial_+\partial_-\mathbf{f}=0\ , \tag{4.5}$$

where ρ is the CGHS Liouville field ($\rho=\hat{\rho}+\phi$).

The kinetic term in these equations is nonsingular, so long as $2\kappa+D'<0$ for all real values of ϕ. For example, in the model of Sec. III, this requires $\kappa<3(\gamma/2)^{2/3}$. This restriction ensures that for all real values of ϕ, the field space metric of the leading-order large N effective Lagrangian for graviton and dilaton, is nondegenerate. Thus, with this restriction, we do not expect to find the kind of singularities that plague the large N equations of CGHS [3,20].

In [3,20] a signal of the possibility of singularity for generic initial conditions was found by fashioning the static equations into the form

$$e^{-2\rho}\ddot{\rho}\sim\frac{2(\dot{\phi}^2e^{-2\rho}-\lambda^2)}{1-\kappa e^{2\phi}}\ , \tag{4.6}$$

which indicates the possibility of a curvature singularity at $\phi=\frac{1}{2}\ln(\kappa)$. It turned out that the static solutions, quantum kinks of [16,18,17], do not have a singularity at this value of ϕ, they bounce from ϕ just below the critical value and have a weak-coupling singularity. However, the value $\phi=\frac{1}{2}\ln(\kappa)$ is the position of the singularity in solutions representing gravitational collapse. In our system, the corresponding equation is

$$e^{-2\rho}\ddot{\rho}\sim\frac{(W'/2+W)e^{-2\phi}-D''\dot{\phi}^2e^{-2\rho}}{D'+2\kappa}\ . \tag{4.7}$$

This expression exhibits no singular value of ϕ, provided that κ obeys the restriction $2\kappa+D'<0$, for all ϕ.

At the moment, we see no other recourse for understanding the solutions of these equations than numerical work. The equations describing Hawking evaporation are hyperbolic partial differential equations of a type notoriously resistant to accurate numerical analysis. The literature on numerical analysis of the related CGHS equations [16–18,21,22] is full of controversy, rather than consensus. We will attempt to numerically simulate the full nonlinear infall problem, but there are several less ambitious things that can be done as well. We believe that it is reasonably easy to obtain very accurate semianalytical solutions to the static equations. One can then study small fluctuations around these solutions. What we hope to find is that the small fluctuation problem around static solutions with ADM mass larger than the extremal value will contain negative modes corresponding to the Hawking decay of these black holes, while small fluctuations around the extremal solution are stable. This would be evidence that the extremal solution is a basin of attraction for some region of initial conditions with ADM mass near the extremal value. It would show that our candidate remnants are the end point of Hawking evaporation for at least some region of black hole parameters. We are presently engaged in setting up the numerical analysis of these equations and hope to report on its results at an early date.

V. CONCLUSIONS

We have shown that a large class of Lagrangians of the form (1.1) have nonsingular black hole solutions with an infinite internal spacetime hidden behind the horizon. The essential criterion for this to occur is that the kinetic term of the Lagrangian be nonsingular, which requires that the function $D'(\phi)$ is nowhere vanishing. We must also impose certain restrictions on the behavior of the Lagrangian in the asymptotic regions $\phi \rightarrow \pm\infty$. These ensure correspondence with the CGHS model in the weak-coupling regime, boundedness of the curvature in all regions of spacetime, and the infinite extent of the internal universe.

If the potential term H has no zeros, we find nonsingular solutions for all values, both positive and negative, of the ADM mass, with no dramatic change in behavior as the mass is varied. We suspect that when such models are coupled to quantum-mechanical matter fields, they will suffer (at least in the mean-field approximation) from the problem of unending Hawking radiation encountered in [15]. If H has some number of simple zeros, this problem may be avoided. There is then an *extremal* value of the ADM mass for which the horizon nearest to the weak-coupling asymptotic region moves off to infinite distance, leaving behind a nonsingular spacetime with zero Hawking temperature. It is plausible that when these models are coupled to matter, this extremal spacetime (or a slight quantum deformation of it) will be a natural end point for Hawking evaporation.

Much work remains to be done to verify this conjecture. We have not yet even solved the classical equations for infalling matter in these systems, but are instead engaged in a numerical study of the large N equations, which include both infall and back reaction. Preliminary analysis suggests that these equations have static solutions corresponding to nonsingular quantum deformations of the solutions studied in this paper, and that they do not suffer from the singularities discovered in [3,20]. We are quite concerned, however, about another sort of potential singularity. As far as we can tell, all of the spacetimes which arise in models in which H has zeros, have Cauchy horizons. It is widely believed that Cauchy horizons become singular when subjected to small perturbations [23]. This suggests that generic dynamical solutions of the mean-field equations might have singularities. Thus, it is possible that our attempt to find a nonsingular semiclassical description of black hole evaporation may fail.

There is a bright side to this dismal conclusion. Hawking's information paradox was supposed to provide a first insight into the conceptual problems of quantum gravity. Its resolution by the agency of cornucopions, or other large remnants with small throats, is in some ways disappointingly semiclassical. It demonstrates once again[7] that a Hilbert space description of quantum gravity must involve the notion of a changeable number of states. It may also lead to an argument that wormhole processes *must* be included in any sensible theory of quantum gravity.[8] However, the remnant scenario does not seem to require us to understand the mind-boggling prospects of large quantum fluctuations in the geometry of spacetime. Nor does it require us to understand the generalization of geometry provided by string theory. Perhaps our (possible) failure to find an adequate semiclassical description of black hole *singularities* will force us to come to grips with these deep issues.

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[7]The first demonstration was the inflationary universe.

[8]The cornucopion scenario requires the description of the *asymptotic* states of geometry to include disconnected spatial universes. It is plausible that unitarity and locality then require such disconnected universes to appear in intermediate states as well.

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