Geometric phase in vacuum instability: Applications in quantum cosmology

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(Received 6 July 1993)

Three different methods, viz., (i) a perturbative analysis of the Schrödinger equation, (ii) an abstract differential geometric method, and (iii) a semiclassical reduction of the Wheeler-Dewitt equation, relating the Pancharatnam phase to vacuum instability are discussed. An improved semiclassical reduction is also shown to yield the correct zeroth-order semiclassical Einstein equations with back reaction. This constitutes an extension of our earlier discussions on the topic.

PACS number(s): 03.65.−w, 04.60.+n

I. INTRODUCTION

The study of geometric phases [1–3] seems to offer important insights into having a better understanding for a large class of physical problems. In quantum field theory, for instance, the Berry phase appears to play a significant role in elucidating several conceptual issues relating to anomalies and associated problems. It is shown [4,5] that various gauge anomalies can be interpreted as due to a nontrivial holonomy on the second quantized (chiral) fermion Hilbert bundle over background static gauge fields. The nontrivial holonomy arises as a measure of topological obstructions in projecting the Fock vacuum in the physical sector of the gauge manifold (static gauge fields mod local gauge group). This in turn implies a loss of gauge invariance (global and non-Abelian anomalies) and/or an induced symmetry breaking (axial anomaly).

Now the breakdown of the global U(1) axial symmetry via an anomalous divergence of the axial-vector current induces axial baryon-lepton nonconserving processes through the production of massless fermion excitations [6]. Nelson and Alvarez-Gaume [4] have further shown that even the global and non-Abelian anomalies could be explained in terms of pair productions. Although nongeneric, the production occurs at the points of degeneracies of the background field-dependent Dirac Hamiltonian, inducing a twist in the pertinent Hilbert bundle.

Recently some applications of the Berry phase were also discussed [7,8] in the semiclassical gravity in the framework of a minisuperspace cosmological model. An improved Born-Oppenheimer analysis in the Wheeler-Dewitt (WD) equation is shown to yield the correct zeroth-order semiclassical Einstein equations. The functional Schrödinger equation describing quantized matter fields in a background curved space is obtained at the next order of approximation. Further, the semiclassical back reaction of the matter fields is shown to be determined by the U(1) Berry connection on the gravitational sector of the minisuperspace. An interesting consequence emerges in the Robertson-Walker (RW) minisuperspace which is one-dimensional with a trivial R topology. The relevant Hilbert bundle turns out to also be trivial, thereby reducing the induced Berry connection essentially to zero. As a consequence, the WD equation corresponding to a gravitational action without a cosmological Λ term yields at the semiclassical regime a matter Schrödinger equation essentially in Minkowski space [8]. However, for an action with a nonzero Λ, one gets a matter equation in the de Sitter (dS) universe, although a zeroth-order analysis does not yield a suitable back reaction. One, however, expects a finite rate of particle production in the dS background. It is, therefore, of interest to see how the semiclassical Einstein equations with a reasonable back reaction can be obtained through some modifications of the arguments in Refs. [7,8].

It is well known that the particle production in quantum field theory (QFT) in the presence of a classical external field is associated with the vacuum decay, which is essentially a nonperturbative effect. Under the influence of a time-varying external field the otherwise stable initial vacuum evolves into an admixture of multiparticle states, thereby reducing the vacuum transition probability amplitude to a value less than the initially normalized value one.

Now one naturally feels tempted to see if there is some intrinsic relationship between the particle production through vacuum decay and the particle production via symmetry breaking due to an anomaly. It is therefore of interest to look for a description of particle creation through vacuum decay in the language of the geometric phase. This will also instill one with important insight as to how the modifications in the Born-Oppenheimer analysis are to be incorporated to get a consistent set of semiclassical Einstein equations.

The motivation of the present paper is exactly this. We discuss some well-known examples of vacuum instability and show how a geometric phase can be associated with the decay width of the state. In Sec. II we show in the context of a quantum-mechanical decay model that the decay width Γ is related to the Pancharatnam phase [3] between the initial and the final states. The Pan-

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charatnam phase is a generalized geometric phase which may be obtained even for a nonunitary noncycle evolution. In particular, the Pancharatnam phase may be nonzero even for a case where the Berry phase is zero or not sensible. Our method uses a perturbative argument although the result is exact and nonperturbative. The result also agrees with more abstract formulations [5] of the geometric phase (and anomaly). However, we discuss this result here as a prelude to our main result (Sec. III). (The author is, however, unaware of any prior explicit discussion of this example in the literature.) In Sec. III, we present an extension of our earlier derivation [7,8] of the semiclassical Einstein equations. This frees the earlier discussions from the necessity of a cyclic evolution. We show that for a noncyclic evolution the back reaction can be related to the Pancharatnam phase. For topologically trivial minisuperspace where the Berry phase is zero, this yields a set of semiclassical equations which describe gravity-induced instabilities in the matter Fock vacuum. The method also offers another proof for the formula relating the Pancharatnam phase and the vacuum decay width.

II. VACUUM INSTABILITY IN QUANTUM MECHANICS

We consider the quantum-mechanical decay of the ground state in the hump potential

$$V=x^2-\lambda x^4, \quad \lambda>0 . \tag{1}$$

This potential has a "bounce" solution with a single negative mode in the Euclidean time $t_E=-it$. The standard instanton calculation [9] yields the vacuum-vacuum amplitude:

$$\langle f|i\rangle \equiv \int \mathcal{D}x \exp\left[-i\int_0^T \left[\frac{1}{2}\dot{x}^2-V\right]dt\right] \simeq e^{-\Gamma T} , \tag{2}$$

where

$$\Gamma=|K|\sqrt{S_0}e^{-S_0} \tag{3}$$

is the decay width of the state, S_0 the Euclidean bounce action, and K is a constant determinant factor. (By a suitable redefinition we absorb the harmonic-oscillator ground-state energy in the potential.) The essential feature of the expression (2) is that the ground-state energy of the corresponding Hamiltonian, which is defined via a suitable analytic continuation $-\lambda \to \lambda e^{+i\pi}$, picks up a small imaginary part Γ signaling the instability. In the instanton calculation this is taken care of by the negative mode in the bounce solution. Moreover, the basic object being the transition probability amplitude, the inquiry into the existence of an extra phase was not necessary in the standard discussion of the problem. However, we are here primarily interested in calculating the nontrival phase of $\langle f|i\rangle$, if any.

For this purpose we use an adiabatic perturbation method to analyze the issue. Let us denote the relevant Hamiltonian by $H(\lambda)$ and the corresponding ground state $\psi(\lambda)$. Introduce a Euclidean parameter τ periodic in $0\le\tau\le 2$ and denote by $\lambda_\tau=\lambda(\tau)$ a slowly varying periodic function so that $\lambda(0)=0$, $\lambda(1)=\lambda$. By slowly varying we mean $\lambda(\tau)\simeq\lambda$ almost everywhere in [0,1]. We now write

$$\psi=e^{-\Gamma t}\phi \tag{4}$$

in the real-time Schrödinger equation

$$i\frac{\partial}{\partial t}\psi=H\psi \tag{5}$$

so that

$$H(\lambda)\phi(\lambda)=-i\Gamma\phi(\lambda) . \tag{6}$$

As already stated the energy is purely imaginary due to the analytic continuation $-\lambda\to\lambda e^{+i\pi}$. The important fact to note is that Eq. (6) can be obtained as well by treating λ_τ perturbatively via the Euclidean equation

$$H(\lambda_\tau)\chi_\tau=\frac{\partial}{\partial\tau}\chi_\tau . \tag{7}$$

The ansatz

$$\phi_\tau=\exp\left[-i\int_0^\tau \Gamma_\tau d\tau\right]\chi_\tau \tag{8}$$

then yields in the limit $\tau\to 1$, Eq. (6). The mechanism, however, defines a parallel transport which generates a phase Γ [for almost constant τ dependence in Γ_τ for the state ϕ ($\equiv\phi_1$)]. Equation (4) then gives the intended phase relation[1]

$$\psi=e^{-\Gamma t}e^{-i\Gamma}\chi . \tag{9}$$

Note that the states ψ and χ belong to different rays. The perturbatively generated phase Γ between them is by definition the Pancharatnam phase [3] which signals an induced twist in the line bundle of states due to the perturbing potential λx^4.

Further insight into the phase relation (9) can be obtained by letting τ make a complete circuit in $0\le\tau\le 2$. After a complete cycle through the classically forbidden region the final oscillator ground state $\psi(2)$ returns to the initial oscillator ground state $\psi(0)$ with, however, an irreducible phase (-2Γ):

$$\psi(2)=e^{-i2\Gamma}\psi(0) . \tag{10}$$

Although both the states are stable, the phase 2Γ carries an imprint of the twists in the perturbed line bundle. [The amplitude of ψ in Eq. (7) drops out since $\Gamma_\tau\to 0$ as $\tau\to 2$.] Stretching the analogy too far, in field theory language, the vacuum decays with associated particle creation in the first half of the circuit $0\le\tau\le 1$. However, in the other half $1\le\tau\le 2$ the annihilation of particles occurs restoring the initial vacuum. The whole process, however, leaves an imprint in the form of a nontrivial

[1]The phase $-\Gamma$ is dimensionless. The period of the Euclidean parameter τ is determined by the intrinsic time scale of the problem fixed by the harmonic-oscillator ground-state energy. We also set $\hbar=1$.

phase shift indicating particle creation (annihilation) in the intermediate stages. In most of the physical situations, though, the two way processes cannot be realized leading to genuine particle production. In the case of gauge theories with chiral fermions the above cyclic process appears to occur; the final irreducible phase indicates the absence of a global symmetry and/or gauge invariance.

We also note that the introduction of the Euclidean parameter τ along with the analytic continuation in the Schrödinger equation (5) provides a complex structure [5] in the quantum system. No such natural complex structure is available in the case of tunneling between degenerate vacua. So one does not expect a geometric phase in this case.

It is comforting to see that the nontrivial phase in Eqs. (9) and (10) can also be obtained from a more abstract formalism [5]. The present quantum system actually corresponds to a Hermitian holomorphic bundle over the punctured complex plane $C-\{0\}$: $z=\pi(t+i\tau)$. The Hermitian metric on this bundle is defined by the norm of the state ψ:

$$\gamma \equiv \langle \psi|\psi\rangle = \exp\left[-\frac{1}{\pi}\int \Gamma(dz+d\bar{z})\right] . \tag{11}$$

The unique holomorphic connection corresponding to the metric (11) is given by

$$A=\gamma^{-1}\partial\gamma=-\frac{1}{\pi}\Gamma-\frac{1}{\pi}\int\frac{\partial\Gamma}{\partial z}\,d\bar{z} . \tag{12}$$

The corresponding curvature $F=\bar{\partial}\gamma^{-1}\partial\gamma$ vanishes identically because of the t independence of Γ. However, the connection has a nonvanishing holonomy

$$\oint A\,dz=-\oint\Gamma\frac{dz}{\pi}-\frac{1}{\pi}\oint\int\frac{d\Gamma}{dz}\,dz\,d\bar{z} . \tag{13}$$

The second integral vanishes for a suitable choice of $\Gamma(\tau)$ ($\Longrightarrow$ vanishing residue of $\partial\Gamma/\partial z$ at $z=0$). Again for almost constant Γ along the cycle we have the desired phase (-2Γ).

We close this section with the following remark. The general method of holomorphic line bundle can be applied to the QFT vacuum instabilities. As in the quantum-mechanical model, the probability of pair creations in a finite volume is equal to the Pancharatnam phase between the out and in vacuum states. An application of the phase in gravity is discussed in the next section.

III. BACK REACTION AND PARTICLE PRODUCTION IN GRAVITY

We consider a minisuperspace gravity-matter system described by the Hamiltonian [7,8,10]

$$H=H_g+H_m , \quad H_g=-\frac{1}{2M}\nabla_g^2+MV(g) , \tag{14}$$

where H_g stands for the gravitational and H_m for the matter Hamiltonian. We represent the matter fields by the symbol φ. The WD equation assumes the form

$$\left[-\frac{1}{2M}\nabla_g^2+MV(g)+H_m\right]\Psi(g,\varphi)=0 . \tag{15}$$

In Refs. [7,8], it is shown that an improved Born-Oppenheimer approximation [11] with the inclusion of a nontrivial Berry phase yields the effective gravitational equation

$$\left[-\frac{1}{2M}D_g^2+MV(g)+\langle\psi|H_m|\psi\rangle\right]\phi(g)=0 , \tag{16}$$

where

$$\Psi(g,\varphi)=\phi(g)\psi(g,\varphi) . \tag{17}$$

$D_g=\nabla_g-iA$ denotes the covariant derivative due to the induced U(1) adiabatic connection

$$A=i\langle\psi|\nabla_g\psi\rangle . \tag{18}$$

Further, using standard semiclassical analysis [10] around the expanding WKB state

$$\phi(g)\sim\exp[-iS(g)]$$

the curved space equation is obtained in the Schrödinger picture at the $O(M^0)$:

$$i\frac{d}{dt}\psi=-H_m\psi , \tag{19}$$

where the "WKB time" t is defined by

$$\frac{d}{dt}=\nabla_g S\cdot\nabla_g . \tag{20}$$

The zeroth-order Einstein equation is retrieved to $O(M)$:

$$\frac{1}{2M}P_{\text{eff}}^2+MV(g)+\langle\psi|H_m|\psi\rangle=0 , \tag{21}$$

where P_{eff} is the effective gravitational momentum. We also note the relation

$$\nabla_g S\cdot A=-\langle\psi|H_m|\psi\rangle . \tag{22}$$

It thus follows that the back reaction in the form of an energy expectation value gets determined by the Berry connection A. However, for a simply connected minisuperspace with flat geometry the connection A can be gauged away $A\equiv 0$, yielding instead of Eq. (21) the source-free equation [8]

$$\frac{1}{2M}P_{\text{free}}^2+MV(g)=0 . \tag{23}$$

Thus the zeroth-order back reaction cannot be obtained from the above argument. For example, in a RW minisuperspace without a Λ term, $V(g)=g^2$ (g = scale factor) and a self-consistent solution of Eqs. (15) and (23) is a flat Minkowski space obtained via a Euclidean continuation [8]. This means that the semiclassical reduction of WD equation for a Friedmann-like model yields only a Minkowski space matter Schrödinger equation. However, for a nonzero Λ term, the reduction yields a dS space as a solution of Eq. (23). The matter equation (18) is therefore a dS space Schrödinger equation. The exponential expansion of the scale factor must therefore produce particles through a dS vacuum instability which, it is expected, should back react to gravity.

One must therefore look for this back reaction either at a higher order of the semiclassical approximation or by a modification of the Born-Oppenheimer scheme applied to Eq. (15). We prefer the latter to get a zeroth-order back reaction even for a simply connected flat minisuperspace.

We again start from Eq. (17):

$$\Psi(g,\phi)=\phi(g)\psi(g,\varphi) . \tag{24}$$

Choose two different values of g, g_i and g_f, corresponding to two "instants" of quasiclassical evolution. Define, for convenience, $|\psi_i\rangle=\psi(g_i,\varphi)$ the initial normalized Fock vacuum: $\langle\psi_i|\psi_i\rangle=1$. We also assume that all excited states in the initial Fock column are empty: $|\psi_i\rangle\langle\psi_i|=\mathbb{1}$. Now, instead of projecting the whole WD equation (15) on $|\psi_i\rangle$ itself, we choose to project on a final Fock state $|\psi_f\rangle=\psi(g_f,\varphi)$:

$$\left[-\frac{1}{2M}(\nabla_g-iA)^2+MV+\frac{\langle\psi_f|H_m|\psi_i\rangle}{\langle f_f|\psi_i\rangle}\right]\phi=0 , \tag{25}$$

where A is given by

$$A=i\langle\psi_i|\nabla_g\psi_i\rangle . \tag{26}$$

For an adiabatically evolving gravitational mode, A is the Berry connection (18). However, the derivation of Eq. (25) does not require the need of a cyclic evolution and may be applied even to a noncyclic case. Using the definition of time, Eq. (20), one gets

$$\nabla_g S\cdot A=i\left\langle\psi_i\left|\frac{d}{dt}\psi_i\right.\right\rangle . \tag{27}$$

Further using the Schrödinger equation (19) and the inverse of the time definition $dg/dt=\nabla_g S$, one obtains the Pancharatnam phase [12] for the transition $|\psi_i\rangle\rightarrow|\psi_f\rangle$ in the form

$$\int_i^f A\cdot dg=-\int_{t_i}^{t_f}\frac{\langle f|H_m|i\rangle}{\langle f|i\rangle}dt . \tag{28}$$

A comment is in order here.

(i) The unitarity condition $|\psi\rangle\langle\psi|=\mathbb{1}$ asserts that the parallel transport of states is to be done along the horizontal subspace [2,3] only. The condition for such a horizontal transport is

$$\left\langle\psi\left|\frac{d}{dt}\psi\right.\right\rangle=0\Longrightarrow A=0$$

in the intermediate stage $t_i\le t<t_f$. However, the condition of horizontal transport fails at $t=t_f$. Consequently, a mixing of the initial Fock states is allowed in the final Fock vacuum yielding a nonzero value for the phase integral (28).

Continuing with the discussion of the phase integral (28) we note that the instantaneous vacuum energy of the Hamiltonian H_m in the presence of an induced instability assumes a small imaginary part $E_0+i\Gamma$. Here E_0 is a possible nonzero energy due to vacuum polarization and Γ is related to the vacuum decay width. The integral in the right-hand side (RHS) of Eq. (28) has the formal expression $\int_i^f(E_0+i\Gamma)dt$. To get a real value one must evaluate the integral for Γ along the Euclidean time $\tau=it$. Further, in the case of a simply connected flat minisuperspace the integral $\int E_0 dt$ can be safely gauged away. Equation (28) together with the remark (i) therefore yield the Pancharatnam phase associated with an instability:

$$\int_i^f A\cdot dg=-\Gamma , \tag{29}$$

which agrees with the phase obtained in Sec. II. In fact, Eq. (29) is another derivation of the fact that the Pancharatnam phase, indicating an instability, has to be pinned via a transport along a Euclidean time [13]. The result is exact (modulo adiabatic condition) and appears to have a general validity.

In the absence of an instability Eq. (29) yields a vanishing Pancharatnam phase. This agrees with the result of Ref. [8] that for a Lorentzian evolution emerging from a flat simply connected minisuperspace the corresponding Berry phase (in fact, connection) is zero. [For a curved minisuperspace the real-time energy integral (28) might yield a meaningful geometric phase. This issue will be taken up separately.]

The semiclassical back-reaction equation corresponding to Eq. (25) thus assumes the form

$$\frac{P_g^2}{2M}+MV(g)+\frac{\langle f|H_m|i\rangle}{\langle f|i\rangle}=0 , \tag{30}$$

where P_g is the source-free gravitational momentum. Note, however, that the gravitational component $P_g^2/2M+MV(g)$ in Eq. (30) is of $O(M)$ whereas that of the back reaction is one order less: $O(M^0)$. Thus a reasonable set of Einstein equations obtained via a semiclassical reduction must assume the iterative form

$$O(M):\ \frac{P_{g_0}^2}{2M}+MV(g_0)=0 , \tag{31a}$$

$$O(M^0):\ i\frac{d}{dt}\psi(g_0,\varphi)=-H_m\psi(g_0,\varphi) , \tag{31b}$$

and the back reaction, Eq. (30), is obtained only as a second-order iterated equation:

$$\frac{P_g^2}{2M}+MV(g)=-\mathrm{Re}\frac{\langle f|H_m|i\rangle_{g_0}}{\langle f|i\rangle_{g_0}} , \tag{31c}$$

the imaginary part of the RHS, being exponentially small, is neglected in the adiabatic approximation.

In the case of a closed RW minisuperspace with a pure Einstein-matter action, Eq. (31a) does not have a reasonable solution in the Lorentzian sector (because momentum P_{g_0} becomes imaginary). However, the equation yields a flat Euclidean solution which via an analytic continuation implies in turn that the evolution of matter is essentially described by a Minkowski space Schrödinger equation. In this case, no gravitationally induced instability is possible and hence Eq. (31c) reduces to Eq. (31a) [normal-ordered vacuum energy expectation value vanishes in Minkowski space].

However, for an action with a positive cosmological term, Eq. (31a) yields a dS space as a solution. The matter equation (31b) is thus a dS vacuum Schrödinger equation which is supposed to produce particles (modulo technicalities in defining appropriate Fock states) because of an instability [14] induced by the exponential expansion. Equation (31c) then describes a possible modification in the dS metric by an appropriate back reaction. Before closing the discussion we note that the relevant phase integral in the presence of a Euclidean wormhole structure assumes the form

$$\int_i^f A\cdot dg = i\int_{\tau_i}^{\tau_f} \frac{\langle f|\tilde{H}_m|i\rangle}{\langle f|i\rangle} d\tau \,. \tag{32}$$

Here τ is a Euclidean time parametrizing the wormhole handle and $\tilde{H}_m$ denotes an appropriate matter Hamiltonian. It is well known [15] that the occurrence of a wormhole needs a complex matter field. The existence of a nontrivial geometric phase along a noncontractible wormhole handle now suggests that $\tilde{H}_m$ must be realizable as an anti-Hermitian operator on a Euclidean Schrödinger energy eigenstate. (This particular point has not been explicitly stated in Ref. [7].)

IV. FINAL REMARKS

Three different methods are shown to yield the same phase relation between the in and out vacua in the presence of an instability. It is clear that an instability occurs whenever the Hilbert bundle associated with a given quantum system has a natural complex structure. In the examples discussed here the complex structure arises from the punctured complex plane of the analytically continued physical time.

The present discussion also suggests a general unambiguous method of obtaining semiclassical Einstein equations from a fully quantized system. It is, however, unclear the precise sense of how the energy expectation values capture the back reaction of the particles produced in the cosmological background. In any case, it is of much interest to see how this argument applies to a more general superspace. It is also of interest to substantiate the general results discussed here by explicit calculations.

Note added in proof. The geometric "electric" field [M. Berry and J. M. Robbins, Proc. R. Soc. London **A442**, 641 (1993) and G. Venturi, in *Differential Geometric Methods in Theoretical Physics,* edited by L. L. Chau and W. Nahm (Plenum, New York, 1990)], although of a higher adiabatic order, seems to affect the back reaction Eq. (31c) nontrivially. The study of its consequences in semiclassical cosmology will be taken up separately.

ACKNOWLEDGMENTS

It is a pleasure to thank Inter-University Centre for Astronomy and Astrophysics, Pune for kind hospitality under its Associateship program. I am particularly thankful to Professor J. V. Narlikar, Director, IUCCA, Professor N. Dadhich, Dr. V. Sahni, and Dr. A. Kshirsagar for stimulating discussions.

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