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Weyl cosmic strings and their consequences

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Cosmic strings are discussed from the viewpoint of the Weyl metric in general relativity and its Newtonian limit. The hypothesis is explored that the observed filamentary clustering of galaxies is due to localization about Weyl cosmic strings. If the mass density is determined by a grand unification mass, then the magnitude of observed asymptotic velocities of stars in galaxies can be explained without recourse to "dark matter."

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The notion of cosmic strings has been of importance in cosmology since the investigations of Kibble [1]. He noted that the condition for the existence of strings is that the unbroken symmetry subgroup of a unified gauge theory be disconnected when the symmetry is spontaneously broken. Standard grand unified models, such as supersymmetric (SUSY) SU(5), SO(10), E6, . . . , are thought to satisfy the Kibble criteria. However, for the considerations which follow, the microscopic details of the string core (apart from the mass per unit length) are not crucial. The length scale L of grand unification and the mass density μ per unit length of string are related by

$$\mu=\hbar/2cL^2 . \tag{1}$$

The Vilenkin metric [2] for a cosmic string is that of "locally flat" space-time,

$$c^2d\tau^2=c^2dt^2-dz^2-d\rho^2-\rho^2d\phi^2 , \tag{2}$$

with (however) a small "angular deficit" β,

$$-\pi+\beta<\phi<\pi-\beta , \tag{3}$$

$$\beta=2G\mu/c^2=(\Lambda/L)^2 . \tag{4}$$

In Eq. (4), Λ is the Planck length. The Vilenkin metric implies (due to local flatness) that a test mass placed at rest near the string would not be accelerated toward the string. (This conclusion was modified in more recent work [2] in which a small gravitational mass is induced by vibrations in the string.) It is of interest to compare the Vilenkin string with the Newtonian string.

In Newtonain gravitational theory, a strong with mass density μ per unit length produces a gravitational potential

$$\Phi(\rho)=(2G\mu)\ln(\rho/L) , \tag{5}$$

and thereby a gravitation acceleration

$$g=-d\Phi(\rho)/d\rho=-2G\mu/\rho=-\beta c^2/\rho . \tag{6}$$

Thus a test particle moving in a circular orbit about a Newtonian string with velocity v_∞ accelerates at $g=-v_\infty^2/\rho$ which together with Eq. (6) implies that

$$v_\infty=c\sqrt{\beta}=c(\Lambda/L) . \tag{7}$$

Taking the ratio of the Planck length to the unification length, as suggested by the most recent data [3] from the CERN e^+e^- collider LEP,

$$(\Lambda/L)\sim 10^{-3} , \tag{8a}$$

yields

$$v_\infty\sim 300 \text{ km/sec} , \tag{8b}$$

which is roughly the rotational velocity of stars near the outside of galaxies [4]. This was also discussed by I. G. Moss (unpublished).

This relationship between grand unification length scales (coupling $\beta\sim 10^{-6}$) and the asymptotic rotational velocities of stars in many galaxies suggests the following hypothesis: Galaxies are localized about cosmic strings due to the gravitational attraction between visible galactic matter and the string mass density. The asymptotic velocity v_∞ is related to the string mass density μ as in Eqs. (7) and (8). No recourse is made to the notion of "dark matter" producing v_∞ beyond the string mass density μ. (Many scenarios for galaxy formation require both strings and dark matter. The above renders the latter superfluous.)

One consquence of the above hypothesis is that galaxies should have a filamentary distribution. There is a considerable literature on this subject, but the final results indicate that filamentary distributions are very common [5–7].

Of course the idea of a Newtonian string is simple but would not be of much theoretical importance were there not a general relativistic metric with an appropriate Newtonian limit. Let us here discuss the metric of interest. The Weyl cosmic string can be described by the metric [8,9]

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$$c^2d\tau^2=(\rho/L)^{2\beta}c^2dt^2-(L/\rho)^{2\beta}\rho^2d\phi^2 -(L/\rho)^{2\beta(1-\beta)}(d\rho^2+dz^2)\ . \tag{9}$$

To see how the Weyl string of Eq. (9) is related to the Newtonian string of Eqs. (5) and (6) we note that the energy of a test mass m at rest in the Weyl metric is given by

$$U=mc^2(\rho/L)^{\beta}\ , \tag{10}$$

so that the weight of the test mass due to the attraction of the string is given by

$$W=-dU/d\rho=-(\beta mc^2/\rho)(\rho/L)^{\beta}=mg(\rho/L)^{\beta}\ ,$$

where g is the gravitational accerelation calculated on the basis of Newtonian potential theory and

$$(\rho/L)^{\beta}=e^{\beta\ln(\rho/L)} \tag{11}$$

describes the correction due to general relativity. Even though the ratio of the galactic radius to the grand unification length (ρ/L) is truly enormous, the weak coupling $(\beta\sim10^{-6})$ still ensures that the factor in Eq. (11) giving the correction to the Newtonian string from general relativity is very close to unity. Thus the Newtonian limit of the Weyl string $(\beta\rightarrow0)$ is achieved to a high degree of accuracy.

The Weyl string will blend light much as the Schwarzschild metric yields the bending of light around a central mass (e.g., the Sun). For a light ray directed normal to the string axis, the scattering angle

$$\Theta=\pi+\Delta\Theta \tag{12}$$

is given by

$$\Theta=(\rho_{\min}/L)^{\beta^2}f(\beta)\ , \tag{13a}$$

$$f(\beta)=[\sqrt{\pi}/(1-2\beta)]\frac{\Gamma((1-2\beta-\beta^2)/(1-4\beta))}{\Gamma((2-4\beta-\beta^2)/(2-4\beta))}\ , \tag{13b}$$

where $\Gamma(z)$ is the Euler gamma function and $\rho_{\min}$ is the radial coordinate of closest approach to the string. For $\beta\sim10^{-6}$ one finds (to a sufficient degree of accuracy) the charge in angle

$$\Delta\Theta=2\pi\beta+\cdots\ , \tag{14}$$

in qualitative agreement with the Vilenkin string angular deficit in Eq. (3). However, although the Weyl string bends light, there is no angular deficit. (This has been discussed by Hindmarsh and Wray [10] using approximate solutions to the light scattering problem.)

In contrast with massless particles (such as the photon) which do scatter and hence escape away from the string, massive particles always remain bound to the string. Let us now discuss whether Weyl strings have direct measurable consequences.

Because the string's cross section for scattering photons is so small, optical detection could be difficult but perhaps not impossible. Both the Aharonov-Bohm-type phase shift and gravitational lensing, predicted for Vilenkin strings [2], would also occur in the Weyl case.

As stated above, massive particles are actually bound to the string by an approximately logarithmic potential. Therefore, it is in principle possible to detect Weyl strings by their effects on the interstellar medium. Specifically, particles in planar orbits around a string would begin to move helically down the string's length when subjected to a perturbation in the axial direction. This effect could result in bulk flows of interstellar matter along well defined, comparatively narrow paths between galaxies threaded by a string. In fact, more than twenty such intergalactic "bridges" or "tails" have been observed [11,12], the most famous being the Magellanic Stream in our own local group [13].

It is also interesting to note that in recent years over one hundred small, mostly parallel, linear features have been discovered *inside* the Milky Way, aligned with and concentrated near the galactic axis [14,15]. One such structure (a thirty parsec "threadlike" gas arc and "double jet" parallel to the galactic axis) is situated at Sagittarius A, apparently at the exact center of the galaxy [16,17]. Also, the recent Cosmic Background Explorer (COBE [18]) experiment on the background cosmic radiation temperature anisotropy is consistent with the estimate $(\Delta T/T)\sim\beta$ which follows from the scattering of radiation from Weyl strings.

Finally, the model discussed above is by no means complete. There are many unsolved theoretical problems associated with the viewpoint here presented. (i) There is presently no detailed theoretical microscopic model in the standard "big bang" cosmology for how the strings would at first be formed. Present "wiggly strings" do grow an inertial mass per unit length, but this is not sufficiently large to obtain for the tension $\tau\ll\mu c^2$, and this inequality is implicit in Eq. (8b). (ii) Again, within the context of the standard cosmological model, as the Universe expands there should be a mechanism for energy losses. Presently there is no microscopic theory of how this might happen. (iii) The above radiation anisotropy in the COBE experiment was based on the estimate that the small angle electromagnetic scattering gives rise to background radiation anisotropy about equal to β. To be convincing, the open theoretical problem of how random string networks localize radiation should be solved in more detail.

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