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Dilatonic black holes in theories with moduli fields

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We discuss the low-energy effective string theory when moduli of the compactified manifold are present. Assuming a nontrivial coupling of the moduli to the Maxwell tensor, we find a class of regular black-hole solutions. Both the thermodynamical and the geometrical structure of these solutions are discussed.

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I. INTRODUCTION

Electrically and magnetically charged black-hole solutions arising in the context of effective low-energy string theories exhibit properties which make them drastically different from the usual Reissner-Nordström black holes [1,2]. When expressed in terms of the "string" metric, these solutions are characterized by a geometry which in the extremal limit is singularity-free. Furthermore, the rather unusual thermodynamical properties seem to indicate that black holes behave much more like elementary particles [3]. This situation is therefore very promising for trying to understand long-standing puzzles of black-hole physics such as the loss of quantum coherence in black-hole evaporation.

Most of the models considered until now work with the Einstein action modified with the introduction of the dilaton field. This is, from the point of view of the low-energy string action, just a first approximation. For large mass holes ($M \gg M_{\rm Pl}$) this may be a good one. However, if string theory has to solve the puzzles of quantum gravity one needs to go beyond this approximation. Various directions can be followed. First, the low-energy string action has the form of an expansion in the inverse string tension α'. One should be able to take into account terms of the action proportional to higher powers in the curvature tensors. Progress in this direction has been made in Ref. [4]. Second, even though one considers only the massless sector of the string spectrum, there are other fields, such as the moduli of the compactified manifold, which should come into play. Third, a correction to the effective low-energy action may appear at the one-loop string level. Moreover nonperturbative effects should be considered.

The important question is as follows: Are the above-mentioned properties of dilatonic black holes shared by more general situations? Are they an artifact of the approximation or are they related to general features of string theory? In this paper we will move a step further in this direction. We will derive and study black-hole solutions of the low-energy string action coming from dimensional reduction from ten to four dimensions, retaining one single modulus which describes the radius of the compactified space. By invoking one-loop string effects we will couple this field to the Maxwell tensor in order to get new black-hole solutions.

The structure of this paper is as follows. In Sec. II we present the low-energy effective string action we want to discuss. In Sec. III we derive the corresponding black-hole solutions. A generalized version, together with its black-hole solutions, of our action is studied. Both the thermodynamical and geometrical features of the solutions are discussed. In Sec. IV we consider the duality symmetries of our theory and show how dual dyonic solutions can be generated.

II. THE ACTION

We start from the four-dimensional low-energy action of heterotic superstring theory derived by Witten in Ref. [5]. The effective action was obtained through dimensional reduction of ten-dimensional supergravity using a suitable truncation of the spectrum. In the following we set to zero all the fields but the graviton, the electromagnetic field, the dilaton ϕ and the scalar σ arising from the ansatz $g_{IJ}=e^{\sigma}\delta_{IJ}$ for the metric of the extra six dimensions. The action reads

$$A=\int d^4x\sqrt{-g}\,[\mathcal{R}-6(\nabla\sigma)^2-8(\nabla\phi)^2-e^{-2\phi+3\sigma}F^2]\ . \tag{1}$$

The corresponding field equations have regular black-hole solutions only for $\sigma=$const [these are the Garfinkle-Horowitz-Strominger (GHS) solutions]. In fact, performing the redefinitions

$$\Phi=\phi-\tfrac{3}{2}\sigma,\quad \Sigma=-\tfrac{3}{2}\sigma-3\phi\ , \tag{2}$$

one can easily see that the action (1) becomes the one of Ref. [1] plus a kinetic term for the field Σ. In particular, there is no coupling of Σ to F^2. This prevents the appearance of new black-hole solutions. Note that Φ and Σ rather than ϕ and σ are the physical dilaton and "compacton" fields of the four-dimensional supergravity theory.

At this stage one can ask himself if a coupling of the field Σ to F^2 could be justified. It is well-known that at the string tree level such coupling does not exist. However, it can be generated at the one-loop level. A correction to the action (1) of the form

$$\delta\mathcal{L} \sim e^{-2\Sigma/3}F^2 \tag{3}$$

was considered some time ago by Ibañez and Nilles, using arguments based upon the supersymmetrization of anomaly canceling terms [6]. More generally, terms of the type $\Delta(e^{-2\Sigma/3})F^2$ are expected to arise from integrating out heavy modes of the string spectrum [7,8]. Furthermore, the appearance of such terms in the effective supergravity Lagrangian seems to be crucial in order to get dynamical supersymmetry breaking [8].

For the moment we consider the "minimal" coupling defined by (3). Later, we will study a more general situation. The form of the action we are lead to study is then

$$A = \int d^4x\sqrt{g}\,[\mathcal{R} - 2(\nabla\Phi)^2 - \tfrac{2}{3}(\nabla\Sigma)^2 - e^{-2\Phi}F^2 - e^{-2\Sigma/3}F^2] \,. \tag{4}$$

III. THE BLACK-HOLE SOLUTIONS

The field equations coming from the action (4) are

$$\begin{aligned} \mathcal{R}_{\mu\nu} &= 2\nabla_\mu\Phi\nabla_\nu\Phi + \tfrac{2}{3}\nabla_\mu\Sigma\nabla_\nu\Sigma + 2(e^{-2\Phi} + e^{-2\Sigma/3})(F_{\mu\rho}F_{\nu\rho} - \tfrac{1}{4}F^2 g_{\mu\nu}) \,, \\ \nabla^2\Phi &= -\tfrac{1}{2}e^{-2\Phi}F^2 \,, \\ \nabla^2\Sigma &= -\tfrac{1}{2}e^{-2\Sigma/3}F^2 \,. \end{aligned} \tag{5}$$

A spherically symmetric solution of the field equations can be found by an ansatz which reduces the system to a Toda-lattice form [2]:

$$\begin{aligned} ds^2 &= e^{2\nu}(-dt^2 + e^{4\rho}d\xi^2) + e^{2\rho}d\Omega^2 \,, \\ F_{mn} &= Q\epsilon_{mn} \,, \end{aligned} \tag{6}$$

where ν and ρ are functions of ξ and Q is the magnetic charge.

Defining $\zeta = \nu + \rho$, the field equations are given by $({}' = d/d\xi)$:

$$\begin{aligned} \zeta'' &= e^{2\zeta} \,, \\ \phi'' &= -Q^2 e^{-2(\Phi-\nu)} \,, \\ \Sigma'' &= -Q^2 e^{-2(\Sigma/3-\nu)} \,, \\ \nu'' + \Phi'' + \Sigma'' &= 0 \,, \end{aligned} \tag{7}$$

with the constraint

$$\zeta'^2 - \nu'^2 - \Phi'^2 - \tfrac{1}{3}\Sigma'^2 + Q^2 e^{-2(\Phi-\nu)} + Q^2 e^{-2(\Sigma/3-\nu)} - e^{2\zeta} = 0 \,. \tag{8}$$

An exact solution can be found if $\Sigma' = 3\Phi'$. By introducing a new variable, such that $e^{2\zeta} = \eta^2 - \eta_0^2$, the only asymptotically flat solution with a regular horizon can be written as

$$\begin{aligned} e^{2\nu} &= (\eta - \eta_0)^{3/5}(\eta + \eta_0)(\eta + \eta_1)^{-8/5} \,, \\ e^{2\rho} &= (\eta - \eta_0)^{2/5}(\eta + \eta_1)^{8/5} \,, \\ e^{-2\Phi} &= (\eta - \eta_0)^{2/5}(\eta + \eta_1)^{-2/5} \,, \end{aligned} \tag{9}$$

where η_0 and η_1 are integration constants.

The solution assumes a neater form by defining a new variable r such that $r = \eta + \eta_1$,

$$r - r_+ = \eta + \eta_0, \quad r - r_- = \eta - \eta_0 \,.$$

As we shall see, the constants r_+ and r_- are simply related to the physical mass M and charge Q of the black hole. One now has

$$ds^2 = -\lambda^2 dt^2 + \lambda^{-2}dr^2 + R^2 d\Omega^2 \,, \tag{10}$$

with

$$\begin{aligned} \lambda^2 &= \left[1 - \frac{r_+}{r}\right]\left[1 - \frac{r_-}{r}\right]^{3/5} \,, \\ R^2 &= r^2\left[1 - \frac{r_-}{r}\right]^{2/5} \,, \end{aligned} \tag{11a}$$

and

$$e^{-2\Phi} = \left[1 - \frac{r_-}{r}\right]^{2/5} \,. \tag{11b}$$

The action (4) corresponds to the "minimal" coupling (3). However, one can also consider more general couplings:

$$\delta\mathcal{L} = \exp\left[-\frac{2q}{3}\Sigma\right], \quad q \in R \,. \tag{12}$$

These terms may be viewed as the building blocks of a series expansion of the general term $\Delta(e^{-2\Sigma/3})F^2$. The action is now

$$A = \int d^4x\sqrt{-g}\,[\mathcal{R} - 2(\nabla\Phi)^2 - \tfrac{2}{3}(\nabla\Sigma)^2 - e^{-2\Phi}F^2 - e^{-2q\Sigma/3}F^2] \,. \tag{13}$$

Proceeding as before, one readily obtains a regular black-hole solution for $\Sigma' = 3q^{-1}\Phi'$. The solutions are

$$\lambda^2 = \left[1 - \frac{r_+}{r}\right]\left[1 - \frac{r_-}{r}\right]^{3/(2q^2+3)} \,, \tag{14a}$$

$$R^2 = r^2\left[1 - \frac{r_-}{r}\right]^{2q^2/(2q^2+3)} \,, \tag{14b}$$

$$e^{-2\Phi} = \left[1 - \frac{r_-}{r}\right]^{2q^2/(2q^2+3)} \,. \tag{14c}$$

The metric functions of our solution coincide, after the redefinition $a^2 = q^2(q^2+3)^{-1}$, with that found in Ref. [1] in the context of a generalized model for dilaton gravity. The expression (14c) for the dilaton, however, is not the same. As we shall see later, this has important consequences for the form of the "string" metric.

The physical mass and charge of the hole are given by

$$\begin{aligned} 2M &= r_+ + \frac{3}{2q^2+3}r_- \,, \\ Q^2 &= \frac{q^2}{2q^2+3}r_+ r_- \,. \end{aligned} \tag{15}$$

Using well-known formulas one can easily calculate the temperature and the entropy of the black hole. We have

$$T=\frac{1}{4\pi r_+}\left[1-\frac{r_-}{r_+}\right]^{3/(2q^2+3)}, \tag{16}$$

$$S=\pi r_+^2\left[1-\frac{r_-}{r_+}\right]^{2q^2/(2q^2+3)}. \tag{17}$$

We see that for extremal black holes ($r_+\to r_-$) both T and S approach monotonically to zero. This behavior is different from that of GHS-dilatonic black holes for which the temperature approaches a constant value in the extremal limit. Thus the interpretation of the final state of the black holes described by (14) is straightforward: zero entropy at zero temperature indicate a nondegenerate ground state, which naturally does not radiate. In the extremal limit the spacetime still displays a naked singularity. However, the metric appearing in the string σ model is not $g_{\mu\nu}$ but rather $e^{2\Phi}g_{\mu\nu}$. This is the metric to which strings couple. The charged extremal black-hole metric (10) now becomes

$$ds^2_{\rm string}=-\left[1-\frac{r_+}{r}\right]^{6/(2q^2+3)}dt^2+\left[1-\frac{r_+}{r}\right]^{-2}dr^2+r^2d\Omega^2 . \tag{18}$$

The geometry of the $t=$const surfaces is identical to that of the extremal GHS-dilatonic black hole. There is a semi-infinite throat attached to an asymptotic flat region. The only difference resides in the form of the g_{tt} component of the metric. Whereas in the GHS case $g_{tt}=$const in Eq. (18) $g_{tt}\to 0$ as $r\to r_+$. This indicates that the horizon though infinitely far away along a spacelike geodesic is not such for a timelike one.

IV. DUAL SOLUTIONS

It is well known that in the case of GHS-dilatonic black holes one can exploit the invariance of the field equation under SL$(2,R)$ to generate dual dilaton dyons [9,10]. This invariance holds, though in a restricted sense, also for the equation of motion coming from an improved form of the action (4). To show this let us introduce in the action (4) the two axion fields a and D coming from the ten-dimensional axion B_{IJ}, together with the couplings of these fields to $F\tilde{F}$ (see Ref. [5]). Defining the complex scalar fields

$$S=3\sqrt{2}D+ie^{-2\Phi}, \quad T=\sqrt{2}a+ie^{-2\Sigma/3}, \tag{19}$$

the action can be written as

$$A=\int d^4x\sqrt{-g}\left\{\mathcal{R}-G_{i\bar{k}}\partial_\mu Z^i\partial_\nu\bar{Z}^{\bar{k}}g^{\mu\nu}+(i/4)[(S+T)F_+^2-(\bar{S}+\bar{T})F_-^2]\right\}, \tag{20}$$

where $G_{i\bar{k}}$ is the Kähler metric of

$$\mathrm{SL}(2,R)/\mathrm{U}(1)\times\mathrm{SL}(2,R)/\mathrm{U}(1)$$

corresponding to the Kähler potential

$$G=-\ln i(S-\bar{S})-3\ln i(T-\bar{T}),$$

$Z_i\doteq\{S,T\}$, $i=1,2$, and $F_\pm=F\pm i\tilde{F}$.

The σ-model kinetic terms of the action (20) are invariant under the nonlinearly realized SL$(2,R)\times$SL$(2,R)$ group. The last two terms of the action however break this symmetry. Nevertheless, one can easily verify, by writing down the field equations, that along the orbit of solutions verifying $T=3S$, the field equations are invariant under the SL$(2,R)$ transformations

$$S\to\frac{aS+b}{cS+d},\quad F_+\to-(cS+d)F_+,\quad ad-bc=1 . \tag{21}$$

Using the invariance of the field equations under the former transformations, one can generate from the solutions (11) dual dyon solutions. These are similar to the ones obtained in Ref. [10]. One needs just to use there the expression (11b) for the dilaton and $\Phi'=3\Sigma'$ to get the corresponding expression for Σ.

V. CONCLUSIONS

In this paper we have found and analyzed black-hole solutions of dilaton gravity when a nontrivially coupled modulus of the compactified manifold is present. Our solutions share common properties with the GHS-dilatonic black holes. In particular, the spatial geometry associated with the "string" metric in the extremal limit is the same. The full spacetime geometry and the thermodynamical properties are, however, different. A deeper insight into the whole subject could be achieved by studying the corresponding two-dimensional gravity theory. We plan to discuss this topic in a forthcoming paper.

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