PHYSICAL REVIEW D VOLUME 48, NUMBER 12 15 DECEMBER 1993

Consequences of twisting solar magnetic fields in solar neutrino experiments

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(Received 25 June 1993)

The transverse component of the solar magnetic field may change its direction along the path of neutrinos. We examine possible consequences of such a twisting magnetic field in the proposed Sudbury Neutrino Observatory and BOREXINO experiments. Although for certain magnetic field configurations twisting can give rise to the production of electron antineutrinos, we show that such a process is sensitive to the detailed structure of the magnetic field.

PACS number(s): 96.60.Kx, 14.60.Gh, 96.60.Hv

I. INTRODUCTION

The observed solar neutrino flux [1–4] is deficient relative to that predicted by the standard solar model [5,6]. It was recognized that, if neutrinos observed in physical processes are mixtures of mass eigenstates, coherent forward scattering of neutrinos in electronic matter can cause an almost complete conversion of electron neutrinos into neutrinos of a different flavor [7]. This scenario, dubbed the Mikheyev-Smirnov-Wolfenstein (MSW) effect, does not require fine-tuning of the mass differences and mixing angles and could possibly provide an elegant solution to the solar neutrino problem within current theoretical prejudices [8,9].

In addition, the flux observed by the chlorine experiment appears to anticorrelate with the number of sunspots [10], although the flux measured by Kamiokande does not [2]. Insufficient data exist for ascertaining a time dependence of the flux observed by the gallium experiments. Since sunspots are regions where "ropes" of magnetic field dive out of and back into the Sun, this possible anticorrelation suggests that neutrinos may have a nonzero magnetic moment. Upon passage through the solar magnetic field, the left-handed electron neutrinos could flip their spin resulting in antineutrinos of a different flavor (for Majorana neutrinos with transition magnetic moments), or sterile right-handed neutrinos (for Dirac neutrinos with diagonal or transition moments) [11–13]. Neither of these types would be detected in the chlorine or gallium experiments, and would result in a smaller than predicted counting rate for Kamiokande.

The combined effect of matter and magnetic fields on neutrino spin and flavor precession was examined by Lim and Marciano [14] and Akhmedov [15]. They pointed out that simultaneous presence of flavor mixing and magnetic (diagonal or transition) moments can give rise to two new resonances in addition to the MSW resonance. This possibility was studied by several authors [16–18] who concluded that one needs a perhaps unrealistically large magnetic field and magnetic moment combination for an emphatic effect.

It is interesting to note that even with a moment too small for any anticorrelation effect there could be other consequences of the neutrino magnetic moment—namely, possible detection of solar *antineutrinos.* As has been pointed out [19–22], if neutrinos are Majorana fermions the spin-flavor precession of $\nu_e \to \overline{\nu}_\mu$ via the Lim-Akhmedov-Marciano (LAM) mechanism followed by vacuum oscillations into $\overline{\nu}_e$ can yield counting rates detectable at the BOREXINO detector and at the Sudbury Neutrino Observatory (SNO). In addition, if the transverse component of the solar magnetic field changes its direction along the neutrino's path (twisting magnetic field), the resonant structure of the magnetic transitions are altered by the rate of this twisting. The change in the resonant structure of the evolution due to the twisting rate was first recognized by Smirnov [23], and has been discussed by other authors [24,25]. For certain twisting rates the Sun may again emit electron antineutrinos for a range of parameters. In this second scenario, electron neutrinos first undergo an MSW transition into muon neutrinos followed by a transition of the muon neutrinos into electron antineutrinos. This latter resonant transition is made possible by the alteration of the resonant structure by the twisting fields.

In this paper we consider the resonant production of electron antineutrinos in a twisting solar magnetic field using a neutrino magnetic moment and solar magnetic field combination which is too small to affect the counting rates of the current detectors. In Sec. II we briefly discuss the limits on the neutrino magnetic moment and the size and possible twisting of the solar magnetic field. Section III gives the evolution equations for combined spin-flavor precession in a twisting magnetic field and determines the size of the twisting necessary to create resonant electron antineutrinos for the regions of the MSW parameter space allowed at the 2σ level for the currently running experiments. In Sec. IV we discuss the artificial, although instructive, case of a constant twisting rate, and also the model of Kubota *et al.* [26] which is based on the work of Yoshimura [27]. Section V contains our concluding remarks.

II. LIMITS ON SOLAR MAGNETIC FIELD AND NEUTRINO MAGNETIC MOMENT

Since there is essentially nothing known about a possible magnetic field in the solar interior ($r \leq 0.7R_\odot$), and since convection is thought to be responsible for the twisting, we consider only a convective zone magnetic field. The convective zone, which is about 0.3 $R_\odot$ thick, has a large toroidal magnetic field on the order of 10^2–10^4 G. This field changes its direction every 11 yr. It seems possible that the magnitude of the magnetic field can reach values of $\sim 10^5$ G near the bottom of the convective zone [28], while the magnitude of the field observed in the sunspots is a few kG.

In addition to the speculation of the approximate magnitude of the magnetic field in the convective zone, Yoshimura [27], by solving the dynamo equation, showed that the toroidal magnetic fields twist, and further, that the twisting may be responsible for the 11-yr cycle of magnetic polarity reversals. The twist in this model is generated by the differential rotation of the Sun and global plasma convection. While the size of the twist is unknown, it seems probable that such a global twist exists for the magnetic field in the convective zone of the Sun.

Direct experimental bounds on the magnetic moments are rather weak. Antineutrino electron scattering data yield [29] a bound of

$$|\mu_{\nu_e}| \leq 4 \times 10^{-10}\mu_B. \tag{1}$$

Astrophysical and cosmological arguments are more restrictive. Among these stellar cooling (energy loss due to neutrino pair emission) provides [30] a bound of

$$|\mu_{\nu_e}| \leq 1 \times 10^{-11}\mu_B. \tag{2}$$

Perhaps the most restrictive astrophysical bound was given by Raffelt who considered [31] the increase of the core mass of red giant stars at helium flash due to anomalous neutrino dipole moments (magnetic or electric) increasing plasmon decay and obtained a limit of

$$|\mu_{\nu_e}| \leq 3 \times 10^{-12}\mu_B \quad (3\sigma). \tag{3}$$

The plasmon decay limit applies to both diagonal and transition magnetic or electric dipole moments of Dirac neutrinos, or to the transition moments of Majorana neutrinos. For Dirac transition moments, the bound is more restrictive than that given in Eq. (3) by a factor of $1/\sqrt{2}$.

III. SPIN-FLAVOR PRECESSION IN MATTER

A. Evolution of chiral components

For the electron neutrino the chiral components ν_{e_L} and ν_{e_R} interact differently with matter. This difference tends to suppress precession by splitting their degeneracy. The equation describing the propagation through matter of the two helicity components of a Dirac neutrino with mass m_ν and magnetic moment μ in a general transverse field is [12,13]

$$i\frac{d}{dt}\begin{pmatrix}\nu_{e_L}\\ \nu_{e_R}\end{pmatrix} = \begin{pmatrix} a_e(t) & \mu B e^{-i\phi} \\ \mu B e^{i\phi} & -a_e(t)\frac{m_\nu^2}{2p^2}\end{pmatrix}\begin{pmatrix}\nu_{e_L}\\ \nu_{e_R}\end{pmatrix}, \tag{4}$$

where $B = B(t) = \sqrt{(B_x^2 + B_y^2)}$ is the magnitude of the transverse magnetic field, $\phi = \phi(t) = \arctan(B_y/B_x)$ is the phase of the transverse field, and a_e is the contribution of matter to the effective mass. If one performs the phase rotation

$$\begin{pmatrix}\nu_{e_L}\\ \nu_{e_R}\end{pmatrix} = \begin{pmatrix} e^{-i\frac{\phi}{2}} & 0 \\ 0 & e^{i\frac{\phi}{2}}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}, \tag{5}$$

one gets an evolution equation for the rotating frame of

$$i\frac{d}{dt}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix} = \begin{pmatrix} a_e(t) - \frac{1}{2}\phi' & \mu B \\ \mu B & -a_e(t)\frac{m_\nu^2}{2p^2} + \frac{1}{2}\phi'\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}, \tag{6}$$

where the prime denotes a derivative. In the standard model (for an unpolarized neutral medium) one gets

$$a_e(t) = \frac{G_F}{\sqrt{2}}(2N_e - N_n), \tag{7}$$

where N_e and N_n are electron and neutron number densities, respectively. In the limit $m_\nu/p \to 0$, which we consider hereafter, the right-handed neutrino does not interact with matter.

Writing the magnetic moment in terms of Bohr magnetons,

$$\mu = g\mu_B, \tag{8}$$

where g is a small number, it is possible to calculate the survival probability of the left-handed component (in moderate magnetic fields) using the perturbation theory. For the survival probability at time T we find

$$P(\nu_{e_L} \to \nu_{e_L}) = 1 - \left[g^2\mu_B^2\left|\int_0^T dt\, B(t)\, e^{iQ_e(t)}\right|^2 + O(g^4)\right], \tag{9}$$

where

$$Q_e(t) = \frac{G_F}{\sqrt{2}}\int_0^t dt\,(2N_e - N_n) - \phi(t). \tag{10}$$

The resonance condition is $2N_e = N_n + \phi'$, where ϕ' is the derivative of the magnetic field phase. For $\phi' = 0$ it is unlikely that this resonance condition can be achieved in the Sun where the neutron density is $N_n \approx N_e/6 \sim N_e/3$. It can, however, be achieved in a supernova [30]. For $\phi' \neq 0$, it is possible to achieve such a resonance even for small values of ϕ' since the number densities fall toward zero at the surface of the Sun. This is an example of how a twisting field can alter the resonance structure of the evolution equations.

B. Spin-flavor precession in the Sun

In this case we consider two generations of Majorana neutrinos. The neutrino evolution equation is [14,20]

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \overline{\nu}_e\\ \overline{\nu}_\mu\end{pmatrix}=\begin{pmatrix}H_\nu & Be^{-i\phi}M^\dagger\\ Be^{i\phi}M & H_{\overline{\nu}}\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \overline{\nu}_e\\ \overline{\nu}_\mu\end{pmatrix}. \tag{11}$$

If one again performs a phase rotation similar to Eq. (5) one obtains the time derivative of half the magnetic phase subtracted from the diagonal terms of H_ν, and added to the diagonal terms of $H_{\overline{\nu}}$. We maintain the ν_e notation for simplicity, since such a phase rotation leaves the probabilities unchanged. The 2×2 submatrices are

$$H_\nu=\begin{pmatrix}\frac{\Delta m^2}{2E}\sin^2\theta+a_e-\frac{1}{2}\phi' & \frac{\Delta m^2}{4E}\sin 2\theta\\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{2E}\cos^2\theta+a_\mu-\frac{1}{2}\phi'\end{pmatrix}, \tag{12}$$

$$M=\begin{pmatrix}0 & -\mu\\ \mu & 0\end{pmatrix}, \tag{13}$$

and $H_{\overline{\nu}}=H_\nu(\{a_e,a_\mu,\phi\}\to\{-a_e,-a_\mu,-\phi\})$. In the above equations θ is the mixing angle, Δm^2 is the difference of the squares of the masses, and E is the neutrino energy. For a neutral, unpolarized medium, the matter potentials are

$$a_e=\frac{G_F}{\sqrt{2}}(2N_e-N_n)\quad,\quad a_\mu=-\frac{G_F}{\sqrt{2}}N_n\,. \tag{14}$$

For the case of no twisting ($\phi=0$), there are two possible crossing resonances: the MSW resonance

$$G_F\sqrt{2}N_e=\frac{\Delta m^2}{2E}\cos 2\theta, \tag{15}$$

and the $\nu_e\to\overline{\nu}_\mu$ resonance

$$G_F\sqrt{2}(N_e-N_n)=\frac{\Delta m^2}{2E}\cos 2\theta. \tag{16}$$

For the Sun, where the neutron density is only about 20% of the electron density, the $\nu_\mu\to\overline{\nu}_e$ cannot occur since it would require negative densities. In the presence of a nonzero twisting rate, the MSW resonance is unchanged while the magnetic transitions are altered. The $\nu_e\to\overline{\nu}_\mu$ resonance becomes

$$G_F\sqrt{2}(N_e-N_n)-\phi'=\frac{\Delta m^2}{2E}\cos 2\theta\ , \tag{17}$$

and the $\nu_\mu\to\overline{\nu}_e$ resonance,

$$G_F\sqrt{2}(N_n-N_e)+\phi'=\frac{\Delta m^2}{2E}\cos 2\theta\ , \tag{18}$$

is now possible for positive ϕ'. One should also note that a positive twist will push the location of the $\nu_e\to\overline{\nu}_\mu$ resonance deeper into the Sun.

C. Determination of the necessary twisting rates

The range of twisting rates necessary to obtain the resonant transition $\nu_\mu\to\overline{\nu}_e$ can be determined for the given values of $\Delta m^2,\sin^2 2\theta$, and E by Eq. (18). The minimum twisting rate for this resonance to occur in the Sun is $\phi'=\frac{\Delta m^2}{2E}\cos 2\theta$. At this value of the twisting rate, the resonance occurs at the surface of the Sun. An approximate maximum value of the twisting rate leading to a resonant transition can be determined by noting that the position of the twisting resonance must lie after the MSW resonance region which produces the muon neutrinos from which the transition occurs. The maximum twisting rate is then given by

$$\phi'_{\max}\simeq\frac{\Delta m^2}{2E}\cos 2\theta+\sqrt{2}G_F[N_e(d)-N_n(d)], \tag{19}$$

with

$$d=r_{\rm MSW}-\delta r/2,$$

where the width of the MSW resonance region is given by [9]

$$\delta r=\left|\frac{1}{N_e}\frac{dN_e}{dr}\right|^{-1}2\tan 2\theta, \tag{20}$$

and the MSW resonant point is given implicitly by

$$\sqrt{2}G_F N_e(r_{\rm MSW})=\frac{\Delta m^2}{2E}\cos 2\theta. \tag{21}$$

If one uses the fact that $N_n\sim N_e/6$ and the approximate expression of the electron density from Bahcall [5],

$$N_e(r)=245\exp(-10.54r/R_\odot)N_A\,{\rm cm}^{-3}, \tag{22}$$

one obtains

$$\phi'_{\max}\simeq\frac{\Delta m^2}{2E}\cos 2\theta(1+\tfrac{5}{6}e^{\tan 2\theta}). \tag{23}$$

This expression for the maximum twisting rate holds only for the case that the beginning of the MSW resonance region falls within the convective zone. If it falls inside the convective zone ($r<0.7R_\odot$), there is no magnetic field (or at least no twisting) at this location, and therefore no possibility of a transition. There are two prominent regions of overlap at the 2σ level of the chlorine, Kamiokande, and GALLEX experiments in the $\sin^2 2\theta,\Delta m^2$ parameter space as calculated, including Earth effects, in Ref. [4]. They are centered at $\Delta m^2\simeq 7\times 10^{-6}{\rm eV}^2,\sin^2 2\theta\simeq 6\times 10^{-3}$, and $\Delta m^2\simeq 3\times 10^{-5}{\rm eV}^2,\sin^2 2\theta\simeq 0.8$. The minimum twisting rate for neutrinos of energy 10 MeV are 196 and 376 rev/$R_\odot$ (revolutions per solar radius), respectively. These are extremely large rates compared to the largest rate proposed by Kubota *et al.* (Ref. [26]), who considered a model based on the work of Yoshimura where the maximum twisting rate used was slightly less than 1 rev/$R_\odot$. There is also a small region of overlap at 2σ due to the Earth effect for the GALLEX detector. The twisting rate for this region, centered at $\Delta m^2\simeq 1.5\times 10^{-7}{\rm eV}^2,\sin^2 2\theta\simeq 0.7$, is 2.3 rev/$R_\odot$ and is much more realistic.

IV. ANTINEUTRINOS AT SNO AND BOREXINO

It is unknown whether there exists a global magnetic field in the solar convective zone. Sunspots, being essentially ropes of magnetic field which exit and reenter the surface of the Sun, suggest that, at least near the solar surface, the field is not global. In the interest of making a prediction of the upper limit on the flux of antineutrinos, we take the solar magnetic field to be nonzero (i.e., extensive) throughout the convective zone. Given that the mechanism generating the magnetic field in the Sun is at best poorly understood, we realize that an extensive field may be an oversimplification. We choose the magnitude of the field as a function of the solar radius to be consistent with the numerical work of Sofia and collaborators [28,33]. Merely for calculational simplicity we take the magnetic field to change from zero at $0.7R_{\odot}$ to 10^5 G at $0.75R_{\odot}$. It remains constant until $0.8R_{\odot}$, after which it falls linearly to a few kilogauss at the solar surface. Such a field is consistent with the observed variations in the solar luminosity and the solar radius [28,33]. Again for calculational simplicity we take the magnetic field near the surface to be extensive as opposed to being confined to sunspots. This simplification does not alter the results in a significant way, since the contribution of the surface region to the antineutrino production is entirely negligible. We use a magnetic moment throughout of $10^{-12}\mu_B$. Since only the combination μB appears in the Hamiltonian, had we taken the magnetic moment to be the maximum value allowed by the astrophysical considerations ($3 \times 10^{-12}\mu_B$), the central magnitude of the magnetic field could be reduced without altering the results. In our plots, we have used the standard solar model of Bahcall and collaborators [5].

Electron antineutrinos will be detected at SNO through the reaction $\overline{\nu}_e + d \to n + n + e^+$. The produced neutrons will be detected using (n, γ) reactions on nuclear targets or by using ^{3}He proportional counters. For this reaction, we use the cross sections calculated by Ying *et al.* [34]. (The small discrepancy between the earlier results of Ying *et al.* [35] and Tatara *et al.* [36] has been eliminated.) Electron antineutrinos will be detected at BOREXINO via inverse β decay on the protons in the trimethylborate [19]. The positron from the inverse β decay ($E_{e^+} = E_{\overline{\nu}_e} - 0.78$ MeV) can give the antineutrino spectrum. The neutron from the β decay will be rapidly absorbed by the ^{10}B in the scintillator and will emit a 0.48-MeV γ ray setting up an $e^+ - n$ delayed coincidence tag for the electron antineutrinos with negligible background. Full details of the calculation are given in Ref. [37].

Since vacuum oscillations from muon to electron antineutrinos can occur independent of the twisting of the magnetic field, we first show the expected counts per year of solar electron antineutrinos at SNO and BOREXINO, for the region of the MSW parameter space centered at $\Delta m^2 \simeq 1.5\times10^{-7}\mathrm{eV}^2, \sin^2 2\theta \simeq 0.7$, in Fig. 1. The number of counts per year in these figures are plotted as a function of the neutrino magnetic moment, for the maximum values of the magnetic field of 10^5 and 10^4 G. One can observe that, for a moment of $10^{-12}\mu_B$ and a field *maximum* of 10^5 G, SNO and BOREXINO could measure about 25 and 17 electron antineutrino counts per year, respectively, when the Sun is in its active period.

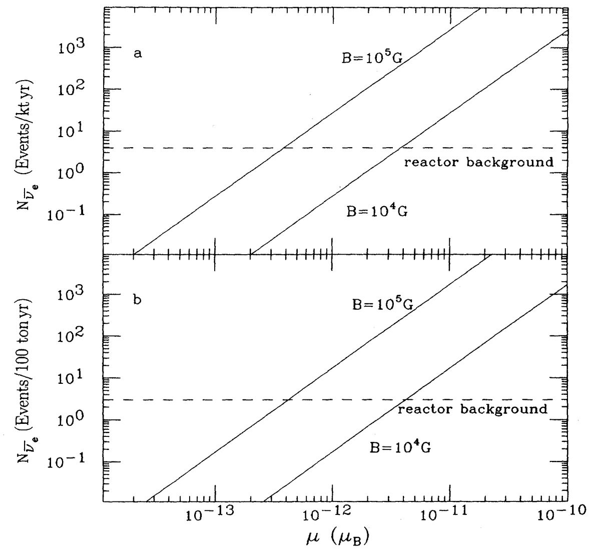

FIG. 1. (a)The antineutrino count rates at SNO per one kiloton of D_2O per year as a function of transition magnetic moment. The upper line corresponds to a maximum value of the convective zone magnetic field of 10^5 G and the lower one of 10^4 G. The mixing angle and mass difference correspond to the small region at $\Delta m^2 = 1.5\times10^{-7}, \sin^2 2\theta = 0.7$, identified in Ref. [4], where the Cl, Kamiokande, and GALLEX counting rates are simultaneously realized at the 2σ level. The dashed line shows the background from the power reactors. (b) The same as the upper panel but for the BOREXINO detector.

We now consider the case in which the magnetic field twists resulting in a resonant conversion of muon neutrinos into electron antineutrinos. Such a resonant conversion will not discernibly alter the electron neutrino flux, and cannot therefore cause a measurable anticorrelation, nor would it cause a detectable change in the counting rate of recoil electrons in the Kamiokande experiment since the cross sections for muon neutrinos and electron antineutrinos in the energy range of 6–15 MeV are nearly equal. Allowed values of the twisting of the solar magnetic field satisfy several restrictions as discussed by Yoshimura [27]. As it was previously mentioned the twisting is thought to arise from the global convection, hence we restrict the twisting to the solar convective zone (r $\geq 0.7R_{\odot}$). Second, the twisting is thought to be due to the field lines wrapping around the toroidal "ropes" of magnetic field. These "ropes" are thought to be shifted away from the solar equator, in both the solar northern and southern hemispheres, by approximately 15 degrees and the direction of the twist is thought to have opposite signs in each hemisphere [27]. Since the plane of the Earth's orbit lies at an angle of 7.25 degrees with respect to the solar equator, there will be only a period of at most several months in which such a twist could create a resonant electron antineutrino flux (recall that the twisting rate must be positive for the resonance to occur). While the size of the magnetic field in the toroidal "ropes" and the radius of the "ropes" are unknown, we will assume for the sake of argument that the magnitude of the field in the ropes can be given by the previously used field suggested by Sofia and collaborators and that the radius of the "ropes" is greater than or equal to the size of the resonance region. These assumptions will be evaluated after determining the value of the twisting rate which maximizes the counting rates at SNO and BOREXINO, for the region of the MSW parameter space located at $\Delta m^2 \simeq 1.5 \times 10^{-7}\mathrm{eV}^2, \sin^2 2\theta \simeq 0.7$

In Fig. 2 we plot the probability of producing resonant electron antineutrinos as a function of the twisting rate for the above values of the mass squared difference and mixing angle assuming a neutrino energy of 10 MeV. We have taken the twisting rate to be constant over the entire convective zone of the Sun, used the magnetic field inspired by the work of Sofia and collaborators, and have taken the neutrino magnetic moment to be $10^{-12}\mu_B$. One observes that the probability peaks at about 6 rev/$R_\odot$. The minimum twisting rate for this resonance to occur is about 2.3 rev/$R_\odot$ and the maximum rate is about 11 rev/$R_\odot$ from Eq. (19), and the probability peak is centrally located within these limits. In order to assess the width of the $\nu_\mu \to \overline{\nu}_e$ resonance region, we plot the probability of producing a resonant electron antineutrino as a function of the solar radius for the above values of the mass squared difference, mixing angle, and magnetic moment for the twisting rate of 6.1 rev/$R_\odot$, the rate which gives the maximum probability, in Fig. 3. One observes the probability begins its rise at about 0.75 $R_\odot$ (where the magnetic field has reached its maximum of 100 kG) and it continues to rise until 0.9 $R_\odot$ when vacuum oscillations from the muon antineutrino begin to take effect. This gives a resonance region width of about 0.13 $R_\odot$. This is smaller than the distance for which the neutrinos are within the "rope" of magnetic field (0.23 $R_\odot$) in the model of Kubota *et al.* [26]. Using these same parameters and a twisting rate of 6.1 rev/$R_\odot$, we plot the probability of producing a resonant electron antineutrino as a function of neutrino energy in Fig. 4. One observes a pronounced peak and a reasonably large energy width in the energy region over which the antineutrinos can be detected.

We now consider the counting rate of electron antineutrinos which would be produced at SNO for the above parameters and magnetic field. In Fig. 5(a) we plot the differential counting rate (counts/kton month MeV) as a function of neutrino energy for a twisting rate of 6.1 rev/$R_\odot$. One sees that the differential counting rate peaks at about 10 MeV as selected by our choice of the twisting rate. If one integrates this rate one obtains a

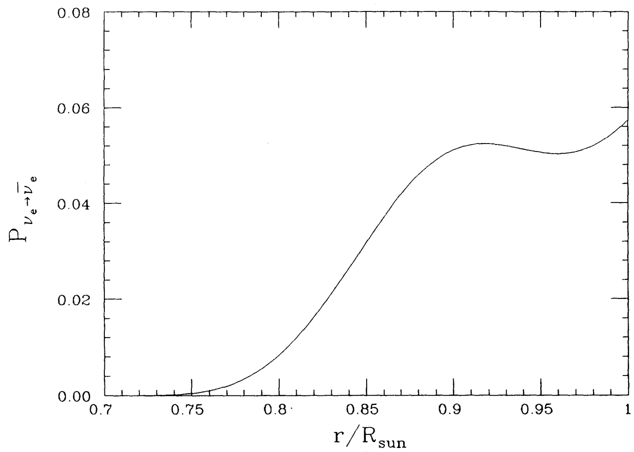

FIG. 3. The probability of producing a resonant electron antineutrino as a function of the solar radius using the same parameters and magnetic field as Fig. 2. The twisting rate is chosen to be 6.1 rev/$R_\odot$, the rate at which the probability in Fig. 2 is maximum.

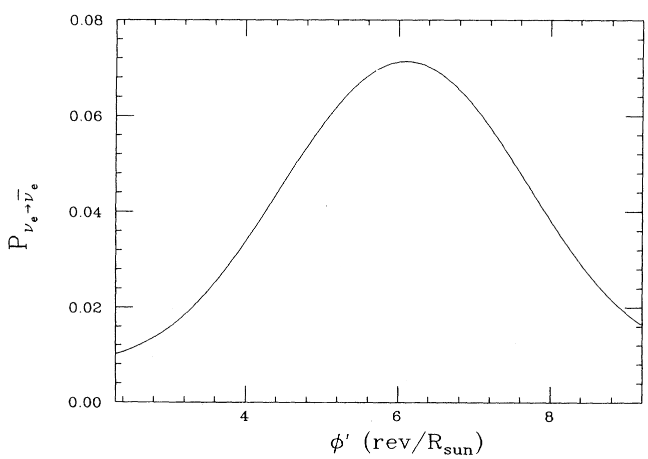

FIG. 2. The probability of producing a resonant electron antineutrino as a function of the magnetic field's (constant) twisting rate. The mass and mixing angle are chosen to be $\Delta m^2 = 1.5 \times 10^{-7}, \sin^2 2\theta = 0.7$. The magnetic moment is $10^{-12}\mu_B$, and the maximum value of the magnetic field is 10^5G.

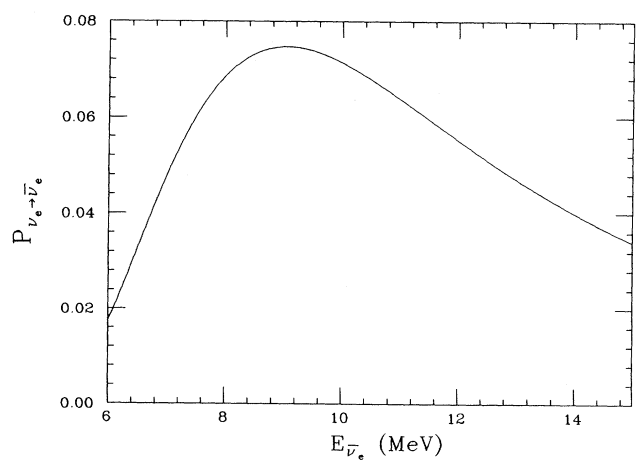

FIG. 4. The probability of producing a resonant electron antineutrino as a function of the neutrino energy using the same parameters and magnetic field as Fig. 2. The twisting rate is chosen to be 6.1 rev/R$_\odot$, the rate at which the probability in Fig. 2 is maximum.

number of counts per kiloton per month of 25.1. Thus, if such a twisting rate and magnetic field can be realized in the Sun during the month or so around the time the Earth is near its maximum distance from the solar equatorial plane, a significant number of electron antineutrino counts can be measured. This is far above the background of 1/3 counts per kiloton per month expected from the power reactors [22]. Such a high $\bar{\nu}_e$ count rate and the shape of the energy spectrum at SNO could provide a signature of the twisting of the solar magnetic field. Figure 5(b) exhibits the differential count rate for the BOREXINO detector. In Fig. 6(a) we plot the counting rate at SNO for the above parameters and magnetic field as a function of the twisting rate. The counts per kiloton per month peak at 26 for a twisting rate 6.6 rev/$R_\odot$ and drop to the level of $\sim$ 3 for a twisting rate of 1 rev/$R_\odot$. For zero twist there will still be the counting

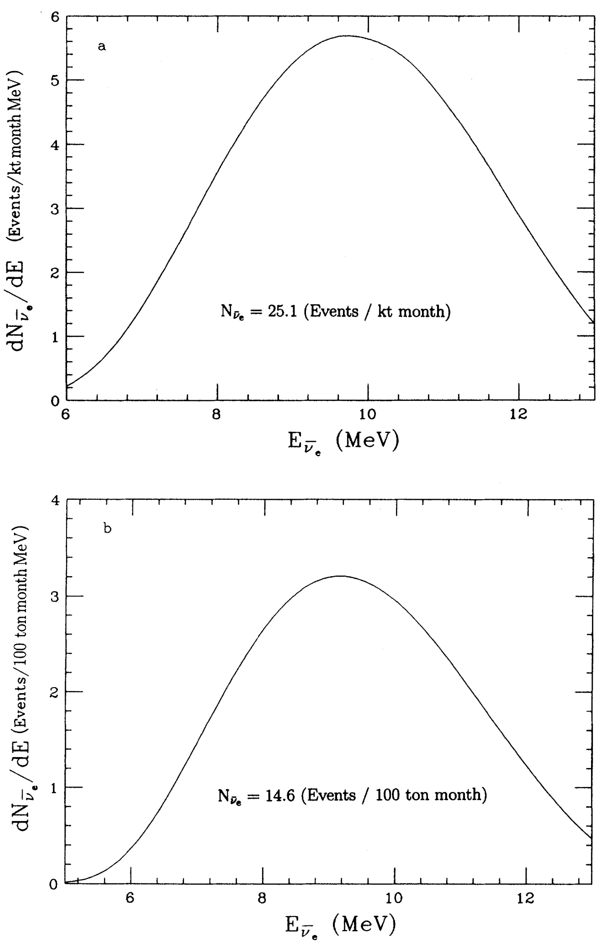

FIG. 5. (a) The counting rate of electron antineutrinos at SNO per one kiloton D_2O per month per MeV as a function of neutrino energy. The mass and mixing angle are chosen to be $\Delta m^2 = 1.5 \times 10^{-7}, \sin^2 2\theta = 0.7$. The magnetic moment is $10^{-12}\mu_B$, and the maximum value of the magnetic field is 10^5G. The twisting rate is 6.1 rev/$R_\odot$. The total number of counts per month per kiloton D_2O is 25.1. (b) The same as (a) but for the BOREXINO detector consisting of 100 tons of trimethylborate.

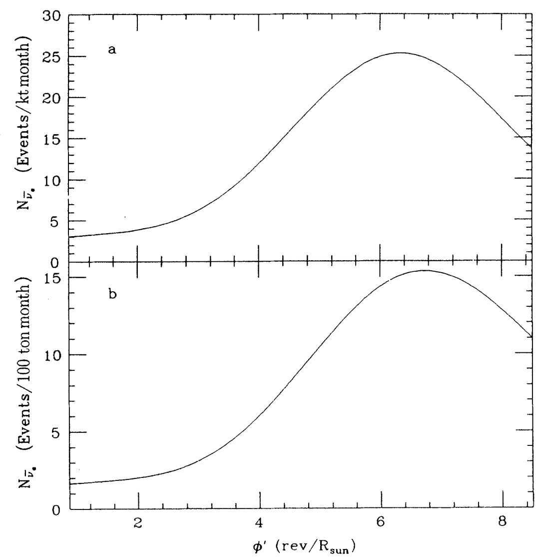

FIG. 6. (a) (upper panel) The counting rate of electron antineutrinos with E $\geq$ 6 MeV at SNO per one kiloton D_2O per month as a function of the twisting rate. The mass and mixing angle are chosen to be $\Delta m^2 = 1.5\times10^{-7}, \sin^2 2\theta = 0.7$. The magnetic moment is $10^{-12}\mu_B$, and the maximum value of the magnetic field is 10^5G. (b) (lower panel) The counting rate of electron antineutrinos with E $\geq$ 5 MeV at the BOREXINO detector consisting of 100 tons of trimethylborate.

rate from the vacuum oscillations discussed at the beginning of this section, which for this region give a monthly counting rate of about 2. Thus, the twisting field can for twisting rate values of about 6 rev/$R_\odot$ increase the counting rate of electron antineutrinos by about one order of magnitude. Such a twisting may not be unreasonably large. The region in which the resonance occurs is actually smaller than the region for which the neutrino is within the "rope" of magnetic field considered in Ref. [26], and therefore the fact that our choice of magnetic field extends over the entire convective zone does not appear to be significant. Figure 6(b) shows the same plot as Fig. 6(a) but for the BOREXINO detector.

We next consider the twisting field model of Kubota *et al.*, which is based on the work of Yoshimura. These authors considered the effects of twisting on the transitions of electron neutrinos into sterile neutrinos. We wish to use their more realistic magnetic field model to investigate resonant production of electron antineutrinos of Majorana type. In their model the magnetic field is given by

$$\begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = B(z) \begin{pmatrix} n_x \cos[\gamma(z)] \\ n_y \sin[\gamma(z)] \cos[\alpha(z)] \\ n_z \sin[\gamma(z)] \sin[\alpha(z)] \end{pmatrix}, \tag{24}$$

where the angles γ and α are given by

$$\gamma(z) = \arctan\left(\frac{2\pi R(z)}{X}\right), \tag{25}$$

$$\alpha(z) = \frac{b\cos(\Lambda - \lambda) - z}{R(z)}, \tag{26}$$

and $R(z)$ is given by

$$R(z) = \sqrt{[b\sin(\Lambda - \lambda)]^2 + [b\cos(\Lambda - \lambda) - z]^2}. \quad (27)$$

Here, Λ and λ are the angles of the center of the magnetic field torus and the neutrino path, respectively, above the solar equator. The neutrino travels along the z axis and $R(z)$ is the distance of the neutrino from the center of the magnetic field torus. The parameter b is the distance from the center of the Sun to the center of the magnetic field torus and α is the angle that the line from the neutrinos path to the torus center makes with the z axis. X is the distance it takes the magnetic field lines to wrap once around the torus. n_x, n_y, and n_z are sign factors that specify the solar hemisphere, and the sense of the twist (right handed or left handed). For a torus radius of a, the position for the neutrino to enter and exit the magnetic field is given by

$$z_{0,1} = b\cos(\Lambda - \lambda) \pm \sqrt{a^2 - b^2\sin^2(\Lambda - \lambda)}, \quad (28)$$

and the twisting rate is given by

$$\phi'(z) = \frac{2\pi X^{-1}}{1 + (2\pi X^{-1})^2[b\cos(\Lambda - \lambda) - z]^2}. \quad (29)$$

We take $B(z)$ to be given by the previously used field suggested by the work of Sofia and collaborators. For the small region of the MSW parameter space allowed at the 2σ level, we again plot the electron antineutrino probability as a function of the maximum twisting rate in Fig. 7. We have chosen $b = 0.8831 R_\odot$, $a = 0.1679 R_\odot$, $\Lambda = 15°$, and $\lambda = 7.25°$, identical to Ref. [26]. The first thing to note is that the nonuniform twisting rate gives no enhancement of the electron antineutrino probability. In fact, contrary to what one may expect, there is actually a *decrease* in the probability for a nonzero maximum twisting rate. To explain these results we plot the effective density for constant and nonconstant twisting rates in Fig. 8. The dashed line, labeled MSW, is the density corresponding to the MSW transition, N_e [cf. Eq. (15)], the solid lines correspond to the effective $\nu_\mu \to \overline{\nu}_e$ density for the constant (upper) and the nonconstant (lower) twisting rates, $(N_n - N_e) + \phi'/\sqrt{2}G_F$ [cf. Eq. (18)], and the dashed horizontal line corresponds to the quantity $\Delta m^2 \cos 2\theta / 2\sqrt{2} E G_F$ at the right-hand side of the Eqs. (15) and (18) for the parameters of the allowed region, $\Delta m^2 \simeq 1.5\times 10^{-7}\text{eV}^2, \sin^2 2\theta \simeq 0.7$, $E = 10$ MeV. In this figure we have chosen the constant twisting rate and the maximum value of the nonconstant twisting rate to be $\phi' = 6\,\text{rev}/R_\odot$. The lack of enhancement of the nonconstant twisting case compared with the constant case is explained by the position of the $\nu_\mu \to \overline{\nu}_e$ resonance in each case (where the upper and lower solid lines intersect the horizontal dashed line). For the constant twisting, this occurs at about 0.76 $R_\odot$ where the magnetic field has reached it's maximum value of 10^5 G, whereas the nonconstant case has resonance points at about 0.86 $R_\odot$ and 0.91 $R_\odot$, where the magnetic field is smaller by an order of magnitude. Secondly, since the transition probability is inversely proportional to the slope of the effective density [17], one can see that the transition probability will be smaller for the nonconstant twisting case compared to the constant twisting case. Lastly, to see why the transition probability actually decreases with the maximum twisting rate in the nonconstant case, note that a positive ϕ' pushes the $\nu_e \to \overline{\nu}_\mu$ resonance deeper into the Sun. Unlike the previous case where the magnetic field extended over the whole convective zone, the field of Ref. [26] is confined to a torus. Thus, by having a positive ϕ', one moves the $\nu_e \to \overline{\nu}_\mu$ to the inside edge of the

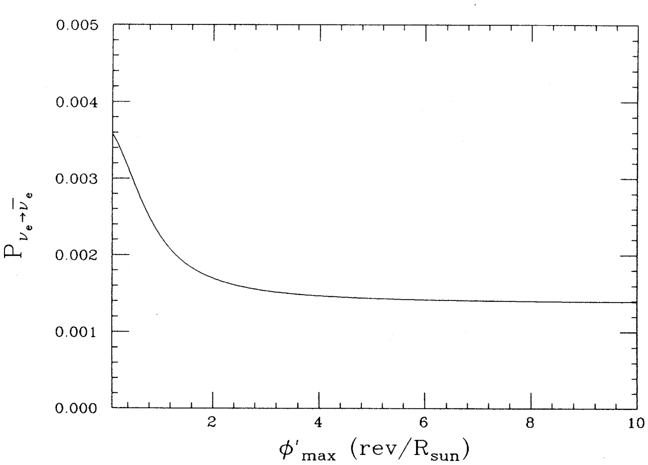

FIG. 7. The probability of producing a resonant electron antineutrino as a function of the maximum twisting rate, using the magnetic field of Ref. [26], except that the magnitude of the field, $B(z)$, is suggested by the work of Sofia and collaborators as noted in the text. The mass and mixing angle are chosen to be $\Delta m^2 = 1.5 \times 10^{-7}, \sin^2 2\theta = 0.7$. The magnetic moment is $10^{-12}\mu_B$, and the maximum value of the magnetic field is 10^5 G.

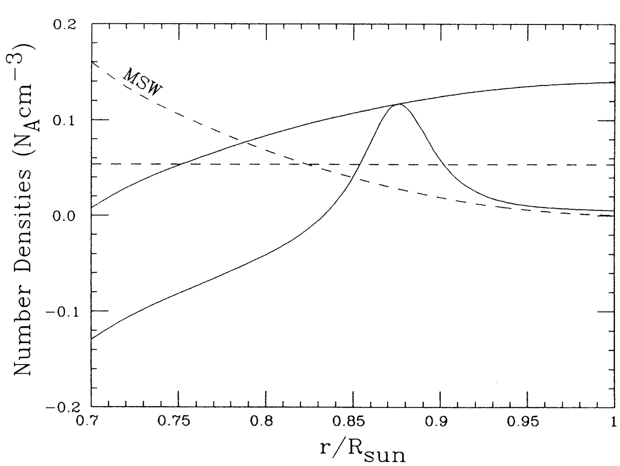

FIG. 8. A comparison of the effective density for the MSW resonance (N_e, the dashed line labeled as MSW), and the $\nu_\mu \to \overline{\nu}_e$ resonance, $(N_n - N_e + \phi'/\sqrt{2}G_F)$, for the constant (the upper solid line) and the nonconstant (the lower solid line) twisting rates. The constant and the maximum of the nonconstant rate are chosen to be 6 rev/$R_\odot$. The other dashed line is the density at which a neutrino with $\Delta m^2 = 1.5 \times 10^{-7}, \sin^2 2\theta = 0.7$ will be at resonance.

magnetic field torus, thereby decreasing the size of the resonance region which is inside the magnetic field torus and consequently decreasing the probability of producing muon antineutrinos. With a smaller $\overline{\nu}_\mu$ probability, the $\overline{\nu}_e$ probability obtained by vacuum oscillations is also smaller. If one plots the $\nu_\mu \to \overline{\nu}_e$ transition probability without the vacuum oscillations obtained on the way to Earth, there is a very slight increase in the probability compared to the zero twisting case for maximum twisting rates around $\phi'_{\max} \simeq 1 - 2$ rev$/R_\odot$.

V. CONCLUSIONS

In addition to producing electron antineutrinos by vacuum oscillations from muon antineutrinos, they can be produced in the Sun by resonance from MSW-produced muon neutrinos for a twisting solar magnetic field. If one limits oneself to the regions of the $\Delta m^2, \sin^2 2\theta$ parameter space allowed at the 2σ level by the chlorine, Kamiokande, and GALLEX experiments, one finds a very small region located at $\Delta m^2 \simeq 1.5 \times 10^{-7}, \sin^2 2\theta \simeq 0.7$ where a reasonable twisting can produce an easily measurable flux of electron antineutrinos, again for a transition moment of $\mu = 10^{-12}\mu_B$, and a magnetic field confined to the convective zone which reaches a maximum of 10^5 G. For simplicity we first examined a scenario where the twisting rate was constant over the entire convective zone. Such a case is probably not realistic. According to the model of Yoshimura, the magnetic field would be confined to toroidal "ropes," centered at an angle of 15 degrees above and below the solar equator, of opposite polarity and twisting, and of a radius about equal to 0.17 $R_\odot$. The twist is thought to increase toward the center of the "rope." Although our field extends throughout the convective zone, we have shown that the width of the resonance region is approximately 0.13 $R_\odot$, and so would fit within that part of the "rope" which lies along a radius at the $\sim \pm 7$ deg, the maximum latitude of Earth's orbit. For a twisting rate of $\phi' \simeq 6$ rev$/R_\odot$, approximately 25 electron antineutrino events could be observed at SNO during the month or so Earth is at its maximum latitude above (or below) the solar equatorial plane. This twisting rate is about 6 times greater than the largest considered by Kubota *et al.* in their model based on Yoshimura's work.

If one uses the more realistic magnetic field model of Kubota *et al.* with our choice of the profile of the overall magnitude of the magnetic field, there is a reduction in the counting rates of electron antineutrinos as the twisting is turned on. For the case of no twisting, the maximum monthly electron antineutrino counts at SNO or BOREXINO are again ~ 2, for a magnetic moment of $10^{-12}\mu_B$. Therefore, if twisting magnetic fields exist in the Sun and are similar to this case, the result would be to reduce count rates which would have been already very low without the twisting.

From our analysis we conclude that one needs the twisting to occur where the magnetic field is large, and that the effective density must be rather flat in order to obtain a large resonance region and therefore an emphatic effect. The resonant conversion of muon neutrinos to electron antineutrinos requires, for reasonable twisting rates, a small value of $\Delta m^2 \cos 2\theta$. The parameters we considered, $\Delta m^2 = 1.5 \times 10^{-7}\mathrm{eV}^2, \sin^2 2\theta = 0.7$, though allowed at the 2σ level are excluded at the 90% confidence level. As the GALLEX detector continues operation this region of the parameter space may be completely eliminated. Twisting rates for the other two allowed regions, in the hundreds of revolutions per solar radius, seem unrealistically large.

Even though our conclusions are mostly negative regarding the magnetic effects, we should emphasize that the detection of neutrons at SNO, especially with ^{3}He proportional counters, would facilitate assessment of neutrino magnetic interactions in the Sun. Even if the results from such a search for solar antineutrinos are negative, the experimental limit on the neutrino magnetic moment and solar magnetic field combination would be pushed down by another order of magnitude as compared to GALLEX and Kamiokande results.

ACKNOWLEDGMENTS

This research was supported in part by the U.S. National Science Foundation Grant No. PHY-9015255.

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