Supersymmetric QCD contributions to the top quark width

Chong Sheng Li,[1–3] Jin Min Yang,[1,3] and Bing Quan Hu[1,2]
[1]CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, People's Republic of China
[2]Department of Applied Physics, Chongqing University, Chongqing, Sichuan 630044, People's Republic of China*
[3]Theoretical Physics Institute, Academia Sinica, Beijing 100080, People's Republic of China
(Received 11 February 1993)

Virtual contributions of squark-gluino coupling to the top quark decay width $\Gamma(t\to W^+b)$ and $\Gamma(t\to H^+b)$ are calculated to one-loop order in the minimal supersymmetric standard model. The corrections are found to reduce the top quark width. If the squarks and gluinos take their lowest allowed masses, the correction to $\Gamma(t\to W^+b)$ can reach the level of 1% and the correction to $\Gamma(t\to H^+b)$ is typically a few percent.

PACS number(s): 14.80.Dq, 12.10.Dm, 12.38.Bx, 14.80.Ly

From the direct top quark search experiment at the Fermilab Tevatron [1] and from the indirect measurement of top quark induced radiative correction effects at the CERN e^+e^- collider LEP [2], a top quark mass range of 100–200 GeV is expected, with a central value of about 150 GeV. Based on these estimates there is some reasonable hope that the top quark can be discovered at the Tevatron. With the operation of CERN Large Hadron Collider (LHC) and Superconducting Super Collider (SSC) one expects to obtain a sufficiently large number of top quarks so that both the mass and the width can be measured with good accuracy. This will enable one to search for new physics beyond the standard model (SM) in the top decay. Within the framework of the SM, $t\to W^+b$ is the dominant decay mode, since other models are suppressed by the small mixing angles. Beyond the SM, a typical nonstandard model top decay channel $t\to H^+b$ could open if the mass of the charged Higgs boson [3], which appears in the two-Higgs-doublet model (2HDM) [4] such as the minimal supersymmetric extension of the standard model (MSSM) [5], is lighter than the top mass. QCD and electroweak (EW) radiative corrections to these two top decay channels have been calculated [6–9]. Recently, Gazadkowski and Hollik [10] calculated the corrections to $\Gamma(t\to W^+b)$ from the extra virtual Higgs bosons in the general 2HDM and in the MSSM. However, looking at the Higgs sector of the MSSM only gives a partial answer for the virtual effects since other virtual supersymmetric (SUSY) particles can also enter the loop diagrams and have to be considered as well. Since the gluinos and squarks are strongly interacting superparticles in MSSM and their one-loop correction to top decay width is of $O(\alpha_s)$, the virtual gluino effects may be comparable to the contribution from the Higgs sector. In this Brief Report we calculate the virtual gluino contributions to $t\to W^+b$ and $t\to H^+b$.

The diagrams contributing to the one-loop virtual gluino corrections to $t\to H^+b$ and $t\to W^+b$ are shown in Fig. 1. The particle labeled $\tilde{g}$ is a gluino and $\tilde{t}$ and $\tilde{b}$ are scalar top and bottom quarks, respectively. In our calculation, neglecting the b quark mass, we use dimensional regularization to regulate all the ultraviolet divergences in the virtual loop corrections and we adopt the on-mass-shell renormalization scheme. The straightforward calculation for these diagrams results in an effective tW^+b vertex of the form

$$-i\frac{g}{\sqrt{2}}\gamma_\mu P_L\left[1+\frac{1}{2}\delta Z_b^L+\frac{1}{2}\delta Z_t^L+\delta Z_1\right] +i\frac{g}{\sqrt{2}}(\gamma_\mu F_2+\gamma_\mu\gamma_5F_3+ik^\nu\sigma_{\mu\nu}F_4+ik^\nu\sigma_{\mu\nu}\gamma_5F_5)\,, \quad (1)$$

and an effective tH^+b vertex of the form

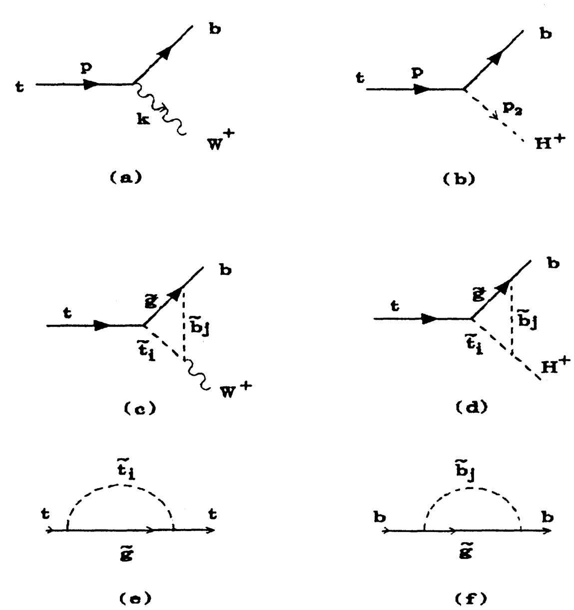

FIG. 1. Feynman diagram: (a), (b) tree level; (c), (d) one-loop vertex diagrams; (e), (f) self-energy diagrams.

*Mailing address.

$$i\frac{g\cot\beta m_t}{\sqrt{2}m_W}P_R\left[1+\tfrac{1}{2}\delta Z_b^L+\frac{1}{2}\delta Z_t^R+\frac{\delta m_t}{m_t}+\delta\Gamma_{\rm irr}\right], \tag{2}$$

with $P_{R,L}=\frac{1}{2}(1\pm\gamma_5)$. Here δZ_1, which is the singular part of the vertex correction, is given by

$$\delta Z_1=\frac{\alpha_s C_F}{4\pi}\Delta\ , \tag{3}$$

and the renormalization constants are given by

$$\delta Z_t^L=\frac{\alpha_s}{4\pi}C_F\left[-\Delta+\sum_{i=1}^{2}\left[(A_{ii}-A'_{ii})\bar{B}_1(-k,m_{\tilde g},m_{\tilde t_i})+2A_{ii}m_t^2\frac{\partial B_1(-k,m_{\tilde g},m_{\tilde t_i})}{\partial k^2}+2m_{\tilde g}m_tB_{ii}\frac{\partial\bar{B}_0(-k,m_{\tilde g},m_{\tilde t_i})}{\partial k^2}\right]\Bigg|_{k^2=m_t^2}\right], \tag{4}$$

$$\delta Z_t^R=\frac{\alpha_s}{4\pi}C_F\left[-\Delta+\sum_{i=1}^{2}\left[(A_{ii}+A'_{ii})\bar{B}_1(-k,m_{\tilde g}m_{\tilde t_i})+2A_{ii}m_t^2\frac{\partial B_1(-k,m_{\tilde g},m_{\tilde t_i})}{\partial k^2}+2m_{\tilde g}m_tB_{ii}\frac{\partial\bar{B}_0(-k,m_{\tilde g},m_{\tilde t_i})}{\partial k^2}\right]\Bigg|_{k^2=m_t^2}\right], \tag{5}$$

$$\delta Z_b^L=\frac{\alpha_s}{4\pi}C_F\left[-\Delta+\sum_{j=1}^{2}(A_{jj}-A'_{jj})\bar{B}_1(-k,m_{\tilde g},m_{\tilde b_j})|_{k^2=0}\right], \tag{6}$$

$$\frac{\delta m_t}{m_t}=\frac{\alpha_s}{4\pi}C_F\left[\Delta-\sum_{i=1}^{2}\left[\frac{m_{\tilde g}}{m_t}B_{ii}\bar{B}_0(-k,m_{\tilde g},m_{\tilde t_i})+A_{ii}\bar{B}_1(-k,m_{\tilde g},m_{\tilde t_i})\right]\Bigg|_{k^2=m_t^2}\right]. \tag{7}$$

Here,

$$\Delta\equiv\frac{1}{\varepsilon}-\gamma_E+\ln4\pi\ ,$$

where $D=4-2\varepsilon$ is the space-time dimension and γ_E is Euler's constant. The functions F_i in Eq. (1) are given by

$$F_2=\frac{\alpha_sC_F}{4\pi}\beta_{ij}[m_tm_{\tilde g}B_{ij}(c_0+c_{11})+m_t^2A_{ij}(c_{12}+c_{23}-c_{11}-c_{21})-2A_{ij}\bar{c}_{24}]\ , \tag{8}$$

$$F_3=\frac{\alpha_sC_F}{4\pi}\beta_{ij}[m_tm_{\tilde g}B'_{ij}(c_0+c_{11})+m_t^2A'_{ij}(c_{12}+c_{23}-c_{11}-c_{21})-2A'_{ij}\bar{c}_{24}]\ , \tag{9}$$

$$F_4=\frac{\alpha_sC_F}{4\pi}\beta_{ij}[m_{\tilde g}B_{ij}(c_0+c_{11})+m_tA_{ij}(c_{12}+c_{23}-c_{11}-c_{21})]\ , \tag{10}$$

$$F_5=\frac{\alpha_sC_F}{4\pi}\beta_{ij}[-m_{\tilde g}B'_{ij}(c_0+c_{11})-m_tA'_{ij}(c_{12}+c_{23}-c_{11}-c_{21})]\ , \tag{11}$$

where

$$c_0,c_{ij}\equiv c_0,c_{ij}(-p,k,m_{\tilde q},m_{\tilde t_i},m_{\tilde b_j})\ ,$$

and

$$\beta_{11}=\cos^2\theta\ ,\quad \beta_{22}=\sin^2\theta,\quad \beta_{12}=\beta_{21}=-\sin\theta\cos\theta\ . \tag{12}$$

$\delta\Gamma_{\rm irr}$ in Eq. (2) is given by

$$\delta\Gamma_{\rm irr}=-\frac{\alpha_sC_F}{4\pi m_t\cot\beta}\sum_{i,j=1}^{2}\alpha_{ij}[(B_{ij}-B'_{ij})m_{\tilde g}c_0-m_t(A_{ij}-A'_{ij})(c_{11}-c_{12})]\ , \tag{13}$$

where $c_0,c_{ij}\equiv c_0,c_{ij}(-p,p_2,m_{\tilde g},m_{\tilde t_i},m_{\tilde b_j})$, and

$$\begin{aligned}\alpha_{11}&=C_1\cos^2\theta+C_3\sin\theta\cos\theta\ ,\\ \alpha_{12}&=-C_1\sin\theta\cos\theta-C_3\sin^2\theta\ ,\\ \alpha_{21}&=C_3\cos^2\theta-C_1\sin\theta\cos\theta\ ,\\ \alpha_{22}&=-C_3\sin\theta\cos\theta+C_1\sin^2\theta\ ,\end{aligned} \tag{14}$$

with

$$C_1=m_t^2\cot\beta-m_W^2\sin2\beta\ ,\quad C_3=m_t(A_t\cot\beta-\mu)\ .$$

Note that in the above expressions $\bar{B}_0\equiv\Delta-B_0$, $\bar{B}_1\equiv\frac{1}{2}\Delta+B_1$, and $\bar{c}_{24}\equiv-\frac{1}{4}\Delta+c_{24}$. B_0, B_1, c_0, and c_{ij} functions result from the evaluation of certain loop integrals, expressions for which can be found in Ref. [11]. θ is the mixing angle of L-R squarks, and A_{ij}, B_{ij}, ..., are the functions of θ:

$$\begin{aligned}A_{ij}&=a_ia_j+b_ib_j\ ,\quad A'_{ij}=a_ib_j+b_ia_j\ ,\\ B_{ij}&=a_ia_j-b_ib_j\ ,\quad B'_{ij}=a_ib_j-a_jb_i\ ,\end{aligned} \tag{15}$$

with

$$a_{1,2}=(1/\sqrt{2})(\cos\theta\pm\sin\theta),\quad b_{1,2}=(1/\sqrt{2})(\sin\theta\mp\cos\theta)\ .$$

The renormalized decay rates are given by

$$\Gamma(t\to W^+b)=\Gamma_0(t\to W^+b)\left[1+2\,{\rm Re}\left[\frac{1}{2}\delta Z_b^L+\frac{1}{2}\delta Z_t^L+\delta Z_1+F_3-F_2+\frac{3m_tm_W^2}{m_t^2+2m_W^2}(F_4+F_5)\right]\right], \tag{16}$$

$$\Gamma(t\to H^+b)=\Gamma_0(t\to H^+b)\left[1+2\,{\rm Re}\left[\frac{1}{2}\delta Z_b^L+\frac{1}{2}\delta Z_t^R+\frac{\delta m_t}{m_t}+\delta\Gamma_{\rm irr}\right]\right], \tag{17}$$

respectively. All the ultraviolet divergences and the μ-dependent terms have canceled in Eqs. (16) and (17), as they should.

In our numerical calculations, we consider only the case of unmixed squarks and assume $m_{\tilde t_1}=m_{\tilde t_2}=m_{\tilde b_1}=m_{\tilde b_2}=m_{\tilde q}$. We use the set of independent parameters which is currently known with the highest experimental accuracy [12]:

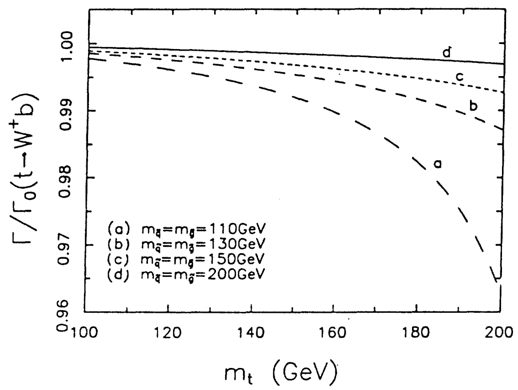

FIG. 2. Plot of Γ/Γ_0 versus m_t for $t \to W^+ b$.

$$
\alpha = \frac{1}{137.036}\,, \quad \alpha_s = 0.108,
$$

$$
G_F = 1.166\,372 \times 10^{-5}\ (\mathrm{GeV})^{-2}\,, \quad M_Z = 91.175\ \mathrm{GeV}\,, \tag{18}
$$

and m_W is determined through [13]

$$
m_W^2 \left[1 - \frac{m_W^2}{m_Z^2}\right] = \frac{\pi\alpha}{\sqrt{2}G_F} \frac{1}{1-\Delta r}\,, \tag{19}
$$

where Δr is the effect of radiative corrections. The quantity Δr as a function of the input parameters has been studied in great detail [13]. For a heavy top quark, Δr is given by [14]

$$
\Delta r \sim -\frac{\alpha N_c c_W^2 m_t^2}{16\pi s_W^4 m_W^2}\,. \tag{20}
$$

Note that the additional contributions to Δr in the MSSM, $\overline{\Delta r}$, may be quite significant for large charged Higgs boson masses [15], although SUSY QCD does not contribute to Δr. One should take into account such additional contributions $\overline{\Delta r}$ in the calculation involving exceptionally heavy Higgs particles. In this Brief Report we, approximately, neglect such additional contributions $\overline{\Delta r}$.

The results are summarized in Figs. 2–5. For $t \to W^+ b$ we find from Figs. 2 and 3 that the corrections

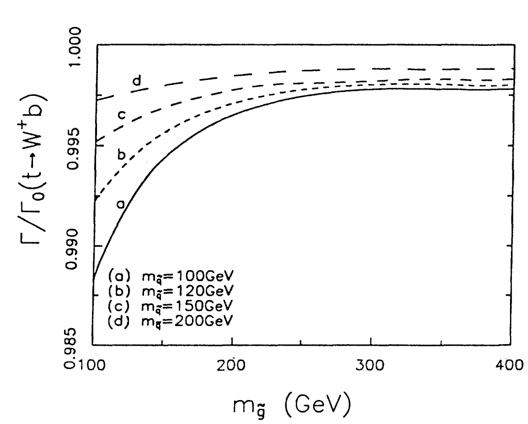

FIG. 3. Plot of Γ/Γ_0 versus $m_{\tilde g}$ for $t \to W^+ b$ with $m_t = 150$ GeV.

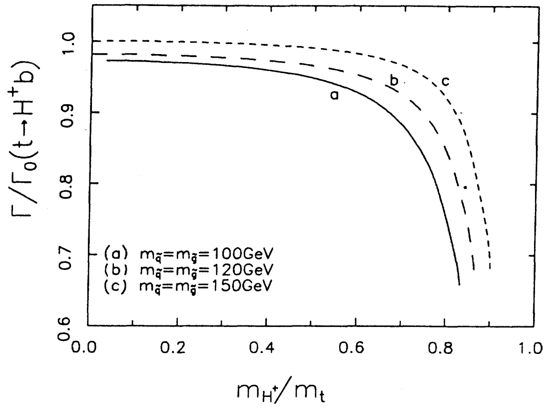

FIG. 4. Plot of Γ/Γ_0 versus m_{H^+}/m_t for $t \to H^+ b$ with $m_t = 150$ GeV.

to the partial width depend strongly on the masses of the squarks, gluinos, and the top quark. If $m_t = 150$ GeV, only for $m_{\tilde q} = m_{\tilde g} < 110$ GeV the corrections can reach the level of 1%, otherwise they are negligibly small. For $t \to H^+ b$ the corrections are related to more unknown quantities and parameters. In Fig. 4 we assume $A_t = 0.1$, $\mu = 30$ GeV [16] and $\tan\beta = 1$, $m_t = 150$ GeV. From Fig. 4 we see that the corrections are typically a few percent for $m_{H^+}/m_t < 0.8$ if $m_{\tilde q} = m_{\tilde g} < 120$ GeV. Figure 5 shows the dependence of $\Gamma/\Gamma_0(t \to H^+ b)$ on $\tan\beta$. Our calculations also show that the result is not sensitive to the parameters A_t and μ, and we do not present the dependence on them.

The recent Collider Detector at Fermilab (CDF) limits [17] on the masses of squarks and gluinos are $m_{\tilde g} > 150$ GeV (independently of $m_{\tilde q}$) and $m_{\tilde q} > 150$ GeV (for $m_{\tilde g} < 400$ GeV). However, the above CDF limits rely on some assumptions not supported by MSSM, one of which is that squarks and gluinos are supposed to decay directly to the lightest supersymmetric particle without intermediate decays to charginos or neutralinos [18]. An analysis [19] without such an assumption allows one to estimate that the Collider Detector at Fermilab (CDF) limits should be lowered by about 30 GeV. Moreover,

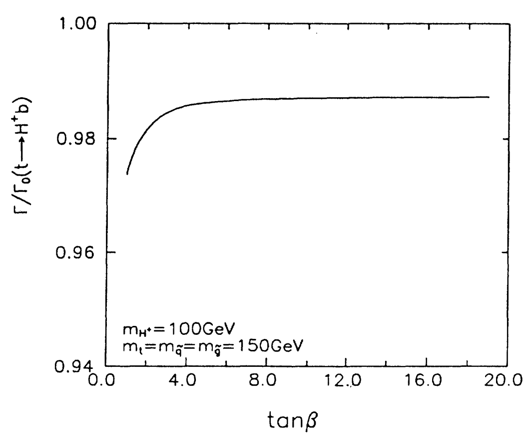

FIG. 5. Plot of Γ/Γ_0 versus $\tan\beta$ for $t \to H^+ b$.

analyzing the Tevatron dilepton data in terms of the R-parity-violating SUSY model yield a mass limit $m_{\tilde{g}} > 100$ GeV, $m_{\tilde{q}} > 100$ GeV [20]. So, if squarks and gluinos take their lowest allowed masses, their virtual effects in $\Gamma(t \to W^+ b)$ and $\Gamma(t \to H^+ b)$ can both reach the level of 1%. For the SM top decay $t \to W^+ b$, such effects are comparable to the virtual effects of the extra Higgs bosons in the MSSM given in Ref. [9], which only for large $\tan\beta$ and $m_{H^+} \simeq 100$ GeV can reach the level of 1%. The standard one-loop EW corrections [9] to $\Gamma(t \to W^+ b)$ also proved to be small (1–2 %) and practically independent of the top mass. Once the top quark is observed, radiative corrections to $t \to W^+ b$ should be experimentally determined. At the 1% level of experimental accuracy, both the QCD correction [6,7] and the EW correction [9] to $t \to W^+ b$ can be detected. As an indirect test for the existence of SUSY, one should consider the virtual effects of gluinos and squarks as well as the virtual effects of extra Higgs bosons in the precision measurements of $\Gamma(t \to W^+ b)$.

It is of interest to compare the corrections to $\Gamma/\Gamma_0(t \to W^+ b)$ and $\Gamma/\Gamma_0(t \to H^+ b)$ in the heavy top limit. For simplicity, we fixed $m_{\tilde{q}} = m_{\tilde{g}} = 130$ GeV, $m_{H^+} = 100$ GeV, and $\tan\beta = 1$. In Fig. 6 we show that with the increase of the top quark mass ($m_t > 1$ TeV), the difference of these corrections is getting negligibly small ($< 1\%$), as expected by the equivalence theorem [21]. This is also consistent with QCD corrections [7].

In conclusion, our calculations show that if the squarks and gluinos take their lowest allowed masses, the supersymmetric QCD corrections to the partial width reach the level of 1% for $\Gamma(t \to W^+ b)$ and are typically a few percent for $\Gamma(t \to H^+ b)$. Such corrections together with the contribution of extra Higgs bosons may be detectable in the future high precision experiments for top physics if SUSY exists.

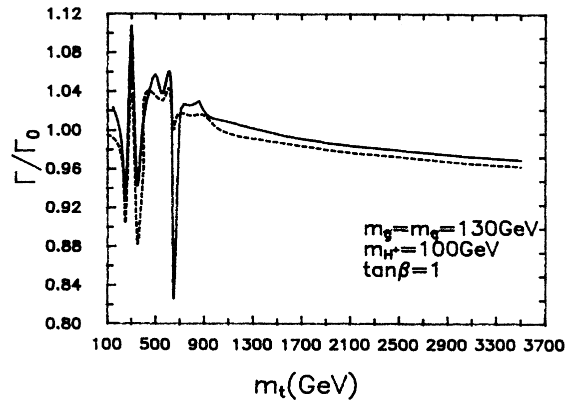

FIG. 6. Plot of Γ/Γ_0 versus $m_t = m_{\tilde{g}} = 130$ GeV, $m_{H^+} = 100$ GeV, and $\tan\beta = 1$. The solid line is the result of $\Gamma/\Gamma_0(t \to H^+ b)$, the dashed one is the result of $\Gamma/\Gamma_0(t \to W^+ b)$.

We thank C. H. Chang for discussions and T. C. Yuan for communication. This work was supported by the Natural Science Foundation of China, the State Education Committee of China, and the Applied Fundamental Research Foundation of Sichuan province.

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