# Chiral symmetry breaking and the pion wave function

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We consider here chiral symmetry breaking through a nontrivial vacuum structure with quark-antiquark condensates. We then relate the condensate function to the wave function of the pion as a Goldstone mode. This simultaneously yields the pion also as a quark-antiquark bound state as a localized zero mode in vacuum. We illustrate the above through the Nambu–Jona-Lasinio model to calculate different pionic properties in terms of the vacuum structure for breaking of exact or approximate chiral symmetry, as well as the condensate fluctuations giving rise to $\sigma$ mesons.

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## I. INTRODUCTION

Nambu and Jona-Lasinio (NJL) have linked chiral symmetry breaking [1] to properties of hadrons with the pion appearing as the Goldstone boson [2, 3]. However, the pion is also thought to be a quark-antiquark bound state. Hence, through the Goldstone theorem it ought to be possible to relate the wave function of the pion as a quark-antiquark pair to the vacuum structure. It is surprising that this aspect is absent in the very extensive literature on the topic [1–4].

We consider the phase transition as a vacuum realignment with an explicit structure. Using techniques developed earlier [5] we demonstrated [6] the gap equation for potential models to be the same as the one derived through the Schwinger-Dyson equation. We also demonstrated that the Goldstone theorem [7] led one to a pion state as a localized quark-antiquark zero mode of the destabilized vacuum [6, 8]. We further discussed the effects of *approximate* symmetry by relating changes in the gap equation to changes in the pion wave function. In this paper we discuss the same physics in the Nambu–Jona-Lasinio model. The reason for doing so is the mathematical simplicity of the NJL model and its relevance in the context of Salam-Weinberg symmetry breaking and the top quark mass [9, 10].

We organize the paper as follows. In Sec. II we consider the vacuum structure with quark-antiquark pairs using an ansatz for the same by minimizing the energy density. This gives rise to the conventional gap equation and involves a new description of the phase transition with an explicit construct for the destabilized vacuum [5]. In Sec. III we identify the pion as a Goldstone mode and relate its wave function with functions associated with the vacuum structure. In Sec. IV we consider the vacuum structure for the NJL model when chiral symmetry is approximate. We also derive here some familiar results of current algebra in the present framework. In Sec. V we calculate the pion charge radius using the wave function determined from the vacuum structure. In Sec. VI we consider the fluctuation of the condensate mode to give a qualitative identification of the $\sigma$ meson. Section VII consists of discussions.

The method here consists of using equal time algebra [11–13] along with the construction of the ground state through a variational principle [5, 6, 14].

## II. CHIRAL SYMMETRY BREAKING AND VACUUM REALIGNMENT

We shall now proceed in the same manner as earlier [6] for the NJL model. Let us start with the effective Hamiltonian

$$\mathcal{H}(\mathbf{x}) = \psi(\mathbf{x})^{i\dagger}(-i\boldsymbol{\alpha}\cdot\nabla)\psi(\mathbf{x})^i + \int d\mathbf{y}\,\psi_\alpha^{i\,\dagger}(\mathbf{x})\psi_\beta^j(\mathbf{x})V_{\alpha\beta,\gamma\delta}^{ij,kl}(\mathbf{x}-\mathbf{y})\psi_\gamma^k(\mathbf{y})^\dagger\psi_\delta^l(\mathbf{y}), \tag{1}$$

which has chiral invariance. In the above $i, j$ stand for color indices, $\alpha, \beta$ stand for the spinor indices, and $V_{\alpha\beta,\gamma\delta}^{ij,kl}(\mathbf{x}-\mathbf{y})$ is the potential. For the effective QCD-based vector potential we may take

$$V_{\alpha\beta,\gamma\delta}^{ij,kl}(\mathbf{x}-\mathbf{y}) = \delta_{\alpha\beta}\delta_{\gamma\delta}\left(\frac{\lambda^a}{2}\right)_{ij}\left(\frac{\lambda^a}{2}\right)_{kl}V(|\,\mathbf{x}-\mathbf{y}\,|), \tag{2a}$$

where $\lambda^a$ are the Gell-Mann matrices. We may also have the NJL model when we take the contact potential as

$$V_{\alpha\beta,\gamma\delta}^{ij,kl}(\mathbf{x}-\mathbf{y}) = G\left[(\gamma^0)_{\alpha\beta}(\gamma^0)_{\gamma\delta}\delta^{ij}\delta^{kl} - (\gamma^0\gamma^5)_{\alpha\beta}(\gamma^0\gamma^5)_{\gamma\delta}(\tau^a)_{ij}(\tau^a)_{kl}\right]\delta(\mathbf{x}-\mathbf{y}). \tag{2b}$$

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Here $G$ is the dimensional interaction coupling constant and the $\tau^a$'s are the isospin matrices.

The field operators $\psi(\mathbf{x})$ may be expanded as

$$\psi(\mathbf{x}) = \frac{1}{(2\pi)^{3/2}} \int [U_r(\mathbf{k})c_{Ir}(\mathbf{k}) + V_s(-\mathbf{k})\tilde{c}_{Is}(-\mathbf{k})] \times e^{i\mathbf{k}\cdot\mathbf{x}} d\mathbf{k} , \tag{3}$$

where $U$ and $V$ are given by

$$U_r(\mathbf{k}) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \boldsymbol{\sigma}\cdot\hat{\mathbf{k}} \end{pmatrix} u_{Ir}; \quad V_s(-\mathbf{k}) = \frac{1}{\sqrt{2}} \begin{pmatrix} -\boldsymbol{\sigma}\cdot\hat{\mathbf{k}} \\ 1 \end{pmatrix} v_{Is} \tag{4}$$

for *free* chiral fields. The perturbative vacuum is defined by this basis when we have $c_I \mid \text{vac}\rangle = 0 = \tilde{c}_I^\dagger \mid \text{vac}\rangle$. We next consider a trial vacuum state given as [5, 6]

$$\mid \text{vac}'\rangle = U \mid \text{vac}\rangle \equiv \exp(B^\dagger - B) \mid \text{vac}\rangle , \tag{5a}$$

with

$$B^\dagger = \int c_{Ir}(\mathbf{k})^\dagger u_{Ir}{}^\dagger(\boldsymbol{\sigma}\cdot\hat{\mathbf{k}}) v_{Is}\tilde{c}_{Is}(-\mathbf{k}) f(\mathbf{k}) d\mathbf{k}. \tag{5b}$$

Here $f(\mathbf{k})$ is a trial function associated as above with the quark-antiquark condensate. We may recall a similar construction in the Bogoliubov-Valatin approach [1, 2, 4]. We shall minimize the energy density of $\mid \text{vac}'\rangle$ to analyze the possibility of a phase transition [5] from $|\text{vac}\rangle$ to $|\text{vac}'\rangle$. For this purpose we first note that with the above transformation the operators which annihilate $\mid \text{vac}'\rangle$ are given as

$$b_I(\mathbf{k}) = U c_I(\mathbf{k}) U^{-1}, \tag{6}$$

which with an explicit calculation yields the Bogoliubov transformation

$$\begin{pmatrix} b_{Ir}(\mathbf{k}) \\ \tilde{b}_{Is}(-\mathbf{k}) \end{pmatrix} = \begin{pmatrix} \cos f & -\frac{f}{|f|}\sin f a_{rs} \\ \frac{f^*}{|f|}\sin f (a^\dagger)_{sr} & \cos f \end{pmatrix} \times \begin{pmatrix} c_{Ir}(\mathbf{k}) \\ \tilde{c}_{Is}(-\mathbf{k}) \end{pmatrix}. \tag{7}$$

Here $a_{rs} = u_{Ir}{}^\dagger(\boldsymbol{\sigma}\cdot\hat{\mathbf{k}})v_{Is}$. Using the above transformation (6) or (7) the expectation value of the Hamiltonian with respect to $\mid \text{vac}'\rangle$ is given as

$$\mathcal{E} = \langle \text{vac}' \mid \mathcal{H}(x) \mid \text{vac}'\rangle \equiv T + V, \tag{8}$$

where $T$ and $V$ are the expectation values corresponding to the kinetic and the potential terms in Eq. (1). With a straightforward evaluation we then obtain that

$$T = \langle \text{vac}' \mid \psi^i(\mathbf{x})^\dagger(-i\alpha\cdot\nabla)\psi^i(\mathbf{x}) \mid \text{vac}'\rangle = -\frac{2N}{(2\pi)^3}\int d\mathbf{k} \mid \mathbf{k} \mid \cos 2f(k), \tag{9}$$

where $N = N_c \times N_f$ is the total number of quarks. Similarly the potential term is given as

$$V = \frac{1}{(2\pi)^6}\int \tilde{V}^{ij,kl}_{\alpha\beta,\gamma\delta}(\mathbf{k}_1 - \mathbf{k}_2) \times [\Lambda_+(\mathbf{k}_1)]_{\beta\gamma}[\Lambda_-(\mathbf{k}_2)]_{\delta\alpha} d\mathbf{k}_1 d\mathbf{k}_2, \tag{10}$$

where $\tilde{V}(\mathbf{k})$ is the Fourier transform of the potential $V(\mathbf{r})$ given as

$$\tilde{V}(\mathbf{k}) = \int V(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}} d\mathbf{k}, \tag{11}$$

and $\Lambda_\pm$ are

$$\Lambda_\pm(\mathbf{k}) = \tfrac{1}{2}[1 \pm \gamma^0 \sin 2f(k) \pm \boldsymbol{\alpha}\cdot\hat{k}\cos 2f(k)]. \tag{12}$$

The expression for $V$ as in Eq. (10) can be calculated for Eq. (1) or (2a) for an effective potential [2]. We shall however now illustrate the method with Eq. (2b) for the NJL model corresponding to the contact potential. The total energy density then becomes

$$\mathcal{E}\{f\} = \mathcal{E} = -\frac{2N}{(2\pi)^3}\int d\mathbf{k} \mid \mathbf{k} \mid \cos 2f - 2GN(2N+1)I^2 , \tag{13}$$

with

$$I = \frac{1}{(2\pi)^3}\int \sin 2f(k) d\mathbf{k}. \tag{14}$$

The leading order in $N$ here corresponds to the Hartree approximation [4]. The energy functional $\mathcal{E}\{f\}$ here is to be determined by minimizing the energy density. This yields

$$\tan 2f(k) = \frac{2GI(2N+1)}{k} \equiv M/k, \tag{15}$$

where $M \equiv 2GI(2N+1)$ is the dynamically generated mass. Further, substituting the above in Eq. (14) yields the self-consistency relation

$$M = \frac{2G(2N+1)}{(2\pi)^3}\int^\Lambda \frac{M}{\sqrt{M^2+k^2}} d\mathbf{k}, \tag{16}$$

with $\Lambda$ above as the ultraviolet cutoff for the NJL model. Equation (16) is usually derived through an approximate solution to Schwinger-Dyson equation [3]. We followed here an alternative variational method with the phase transition as a *vacuum realignment* as in Eq. (5), determined through the *minimizing energy density functional* [5].

The above equation has a solution with $M \neq 0$ (Goldstone phase) provided

$$G\Lambda^2(2N+1) > 2\pi^2. \tag{17}$$

The energy density of $\mid \text{vac}'\rangle$ with respect to the perturbative vacuum $\mid \text{vac}\rangle$ may be evaluated to be

$$\Delta\mathcal{E} = \mathcal{E}\{f\} - \mathcal{E}\{f=0\} = \frac{2N}{(2\pi)^3}\int^\Lambda (k - \sqrt{k^2+M^2}) d\mathbf{k} + \frac{N}{2G(2N+1)}M^2 , \tag{18}$$

which is negative when a nontrivial solution to Eq. (16) exists or Eq. (17) is satisfied. The state with condensates $\mid \text{vac}'\rangle$ then becomes the physical vacuum. One may also calculate the order parameter $\langle\bar{\psi}\psi\rangle$ given as

$$\langle \text{vac}' \mid \bar{\psi}\psi \mid \text{vac}' \rangle = -\frac{1}{(2\pi)^3} 2NM \int^{\Lambda} \frac{d\mathbf{k}}{\sqrt{k^2+M^2}} \, . \tag{19}$$

## III. GOLDSTONE THEOREM AND PION WAVE FUNCTION

We shall now recapitulate [6] that the present description of the phase transition permits us to define the pion also as a quark-antiquark pair. From the gap equation we obtained two solutions for the field operators corresponding to $\sin 2f(k) = 0$ or $\sin 2f(k) \neq 0$ along with the corresponding ground state as $\mid \text{vac}\rangle$ or $\mid \text{vac}'\rangle$, respectively.

For the case of chiral symmetry breaking, we have the gap equation

$$1 = \frac{2G(2N+1)}{(2\pi)^3} \int^{\Lambda} \frac{1}{\sqrt{M^2+k^2}} d\mathbf{k}, \tag{20}$$

which determines the value of the mass parameter $M$. Once $M$ is determined, the function $f(\mathbf{k})$ becomes known and hence the condensate structure of the vacuum becomes known. However, the Hamiltonian of Eq. (1) had chiral symmetry, which through Eq. (19) or otherwise is now seen to be broken. Hence we should have a Goldstone mode corresponding to a zero mass particle [7]. We shall approach this theorem in a modified manner to obtain the wave function as a quark-antiquark pair [8]. When chiral symmetry remains good,

$$Q_5{}^a \mid \text{vac}\rangle = 0 \, , \tag{21}$$

where $Q_5{}^a$ is the chiral charge operator given as

$$Q_5{}^a = \int \psi(\mathbf{x})^\dagger \frac{\tau^a}{2} \gamma^5 \psi(\mathbf{x}) d\mathbf{x}. \tag{22}$$

For the symmetry broken case, however,

$$Q_5{}^a \mid \text{vac}'\rangle \neq 0. \tag{23}$$

We expect that this will describe a pion of zero total momentum. Since it will be massless, it will also have zero energy corresponding to the pion state. To show this we first note that

$$[Q_5{}^a, H] = 0, \tag{24}$$

irrespective of whether $Q_5{}^a$ and $H$ are written in terms of field operators corresponding to $\sin 2f = 0$ or $\sin 2f \neq 0$ since the anticommutation relations between the operators remain unchanged by the Bogoliubov transformation. Clearly, for the Goldstone phase, $|\text{vac}'\rangle$ is an approximate eigenstate of $H$ with $\mathcal{E}V$ as the approximate eigenvalue ($V$ being the total volume). With $H_{\text{eff}} = H - \mathcal{E}V$, we then obtain from Eq. (23) that

$$H_{\text{eff}} Q_5{}^a |\text{vac}'\rangle = 0 \, ; \tag{25}$$

i.e., the state $Q_5{}^a \mid \text{vac}'\rangle$ with zero momentum has also zero energy, thus corresponding to the massless pion. Explicitly, using Eqs. (3) and (7), we then obtain, with $q_I$ now as a two-component isospin doublet corresponding to $(u,d)$ quarks above,

$$\mid \pi^a(\mathbf{0})\rangle = N_\pi \int q_I(\mathbf{k})^\dagger \left(\frac{\tau^a}{2}\right) \tilde{q}_I(-\mathbf{k}) \sin 2f(k) d\mathbf{k} \mid \text{vac}'\rangle, \tag{26}$$

where $N_\pi$ is a normalization constant. The wave function $\tilde{u}(\mathbf{k})$ for the pion thus is given as proportional to $\sin 2f(k)$. The operators $q_I^\dagger(\mathbf{k})$ and $\tilde{q}_I(-\mathbf{k})$ above are respectively the *two-component* creation operators for quarks and antiquarks with the spin and isospin indices of the quarks and antiquarks having been suppressed. Clearly the above state has an odd parity, is an isospin triplet, and, as stated, corresponds to the pion. Using the normalization

$$\langle \pi^a(\mathbf{0}) \mid \pi^b(\mathbf{p})\rangle = \delta^{ab}\delta(\mathbf{p}), \tag{27}$$

the constant $N_\pi$ is given by

$$N_\pi^2 \frac{N_c N_f}{2} \int \sin^2 2f(k) d\mathbf{k} = 1. \tag{28}$$

Further, the state as in Eq. (26) as the Goldstone mode will be accurate to the extent we determine the vacuum structure sufficiently accurately through variational or any other method, so that $|\text{vac}'\rangle$ is an eigenstate of the Hamiltonian. The above results yield the derivation of the pion wave function from the vacuum structure for chiral symmetry breaking through Eq. (25).

We may note that we could relate the pion wave function to the vacuum structure since the vacuum had an explicit structure as in Eqs. (5). In fact, the two-body condensate as in Eqs. (5) for the destabilized vacuum is strictly related to the presence of the zero mode as seen here.

## IV. APPROXIMATE CHIRAL SYMMETRY

While considering chiral symmetry breaking, we often use results from current algebra so that we may obtain numbers for approximate chiral symmetry breaking. For the sake of completeness, with the present mechanism, we elaborate [6] these results so as to use the same for the NJL model. For this purpose we may add a small mass term to the Hamiltonian that breaks the chiral symmetry explicitly. Then $Q_5{}^a \mid \text{vac}'\rangle$ will not be a zero mode and will have a finite mass. In fact the mass of the pion in the lowest order will now be $m_\pi$, formally given as

$$\langle \pi^a(\mathbf{0}) \mid H_{\text{SB}} \mid \pi^a(\mathbf{0})\rangle = m_\pi \delta(\mathbf{0}), \tag{29}$$

where $H_{\text{SB}}$ is the symmetry breaking part of the Hamiltonian corresponding to the Hamiltonian density $\mathcal{H}_{\text{SB}} = m\bar{\psi}\psi$, $m$ being the current quark mass. The above may be related to $N_\pi$ and the pion decay constant as follows. First we note that the identity for the pion decay constant is [15]

$$\langle 0 \mid J_5{}^{0a} \mid \pi(\mathbf{p})\rangle = \frac{i}{(2\pi)^{3/2}} \frac{c_\pi p_0}{\sqrt{2p_0}} e^{i\mathbf{p}\cdot\mathbf{x}}, \tag{30}$$

where $c_\pi = 94$ MeV. The normalization constant $N_\pi$ in

Eq. (26) is then given by using

$$N_\pi^{-2} \times \delta(\mathbf{0}) = \langle \text{vac}' \mid Q^a{}_5 Q^a{}_5 \mid \text{vac}' \rangle = \int \langle \text{vac}' \mid Q^a{}_5 \mid \pi^b(\mathbf{p}) \rangle d\mathbf{p} \times \langle \pi^b(\mathbf{p}) \mid Q^a{}_5 \mid \text{vac}' \rangle, \quad (31)$$

where we have saturated the intermediate states with pions. The index $b$ is summed and there is no summation over the index $a$. With Eqs. (30) and (31) we then have

$$N_\pi^{-2} = \tfrac{1}{2}(2\pi)^3 m_\pi c_\pi^2, \quad (32)$$

which links $N_\pi$ of the vacuum structure with the pion mass and pion decay constant. We shall now substitute explicitly the pion state as in Eq. (26) in Eq. (29). We shall also substitute the value of normalization constant in Eq. (26) by Eq. (32). On using straightforward commutation relations, we then obtain that

$$\langle \pi^a(\mathbf{0}) \mid H_{\text{SB}} \mid \pi^a(\mathbf{0}) \rangle = \frac{2}{m_\pi {c_\pi}^2} \frac{1}{(2\pi)^3} \langle \text{vac}' \mid Q^a{}_5 H_{\text{SB}} Q^a{}_5 \mid \text{vac}' \rangle$$

$$= \frac{2}{m_\pi c_\pi^2} \frac{1}{(2\pi)^3} \frac{1}{2} \langle \text{vac}' \mid [[Q^a{}_5, H_{\text{SB}}], Q^a{}_5] \mid \text{vac}' \rangle \quad (33)$$

$$= \frac{2}{m_\pi c_\pi^2} \left(-\frac{m}{2}\right) \langle \text{vac}' | \bar{\psi}\psi \mid \text{vac}' \rangle \delta(\mathbf{0}). \quad (34)$$

From Eqs. (34) and (29) we then obtain that

$$m_\pi^2 = -\frac{m}{c_\pi^2} \langle \bar{\psi}\psi \rangle , \quad (35)$$

which is the familiar result for current algebra.

## V. CHARGE RADIUS OF PION

With the wave function of the pion as above, we may next estimate the size of the Goldstone pion as related to the vacuum structure. For this purpose we first construct the positively charged pion state with momentum $\mathbf{p}$ from Eq. (26) by using translational invariance. With the same notation as earlier, taking now the appropriate isospin combination, this state is given as

$$|\pi^+(\mathbf{p})\rangle = N_\pi \int d\mathbf{k} q_I^i \left(\mathbf{k} + \frac{\mathbf{p}}{2}\right)^\dagger (\tau^+)_{ij} \times \tilde{q}_I^j \left(-\mathbf{k} + \frac{\mathbf{p}}{2}\right) \tilde{u}(\mathbf{k}) |\text{vac}'\rangle. \quad (36)$$

In the Breit frame the electric form factor is given by [11]

$$G_E(t) = (2\pi)^3 \langle \pi^+(-\mathbf{p}) | J_0 | \pi^+(\mathbf{p}) \rangle , \quad (37)$$

where $t = -4p^2$ and $J_0 = e\psi^\dagger\psi$. This may be evaluated directly as

$$G_E(t) = eN_p i^2 \int d\mathbf{k} \tilde{u} \left(\mathbf{k} - \frac{\mathbf{p}}{2}\right)^* \tilde{u} \left(\mathbf{k} + \frac{\mathbf{p}}{2}\right) \times \Big\{ u_1(\mathbf{k} - \mathbf{p}) u_1(\mathbf{k} + \mathbf{p}) + (k^2 - p^2) u_2(\mathbf{k} - \mathbf{p}) u_2(\mathbf{k} + \mathbf{p}) \Big\}. \quad (38)$$

In the above

$$u_1(\mathbf{k}) = \sqrt{\frac{1 + \sin 2f(|\mathbf{k}|)}{2}}, \qquad u_2(\mathbf{k}) = \frac{1}{|\mathbf{k}|} \sqrt{\frac{1 - \sin 2f(|\mathbf{k}|)}{2}}. \quad (39)$$

To calculate the charge radius we expand the above in powers of $\mathbf{p}$ and the coefficient of $p^2$ will be related to the charge radius through

$$G_E(t) = e(1 + \tfrac{1}{6} R_{\text{ch}}^2 t + \cdots). \quad (40)$$

With $G_E(t)$ as in Eq. (38) we then obtain that

$$\langle R_{\text{ch}}^2 \rangle = \frac{1}{2} \int d\mathbf{k} \bigg[ \frac{1}{4} \left( u_0'(k)^2 - \frac{2}{k} u_0' u_0 - u_0'' u_0 \right) + u_0^2 \bigg\{ \left( u_1'^2 - \frac{2}{k} u_1' u_1 - u_1'' u_1 \right) + 3u_2^2 + k^2 \left( u_2'^2 - \frac{2}{k} u_2' u_2 - u_2'' u_2 \right) \bigg\} \bigg], \quad (41)$$

where we have substituted

$$u_0(k) = \frac{1}{\sqrt{\int \sin^2 2f(k) d\mathbf{k}}} \tilde{u}(k), \quad (42)$$

and primes denote differentiation with respect to $k$.

The above formula applies for any known vacuum realignment with condensates. Let us now estimate the charge radius in the Nambu–Jona-Lasinio model. We shall also use Eq. (34) for the pion decay constant so that chiral symmetry is approximately true. With $\mathcal{H}_{\text{SB}} = m\bar{\psi}\psi$, the extra contribution to the energy density is $-m \times 2NI$. On extremization the modified gap function is given by

$$\tan 2f(k) = \frac{2G(2N+1)I + m}{|k|} = \frac{M'}{k}, \tag{43}$$

where $M' = 2G(2N+1)I + m$ satisfies the equation parallel to Eq. (16) given as

$$1 = \frac{2G(2N+1)}{(2\pi)^3} \int^{\Lambda} \frac{d\mathbf{k}}{\sqrt{k^2 + M'^2}} + m/M'. \tag{44}$$

Thus here we may also have a vacuum realignment.

We shall now choose an optimal set of parameters for $G\Lambda^2$, $\Lambda$, and $m$. Then, for example, with $\Lambda = 420$ MeV, $G\Lambda^2$=2.24, and $m$ = 16 MeV, we get $M$=305 MeV, $\langle -\bar{\psi}\psi \rangle^{1/3}$= 220 MeV, $R_\pi^2$=0.25 fm$^2$, and $c_\pi$= 94 MeV. Here we have taken $m_\pi$ = 138 MeV. As a further illustration to see how the the corresponding quantities change with parameters, for $\Lambda$=500 MeV, $G\Lambda^2$=2.15, $m$=10 MeV, we have, $M$=320 MeV, $\langle -\bar{\psi}\psi \rangle^{1/3}$=255 MeV, $R_\pi^2$=0.20 fm$^2$, and $c_\pi$=93 MeV. We note that the pion structure as arising from vacuum realignment appears to give a smaller value of the charge radius than would be expected. In fact, with $G\Lambda^2$ = 2.0, $\Lambda$ = 700 MeV, and $m$= 5 MeV [3], we obtain that $M$= 360 MeV, $\langle -\bar{\psi}\psi \rangle^{1/3}$= 342 MeV, $R_\pi^2$= 0.13 fm$^2$, and $c_\pi$=103 MeV. Thus the above set of parameters does not appear to be acceptable [4, 16]. We may also note that the four-component Dirac field operators for the quarks will change the above numbers as examined elsewhere [12], which however does not change the above remarks. The above illustrates the nature of constraints derived for symmetry breaking through determination of the pion wave function. A parallel approach [17] with Bogoliubov transformations and the Schwinger-Dyson equation has been used to obtain the Salpeter wave function for the pion, which, however, does not permit the definition of the pion as a state since the wave function is not normalizable and therefore does not give rise to such constraints.

## VI. NEW MODES IN VACUUM

When the vacuum has a structure, there can be excitations present due to such a structure. For chiral symmetry breaking, let us substitute

$$\begin{aligned}\bar{\psi}(\mathbf{x})\psi(\mathbf{x}) &= \langle \bar{\psi}(\mathbf{x})\psi(\mathbf{x})\rangle + M_{sc}^2\sigma(\mathbf{x}) \\ &\equiv \mu^3 + M_{sc}^2\sigma(\mathbf{x}),\end{aligned} \tag{45}$$

where $M_{sc}$ is a mass parameter and $\sigma(\mathbf{x})$ may represent the scalar field of vacuum fluctuations. Then $\sigma(\mathbf{x})$ can represent quantum fluctuations of the condensate. In fact, we may evaluate

$$\begin{aligned}\langle \text{vac}'|[\bar{\psi}(\mathbf{x})\psi(\mathbf{x}) - \mu^3][\bar{\psi}(\mathbf{y})\psi(\mathbf{y}) - \mu^3]|\text{vac}'\rangle &= M_{sc}^4 \langle \text{vac}'|\sigma(\mathbf{x})\sigma(\mathbf{y})|\text{vac}'\rangle \\ &\simeq \frac{M_{sc}^4}{(2\pi)^3} \int \frac{e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}}{2(\mathbf{k}^2 + m_\sigma^2)^{1/2}} d\mathbf{k},\end{aligned} \tag{46}$$

where we approximate $\sigma(\mathbf{x})$ by a free field of mass $m_\sigma$. Let us define

$$I(\mathbf{k}) = \int d\mathbf{x} \exp(-i\mathbf{k}\cdot\mathbf{x}) \langle \text{vac}'|[\bar{\psi}(\mathbf{x})\psi(\mathbf{x}) - \mu^3][\bar{\psi}(\mathbf{0})\psi(\mathbf{0}) - \mu^3]|\text{vac}'\rangle . \tag{47}$$

In that case, clearly the free field approximation for $\sigma(\mathbf{x})$ corresponds to

$$I(\mathbf{k}) = \frac{M_{sc}^4}{2\sqrt{m_\sigma^2 + \mathbf{k}^2}}. \tag{48}$$

Explicit evaluation of the right hand side of Eq. (47) in the limit of small $|\mathbf{k}|$ yields

$$\begin{aligned}I(\mathbf{k}) &\simeq \frac{1}{\pi^2}\left[\left\{\frac{\Lambda^3}{3} - M^2\Lambda + M^3 \arctan^{-1}\left(\frac{\Lambda}{M}\right)\right\}\right. \\ &\quad \left. - \mathbf{k}^2\left\{\frac{\Lambda^5}{6(\Lambda^2+M^2)^2} + \frac{1}{8}\frac{\Lambda^3 M^2}{(\Lambda^2+M^2)^2} + \frac{3}{16}\frac{\Lambda M^2}{(\Lambda^2+M^2)} - \frac{3}{16} M \arctan\left(\frac{\Lambda}{M}\right)\right\}\right] \\ &= \frac{1}{8\pi^3}\left[\frac{M_{sc}^4}{2m_\sigma} - \mathbf{k}^2 \frac{M_{sc}^4}{4m_\sigma^3}\right],\end{aligned} \tag{49}$$

where in Eq. (48) we have kept terms up to $\mathbf{k}^2$. Equating equal powers of $\mathbf{k}$ in Eq. (49) and eliminating $M_{sc}$ in favor of $m_\sigma$ yields, with $x = M/\Lambda$,

$$m_\sigma^2 = \frac{\Lambda^2}{2}\left[\frac{\frac{x^3}{3} - x^2 + x^3 \arctan(\frac{1}{x})}{\frac{1}{6}\frac{1}{(1+x^2)^2} - \frac{3x}{16}\arctan(\frac{1}{x}) + \frac{3}{16}\frac{x^2}{(1+x^2)} + \frac{1}{8}\frac{x^2}{(1+x^2)^2}}\right]. \tag{50}$$

We may next estimate the mass of such a mode for different values of $\Lambda$ and $M$ obtained in the previous section. For example, for $\Lambda$ = 420 MeV and $M$ = 305 MeV, $m_\sigma \simeq 2.07M$; for $\Lambda$ = 500 MeV and $M$ = 320 MeV, $m_\sigma \simeq 2.27M$; and for $\Lambda$ = 700 MeV and $M$ = 360 MeV, $m_\sigma \simeq 2.65M$. These may be compared with the mass of $\sigma$ field obtained through an approximate determination of the pole of the propagator with a polarization insertion, which is given as $m_\sigma = (4M^2 + m_\pi^2)^{1/2}$ [4].

## VII. DISCUSSION

We thus consider here chiral symmetry breaking as a vacuum realignment with an explicit construct for a destabilized vacuum. The new feature of this approach [5] is that it enables us to relate the function that describes the vacuum structure determined variationally to the wave function of the pion as the localized Goldstone mode in a straightforward manner. This language is not only physically appealing in reproducing the conventional results but also puts severe constraints on the parameters for symmetry breaking as illustrated here for the NJL model. Some other aspects of low energy hadronic properties as related to the vacuum structure for chiral symmetry breaking have been discussed elsewhere [12].

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[1] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); **124**, 246 (1961).

[2] J. R. Finger and J. E. Mandula, Nucl. Phys. **B199**, 168 (1982); S. L. Adler and A. C. Davis, *ibid.* **B244**, 469 (1984); R. Alkofer and P. A. Amundsen, *ibid.* **B306**, 305 (1988).

[3] S. Li, R. S. Bhalerao, and R. K. Bhaduri, Int. J. Mod. Phys. A **6**, 501 (1991); V. Bernard, Phys. Rev. D **34**, 1601 (1986).

[4] J. da Providencia, M. C. Ruivo, and C. A. de Sousa, Phys. Rev. D **36**, 1882 (1987); B. Haeri and M. B. Haeri, *ibid.* **43**, 3732 (1991); B. Rosenstein, B. J. Warr, and S. H. Park, Phys. Rep. **205**, 59 (1991); U. Vogl and W. Weise, Prog. Part. Nucl. Phys. **27**, 195 (1991); L. N. Chang, N. P. Chang, and K. C. Chou, Phys. Rev. D **43**, 596 (1991); Lay Nam Chang and Ngee Pong Chang, *ibid.* **45**, 2988 (1992); S. P. Klevensky, Rev. Mod. Phys. **64**, 649 (1992).

[5] H. Mishra, S. P. Misra, and A. Mishra, Int. J. Mod. Phys. A **3**, 2331 (1988); A. Mishra, H. Mishra, S. P. Misra, and S. N. Nayak, Phys. Lett. B **251**, 541 (1990); Phys. Rev. D **44**, 110 (1991); Pramana **37**, 59 (1991); Z. Phys. C **57**, 233 (1993); A. Mishra, H. Mishra, and S. P. Misra, Z. Phys. C **59**, 159 (1993).

[6] A. Mishra, H. Mishra, and S. P. Misra, Z. Phys. C **57**, 241 (1993).

[7] J. Goldstone, Nuovo Cimento **19**, 154 (1961); J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. **127**, 965 (1962).

[8] S. P. Misra, talk given at the Symposium on Quantum Field Theory and Statistical Mechanics, Calcutta, India, 1992 (unpublished).

[9] Y. Nambu, in *New Theories in Physics*, Proceedings of the Eleventh International Symposium on Elementary Particle Physics, Kazimierz, Poland, edited by Z. Ajduk, S. Pokoroski, and A. Trautman (World Scientific, Singapore, 1989); W. J. Marciano, Phys. Rev. Lett. **62**, 2793 (1989); Phys. Rev. D **41**, 219 (1990); W. A. Bardeen, C. T. Hill, and M. Linder, *ibid.* **41**, 1647 (1990).

[10] K. S. Babu and Rabindra N. Mohapatra, Phys. Rev. Lett. **66**, 556 (1991).

[11] S. P. Misra, Phys. Rev. D **18**, 1661 (1978); **18**, 1673 (1978).

[12] A. Mishra and S. P. Misra, Z. Phys. C **58**, 325 (1993).

[13] A. Le Yaouanc, L. Oliver, S. Ono, O. Pène, and J. C. Raynal, Phys. Rev. Lett. **54**, 506 (1985).

[14] H. Mishra, S. P. Misra, P. K. Panda, and B. K. Parida, Int. J. Mod. Phys. E **1**, 405 (1992).

[15] J. J. Sakurai, *Currents and Mesons* (The University of Chicago Press, Chicago, 1969), p. 83.

[16] There is a disagreement over the value of $\Lambda$. Further, Eq. (17) and the corresponding equation by Bernard in Ref. [3] have a difference of a factor of 2. We presume that this is a printing error since the $G\Lambda^2$ they have taken does not satisfy their bound whereas it is consistent with Eq. (17).

[17] A. Amer, A. Le Yaouanc, L. Oliver, O. Pène, and J. C. Raynal, Phys. Rev. Lett. **50**, 87 (1983); Phys. Rev. D **28**, 1530 (1983).