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Natural suppression of Higgsino-mediated proton decay in supersymmetric SO(10)

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In supersymmetric grand unified theories, proton decay mediated by the color-triplet Higgsino is generally problematic and requires some fine-tuning of parameters. We present a mechanism which naturally suppresses such dimension-5 operators in the context of SUSY SO(10). The mechanism, which implements natural doublet-triplet splitting using the adjoint Higgs boson, converts these dimension-5 operators effectively into dimension 6. By explicitly computing the Higgs spectrum and the resulting threshold uncertainties we show that the successful prediction of $\sin^2\theta_W$ is maintained *as a prediction* in this scheme. It is argued that only a weak suppression of the Higgsino-mediated proton decay is achievable within SUSY SU(5) without fine-tuning, in contrast to a strong suppression in SUSY SO(10).

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I. INTRODUCTION

The dramatically precise unification of coupling [1] that occurs in the minimal supersymmetric SU(5) model has been much touted, and indeed is striking. A fit [2] to all W, Z, and neutral current data (using $m_t=138$ GeV and $m_H=M_Z$) gives $\sin^2\theta_W(M_Z)=0.2324\pm0.0003$, while in supersymmetric (SUSY) SU(5) one has [2] $\sin^2\theta_W(M_Z)=0.2334\pm0.0036$ [we have combined the uncertainties due to $\alpha_s(M_Z)$, $\alpha(M_Z)$, sparticle thresholds, m_t, m_{h_0}, high-scale thresholds, and nonrenormalizable operators]. As a measure of how significant this agreement is, consider that the addition of just one extra pair of light Higgs doublet supermultiplets would increase the SUSY grand unified theory (GUT) prediction of $\sin^2\theta_W(M_Z)$ to about 0.256, leading to a discrepancy of over six standard deviations.

At the same time SUSY SU(5) suffers from a problem [3] with proton decay arising from dimension-5 operators caused by the exchange of color-triplet Higgsinos in the same SUSY-SU(5) multiplet with the light Higgs doublets H and H'. For "central values" of the model parameters the predicted proton lifetime mediated by the dimension-5 operators [4,5] is shorter than the current experimental limits [6]. A certain amount of adjustment is then required for consistency, which pushes parameters to the corner of their allowed region. For short we will henceforth refer to this as the "Higgsino-mediated proton decay (HMPD) problem."

It would seem that one cannot take seriously the unification of couplings of SUSY GUT's, however impressive, in the absence of satisfactory mechanism that suppresses Higgsino-mediated proton decay. Two requirements for a "satisfactory" mechanism that we will impose are that it involves no "fine-tuning" or artificial adjustments of parameters, and that the unification of couplings is maintained as a *prediction*. (We emphasize that word because a discrepancy in $\sin^2\theta_W$ can often be remedied by introducing *ad hoc* new particles, threshold effects, etc.; but we would not regard the resulting agreement as being in any sense a prediction.)

The Higgsino-mediated proton decay problem [3] is easily described. In SUSY SU(5) models there exists a pair of Higgs supermultiplets, that we will denote $\mathbf{5}_H+\overline{\mathbf{5}}'_H$. Under the standard model group, $G_S=\mathrm{SU(3)}\times\mathrm{SU(2)}\times\mathrm{U(1)}$, these decompose as

$$\mathbf{5}_H=\{(1,2,+\tfrac{1}{2})+(3,1,-\tfrac{1}{3})\}\equiv\{\mathbf{2}_H+\mathbf{3}_H\}\ ,$$

and

$$\overline{\mathbf{5}}'_H=\{(1,2,-\tfrac{1}{2})+(\overline{3},1+\tfrac{1}{3})\}\equiv\{\overline{\mathbf{2}}'_H+\overline{\mathbf{3}}'_H\}\ .$$

The $\mathbf{2}_H$ and $\overline{\mathbf{2}}'_H$ are just the familiar H and H' of the supersymmetric standard model, and their couplings to the light quarks and leptons are therefore fixed in terms of their vacuum expectation values and the light fermion masses. Then by SU(5) the Yukawa couplings of the $\mathbf{3}_H$ and $\overline{\mathbf{3}}'_H$ are fixed as well. If there is a Dirac mass term connecting the Higgsinos in $\mathbf{3}_H$ and $\overline{\mathbf{3}}'_H$ to each other, i.e., a term of the form $M(3_H\overline{3}'_H)$, then the diagram shown in Fig. 1 exists, which depicts a baryon-number-

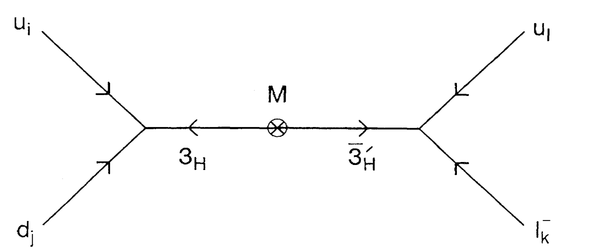

FIG. 1. A diagram that gives a dimension-5 baryon-number-violating operator. The arrows indicate the direction a left-handed chiral superfield is flowing. Chirality shows this to be an F term and hence suppressed by $(\mathrm{mass})^{-1}$. Chirality also implies that there must be a mass insertion (denoted by M) on the colored Higgsino line. The suppression is thus M/M_{GUT}^2 which is naturally of order M_{GUT}^{-1} in most models.

violating process mediated by colored Higgsino exchange. The Higgsino, of course, converts the quarks and leptons to their scale partners, so Fig. 1 needs to be "dressed" for it to correspond to proton decay. Dressing by $\tilde{W}$-ino exchange is by far the most dominant, the resulting lifetime for $p \to K^+ \bar{\nu}_\mu$, for example (the antisymmetry of the relevant operator causes the change in flavor), has been estimated [4] to be

$$\tau(p \to K^+ \bar{\nu}_\mu) = 6.9 \times 10^{28}\ \text{yr} \times \left| \left[\frac{0.01\ \text{GeV}^3}{\beta} \right] \left[\frac{0.67}{A_s} \right] \left[\frac{\sin 2\beta_H}{1+y^{tk}} \right] \left[\frac{M_{H_c}}{10^{16}\ \text{GeV}} \right] \frac{\text{TeV}^{-1}}{f(u,d)+f(u,e)} \right|^2 . \quad (1)$$

Here β is the relevant nuclear matrix element which lies in the range $\beta = (0.003-0.03)$ GeV^3. A_s is the short-distance renormalization factor ($A_s \simeq 0.6$), $\tan\beta_H = \langle H \rangle / \langle H' \rangle$, and y^{tk} parametrizes the contribution of the top family relative to the first two ($0.1 < |y^{tk}| < 1.3$ for $m_t = 100$ GeV). The functions f arise from the one-loop integrals and $f(u,d) \simeq m_{\tilde{W}^\pm} / m_{\tilde{Q}}^2$ for $m_{\tilde{W}^\pm} \ll m_{\tilde{Q}}$.

The prediction (1) is to be compared with the present experimental lower limit

$$\tau(p \to K^+ \bar{\nu}_\mu) \gtrsim 1 \times 10^{32}\ \text{yr}\ [6] .$$

There is already, as can be seen, somewhat of a difficulty reconciling these numbers. At least several of the following conditions should be satisfied: (a) The nuclear matrix element β is near the lower end, $\beta \simeq 0.003$ GeV^3; (b) $\tan\beta_H$ is not too large; (c) either the $\tilde{W}^\pm$ is significantly ligher than the squark $\tilde{Q}$ or vice versa; (d) the colored Higgsino mass should exceed the GUT scale significantly ($M_{H_c} \gtrsim 10^{17}$ GeV); (e) there is some cancellation in $(1+y^{tk})$ between the third family and the first two family contributions. Obviously this pushes almost all parameters to their corners. Although not excluded, the problem begs for a more elegant explanation.

The necessity of the Dirac mass term $M(3_H \bar{3}'_H)$ for obtaining the $D=5$ baryon-violating operator is a crucial point. A quartic term in the chiral superfields will be suppressed by $1/M_{\text{GUT}}$ or $1/M_{\text{GUT}}^2$ depending on whether it is an F term or a D term. To get an F term all the left-handed chiral superfields must be coming into the diagram (or out of it). This requires the chirality-flipping mass insertion coming from the $M(3_H \bar{3}'_H)$ term as shown in Fig. 1. If the mass M vanished, one could only write superfield diagrams such as Fig. 2, which clearly give D terms and therefore are suppressed by $1/M_{\text{GUT}}^2$. (The F terms correspond to Higgsino exchange, the D terms to Higgs boson exchange.)

The foregoing considerations have suggested to several authors [7,8] an approach to suppressing Higgsino-mediated proton decay by imposing a (Peccei-Quinn-type) symmetry that suppresses the Dirac mass between the 3_H and $\bar{3}'_H$. However, this approach leads to a dilemma. In the simplest SUSY SU(5) model it is precisely the $M(3_H \bar{3}'_H)$ Dirac term that gives to the 3_H and $\bar{3}'_H$ the superlarge mass that they must have (else even the $D=6$ operators would cause a disaster). How, then, can the 3_H and $\bar{3}'_H$ be made superheavy and yet not have a large Dirac mass connecting them to each other? To resolve this, one ends up introducing new color-triplet Higgs superfields, a $\bar{3}_H$ to mate with the 3_H and a $3'_H$ to mate with the $\bar{3}'_H$, so that all color triplets become heavy while still leaving the unprimed and primed sectors disconnected. That means having four instead of the minimal two 5-plets of Higgs bosons. The situation can be represented diagrammatically as

$$\begin{bmatrix} 3 \\ 2 \end{bmatrix} - \begin{bmatrix} \bar{3} \\ \bar{2} \end{bmatrix} - - - \begin{bmatrix} 3' \\ 2' \end{bmatrix} - \begin{bmatrix} \bar{3}' \\ \bar{2}' \end{bmatrix}$$
$$\| \qquad \| \qquad \| \qquad \|$$
$$5_H \qquad \bar{5}_H \qquad 5'_H \qquad \bar{5}'_H , \quad (2)$$

where the solid horizontal lines representing superlarge Dirac masses. The 2 and $\bar{2}'$ are the usual light H and H' of the minimal supersymmetric standard model (MSSM). But now it is to be noticed that there are two additional light doublets, $\bar{2}$ and $2'$. As noted above, this is unacceptable if the dramatic unification of gauge couplings is to be preserved as a prediction.

One could remove the extra pair of Higgs boson (Higgsino) doublets to superlarge scales by introducing a mass term $M(\bar{5}_H 5'_H)$, indicated by the dashed line in Eq. (2). This, of course, would reintroduce the Higgsino-mediated proton decay as shown in Fig. 3. The parameter M would then control both the Higgsino-mediated proton decay and the mass of the pair of extra doublets.

As we shall see in the later sections, this situation is typical of mechanisms that naturally suppress Higgsino-mediated proton decay: (a) there is a doubling of the Higgs sector, (b) there is the consequent danger to the

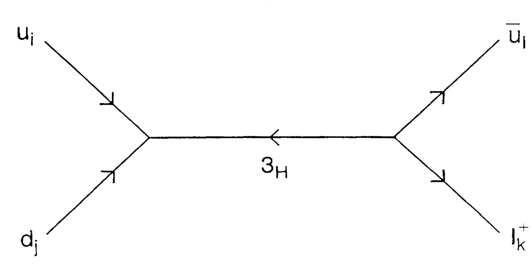

FIG. 2. A diagram without the chirality-flipping mass insertion of Fig. 1 and thus representing a D term. Such a term is effectively of dimension 6 and suppressed by M_{GUT}^{-2}. It corresponds to colored Higgs boson exchange.

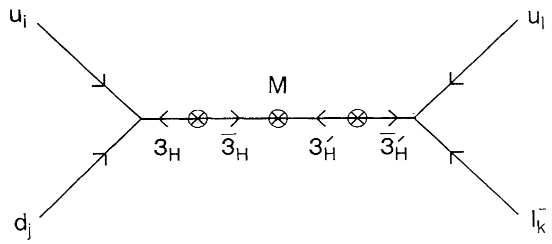

FIG. 3. In the "weak suppression" scheme discussed in Sec. I [see Eq. (2)] the primed and unprimed Higgs (Higgsino) sectors are connected by a Dirac mass, M. This coupling of the two sectors allows a dimension-5, baryon-number-violating operator to arise as shown here.

unification of couplings of extra light fields in incomplete multiplets, and (c) there is a parameter M which controls both proton decay and the mass of these extra fields.

Two approaches, therefore, appear to be possible. (1) *Weak suppression* of Higgsino-mediated proton decay, resulting from M being of order, but slightly less than, $M_{\rm GUT}$. For example, with $M=\frac{1}{10}M_{\rm GUT}$ Higgsino-mediated proton decay is suppressed by a factor of 10^{-2} while at the same time those extra fields whose mass is given by M will only lead to small threshold corrections to $\sin^2\theta_W$. In this case the suppression of Higgsino-mediated proton decay is just numerical; there is no symmetry or other qualitative explanation of it. One would have no *a priori* reason therefore to expect the suppression to be particularly large. A hope would therefore exist that $p\rightarrow K^+\bar{\nu}_\mu$, $n\rightarrow K^0\bar{\nu}$, etc., might be seen at super-Kamiokande. (2) *Strong suppression* of Higgsino-mediated proton decay would result if (due to some approximate symmetry perhaps) M were much less than $M_{\rm GUT}$, for example, $O(M_W)$. In that case it is imperative that there be no "extra" fields (i.e., beyond the minimal supersymmetric standard model) in incomplete SU(5) multiplets whose mass is proportional to M, or else the unification of couplings would be destroyed. To achieve this without fine-tuning turns out to be a nontrivial problem. One of the main conclusions of this paper is that such a strong suppression of proton decay can only be achieved in a satisfactory way in SO(10) (or larger groups).

The whole problem of Higgsino-mediated proton decay is, of course, intimately connected to the well-known question of "doublet-triplet" splitting [9–13]. We will show that the most satisfactory treatments of this problem make use of an old but somewhat neglected idea for doublet-triplet splitting in SO(10) using the adjoint Higgss boson due to Dimopoulos and Wilczek [13].

Our paper is organized as follows. In Sec. II we review the Dimopoulos-Wilczek idea of doublet-triplet splitting and show how both weak suppression and strong suppression of HMPD can be achieved naturally in SUSY SO(10). In Sec. III we consider SUSY SU(5) and show that only weak suppression of HMPD can be achieved without fine-tuning parameters. In Sec. IV we discuss flipped SU(5). In Sec. V a closer examination of the Dimopouloss-Wilczek mechanism in SO(10) is undertaken, and we show that it can be made viable and consistent. Our conclusions are summarized in Sec. VI. In Appendix A we give the details of the minimization of a realistic SO(10) superpotential. There we show that the gauge symmetry breaking can be achieved consistent with supersymmetry without generating pseudo Goldstone bosons. Appendix B deals with the threshold corrections to $\sin^2\theta_W$.

II. SUPPRESSING PROTON DECAY IN SUSY SO(10)

The problem with doublet-triplet splitting arises in SU(5) because of the tracelessness of its irreducible representations. Suppose the superpotential contains the term $\lambda_1\mathbf{\bar{5}}'_H\mathbf{24}_H\mathbf{5}_H$ and

$$\langle \mathbf{24}_H\rangle={\rm diag}(x,x,x,y,y)\ ,$$

then the $\mathbf{3}_H$ and $\mathbf{\bar{3}}'_H$ Higgsinos get a Dirac mass of $\lambda_1 x$ and the doublets $\mathbf{2}_H$ and $\mathbf{\bar{2}}'_H$ get a mass $\lambda_1 y$. We need $\lambda_1 x\sim M_{\rm GUT}$ and $\lambda_1 y\sim M_W$ but this is impossible since, by the tracelessness of **24**, $y=-\frac{3}{2}x$. This can be remedied by introducing a singlet superfield, $\mathbf{1}_H$, with the coupling $\lambda_2\mathbf{\bar{5}}'_H\mathbf{1}_H\mathbf{5}_H$ and $\langle \mathbf{1}_H\rangle\equiv z$ (or equivalently by a bare mass term), but only by tuning the parameters so that

$$(-\tfrac{3}{2}\lambda_1 x+\lambda_2 z)\lesssim 10^{-14}(\lambda_1 x+\lambda_2 z).$$

In SO(10) such fine adjustment of parameters can be avoided because the analogue of the tracelessness condition does not exist [13]. The **24**, which is the adjoint of SU(5), is contained in the **45** which is the adjoint of SO(10). **45** is a rank-2 antisymmetric tensor and the vacuum expectation value (VEV) of $\mathbf{45}_H$ can be brought to the canonical form

$$\langle \mathbf{45}_H\rangle=\eta\otimes{\rm diag}(x_1,x_2,x_3,x_4,x_5)\ ,\qquad \eta\equiv\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}, \tag{3}$$

which corresponds to the U(5) matrix ${\rm diag}(x_1,x_2,x_3,x_4,x_5)$. Because this is a U(5) rather than an SU(5) matrix its trace need not vanish. One can therefore have the VEV of $\langle \mathbf{45}_H\rangle$ take the form

$$\langle \mathbf{45}_H\rangle=\eta\otimes{\rm diag}(a,a,a,0,0)\ . \tag{4}$$

This is just what is needed to give mass to the $SU(3)_C$-triplet Higgs boson (Higgsino) and not the $SU(2)_L$-doublet ones. This is what we call the Dimopoulos-Wilczek mechanism.

There is another group-theoretical explanation for the doublet-triplet splitting in SO(10). Under its maximal subgroup $SU(2)_L\times SU(2)_R\times SU(4)_C$, the standard model singlets of **45** which could acquire GUT-scale VEV's are contained in the (1,1,15) and (1,3,1) multiplets. The **10** of Higgs decomposes as (2,2,1)+(1,1,6). If only the (1,1,15) of **45** acquires a VEV, it gives the color triplets of (1,1,6) a mass and not the doublets of (2,2,1). If the (1,3,1) acquires a VEV, it will supply a super-large mass to the doublets, and not to the triplets. (We shall shortly make use of the second property to suppress proton decay and at the same time preserve $\sin^2\theta_W$ as a prediction.) Such options are not available in SU(5), since SU(5) has no intermediate symmetries, even for the sake of classification.

Consider the following coupling in the superpotential of a SUSY-SO(10) model:

$$W \supset \lambda \mathbf{10}_{1H}\mathbf{45}_H\mathbf{10}_{2H} \ . \tag{5}$$

One must introduce *two* **10**'s of Higgs (Higgsino) fields because with just one the term $\mathbf{10}_H\mathbf{45}_H\mathbf{10}_H$ would vanish by the antisymmetry of the **45**. As we noted in the Introduction, and shall see more clearly later, such a doubling is actually a useful thing from the point of view of suppressing Higgsino-mediated proton decay. This is another appealing feature of SO(10).

When the $\mathbf{45}_H$ gets the VEV shown in Eq. (4) all of the triplet Higgs (Higgsino) fields in $\mathbf{10}_{1H}$ and $\mathbf{10}_{2H}$ get superlarge masses. The situation can be represented schematically as

$$\begin{bmatrix}3_1\\2_1\end{bmatrix} - \begin{bmatrix}\bar{3}_2\\\bar{2}_2\end{bmatrix} \begin{bmatrix}3_2\\2_2\end{bmatrix} - \begin{bmatrix}\bar{3}_1\\\bar{2}_1\end{bmatrix}$$
$$\begin{matrix}\| & \quad\| & \quad\| & \quad\|\\ \mathbf{5}_{1H} & \bar{\mathbf{5}}_{2H} & \mathbf{5}_{2H} & \bar{\mathbf{5}}_{1H}\end{matrix} \ , \tag{6}$$

where under SO(10)→SU(5), $\mathbf{10}_{1H}=\bar{\mathbf{5}}_{1H}+\mathbf{5}_{1H}$ and $\mathbf{10}_{2H}=\bar{\mathbf{5}}_{2H}+\mathbf{5}_{2H}$. By comparison with the scheme shown in Eq. (2) one sees that the "unprimed sector" consists of $\mathbf{5}_{1H}$ and $\bar{\mathbf{5}}_{2H}$, while the "primed sector" consists of $\mathbf{5}_{2H}$ and $\bar{\mathbf{5}}_{1H}$. One can then identify $H\equiv\mathbf{2}_{1H}$ and $H'=\bar{\mathbf{2}}_{1H}$.

Now we face the problem of generating mass for the "extra" doublets which reside in $\mathbf{5}_{2H}+\bar{\mathbf{5}}_{2H}=\mathbf{10}_{2H}$. The simplest possibility is just to introduce into the superpotential W the term $M(\mathbf{10}_{2H}\mathbf{10}_{2H})$, with $M/M_{\rm GUT}$ being less than, but not *much* smaller than, 1. The resulting threshold correction to $\sin^2\theta_W$ is

$$+\frac{3\alpha(M_Z)}{10\pi}\ln(M_{\rm GUT}/M)\approx 10^{-3} \ .$$

For proton decay to be suppressed it is also necessary that only $\mathbf{10}_{1H}$ but not $\mathbf{10}_{2H}$ couple to light quarks and leptons with usual strength. All of this [including the absence of a $(\mathbf{10}_{1H})^2$ term in W] can be enforced by a global symmetry. For example, we have found a Z_6 symmetry [which is also compatible with Eq. (13) below if the masses m_1 and m_2 are replaced by VEV's of singlets].

The above appears to us to be the simplest way of achieving weak suppression of proton decay. [For comparison with SU(5) see the next section.] However, in SO(10), but not in SU(5), it is actually possible to achieve a *strong suppression* in a satisfactory way. To do this we need to give $\mathbf{2}_{2H}$ and $\bar{\mathbf{2}}_{2H}$ a superheavy Dirac mass (so as to not mess up $\sin^2\theta_W$) without having a superheavy Dirac mass connecting $\mathbf{3}_{2H}$ and $\bar{\mathbf{3}}_{2H}$ (which could produce excessive proton decay). But this is just a doublet-triplet splitting problem—but upside down to the familiar one. Here the doublets but not the triplets need a mass term. This will prove to be not doable in SU(5) without tuning, but in SO(10) it can be done. What is required is another $\mathbf{45}_H$ with a VEV:

$$\langle \mathbf{45}'_H\rangle=\eta\otimes{\rm diag}(0,0,0,a',a') \ . \tag{7}$$

As already noted, this is just as achievable as the VEV given in Eq. (4). (See Sec. V for the demonstration.)

There is a slight hitch in that we cannot, because of the antisymmetry of $\mathbf{45}'_H$, simply write down $\mathbf{10}_{2H}\mathbf{45}'_H\mathbf{10}_{2H}$ to give mass to $\mathbf{2}_{2H}$ and $\bar{\mathbf{2}}_{2H}$. However, this can be overcome by introducing an additional 10 of Higgs (Higgsino) fields. Consider a superpotential containing

$$W\supset\lambda\mathbf{10}_{1H}\mathbf{45}_H\mathbf{10}_{2H}+\lambda'\mathbf{10}_{2H}\mathbf{45}'_H\mathbf{10}_{3H}+M\mathbf{10}_{3H}\mathbf{10}_{3H}+\sum_{i,j=1}^{3}f_{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{10}_{1H} \ , \tag{8}$$

with $\langle\mathbf{45}_{Hr}\rangle$ and $\langle\mathbf{45}'_H\rangle$ being given by Eqs. (4) and (7). Then the superheavy mass matrices of the color-triplet and weak-doublet Higgs boson (Higgsino) are

$$(\bar{\mathbf{2}}_1,\bar{\mathbf{2}}_2,\bar{\mathbf{2}}_3)\begin{bmatrix}0&0&0\\0&0&\lambda'a'\\0&-\lambda'a'&M\end{bmatrix}\begin{bmatrix}\mathbf{2}_1\\\mathbf{2}_2\\\mathbf{2}_3\end{bmatrix} \tag{9}$$

and

$$(\bar{\mathbf{3}}_1,\bar{\mathbf{3}}_2,\bar{\mathbf{3}}_3)\begin{bmatrix}0&\lambda a&0\\-\lambda a&0&0\\0&0&M\end{bmatrix}\begin{bmatrix}\mathbf{3}_1\\\mathbf{3}_2\\\mathbf{3}_3\end{bmatrix} \ . \tag{10}$$

The doublet matrix is rank-two leaving a single pair of light doublets $H\equiv\mathbf{2}_{1H}$ and $H'\equiv\bar{\mathbf{2}}_{1H}$. All triplets get superheavy mass; however, there is no mixing between $\mathbf{3}_{1H}$ and $\bar{\mathbf{3}}_{1H}$ that would permit the diagrams shown in Figs. 1 or 3.

There are several questions to be answered concerning the SO(10) approaches to the proton decay problem. (1) Can the VEV's in Eqs. (4) and (7) arise from an actual(super) potential [14]? (2) Can SO(10) be broken all the way to SU(3)×SU(2)×U(1) without destabilizing these VEV's? (3) Are the threshold corrections in such an SO(10) model likely to be small enough not to vitiate the successful prediction of $\sin^2\theta_W$? We will show in Sec. V that the answer to all these questions is "yes." But first we will examine the possibilities that exist in SU(5) and flipped SU(5).

III. SUPPRESSING PROTON DECAY IN SUSY SU(5)

The only viable method of doublet-triplet splitting in SUSY SU(5) that does not involve fine-tuning of parameters is the "missing partner mechanism" [9,10]. The so-called sliding-singlet mechanism [11] has the problem in SU(5) that radiative corrections destroy the gauge hierarchy [12]. For the missing partner mechanism in SU(5) one requires (at least) the set $\mathbf{5}+\bar{\mathbf{5}}+\mathbf{50}+\overline{\mathbf{50}}+\mathbf{75}$ of Higgs supermultiplets. In the $\mathbf{50}(\overline{\mathbf{50}})$ there is a color $\mathbf{3}(\bar{\mathbf{3}})$ but no weak $\mathbf{2}(\bar{\mathbf{2}})$. Thus the couplings

$$\lambda 5_H \overline{50}_H \langle 75_H \rangle + \lambda' \overline{5}_H 50_H \langle 75_H \rangle$$

give mass to the triplets in $5_H + \overline{5}_H$ but not to the doublets. Schematically,

$$\begin{bmatrix} 3 \\ 2 \end{bmatrix} - \begin{bmatrix} \overline{3} \\ \text{other} \end{bmatrix} \begin{bmatrix} 3 \\ \text{other} \end{bmatrix} - \begin{bmatrix} \overline{3} \\ \overline{2} \end{bmatrix}$$
$$\begin{matrix} \| & \| & \| & \| \\ 5_H & \overline{50}_H & 50_H & \overline{5}_H \ . \end{matrix} \tag{11}$$

The horizontal solid lines represent superheavy triplet Higgs (boson) (Higgsino) masses arising from the $\langle 75_H \rangle$. As in the cases considered in previous sections, there is the question of how to make the "other" fields in the $50+\overline{50}$ superheavy. (They contribute to the RGE at one loop the same as a pair of weak doublets.) If one wanted a "strong suppression" of proton decay, it would require giving superlarge masses to all the "other" fields in $50+\overline{50}$ but having the $\overline{3}(\overline{50}_H)$ and $3(50_H)$ not be connected by a large Dirac mass term. There is no analogue of the missing partner mechanism that could accomplish this in a natural way. It could only be done by fine-tuning. For example, two different representations (1_H and 24_H, or 24_H and 75_H) could couple 50 to $\overline{50}$ and be relatively adjusted to give light mass only to the $3(50_H)+\overline{3}(\overline{50}_H)$. However, we have foresworn fine-tuning.

It *is* possible to achieve a natural weak suppression of Higgsino-mediated proton decay in SU(5) by introducing an explicit mass term $M(\overline{50}_H 50_H)$ into the superpotential and having $M/M_{\rm GUT}$ be of order but somewhat smaller than unity. This works well as the weak suppression mechanism in SO(10) discussed in Sec. II. However, in SU(5) there is the necessity of introducing the somewhat exotic high rank representations **50**, $\overline{\mathbf{50}}$, and **75**, whereas in SO(10) only the low rank representations **10**, **45**, and **54** are required. If one were willing to live with multiple fine-tunings of parameters one could do with just doubling the Higgs sector in SU(5) to $5+\overline{5}+5'+\overline{5}'$ as discussed briefly in the Introduction. With *two* fine-tunings one could make all the triplets heavy and achieve weak suppression of proton decay. With a *third* fine-tuning one could give mass to the extra pair of doublets and yet achieve strong suppression of proton decay. There are papers in the literature that take this approach [7,8].

IV. SUPPRESSING PROTON DECAY IN FLIPPED SU(5)

As is well-known, the missing partner mechanism works much more economically in flipped SU(5) [15] than in ordinary SU(5) [16]. One requires for the mechanism the SU(5)×U(1) representations $5_H^{-2}+\overline{5}_H^{2}+10_H^1+\overline{10}_H^{-1}$, and the superpotential couplings $\lambda 5_H^{-2} 10_H^1 10_H^1 + \lambda' \overline{5}_H^2 \overline{10}_H^{-1} \overline{10}_H^{-1}$. The $10_H(\overline{10}_H)$ contains a color $\overline{3}(3)$ but no color-singlet, weak-doublet components. The 10_H and $\overline{10}_H$ get VEV's that break SU(5)×U(1) down to G_S and also mate the triplet Higgs (bosons) (Higgsinos) in the $5_H+\overline{5}_H$ with these in the $10_H+\overline{10}_H$ leaving the doublets in $5_H+\overline{5}_H$ light. Schematically,

$$\begin{bmatrix} 3 \\ 2 \end{bmatrix} - \begin{bmatrix} \overline{3} \\ \text{other} \end{bmatrix} \begin{bmatrix} 3 \\ \text{other} \end{bmatrix} - \begin{bmatrix} \overline{3} \\ \overline{2} \end{bmatrix}$$
$$\begin{matrix} \| & \| & \| & \| \\ 5_H & 10_H & \overline{10}_H & \overline{5}_H \ . \end{matrix} \tag{12}$$

Another beautiful feature of flipped SU(5) is that there is no necessity to do anything else to give mass to the "other" fields in the $10_H+\overline{10}_H$: they are all disposed of by the (super) Higgs mechanism. They are either absorbed or become superheavy with the gauge and/or gaugino particles. Thus, in flipped SU(5) one can strongly suppress Higgsino-mediated proton decay without any fine-tuning and without leaving any "extra" split multiplets lighter than $M_{\rm GUT}$. We found this to be impossible in ordinary SU(5). However, there is one major drawback: the group of flipped SU(5) is really SU(5)×U(1) and so real unification of gauge couplings is not achieved. One has therefore lost, or rather never had, the unification of gauge couplings *as a prediction.*

V. A MORE DETAILED EXAMINATION OF SO(10)

In Sec. II certain ideas were discussed for solving the doublet-triplet splitting problem and for suppressing Higgsino-mediated proton decay in SO(10) that made essential use of specific patterns of VEV's, in particular those shown in Eqs. (4) and (7). The question arises whether such VEV's are natural. In Ref. [14] Srednicki wrote down a superpotential for a **45** and a **54** of Higgs bosons that has both of these forms as possible supersymmetric minima. Let us denote the **45** and **54** by A and S, respectively. Then the most general SO(10)-invariant superpotential involving just these fields has the form

$$W(A,S)=m_1A^2+m_2S^2+\lambda_1S^3+\lambda_2A^2S \ . \tag{13}$$

The equations for a supersymmetric minimum are

$$\begin{aligned} 0&=F_A=2(m_1+\lambda_2 S)A \ , \\ 0&=F_S=(2m_2S+\lambda_2A^2+3\lambda_1S^2) \ . \end{aligned} \tag{14}$$

Suppose we choose

$$\langle S \rangle = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \otimes \mathrm{diag}(s,s,s,-\tfrac{3}{2}s,-\tfrac{3}{2}s)$$

[S is a traceless rank-two symmetric tensor of SO(10)] and

$$\langle A \rangle = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \otimes \mathrm{diag}(a,a,a,b,b) \ .$$

Then Eq. (14) gives two equations:

$$\begin{aligned} &(m_1-\tfrac{3}{2}\lambda_2 s)b=0 \ , \\ &(m_1+\lambda_2 s)a=0 \ . \end{aligned} \tag{15}$$

Either a or b or both must therefore vanish (if $s\neq0$). There are therefore three possible solutions: (1) $b=0$, $s=-m_1/\lambda_2$; (2) $a=0$, $s=\tfrac{2}{3}m_1/\lambda_2$; (3) $a=b=0$.

For doublet-triplet splitting and weak susppression of proton decay we need only the solution (1). For strong suppression we need (at least) two adjoints, one with a VEV corresponding to solution (1) and the other to solution (2). [See Eqs. (4) and (7).] We will examine this latter more complicated case in greater detail. If the required pattern of VEV's can be achieved in a realistic model for that case, then *a fortiori* the simpler requirements for weak suppression can be achieved also. The main issues are whether the VEV's in Eqs. (4) and (7) can be achieved, the group SO(10) broken completely to $SU(3)\times SU(2)\times U(1)$, and Goldstone particles avoided. (The issue of threshold corrections to $\sin^2\theta_W$ is dealt with in Appendix B.)

To begin with we double the superpotential shown in Eq. (13). That is, we have two **45**'s denoted A and A', and two **54**'s, denoted S and S', with superpotential

$$W(A,S;A',S')=m_1A^2+m_2S^2+\lambda_1S^3+\lambda_2A^2S+m_1'A'^2+m_2'S'^2+\lambda_1'S'^3+\lambda_2'A'^2S' \ . \qquad (16)$$

We aim to have the (A,S) sector have VEV's in solution (1), and the (A',S') sector have VEV's in solution (2).

The superpotential in (16) is certainly not enough because, for one thing, nothing determines the relative alignment of the VEV's of the two sectors, and so there are Goldstone modes corresponding to a continuous degeneracy whereby the sectors are rotated in SO(10) space with respect to each other.

A second problem with Eq. (16) is that $\langle A\rangle$ and $\langle S\rangle$ break

$$SO(10)\rightarrow[SU(3)\times U(1)]\times SO(4) \ ,$$

while $\langle A'\rangle$ and $\langle S'\rangle$ break

$$SO(10)\rightarrow SO(6)\times[SU(2)\times U(1)\times U(1)] \ .$$

Altogether, then, the unbroken group is

$$SU(3)\times SU(2)\times U(1)\times U(1)=\text{rank } 5 \ .$$

To break all the way to the standard model further Higgs fields are needed. (They are needed for right-handed neutrino masses in any case.) The simplest choices are $\mathbf{16}+\overline{\mathbf{16}}$ or $\mathbf{126}+\overline{\mathbf{126}}$. Let us call these $C+\overline{C}$ where $C=\mathbf{16}$ or $\mathbf{126}$. One can write a superpotential that gives $C+\overline{C}$ VEV's which break $SU(10)\rightarrow SU(5)$. Together with A, S, A', and S' this will complete the breaking of SO(10) to G_S and allow ν_R masses.

At this point a further somewhat subtle technical problem arises. There are certain generators of SO(10) that are broken both by $C+\overline{C}$ and by the adjoints A and A', but not by the symmetric tensors $S+S'$, specifically the generators in $SO(6)/SU(3)\times U(1)$ and $SO(4)/SU(2)\times U(1)$ [where $SO(10)\supset SO(6)\times SO(4)$]. Thus to avoid residual Goldstone bosons there must be coupling between the $C+\overline{C}$ sector and the adjoints A and A'. The technical problem is that the direct couplings $\overline{C}AC$ and $\overline{C}A'C$ would destabilize the desired VEV's of A and A'. In particular, *all* the "diagonal" components of $\langle A\rangle$ and $\langle A'\rangle$ [written as U(5) matrices, that is] become nonvanishing. [This is because the VEV's of $C+\overline{C}$ that break $SO(10)\rightarrow SU(5)$ couple in $\overline{C}AC$ to the SU(5)-singlet combination $(3a+2b)$, where

$$\langle A\rangle=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\times \text{diag}(a,a,a,b,b) \ .$$

This leads to a mass term proportional to $(3a+2b)^2$ in the ordinary scalar potential, which in turn leads to cross terms of the form ab, destabilizing the solution $a\neq 0$ and $b=0$. The same thing would happen to A'.]

There are various solutions to this technical difficulty. The one we shall study here involves the introduction of a third adjoint, which we will denote A'', that serves as an intermediary between the $C+\overline{C}$ sector and A and A'.

The part of the superpotential that does the complete breaking to the standard model and obviates *all* difficulties is given in full by

$$W=m_1A^2+m_2S^2+\lambda_1S^3+\lambda_2A^2S+m_1'A'^2+m_2'S'^2+\lambda_1'S'^3+\lambda_2'A'^2S'+m_1''A''^2+m_2''\overline{C}C+\lambda_2''\overline{C}A''C+\lambda AA'A'' \ . \qquad (17)$$

There are three sectors, (A,S), (A',S'), and $(A'',\overline{C}+C)$, that are coupled together only by the last term $\lambda AA'A''$.

The term $\lambda_2''\overline{C}A''C$ does serve to give A'' a VEV in the SU(5)-singlet direction:

$$\langle A''\rangle=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\otimes \text{diag}(a'',a'',a'',a'',a'') \ . \qquad (18)$$

But this does not destabilize the VEV's of A and A' which are assumed to be of the forms given in Eqs. (4) and (7). This is easily seen by examining the $\lambda AA'A''$ term, which is the only thing linking A'' to A and A'. Consider the $F_A=0$ equation. $(F_A)^{[ab]}$ is an antisymmetric tensor to which $\lambda AA'A''$ contributes $\lambda(A'A'')^{[ab]}$, which vanishes when the values of $\langle A'\rangle$ and $\langle A''\rangle$ given in Eqs. (7) and (18) are substituted. Similarly, at the desired minimum, $\lambda AA'A''$ gives no contribution to the $F_{A'}$ and $F_{A''}$ equations. In other words it can be neglected in doing the minimization! However, it does contribute to the Higgs boson (Higgsino) masses, and, indeed, removes all of the possible Goldstone modes discussed above.

In Appendix A we present details of the minimization of the superpotential, Eq. (17), assuming $C=\mathbf{16}$. There we show explicitly that SO(10) may be completely broken to the standard model (without breaking SUSY), uneaten Goldstone bosons avoided, and VEV's of the desired form achieved. The masses of the various Higgs (super) multiplets enumerated in Appendix A will be used to estimate threshold corrections in the model.

An interesting question is whether the superpotential given in Eq. (17) (or some other realistic superpotential that could lead to the desired pattern of VEV's) is the most general compatible with some set of discrete symmetries. (It is not necessary that this be the case since in light of the nonrenormalization theorem one is not obliged to write all possible terms. But in light of the problem we are trying to solve it would be desirable.) The answer is "no" for Eq. (17) since no symmetry can forbid terms such as A^2S', A'^2S, S^2S', S'^2S obtained by substituting S for S' in terms in Eq. (17). However, these terms are not dangerous, they do not destabilize the pattern of VEV's, whereas terms that mix A, A' and A'' [such as AA', AA'', and $AA'S$ but *not* $\mathrm{tr}(AA'A'')$] are destabilizing. If we add all terms that mix S and S' to Eq. (17) but continue to forbid those that mix A, A', and A'' (except the $\lambda AA'A''$ term) the resulting superpotential is the most general compatible with a discrete symmetry that takes $(A, A')\rightarrow(-A', A)$. Note that this forbids AA', AA'', $AA'S$, etc., but allows $\mathrm{tr}(AA'A'')$ because

$$\mathrm{tr}(AA'A'') = -\mathrm{tr}(A'AA'') \ .$$

Unfortunately, we have found no discrete symmetry group that works when the coupling of the Higgs superfields to the quarks and leptons is included.

We have found other superpotentials and sets of Higgs fields that allow us to achieve the desired VEV's in a consistent and realistic way. We have presented Eq. (17) as being algebraically simple to analyze. It should also be noted that the implementation of weak suppression of HMPD, where only a single $\mathbf{45}_H$ is needed with VEV of the form in Eq. (4), is a simpler task and fewer fields are required. We have not tried to find the absolutely minimal scheme.

At this point we wish to make an aside. If one is willing to give up $\sin^2\theta_W$ as a *prediction,* there is a much simpler way to simultaneously achieve doublet-triplet splitting without fine-tuning and a strong suppression of Higgsino mediated proton decay. All we need is then one $\mathbf{45}_H$ of Higgs superfield with its VEV as given in Eq. (4). Suppose the relevant superpotential term for doublet-triplet splitting is just $\lambda \mathbf{10}_{1H}\mathbf{45}_H\mathbf{10}_{2H}$ as in Eq. (5). This will make the color triplets heavy, but one is left with two pairs of light doublets. Now, if the $\mathbf{45}_H$ does not couple to the sector that breaks SO(10)$\rightarrow$SU(5) (via $C+\overline{C}$ superpotential), then in addition to the extra pair of doublets, one will have a $\{(3,1,\frac{2}{3})+\text{H.c.}\}$ Goldstone supermultiplet which remains light. [These are the Goldstone bosons corresponding to SO(6)/SU(3)$\times$U(1) mentioned earlier.] The combined effect of having an extra pair of light Higgs doublets and the charge $\frac{2}{3}$ Higgs (super)fields is to alter $\sin^2\theta_W$ prediction to $\simeq 0.215$ at one loop. The unification scale also comes down by an order of magnitude or so. If one "fixes" these features by relying on particle thresholds, such a scenario may not be inconsistent. This scenario can be tested by directly searching for the $(3,1,\frac{2}{3})$-Higgs and Higgsino particles. This situation is somewhat analogous to the case studied in Ref. [17]. We do not advocate this scenario here, since our aim is to preserve the successful unification of couplings as a prediction.

Returning to the superpotential in Eq. (17), it might be imagined that with $\mathbf{3(45)+2(54)+\overline{16}+16}$ the threshold corrections might be fairly large; large enough, perhaps to vitiate the successful "prediction" of $\sin^2\theta_W$. Actually, this is not the case, especially if one assumes SO(10) breaks in two stages to the standard model:

$$\mathrm{SO}(10)\rightarrow\mathrm{SU}(5)\rightarrow\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1) \ .$$

SO(10) is broken to SU(5) by the VEV's of $\overline{C}$, C, and A'' at a scale M_{10}, while SU(5) is broken down to SU(3)$\times$SU(2)$\times$U(1) by the VEV's of A, S, A', and S' at a scale M_5. The masses of particles will be of the form $\alpha M_{10}+\beta M_5$. In the limit that $\beta M_5/\alpha M_{10}\rightarrow 0$ for a given multiplet its one-loop threshold corrections to $\sin^2\theta_W$ will vanish since it will become a complete and degenerate SU(5) multiplet. Thus threshold corrections of complete SU(5) multiplets go as

$$\ln(\alpha M_{10}+\beta M_5)/(\alpha M_{10})\cong\beta M_5/\alpha M_{10}$$

for $M_5 \ll M_{10}$. Thus if M_{10} is assumed to be somewhat larger than M_5, the GUT-scale threshold corrections to $\sin^2\theta_W$ are substantially reduced. These will be discussed explicitly in Appendix B, where it is found that the uncertainties in $\sin^2\theta_W$ due to superheavy thresholds is typically in the range of 3×10^{-3} to 10^{-2}.

VI. CONCLUSIONS

If one seeks a supersymmetric grand unified model in which proton decay mediated by color-triplet Higgsino is strongly suppressed through a mechanism based on symmetry, in which there is no fine-tuning of parameters, and in which the remarkable prediction of $\sin^2\theta_W$ is maintained *as a prediction,* then it seems that one must turn to SO(10). On the other hand, a weak suppression due not to symmetry but to the smallness of a parameter is achievable in both SU(5) and SO(10) without either fine-tuning or sacrificing the $\sin^2\theta_W$ prediction, though we believe SO(10) allows the more economical solution. The SU(5) solution, being based on the "missing-partner mechanism," requires the introduction of Higgs (Higgsino) multiplets in $\mathbf{50}+\overline{\mathbf{50}}+\mathbf{75}$ (which are four and five index tensors), whereas the SO(10) solution requires only the usual (rank one and two) tensors **45**, and **54**, and the spinors $\mathbf{16}+\overline{\mathbf{16}}$.

The advantage of SO(10) is due to the possibility of exploiting the elegant Dimopoulos-Wilczek mechanism of doublet-triplet splitting. We have studied that mechanism and found that it can be implemented in a fully realistic way.

In our view these results constitute yet another argument in favor of SO(10). It is already well known that SO(10) has the advantage over SU(5) of allowing R parity to be a gauge symmetry (that is because Higgs fields are in tensor representations and matter fields are in spinor representations). And, of course, SO(10) achieves greater unification of quarks and leptons and requires the existence of right-handed neutrinos.

In any event, we have shown that Higgsino-mediated proton decay is not a serious difficulty of supersymmetric grand unification as there are quite simple and natural means to suppress it without undercutting the main success of those models. If the suppression is of the "weak" type then there are grounds to hope to see proton decay in super Kamiokande.

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APPENDIX A

In this Appendix, we give details of the minimization of the superpotential of Eq. (17). We shall see explicitly that (a) SO(10) breaks completely to the standard model in the supersymmetric limit, (b) the desired forms of the VEV's of A and A' are achieved, and (c) there are no unwanted pseudo Goldstone modes which could potentially ruin the successful $\sin^2\theta_W$ prediction.

A^2 in Eq. (17) denotes $\mathrm{Tr}(A^2)$, $AA'A''$ denotes $\mathrm{Tr}(AA'A'')$, etc. We shall confine ourselves to the case where $C+\bar{C}\equiv \mathbf{16}+\overline{\mathbf{16}}$. The term $\bar{C}A''C$ is explicitly written down as $\bar{C}\sigma_{\alpha\beta}A''_{\alpha\beta}C/4$, where $\sigma_{\alpha\beta}$ are the generators of SO(10) algebra [18].

The VEV's for the fields are chosen as

$$\begin{aligned}
\langle A\rangle &= \eta\otimes \mathrm{diag}(a,a,a,0,0)\ ,\\
\langle A'\rangle &= \eta\otimes \mathrm{diag}(0,0,0,a',a')\ ,\\
\langle A''\rangle &= \eta\otimes (a'',a'',a'',a'',a'')\ ,\\
\langle S\rangle &= I\otimes \mathrm{diag}(s,s,s,-\tfrac{3}{2}s,-\tfrac{3}{2}s)\ ,\\
\langle S'\rangle &= I\otimes \mathrm{diag}(s',s',s',-\tfrac{3}{2}s',-\tfrac{3}{2}s')\ ,\\
\langle C\rangle &= \langle \bar{C}\rangle = c\ ,
\end{aligned} \tag{A1}$$

where

$$\eta\equiv\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$$

and

$$I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}.$$

The equality $\langle C\rangle=\langle\bar{C}\rangle$ follows from the requirement of D flatness. The vanishing of the F terms lead to the following conditions, corresponding to A,S,A',S',A'',C fields, respectively:

$$\begin{aligned}
0&=m_1+\lambda_2 s\ ,\\
0&=m_2 s-\tfrac{3}{4}\lambda_1 s^2-\tfrac{1}{5}\lambda_2 a^2\ ,\\
0&=m_1'-\tfrac{3}{2}\lambda_2' s'\ ,\\
0&=m_2' s'-\tfrac{3}{4}\lambda_1' s'^2+\tfrac{1}{5}\lambda_2' a'^2\ ,\\
0&=m_1'' a''+\frac{\lambda_2''}{8}c^2\ ,\\
0&=m_2''-\tfrac{5}{2}\lambda_2'' a''\ .
\end{aligned} \tag{A2}$$

Since SUSY is unbroken, it is sufficient to investigate the Higgsino mass spectrum. The multiplets which transform as $\{(3,1,\tfrac{2}{3})+\mathrm{H.c.}\}$ under $SU(3)_C\times SU(2)_L\times U(1)_Y$ have the mass matrix

$$\mathcal{M}_1=\begin{bmatrix}0 & 2\lambda a'' & 0 & 0\\ 2\lambda a'' & -10\lambda_2' b' & -2\lambda a & 0\\ 0 & -2\lambda a & -4m_1'' & -\lambda_2'' c\\ 0 & 0 & -\lambda_2'' c & 2\lambda_2'' a''\end{bmatrix}. \tag{A3}$$

This matrix has one zero eigenvalue by virtue of Eq. (A2). All the other three states become massive.

The mass matrix corresponding to $\{(1,1,+1)+\mathrm{H.c.}\}$ is

$$\mathcal{M}_2=\begin{bmatrix}10\lambda_2 b & 2\lambda a'' & -2\lambda a' & 0\\ 2\lambda a'' & 0 & 0 & 0\\ -2\lambda a' & 0 & -4m_1'' & -\lambda_2'' c\\ 0 & 0 & -\lambda_2'' c & 2\lambda_2'' a''\end{bmatrix}. \tag{A4}$$

Again, $\mathcal{M}_2$ has one zero eigenvalue [using (A2)] and three nonzero eigenvalues.

The mass matrix for $\{(3,2,-\tfrac{5}{6})+\mathrm{H.c.}\}$ is given by

$$\mathcal{M}_3=\begin{bmatrix}5\lambda_2 s & \sqrt{2}\lambda_2 a & 0 & 0 & -\lambda a'\\ \sqrt{2}\lambda_2 a & 2m_2-\tfrac{3}{2}\lambda_1 s & 0 & 0 & 0\\ 0 & 0 & -5\lambda_2' s' & \sqrt{2}\lambda_2' a' & -\lambda a\\ 0 & 0 & \sqrt{2}\lambda_2' a' & 2m_2'-\tfrac{3}{2}\lambda_1' s' & 0\\ -\lambda a' & 0 & -\lambda a & 0 & -4m_1''\end{bmatrix}. \tag{A5}$$

Using (A2) one sees $\mathcal{M}_3$ has one of its eigenvalues equal to zero, while the rest are all nonzero.

The corresponding matrix for $\{(3,2,\tfrac{1}{6})+\mathrm{H.c.}\}$ is given by

$$\mathcal{M}_4=\begin{bmatrix}5\lambda_2 s & \sqrt{2}\lambda_2 a & 2i\lambda a'' & 0 & -i\lambda a' & 0\\ \sqrt{2}\lambda_2 a & 2m_2-\tfrac{3}{2}\lambda_1 s & 0 & 0 & 0 & 0\\ -2i\lambda a'' & 0 & -5\lambda_2' b' & \sqrt{2}\lambda_2' a' & i\lambda a & 0\\ 0 & 0 & \sqrt{2}\lambda_2' a' & 2m_2'-\tfrac{3}{2}\lambda_1' s' & 0 & 0\\ i\lambda a' & 0 & -i\lambda a & 0 & -4m_1'' & \lambda_2'' c\\ 0 & 0 & 0 & 0 & \lambda_2'' c & 2\lambda_2'' a''\end{bmatrix}. \tag{A6}$$

This has one zero and five nonzero eigenvalues.

The Goldstone modes in $\mathcal{M}_1$, $\mathcal{M}_2$, $\mathcal{M}_3$, and $\mathcal{M}_4$ when combined with the zero mass mode corresponding to the phase of the $(C+\bar{C})$ singlet add up to the 33 massless modes needed for the symmetry breaking

$$\mathrm{SO}(10)\to \mathrm{SU}(3)\times \mathrm{SU}(2)\times \mathrm{U}(1)\ .$$

All the remaining fields become massive. Their spectrum looks as follows. From the (A,S) sector, we have

$$\{(6,1,-\tfrac{2}{3})+\mathrm{H.c.}\}=2m_2+6\lambda_1 s\ ,$$

$$\{(1,3,\pm 1);\ (1,3,0)\}=2m_2-9\lambda_1 s\ ,$$

$$\{(1,3,0)+(1,1,0)\}=10\lambda_2 s\ , \tag{A7}$$

$$\{(8,1,0)\}=\begin{bmatrix}0 & 2\sqrt{2}\lambda_2 a\\ 2\sqrt{2}\lambda_2 a & 2m_2+6\lambda_1 s\end{bmatrix},$$

$$\{(1,1,0)\}=\begin{bmatrix}0 & -(4/\sqrt{5})\lambda_2 a\\ -(4/\sqrt{5})\lambda_2 a & 2m_2-3\lambda_1 s\end{bmatrix}.$$

From the (A',S') sector, one finds

$$\begin{aligned}
&\{(6,1,-\tfrac{2}{3})+\mathrm{H.c.},(8,1,0)\}=2m_2'+6\lambda_1 s'\ ,\\
&\{(8,1,0);\ (1,1,0)\}=-4\lambda_2' s'\ ,\\
&\{(1,3,\pm 1)\}=2m_2'-9\lambda_1' s'\ ,\\
&\{(1,3,0)\}=\begin{bmatrix}0 & -2\sqrt{2}\lambda_2' a'\\ -2\sqrt{2}\lambda_2' a' & 2m_2'-9\lambda_1' s'\end{bmatrix},\\
&\{(1,1,0)\}=\begin{bmatrix}0 & \left[\frac{24}{5}\right]^{1/2}\lambda_2' a'\\ \left[\frac{24}{5}\right]^{1/2}\lambda_2' a' & 2m_2'-3\lambda_1' s'\end{bmatrix}.
\end{aligned} \tag{A8}$$

Finally, from the $(A'',C+\bar{C})$ sector, one finds

$$\begin{aligned}
&\{(8,1,0)+(1,3,0)+(1,1,0)\}=-4m_1''\ ,\\
&\{(3,1,-\tfrac{1}{3})+\mathrm{H.c.};\ (1,2,\tfrac{1}{2})+\mathrm{H.c.}\}=2m_2''+3\lambda_2'' a''\ ,\\
&\{(1,1,0)\}=\begin{bmatrix}-4m_1'' & -\left[\frac{5}{2}\right]^{1/2}\lambda_2'' c\\ -\left[\frac{5}{2}\right]^{1/2}\lambda_2'' c & m_2''-\frac{5}{2}\lambda_2'' a''\end{bmatrix}.
\end{aligned} \tag{A9}$$

Now to reduce threshold corrections somewhat (and simplify calculations) we assume the scale of SO(10) breaking, M_{10}, is somewhat greater than the scale of SU(5) breaking, M_5. (This means we assume $m_1'',m_2'',c,a''\gg m_1,m_2,m_1',m_2',a,a',s,s'$.) As explained in the text, the multiplets which have mass $O(M_{10})$ will give contributions to the threshold corrections suppressed by M_5/M_{10}. Thus in Appendix B we will only need the masses of particles which are $O(M_5)$. For sets of particles with the same G_S quantum numbers we only will need to know the products of their masses. These are listed below:

$$\begin{aligned}
&\{(3,2,\tfrac{1}{6})+\mathrm{H.c.}\}=(\tfrac{4}{25})\lambda_2\lambda_2' a^2 a'^2/ss'\ \ (2\ \text{states})\ ,\\
&\{(3,2,-\tfrac{5}{6})+\mathrm{H.c.}\}=(\tfrac{4}{25})(\lambda^2\lambda_2\lambda_2'/\lambda'')(a^2a'a''/ss'c^2)[2aa'(a+a')+25(a's^2+as'^2)]\ \ (3\ \text{states})\ ,\\
&\{(6,1,-\tfrac{2}{3})+\mathrm{H.c.}\}=(\tfrac{15}{2})\lambda_1 s+(\tfrac{2}{5})\lambda_2 a^2/s\ \ (1\ \text{state})\ ,\\
&\{(1,3,\pm 1);\ (1,3,0)\}=-(\tfrac{15}{2})\lambda_1 s+(\tfrac{2}{5})\lambda_2 a^2/s\ \ (1\ \text{state})\ ,\\
&\{(1,3,0)\}=10\lambda_2 s\ \ (1\ \text{state})\ ,\\
&\{(8,1,0)\}=8\lambda_2^2 a^2\ \ (2\ \text{states})\ ,\\
&\{(6,1,-\tfrac{2}{3})+\mathrm{H.c.};\ (8,1,0)\}=(\tfrac{15}{2})\lambda_1' s'-(\tfrac{2}{5})\lambda_2' a'^2/s'\ \ (1\ \text{state})\ ,\\
&\{(8,1,0)\}=-4\lambda_2' s'\ \ (1\ \text{state})\ ,\\
&\{(1,3,\pm 1)\}=-(\tfrac{15}{2})\lambda_1' s'-(\tfrac{2}{5})\lambda_2' a'^2/s'\ \ (1\ \text{state})\ ,\\
&\{(1,3,0)\}=8\lambda_2'^2 a'^2\ \ (2\ \text{states})\ .
\end{aligned} \tag{A10}$$

We have expressed these in terms of the VEV's and Yukawa couplings and eliminated the mass parameters, m_1, m_2, etc., using Eqs. (A2).

APPENDIX B

Here we use the results of Appendix A to compute the threshold corrections to $\sin^2\theta_W$ coming from superheavy fields. We look at the full model with three **45**'s (A, A', A'') and two **54**'s (S, S'), and we assume SO(10) breaks to SU(5) at a scale (M_{10}) which is higher than the scale at which SU(5) breaks to the standard model (M_5).

It will prove convenient to define the parameters $M \equiv \lambda_2 s$, $x \equiv a/s$, $x' \equiv a'/s'$, $y \equiv \lambda_1/\lambda_2$, $y' \equiv \lambda_1'/\lambda_2'$, and $z \equiv \lambda_2' s'/\lambda_2 s$. The correction to $\sin^2\theta_W$ is given at one loop by

$$\Delta \sin^2\theta_W(M_Z) = \frac{\alpha(M_Z)}{30\pi} \sum_j (5b_1^j - 12b_2^j + 7b_3^j) \ln M_j \ , \tag{B1}$$

where the sum is taken over multiplets of SU(3) $\times$SU(2)$\times$U(1). From Eqs. (A10) one obtains

$$\begin{aligned}\Delta \sin^2\theta_W(M_Z) = \frac{\alpha(M_Z)}{30\pi} \{ & -21 \ln(\tfrac{4}{25}x^2 x'^2 z M^2) + 3 \ln(tM^3) + 51 \ln[(\tfrac{15}{2}y + \tfrac{2}{5}x^2)M] \\ & + (-30-24)\ln[(-\tfrac{15}{2}y + \tfrac{2}{5}x^2)M] - 24 \ln(10M) + 21 \ln(8x^2 M^2) \\ & + (51+21)\ln[(\tfrac{15}{2}y' - \tfrac{2}{5}x'^2)zM] + 21 \ln(4zM) - 30 \ln[(-\tfrac{15}{2}y' - \tfrac{2}{5}x'^2)zM] \\ & - 24 \ln(8x'^2 z^2 M^2)\} \ . \end{aligned} \tag{B2}$$

[Here tM^3 is defined to be equal to the complicated expresssion on the right-hand side of the second equation of (A10). t is of order M_5/M_{10}. However, the coefficient of this term is mercifully small, so that its effect is negligible. We have not displayed the threshold effects due to the doublet and triplet fields of Eqs. (9) and (10) as they are negligible.] All logarithms are understood to have an absolute value in their arguments. We have used $b_3(3)=\frac{1}{2}$, $b_3(6)=\frac{5}{2}$, $b_3(8)=3$, $b_2(2)=\frac{1}{2}$, $b_2(3)=2$, $b_1(y/2)=\frac{3}{5}(y/2)^2$.

Collecting terms,

$$\begin{aligned}\Delta \sin^2\theta_W(M_Z) \cong \frac{\alpha(M_Z)}{30\pi} [& (18 \ln 5 - 33 \ln 2) + 3 \ln t - 6 \ln z - 45 \ln(x'^2) + 51 \ln(\tfrac{15}{2}y + \tfrac{2}{5}x^2) - 54 \ln(\tfrac{15}{2}y - \tfrac{2}{5}x^2) \\ & + 72 \ln(\tfrac{15}{2}y' - \tfrac{2}{5}x'^2) - 30 \ln(\tfrac{15}{2}y' + \tfrac{2}{5}x'^2) - 3 \ln M] \ . \end{aligned} \tag{B3}$$

The term $-3\ln M$ is due to the $(3,2,-\frac{5}{6})$+H.c. that get absorbed when SU(5) breaks and is present also in the minimal SU(5) model. If it were not for this absorption, all the superheavy multiplets would be complete SU(5) multiplets and the dependence on M would disappear. (It is only splitting within multiplet that contribute at one loop to $\sin^2\theta_W$. Since all masses are scaled by M, M drops out in the ratios.) In the first term, $18\ln 5 - 33\ln 2 \cong 6.1$. All of the logarithms can have either sign. There are five potentially large terms with coefficients averaging about 50. If we assume the logarithms are of order one with arbitrary signs then the typical threshold correction to $\sin^2\theta_W$ would be expected to be about

$$\pm(\alpha/30\pi)(10^2) \sim \pm 10^{-2} \ .$$

This is to be contrasted with the effect of a pair of extra light Higgs doublets of

$$\Delta \sin^2\theta_W \simeq 2.5 \times 10^{-2} \ .$$

It should also be compared to the theoretical uncertainties in $\sin^2\theta_W$ in minimal SUSY SU(5), referred to in the opening paragraph of this paper, of about 0.36×10^{-2}.

The expression in (B2) simplifies considerably if we assume $y \ll \frac{4}{75}x^2$, $y' \ll \frac{4}{75}x'^2$. Then

$$\Delta \sin^2\theta_W \simeq \left(\frac{\alpha}{30\pi}\right)[3\ln t - 3\ln(x^2) - 3\ln(x'^2) - 6\ln z + (6\ln 2 - 21\ln 5)] \ . \tag{B4}$$

Since $6\ln 2 - 21\ln 5a = -29.4$ one expects the threshold correction to be negative and about -0.3×10^{-2} in this limit. We mention this limit since it is a special solution of the superpotential of Eq. (17), corresponding to setting the parameters λ_1, λ_1' to zero. We note that this limit can be reached naturally without giving rise to any pseudo Goldstone bosons (see Appendix A).

We conclude that the threshold correction uncertainties to $\sin^2\theta_W$ in the kind of SO(10) model we are discussing are likely to be a few times larger than the total theoretical uncertainty in $\sin^2\theta_W$ in minimal SUSY SU(5), but several times smaller than the effect on $\sin^2\theta_W$ of an extra pair of light Higgs doublets. We should emphasize that if one is satisfied only to suppress Higgsino-mediated proton decay *weakly*, a much smaller Higgs sector may be adequate, with correspondingly smaller threshold corrections.

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