Unified universal seesaw models

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A set of grand unified theories based upon the gauge groups $SU(5)_L \times SU(5)_R$, $SO(10)_L \times SO(10)_R$ and $SU(4)_C \times SU(4)_L \times SU(4)_R$ is explored. Several novel features distinguish these theories from the well-known SU(5), SO(10), and $SU(4)_C \times SU(2)_L \times SU(2)_R$ models which they generalize. Firstly, standard model quarks and leptons are accompanied by and mix with heavy $SU(2)_L \times SU(2)_R$ singlet partners. The resulting fermion mass matrices are seesaw in form. Discrete parity symmetries render the determinants of these mass matrices real and eliminate *CP*-violating gauge terms. The unified seesaw models consequently provide a possible resolution to the strong *CP* problem. Secondly, $\sin^2\theta_W$ at the unification scale is numerically smaller than the experimentally measured Z scale value. The weak angle must therefore increase as it evolves down in energy. Finally, proton decay is suppressed by small seesaw mixing factors in all these theories.

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I. INTRODUCTION

Among the many questions left unanswered by the standard model of particle physics, the origin of fermion masses ranks as one of the most intriguing. Details of the fermion mass spectrum remain a perplexing mystery, and even its gross features are not understood. One general characteristic which remains unexplained within the context of the minimal standard model is the disparity between the electroweak scale and quark and lepton masses. This dichotomy can of course be accommodated in the standard model by tuning certain Yukawa couplings to be sufficiently small. However, a more natural explanation for this mass gap would be preferable.

In the past few years, a qualitative explanation has been offered in which the familiar neutrino seesaw mechanism [1] is applied to charged fermions as well [2,3]. This universal seesaw proposal necessitates the introduction of new heavy partners for each of the known standard model fermions with which they mix. The lightness of observed quarks and leptons then results as a consequence of the seesaw mechanism. This scheme obviously works best for the first generation of fermions and worst for the third. In particular, achieving the anomalously large mass for the top quark is problematic. Nonetheless, the basic idea of a universal seesaw mechanism is interesting and sheds some light on the fermion mass puzzle.

A second and much more compelling motivation for studying theories with a universal seesaw mechanism is that they can resolve the strong *CP* problem. Such theories generally possess a parity symmetry which prohibits a *CP*-violating $\theta_{\rm QCD}$ term from appearing in the QCD Lagrangian and renders Yukawa coupling matrices Hermitian. So while the fermion mass matrix can be complex and generate weak *CP* violation, the argument $\theta_{\rm QFD}$ of its determinant is zero. The physically observable parameter $\bar\theta=\theta_{\rm QCD}+\theta_{\rm QFD}$ consequently vanishes at tree order. Universal seesaw models thus offer a solution to the strong *CP* problem which does not involve axions [4,5,6].

The universal seesaw mechanism has been studied in the past mainly within the context of the left-right symmetric $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)$ model. In this paper, we explore a number of possibilities for embedding this mechanism within a unified theory. In particular, we investigate models based upon the gauge groups $SU(5)_L \times SU(5)_R$, $SO(10)_L \times SO(10)_R$, and $SU(4)_C \times SU(4)_L \times SU(4)_R$. As we shall see, such unified theories provide a rationale for the seemingly *ad hoc* introduction of heavy $SU(2)_L \times SU(2)_R$ singlet fermions in their ununified counterparts. Moreover, these particular models generalize the well-known SU(5) [7] and SO(10) [8] grand unified theories (GUT's) and the $SU(4)_C \times SU(2)_L \times SU(2)_R$ Pati-Salam model [9]. So they are of interest in their own right.

To help guide our exploration, we will adopt the following set of unified seesaw model building rules.

(I) The model must reproduce the measured Z scale values for the standard model couplings [10–12]:

$$\sin^2\theta_W(M_z)=0.2325\pm0.0008 \;, \tag{1.1a}$$

$$\alpha_{\rm EM}^{-1}(M_z)=127.8\pm0.2 \;, \tag{1.1b}$$

$$\alpha_s(M_z)=0.118\pm0.007 \;. \tag{1.1c}$$

(II) The model must satisfy other phenomenological constraints such as limits on new particle masses and bounds on proton decay.

(III) The model should incorporate heavy $SU(2)_L \times SU(2)_R$ singlet fermions which mix with standard model quarks and leptons to allow for a seesaw mass matrix whose determinant is real.

(IV) The model should contain fermions in anomaly-free but complex representations in accordance with the "survival hypothesis" [13].

(V) The model preferably maintains left-right symmetry from the unification scale down to the standard model subgroup level.

These requirements are listed in approximate order of importance. The first two experimental constraints are binding and must be satisfied by any realistic GUT. The third point summarizes the distinctive features of universal seesaw models that allow them to resolve the strong *CP* problem. The final two rules represent natural theoretical guidelines which are more negotiable than the first three constraints. In particular, the last item is included only to help restrict the large number of possible symmetry-breaking patterns in the models we shall explore. So we may relax this final aesthetic condition in order to satisfy the other more stringent requirements in this list.

The remainder of our paper is organized as follows. We present the SU(5)×SU(5) and SO(10)×SO(10) models in Secs. II and III. These theories illustrate the basic features of all unified universal seesaw models. They also serve as warmups for the SU(4)×SU(4)×SU(4) model which is discussed in greater detail in Sec. IV. Finally, we close in Sec. V with some indications for possible further investigation of this new class of GUT's.

II. THE PROTOTYPE SU(5)×SU(5) MODEL

The first model that we shall explore is based upon the gauge group $G=\mathrm{SU}(5)_L\times\mathrm{SU}(5)_R$. This theory represents an obvious generalization of the Georgi-Glashow SU(5) model [7] and shares many of its attractive features. It is also the simplest unified seesaw model and has been analyzed in the past [14,15]. While this theory ultimately turns out not to be phenomenologically viable, it is worth reviewing since many of its basic characteristics are common to all unified universal seesaw models.

To begin, we impose a Z_2 symmetry on the chiral theory which combines a spatial inversion with interchanging the SU(5) factors in the product group G. Such a discrete symmetry is needed to ensure the equality of the $\mathrm{SU}(5)_L$ and $\mathrm{SU}(5)_R$ coupling constants above the unification sale. In its absence, the couplings would run differently and diverge even if they were set equal at one particular renormalization point. The generalized parity operation enforces a left-right symmetry on the Lagrangian which may be violated only softly by super renormalizable interactions. It also dictates a one-to-one correspondence among matter field representations of $\mathrm{SU}(5)_L$ and $\mathrm{SU}(5)_R$. The spectrum of this theory consequently exhibits an explicit parity doubling.

We next embed the standard model within the GUT following the Georgi-Glashow model blueprint. Color SU(3) and weak SU(2) are identified with the diagonal $\mathrm{SU}(3)_{L+R}$ subgroup of G and the $\mathrm{SU}(2)_L$ subgroup of $\mathrm{SU}(5)_L$ respectively. $\mathrm{U}(1)_{\mathrm{EM}}$ is generated by the diagonal sum of the familiar $\mathrm{SU}(5)_L$ and $\mathrm{SU}(5)_R$ electric charge generators. The SU(3)×SU(2)×U(1) content of a single fermion family representation,

$$\mathcal{F}\sim(\bar{5}+10,1)+(1,\bar{5}+10)\,, \tag{2.1}$$

is then readily established. The fermions' colors, flavors, and electric charges are indicated by conventional letter names in the matrices below:

$$(\psi_L)_i=\begin{bmatrix} D_1^c \\ D_2^c \\ D_3^c \\ e \\ -\nu \end{bmatrix}_L \sim(\bar{5},1)\,,\quad (\Psi_L)^{ij}=\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & U_3^c & -U_2^c & -u_1 & -d_1 \\ -U_3^c & 0 & U_1^c & -u_2 & -d_2 \\ U_2^c & -U_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -E^c \\ d_1 & d_2 & d_3 & E^c & 0 \end{bmatrix}_L \sim(10,1)$$

$$(\psi_R)_{i'}=\begin{bmatrix} D_1^c \\ D_2^c \\ D_3^c \\ e \\ -\nu \end{bmatrix}_R \sim(1,\bar{5})\,,\quad (\Psi_R)^{i'j'}=\frac{1}{\sqrt{2}}\begin{bmatrix} 0 & U_3^c & -U_2^c & -u_1 & -d_1 \\ -U_3^c & 0 & U_1^c & -u_2 & -d_2 \\ U_2^c & -U_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -E^c \\ d_1 & d_2 & d_3 & E^c & 0 \end{bmatrix}_R \sim(1,10)\,. \tag{2.2}$$

Three generations of families are assumed as in the standard model and assigned to three copies of $\mathcal{F}$.

We now specify a simple symmetry-breaking pattern that starts with the unified chiral gauge group and cascades down to unbroken color and electromagnetism:

$$\begin{gathered} \mathrm{SU}(5)_L\times\mathrm{SU}(5)_R \\ \downarrow M_{\mathrm{GUT}} \\ \mathrm{SU}(3)_L\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_L\times\mathrm{SU}(3)_R\times\mathrm{SU}(2)_R\times\mathrm{U}(1)_R \\ \downarrow \Lambda_{LR} \\ \mathrm{SU}(3)_{L+R}\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{U}(1)_{L+R} \\ \downarrow v_R \\ \mathrm{SU}(3)_{L+R}\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y \\ \downarrow v_L \\ \mathrm{SU}(3)_{L+R}\times\mathrm{U}(1)_{\mathrm{EM}}. \end{gathered} \tag{2.3}$$

A minimal number of fundamental Higgs fields is introduced into the theory to achieve this pattern. As in the Georgi-Glashow model, $SU(5)_L$ and $SU(5)_R$ are broken with scalars $\Phi_L \sim (24,1)$ and $\Phi_R \sim (1,24)$ that transform in their adjoint representations. The fermion families decompose under the resulting $[SU(3)\times SU(2)\times U(1)]^2$ subgroup as

$$\mathcal{F} \sim [(\bar{3},1,1,1)^{1/3,0}+(1,\bar{2},1,1)^{-1/2,0}+(\bar{3},1,1,1)^{-2/3,0}+(3,2,1,1)^{1/6,0}+(1,1,1,1)^{1,0}]_L$$
$$+[(1,1,\bar{3},1)^{0,1/3}+(1,1,1,\bar{2})^{0,-1/2}+(1,1,\bar{3},1)^{0,-2/3}+(1,1,3,2)^{0,1/6}+(1,1,1,1)^{0,1}]_R \ . \tag{2.4}$$

The subsequent breaking of chiral color and chiral hypercharge to their diagonal subgroups is performed at the Λ_{LR} scale by Higgs fields $\omega \sim (\bar{5},5)$ and $\Omega \sim (10,\overline{10})$. If these scalars develop the vacuum expectation values

$$\langle \omega \rangle_1^{1'} = \langle \omega \rangle_2^{2'} = \langle \omega \rangle_3^{3'} = \langle \Omega \rangle_{[1'2']}^{[12]} = \langle \Omega \rangle_{[2'3']}^{[23]}$$
$$= \langle \Omega \rangle_{[3'1']}^{[31]} = \langle \Omega \rangle_{[4'5']}^{[45]} = \Lambda_{LR} \ , \tag{2.5}$$

the chirally colored $(\bar{3},1,1,1)$ and $(1,1,\bar{3},1)$ and chirally hypercharged $(1,1,1,1)^{1,0}$ and $(1,1,1,1)^{0,1}$ fields in (2.4) marry together and acquire Dirac masses through the Yukawa interactions

$$\mathcal{L}_{\text{Yukawa}}(\omega,\Omega) = -\left[f_\omega (\bar{\psi}_L)^i (\omega)_i^{i'} (\psi_R)_{i'} + \frac{f_\Omega}{2} (\bar{\Psi}_L)_{ij} (\Omega)^{ij}_{i'j'} (\Psi_R)^{i'j'} \right] + \text{H.c.} \tag{2.6a}$$

These fourteen $SU(2)_L \times SU(2)_R$ singlet fermions automatically emerge in the unified theory as the heavy seesaw partners that are added by hand in un-unified seesaw models. They are denoted by capital letters in (2.2). The remaining sixteen fields in (2.4) reside within SU(2) doublets and stay massless at the Λ_{LR} scale. They essentially correspond to the known standard model fermions plus a right-handed neutrino and are represented by the lower case letters in (2.2).

The last two steps in pattern (2.3) are accomplished by scalars $\phi_L \sim (5,1)$ and $\phi_R \sim (1,5)$ which break $SU(2)_L$ and $SU(2)_R$ via the vacuum expectation values (VEV's)

$$\langle \phi_{L,R} \rangle = (0,0,0,0,v_{L,R}/\sqrt{2})^T \ .$$

Masses connecting heavy and light fermions are then generated by the Yukawa terms

$$\mathcal{L}_{\text{Yukawa}}(\phi) = f_\phi [(\psi_L^T)_i C (\Psi_L)^{ij} (\phi_L^\dagger)_j + (\psi_R^T)_{i'} C (\Psi_R)^{i'j'} (\phi_R^\dagger)_{j'}]$$
$$+ f'_\phi [\epsilon_{ijklm} (\Psi_L^T)^{ij} C (\Psi_L)^{kl} (\phi_L)^m + \epsilon_{i'j'k'l'm'} (\Psi_R^T)^{i'j'} C (\Psi_R)^{k'l'} (\phi_R)^{m'}] + \text{H.c.} \tag{2.6b}$$

The quark and lepton mass matrices thus assume the seesaw forms

$$\mathcal{L}_{\text{mass}} = (\overline{u_L}\,\overline{U_L}) \begin{bmatrix} 0 & \sqrt{2} f_\phi'^\dagger v_L \\ \sqrt{2} f'_\phi v_R & f_\Omega^T \Lambda_{LR} \end{bmatrix} \begin{bmatrix} u_R \\ U_R \end{bmatrix} + (\overline{d_L}\,\overline{D_L}) \begin{bmatrix} 0 & \frac{1}{2} f_\phi^\dagger v_L \\ \frac{1}{2} f_\phi v_R & f_\omega^T \Lambda_{LR} \end{bmatrix} \begin{bmatrix} d_R \\ D_R \end{bmatrix}$$
$$+ (\overline{e_R^+}\,\overline{E_R^+}) \begin{bmatrix} 0 & \frac{1}{2} f_\phi v_L \\ \frac{1}{2} f_\phi^\dagger v_R & f_\Omega \Lambda_{LR} \end{bmatrix} \begin{bmatrix} e_L^+ \\ E_L^+ \end{bmatrix} + \text{H.c.} \tag{2.7}$$

It is important to recall that the fermion fields are ($N_{\mathcal{F}}=3$)-dimensional vectors in family space. The Yukawa couplings in Eqs. (2.6a) and (2.6b) are consequently $N_{\mathcal{F}} \times N_{\mathcal{F}}$ matrices with generation indices that have been suppressed. Parity constrains f_ω and f_Ω to be Hermitian, while the form of the second term in (2.6b) automatically renders f'_ϕ symmetric. If these Yukawa couplings are approximately comparable in magnitude, then the mass matrices have the well-known seesaw eigenvalues

$$m = -O\left[f \frac{v_L v_R}{\Lambda_{LR}} \right] ,$$
$$M = O(f \Lambda_{LR}) , \tag{2.8}$$

and corresponding eigenvectors

$$\begin{bmatrix} q' \\ Q' \end{bmatrix} = \begin{bmatrix} 1 & -O(v_R/\Lambda_{LR}) \\ O(v_L/\Lambda_{LR}) & 1 \end{bmatrix} \begin{bmatrix} q \\ Q \end{bmatrix} \tag{2.9}$$

provided $v_L v_R \ll \Lambda_{LR}^2$. We thus recover the universal seesaw mechanism in this $SU(5)\times SU(5)$ theory.

The fermion mass matrices in (2.7) are generally complex and induce weak *CP* violation as in the standard model. But their determinants are real. This can be simply verified by rewriting the down-type quark matrix for example as

$$\mathcal{M}_{dD} = \begin{bmatrix} 1 & 0 \\ 0 & v_r/\Lambda_{LR} \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{2} f_\phi^\dagger \Lambda_{LR} \\ \frac{1}{2} f_\phi \Lambda_{LR} & f_\omega^T \Lambda_{LR}^3 / v_L v_R \end{bmatrix}$$
$$\times \begin{bmatrix} 1 & 0 \\ 0 & v_L/\Lambda_{LR} \end{bmatrix} . \tag{2.10}$$

Since the diagonal matrices are real while the middle matrix is Hermitian, we conclude that $\arg(\det \mathcal{M}_{dD}) = 0$. So as a result of the generalized parity symmetry in the $SU(5)\times SU(5)$ model, the complex argument θ_{QFD} of the total mass matrix as well as the $\theta_{\rm QCD}$ term in the QCD Lagrangian vanish at tree order. The seesaw GUT therefore provides a possible solution to the strong CP problem.

Unfortunately, the symmetry breaking pattern in (2.3) is not phenomenologically viable. Recall that once the embedding of the electroweak subgroup inside the gauge group G is specified, the value of $\sin^2\theta_W$ at the unification scale is fixed:

$$\sin^2\theta_W(M_{\rm GUT})=\frac{{\rm Tr}(T_L^3)^2}{{\rm Tr}Q^2}=\frac{3}{16}=0.1875\ . \tag{2.11}$$

In this SU(5)×SU(5) model, there are twice as many electrically charged fermions as in the SU(5) theory but precisely the same number of weak $\mathrm{SU}(2)_L$ doublets. So $\sin^2\theta_W(M_{\rm GUT})$ is half as large as in the Georgi-Glashow model [7] and starts out numerically smaller than $\sin^2\theta_W(M_z)=0.2325$. Moreover, renormalization effects decrease the value of $\sin^2\theta_W(\mu)$ for $\mu<M_{\rm GUT}$ in the SU(5)×SU(5) theory just as in the SU(5) model [16]. Therefore, pattern (2.3) cannot duplicate the Z scale measurement and must be rejected.

One can try to search for alternate breaking patterns in which $\sin^2\theta_W$ increases as it evolves down in energy from the GUT scale. Maximal enhancement is achieved if the first stage of symmetry breaking is taken to be $\mathrm{SU}(5)_L\times\mathrm{SU}(5)_R\to\mathrm{SU}(3)_L\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_L\times\mathrm{SU}(5)_R$ [14,15]. This clearly leads to trouble with proton decay. Moreover, detailed calculation demonstrates that this asymmetrical-breaking pattern still cannot yield the values for the standard model couplings in (1.1) [15]. We therefore conclude that an SU(5)×SU(5) seesaw theory is ruled out.

III. THE SO(10)×SO(10) MODEL

The GUT scale value for $\sin^2\theta_W$ tends to be small in all unified universal seesaw models as we have seen in the particular case of SU(5)×SU(5). So in order for these theories to be phenomenologically viable, we must find some mechanism for enhancing $\sin^2\theta_W$ as it evolves down in energy from the unification scale. We will illustrate a general strategy for overcoming this problem in the context of an SO(10)×SO(10) model.

This second theory represents an obvious generalization of the first considered in the preceding section, and a number of parallel features can immediately be established. For instance, a discrete interchange symmetry must again be imposed on the separate factors in the gauge group $G=\mathrm{SO}(10)_L\times\mathrm{SO}(10)_R$. As a result, particle representations occur in pairs, and fermion families in particular transform as

$$\mathcal{F}\sim(16,1)+(1,16)\ , \tag{3.1}$$

which generalizes the SU(5)×SU(5) assignments in (2.1). There are, however, some significant differences between the two models. Most importantly, the larger size of SO(10)×SO(10) allows several new possibilities for electroweak subgroup embedding and symmetry breaking. As we shall see, this greater flexibility provides the key to increasing $\sin^2\theta_W$ at the Z scale.

Among the different potential breaking schemes, we focus upon the following pattern which maintains explicit left-right symmetry down to the standard model:

$$\begin{array}{c}
\begin{array}{cc}
\multicolumn{2}{c}{\mathrm{SO}(10)_L\times\mathrm{SO}(10)_R}\\
L^\alpha & R^\alpha\\
g_{10L} & g_{10R}
\end{array}\\
\downarrow M_{\rm GUT}\\
\begin{array}{cccccc}
\mathrm{SU}(4)_L\times & \mathrm{SU}(2)_L\times & \mathrm{SU}(2)'_L\times & \mathrm{SU}(4)_R\times & \mathrm{SU}(2)_R\times & \mathrm{SU}(2)'_R\\
U_L^A & T_L^i & T_L^{i\,\prime} & U_R^A & T_R^i & T_R^{i'}\\
g_{4L} & g_{2L} & g'_{2L} & g_{4R} & g_{2R} & g'_{2R}
\end{array}\\
\downarrow \Lambda_{LR}\\
\begin{array}{cccc}
\mathrm{SU}(4)_{L+R}\times & \mathrm{SU}(2)_L\times & \mathrm{SU}(2)_R\times & \mathrm{SU}(2)'_{LR}\\
U^A=U_L^A+U_R^A & T_L^i & T_R^i & T^{i'}=T_L^{i'}+T_R^{i'}\\
g_4 & g_{2L} & g_{2R} & g'_2
\end{array}\\
\downarrow \Lambda_c\\
\begin{array}{cccc}
\mathrm{SU}(3)_{L+R}\times & \mathrm{SU}(2)_L\times & \mathrm{SU}(2)_R\times & \mathrm{U}(1)_{L+R}\\
U^a & T_L^i & T_R^i & S=T^{3'}+(\frac{2}{3})^{1/2}U^{15}\\
g_3 & g_{2L} & g_{2R} & g_1
\end{array}\\
\downarrow v_R\\
\begin{array}{ccc}
\mathrm{SU}(3)_{L+R}\times & \mathrm{SU}(2)_L\times & \mathrm{U}(1)_Y\\
U^a & T_L^i & Y/2=T_R^3+S\\
g_3 & g_{2L} & g'
\end{array}\\
\downarrow v_L\\
\begin{array}{cc}
\mathrm{SU}(3)_{L+R}\times & U(1)_{\rm EM}\\
U^a & Q=T_L^3+Y/2\\
g_3 & e
\end{array}
\end{array} \tag{3.2}$$

We have listed underneath each of the subgroup factors in this pattern our nomenclature conventions for the associated generators and coupling constants.[1]

The generators at each level in (3.2) are linear combinations $H=\sum_i c_i G_i$ of those at the previous level, and the corresponding couplings are related as $h^{-2}=\sum_i (g_i/c_i)^{-2}$. In particular, the electric charge generator

$$Q=T_L^3+T_R^3+T_L^{3'}+T_R^{3'}+(\tfrac{2}{3})^{1/2}U_L^{15}+(\tfrac{2}{3})^{1/2}U_R^{15} \tag{3.3}$$

of the final unbroken $\mathrm{U}(1)_{\rm EM}$ subgroup is a combination of elements in the Cartan subalgebras of $\mathrm{SU}(2)_{L,R}$, $\mathrm{SU}(2)'_{L,R}$, and $\mathrm{SU}(4)_{L,R}$. The corresponding relation among these groups' coupling constants

$$\begin{aligned}e(\mu)^{-2}&=g_{2L}(\mu)^{-2}+g_{2R}(\mu)^{-2}+g'_{2L}(\mu)^{-2}+g'_{2R}(\mu)^{-2}\\&\quad+\tfrac{2}{3}g_{4L}(\mu)^{-2}+\tfrac{2}{3}g_{4R}(\mu)^{-2}\end{aligned} \tag{3.4}$$

fixes the GUT scale value of the weak mixing angle:

$$\sin^2\theta_W(M_{\rm GUT})=\frac{e(M_{\rm GUT})^2}{g_{2L}(M_{\rm GUT})^2}=\frac{3}{16}=0.1875\ . \tag{3.5}$$

[1]The ranges of the SO(10), SU(4), SU(3), and SU(2) generator labels α, A, a, and i are, respectively, 1–45, 1–15, 1–8, and 1–3.

Since the $SU(3)\times SU(2)\times U(1)$ content of the fermion representation (3.1) in the $SO(10)\times SO(10)$ model is identical to that of (2.1) in the $SU(5)\times SU(5)$ theory except for an additional electrically neutral $SU(2)_L\times SU(2)_R$ singlet field, we have again found a value for $\sin^2\theta_W(M_{GUT})$ which is precisely half a large as in the Georgi-Glashow model. But the behavior of $\sin^2\theta_W$ below the unification scale is qualitatively different:

$$\sin^2\theta_W(\mu)=\begin{cases}\dfrac{1}{1+\left[\dfrac{g_{2L}}{g_{2R}}\right]^2+\left[\dfrac{g_{2L}}{g'_{2L}}\right]^2+\left[\dfrac{g_{2L}}{g'_{2R}}\right]^2+\dfrac{2}{3}\left[\dfrac{g_{2L}}{g_{4L}}\right]^2+\dfrac{2}{3}\left[\dfrac{g_{2L}}{g_{4R}}\right]^2}\,, & \Lambda_{LR}\le\mu\le M_{GUT}\,, \qquad (3.6a)\\[2ex] \dfrac{1}{1+\left[\dfrac{g_{2L}}{g_{2R}}\right]^2+\left[\dfrac{g_{2L}}{g'_{2}}\right]^2+\dfrac{2}{3}\left[\dfrac{g_{2L}}{g_{4}}\right]^2}\,, & \Lambda_C\le\mu\le\Lambda_{LR}\,, \qquad (3.6b)\\[2ex] \dfrac{1}{1+\left[\dfrac{g_{2L}}{g_{2R}}\right]^2+\left[\dfrac{g_{2L}}{g_{1}}\right]^2}\,, & v_R\le\mu\le\Lambda_C\,, \qquad (3.6c)\\[2ex] \dfrac{1}{1+\left[\dfrac{g_{2L}}{g'}\right]^2}\,, & v_L\le\mu\le v_R\,. \qquad (3.6d)\end{cases}$$

In (3.6a), the SU(2) couplings g_{2L}, g_{2R}, g'_{2L}, and g'_{2R} are all asymptotically free and increase as they run down in energy. However, the SU(4) couplings g_{4L} and g_{4R} increase even faster. So the denominator in (3.6a) decreases and the total fraction grows larger for $\mu<M_{GUT}$. This rising trend continues until the Λ_C scale is reached. At that point, $\sin^2\theta_W(\mu)$ begins to decrease and continues downward all the way to $\mu=M_Z$. The final sign and magnitude of the net change in $\sin^2\theta_W$ depend in detail upon the numerical values of the various intermediate scales and β functions of the couplings appearing within the multilevel pattern (3.2). But we at least see how an enhancement of the weak mixing angle may be achieved in principle [17,18].

Unification by itself cannot uniquely determine all the symmetry-breaking scales in (3.2). However, a number of phenomenological considerations restrict their values. Firstly, K-$\bar{K}$ mixing places lower mass limits of 1.6–2.5 TeV on $W_R^{\pm}$ gauge bosons in manifestly left-right symmetric $SU(2)_L\times SU(2)_R\times U(1)$ theories [19,20]. Therefore, v_R must lie at least in the multi-TeV region. Secondly, limits on the lepton family number violating decay $K_L\to\mu^+e^-$ provide a bound on the Λ_C scale, for it is mediated by $SU(4)_{L+R}$ gauge boson exchange. Including renormalization effects [21], we estimate that the branching fraction limit [10]

$$\frac{\Gamma(K_L\to\mu^+e^-)}{\Gamma(K^+\to\mu^+\nu_\mu)}<3.54\times10^{-11} \qquad (3.7)$$

restricts $\Lambda_C\gtrsim10^6$ GeV. Finally, the unification scale M_{GUT} must be sufficiently large to allow for an acceptable proton lifetime.

It is useful to imagine constructing a low-energy effective field theory at each symmetry-breaking stage in pattern (3.2) in order to simplify the renormalization-group analysis of coupling constant evolution. Particles that can acquire masses at a certain scale are integrated out together and do not contribute to subsequent renormalization group running. We thus find the following one-loop gauge boson and fermion contributions to the U(1) and SU(n) β functions[2] $\beta(g_n)=b_n g_n^3/16\pi^2$:

$$\begin{aligned} b_Y&=\tfrac{20}{3}N_{\mathcal{F}}\,,\\ b_1&=\tfrac{8}{9}N_{\mathcal{F}}\,,\\ b'_2&=-\tfrac{22}{3}\,,\\ b_n&=-\left[\frac{11n}{3}-\frac{4}{3}N_{\mathcal{F}}\right]. \end{aligned} \qquad (3.8)$$

It is then straightforward to integrate the renormalization-group equations to obtain a linear system of equations that relates the three high-energy quantities $\alpha_{10}(M_{GUT})=g_{10}(M_{GUT})^2/4\pi$, $\ln(M_{GUT}/\Lambda_{LR})$, and $\ln(\Lambda_{LR}/\Lambda_C)$ to the three low-energy parameters $\sin^2\theta_W(M_Z)$, $\alpha_{EM}(M_Z)$, and $\alpha_s(M_Z)$:

[2]Ordinary quarks and leptons are singlets under $SU(2)'_{L+R}$, while their seesaw patterns acquire heavy masses and decouple at the Λ_{LR} scale in (3.2). There is consequently no fermion contribution to the $SU(2)'_{L+R}\beta$-function coefficient b'_2.

$$\begin{bmatrix} \frac{13}{3} & 3b_{2L}+\frac{4}{3}b_{4L} & b_{2L}+b_2'+\frac{2}{3}b_4 \\ 1 & b_{2L} & b_{2L} \\ 2 & 2b_{4L} & b_4 \end{bmatrix} \begin{bmatrix} 2\pi/\alpha_{10}(M_{\rm GUT}) \\ \ln M_{\rm GUT}/\Lambda_{LR} \\ \ln\Lambda_{LR}/\Lambda_C \end{bmatrix} = \begin{bmatrix} 2\pi\cos^2\theta_W(M_Z)/\alpha_{\rm EM}(M_Z)-(b_1+b_2)\ln\Lambda_C/v_R-b_Y\ln v_R/M_Z \\ 2\pi\sin^2\theta_W(M_Z)/\alpha_{\rm EM}(M_Z)-b_{2L}\ln\Lambda_{LR}/M_Z \\ 2\pi/\alpha_s(M_Z)-b_3\ln\Lambda_C/M_Z \end{bmatrix} . \tag{3.9}$$

Unfortunately, no consistent solution to this matrix equation exists which satisfies the phenomenological restrictions on the intermediate scales and reproduces the high precision numbers in (1.1). A fit for the GUT scale parameters based upon the inputs $v_R=5$ TeV, $\Lambda_C=1000$ TeV, and $\Lambda_{LR}=100\,000$ TeV yields the results

$$\alpha_{10}(M_{\rm GUT})=0.025\ , \qquad M_{\rm GUT}=1.3\times10^{15}\ {\rm GeV}\ , \tag{3.10}$$

which imply the Z scale values

$$\sin^2\theta_W(M_Z)=0.197\ , \qquad \alpha_{\rm EM}^{-1}(M_Z)=124.3\ , \qquad \alpha_s(M_Z)=0.137\ . \tag{3.11}$$

The match between these theoretical numbers and the experimental measurements in (1.1) is obviously poor. Nonetheless, we see that the basic strategy of embedding part of the hypercharge generator within an asymptotically free subgroup has led to an increase in $\sin^2\theta_W(M_Z)$ over its unification value [18]. This trick must generally be employed in any unified universal seesaw model.

At this point, we could explore other symmetry-breaking schemes for the $SO(10)\times SO(10)$ theory in which manifest left-right symmetry is broken at an earlier stage than in pattern (3.2) so as to further enhance the value for $\sin^2\theta_W(M_Z)$. Alternatively, we could continue to search for a phenomenologically viable chiral $G_L\times G_R$ model based upon an even larger group such as $E_6\times E_6$. But we turn instead to explore a somewhat different theory with the gauge structure $G_C\times G_L\times G_R$ in the following section.

IV. THE SU(4)×SU(4)×SU(4) MODEL

The prototypical example of a unified $G_C\times G_L\times G_R$ theory is the $SU(3)_C\times SU(3)_L\times SU(3)_R$ model [22]. This amusing "trinification" theory has been studied in the past as an alternative to SU(5) and SO(10) unification. The $SU(3)^3$ model, however, cannot accommodate a heavy $SU(2)_L\times SU(2)_R$ partner for each standard model fermion. So we are led to consider the next simplest possibility based upon the gauge group

$$G=SU(4)_C\times SU(4)_L\times SU(4)_R\ , \tag{4.1}$$

which is supplemented with a cyclic Z_3 symmetry to ensure equality among the separate SU(4) coupling constants. This theory represents an obvious generalization of $SU(3)^3$ trinification as well as the $SU(4)_C\times SU(2)_L\times SU(2)_R$ Pati-Salam model [9]. Indeed, Pati and Salam originally proposed G as a possible global symmetry of nature in which lepton number plays the role of a fourth color. The similarities and differences between our model and these others that have been studied in the past will become evident as we proceed.

Embedding the standard model subgroup within $SU(4)^3$ is straightforward. We take a generalized set of Gell-Mann matrices as generators of $SU(4)_C$. The first eight members of this set are associated with color SU(3), while the fifteenth matrix

$$U_C^{15}=\left[\frac{3}{2}\right]^{1/2}\begin{bmatrix} \frac{1}{6} & & & \\ & \frac{1}{6} & & \\ & & \frac{1}{6} & \\ & & & -\frac{1}{2} \end{bmatrix} \tag{4.2}$$

generates a commuting $U(1)_C$ factor. For $SU(4)_{L,R}$, we use the set of 4×4 Pauli matrices

$$\frac{\sigma^i}{2\sqrt{2}}\ ,\quad \frac{\tau^j}{2\sqrt{2}}\ ,\quad \frac{\sigma^i\tau^j}{2\sqrt{2}}\ ,\quad i,j=1,2,3\ , \tag{4.3}$$

as normalized generators. The linear combinations

$$\begin{aligned} T^i_{L,R}&=\frac{\sigma_i(1+\tau^3)}{4}=\frac{1}{2}\begin{bmatrix}\sigma^i & \\ & 0\end{bmatrix}\ , \\ T^{i'}_{L,R}&=\frac{\sigma^i(1-\tau^3)}{4}=\frac{1}{2}\begin{bmatrix}0 & \\ & \sigma^i\end{bmatrix}\ , \\ S_{L,R}&=\frac{\tau^3}{2\sqrt{2}}=\frac{1}{2\sqrt{2}}\begin{bmatrix}1 & \\ & -1\end{bmatrix} \end{aligned} \tag{4.4}$$

belong to an $SU(2)_{L,R}\times SU(2)'_{L,R}\times U(1)_{L,R}$ subalgebra of $SU(4)_{L,R}$. Weak SU(2) and its right-handed analogue are identified as $SU(2)_L$ and $SU(2)_R$. Finally, we choose

$$Q=T_L^3+T_L^{3'}+T_R^3+T_R^{3'}+\left[\frac{2}{3}\right]^{1/2}U_C^{15} \tag{4.5}$$

as the generator of electromagnetism. This definition implies

$$\sin^2\theta_W(M_{\rm GUT})=\tfrac{3}{14}=0.2143\ .$$

While this GUT scale value is still below the Z scale measurement

$$\sin^2\theta_W(M_Z)=0.2325 ,$$

it is certainly closer than the corresponding

$$\sin^2\theta_W(M_{\rm GUT})=\tfrac{3}{16}=0.1875$$

that we found in the SU(5)×SU(5) and SO(10)×SO(10) models. So we already see one clear advantage of the $SU(4)^3$ theory over its predecessors.

Gauge bosons in this model transform according to the 45-dimensional representation

$$\mathcal{G}\sim(15,1,1)+(1,15,1)+(1,1,15) , \tag{4.6}$$

which automatically remains invariant under cyclic Z_3 permutations. In the fermion sector, a single family of left handed fields is assigned to the anomaly-free but complex representation

$$\mathcal{F}\sim(4,\bar{4},1)+(1,4,\bar{4})+(\bar{4},1,4) . \tag{4.7}$$

One generation of left handed quarks and leptons along with their seesaw partners fit snugly inside $(4,\bar{4},1)$, while conjugate fields appear in $(\bar{4},1,4)$. The remaining $(1,4,\bar{4})$ contains a new set of leptons. All these particles' colors, favors, and electric charges are indicated in the matrices below:

$$\Psi_{CL}(4,\bar{4},1)=\begin{bmatrix} d_1 & u_1 & D_1 & U_1\\ d_2 & u_2 & D_2 & U_2\\ d_3 & u_3 & D_3 & U_3\\ e & \nu & E & N\end{bmatrix}_L , \tag{4.8a}$$

$$\Psi_{LR}(1,4,\bar{4})=\begin{bmatrix} I_0 & I^+ & J^0 & J^+\\ I^- & I^{0c} & J^- & J^{0c}\\ K^0 & K^+ & L^0 & L^+\\ K^- & K^{0c} & L^- & L^{0c}\end{bmatrix}_L , \tag{4.8b}$$

$$\Psi_{RC}(\bar{4},1,4)=\begin{bmatrix} d_1^c & d_2^c & d_3^c & e^c\\ u_1^c & u_2^c & u_3^c & \nu^c\\ D_1^c & D_2^c & D_3^c & E^c\\ U_1^c & U_2^c & U_3^c & N^c\end{bmatrix}_L . \tag{4.8c}$$

There exist a number of potential symmetry-breaking chains that start from the GUT group and end with the standard model. The simplest schemes which retain manifest left-right symmetry down to the SU(3)×SU(2)×U(1) subgroup do not sufficiently enhance $\sin^2\theta_W$ as it runs down in energy to reproduce the measured Z scale values. However, if left-right symmetry is broken either spontaneously or softly at the first stage, then we can find viable breaking patterns that lead to phenomenologically interesting results. One such possibility is

$$\begin{gathered}
SU(4)_C\times SU(4)_L\times SU(4)_R\\
\downarrow\Lambda_L=M_{\rm GUT}\\
SU(4)_C\times SU(2)_L\times SU(2)'_L\times U(1)_L\times SU(4)_R\\
\downarrow\Lambda_R\\
SU(4)_C\times SU(2)_L\times SU(2)'_L\times U(1)_L\times SU(2)_R\times SU(2)'_R\times U(1)_R\\
\downarrow\Lambda_C\\
SU(3)_C\times U(1)_C\times SU(2)_L\times SU(2)'_L\times U(1)_L\times SU(2)_R\times SU(2)'_R\times U(1)_R\\
\downarrow\Lambda_{LR}\\
SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)'_{LR}\times U(1)_C\\
\downarrow u_R\\
SU(3)_{L+R}\times SU(2)_L\times U(1)_Y\\
\downarrow v_L\\
SU(3)_{L+R}\times U(1)_{\rm EM} .
\end{gathered} \tag{4.9}$$

The renormalization-group analysis of coupling constant running in this pattern is similar to that described in the preceding section for the SO(10)×SO(10) model. The only qualitatively new feature that we include in the $SU(4)^3$ analysis is scalar contributions to β functions. These come from the Higgs sector of the theory which we will discuss in detail shortly. The results of the renormalization-group analysis yield a range of values for the symmetry-breaking scales in (4.9) that reproduce the standard model parameters in (1.1) and satisfy all other phenomenological constraints. For simplicity, we merge the intermediate Λ_R and Λ_C thresholds together and quote a set of representative values for these scales:

$$\begin{aligned}
\Lambda_L&=M_{\rm GUT}=6.47\times10^{11}\ \text{GeV} ,\\
\Lambda_R&=\Lambda_C=2.07\times10^{7}\ \text{GeV} ,\\
\Lambda_{LR}&=1.0\times10^{5}\ \text{GeV} ,\\
u_R&=5.0\times10^{3}\ \text{GeV} ,\\
v_L&=2.46\times10^{2}\ \text{GeV} .
\end{aligned} \tag{4.10}$$

The evolution of $\sin^2\theta_W$ for this choice of scales is illustrated in Fig. 1.

We now consider the minimal Higgs content of the $SU(4)^3$ model needed to perform the several stages of symmetry breaking in (4.9) and to provide fermion masses. The first three steps result from vacuum expectation values of the adjoint fields in

$$\Phi=\Phi_C(15,1,1)+\Phi_L(1,15,1)+\Phi_R(1,1,15)\ . \tag{4.11}$$

These scalars' VEV's,

$$\langle\Phi_{L,R}\rangle=\Lambda_{L,R}\begin{bmatrix}1&&&\\&1&&\\&&-1&\\&&&-1\end{bmatrix},$$
$$\langle\Phi_C\rangle=\Lambda_C\begin{bmatrix}1&&&\\&1&&\\&&1&\\&&&-3\end{bmatrix}, \tag{4.12}$$

break the separate SU(4) factors in G as

$$SU(4)_{L,R}\xrightarrow{\langle\Phi_{L,R}\rangle}SU(2)_{L,R}\times SU(2)'_{L,R}\times U(1)_{L,R}\ , \tag{4.13a}$$

$$SU(4)_C\xrightarrow{\langle\Phi_C\rangle}SU(3)_C\times U(1)_C\ . \tag{4.13b}$$

The $SU(2)'_L$ and $SU(2)'_R$ subgroups under which the seesaw fermions transform are subsequently reduced at the Λ_{LR} scale to the diagonal $U(1)'_{LR}$ generated by $S'_{LR}=T_L^{3'}+T_R^{3'}$. We introduce two sets of scalars,

$$\phi^I=\phi^I_{CL}(4,\bar{4},1)+\phi^I_{LR}(1,4,\bar{4})+\phi^I_{RC}(\bar{4},1,4)\ , \tag{4.14}$$

labeled by the flavor index $I=u,d$ to accomplish this

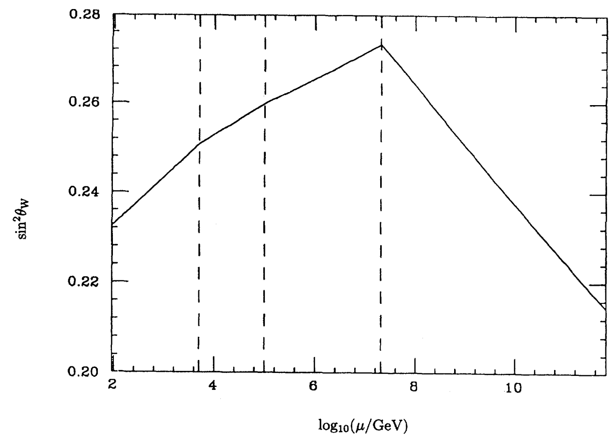

FIG. 1. Evolution of $\sin^2\theta_W(\mu)$ over the range $M_Z\le\mu\le M_{\rm GUT}$ in the $SU(4)^3$ model. Dashed lines mark the locations of the intermediate v_R, Λ_{LR}, and $\Lambda_C=\Lambda_R$ scales.

breaking. The ϕ^u_{LR} and ϕ^d_{LR} fields are presumed to acquire the distinct vacuum expectation values

$$\langle\phi^u_{LR}\rangle=\begin{bmatrix}0&0&0&0\\0&0&0&v_L\\0&0&0&0\\0&v_R&0&\Lambda_{LR}\end{bmatrix},$$
$$\langle\phi^d_{LR}\rangle=\begin{bmatrix}0&0&v_L&0\\0&0&0&0\\v_R&0&\Lambda_{LR}&0\\0&0&0&0\end{bmatrix}. \tag{4.15}$$

Heavy Dirac masses for the U, N, D, and E fermions are then generated via the Yukawa interaction

$$\mathcal{L}_{\rm Yukawa}(\phi^I)=f^I{\rm Tr}[(\Psi^T_{RC})(\bar{4},1,4)C\Psi_{CL}(4,\bar{4},1)\phi^I_{LR}(1,4,\bar{4})+(\Psi^T_{CL})(4,\bar{4},1)C\Psi_{LR}(1,4,\bar{4})\phi^I_{RC}(\bar{4},1,4)$$
$$+(\Psi^T_{LR})(1,4,\bar{4})C\Psi_{RC}(\bar{4},1,4)\phi^I_{CL}(4,\bar{4},1)]+{\rm H.c.} \tag{4.16a}$$

We also give $O(\Lambda_{LR})$ masses to the new leptons in (4.8b) through a second Yukawa term

$$\mathcal{L}_{\rm Yukawa}(\chi)=\frac{g}{2}{\rm Tr}[(\Psi^T_{CL})(4,\bar{4},1)C\Psi_{CL}(4,\bar{4},1)\chi_{CL}(6,6,1)+(\Psi^T_{LR})(1,4,\bar{4})C\Psi_{LR}(1,4,\bar{4})\chi_{LR}(1,6,6)$$
$$+(\Psi^T_{RC})(\bar{4},1,4)C\Psi_{RC}(\bar{4},1,4)\chi_{RC}(6,1,6)]+{\rm H.c.}\ , \tag{4.16b}$$

which antisymmetrically couples fermions to the additional Higgs field

$$X=\chi_{CL}(6,6,1)+\chi_{LR}(1,6,6)+\chi_{RC}(6,1,6)\ . \tag{4.17}$$

The only components of X that may develop nonvanishing vacuum expectation values which do not break color and electromagnetism but do violate $U(1)_{L,R}$ are $(\chi_{LR})^{[12]}_{[12]}$, $(\chi_{LR})^{[12]}_{[34]}$, $(\chi_{LR})^{[34]}_{[12]}$ and $(\chi_{LR})^{[34]}_{[34]}$. We choose these VEV's to all equal Λ_{LR}.

The last two symmetry-breaking steps in (4.9) result from the v_R and v_L entries in (4.15) and

$$\langle \phi^I_{CL} \rangle = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & v_L & 0 & 0 \end{bmatrix} , \qquad \langle \phi^I_{RC} \rangle = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & v_R \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} . \tag{4.18}$$

The Yukawa Lagrangian induces mixing between the heavy seesaw fermions and their light standard model partners. The final forms of the quark and charged lepton mass matrices appear as

$$M_{uU} = \begin{array}{c|cc} & u_L & U_L \\ \hline u^c_L & 0 & f^u v_R \\ U^c_L & f^u v_L & f^u \Lambda_{LR} \end{array} , \qquad M_{dD} = \begin{array}{c|cc} & d_L & D_L \\ \hline d^c_L & 0 & f^d v_R \\ D^c_L & f^d v_L & f^d \Lambda_{LR} \end{array} , \tag{4.19a}$$

and

$$M_{\substack{\text{charged}\\ \text{leptons}}} = \begin{array}{c|cccccc} & e_L & E_L & I^-_L & J^-_L & K^-_L & L^-_L \\ \hline e^c_L & 0 & f^d v_R & (f^u+f^d)v_L & 0 & 0 & 0 \\ E^c_L & f^d v_L & f^d \Lambda_{LR} & 0 & (f^u+f^d)v_L & 0 & 0 \\ I^+_L & (f^u+f^d)v_R & 0 & -g\Lambda_{LR} & 0 & 0 & 0 \\ J^+_L & 0 & 0 & 0 & -g\Lambda_{LR} & 0 & 0 \\ K^+_L & 0 & (f^u+f^d)v_R & 0 & 0 & -g\Lambda_{LR} & 0 \\ L^+_L & 0 & 0 & 0 & 0 & 0 & -g\Lambda_{LR} \end{array} . \tag{4.19b}$$

We refrain from explicitly writing down the neutral lepton matrix since it is larger and more complicated than those exhibited above.

We should recall that the f^u, f^d, and g Yukawa couplings are $N_{\mathcal{F}} \times N_{\mathcal{F}}$ matrices in fermion family space. As we saw before in the $SU(5) \times SU(5)$ theory, it is useful to invoke a parity symmetry P to constrain the forms of these Yukawa matrices. We therefore follow Ref. [23] and promote the discrete Z_3 symmetry in our $SU(4)^3$ model to S_3 through the addition of a parity operation and its two cyclic partners. P performs a conventional spatial inversion and swaps the $SU(4)_L$ and $SU(4)_R$ factors in the gauge group. Its action upon the $SU(4)_C \times SU(4)_L \times SU(4)_R$ gauge fields

$$C^\mu(\mathbf{x},t) \to C_\mu(-\mathbf{x},t) , \quad L^\mu(\mathbf{x},t) \to R_\mu(-\mathbf{x},t) , \quad R^\mu(\mathbf{x},t) \to L_\mu(-\mathbf{x},t) \tag{4.20a}$$

forbids a CP-violating topological term from appearing in the gauge part of the Lagrangian. In the fermion sector, parity maps left handed fields into their right-handed analogues which we express as left-handed conjugates:

$$\begin{aligned} \Psi_{LR}(\mathbf{x},t) &\to -C(\Psi^c_{LR})^*(-\mathbf{x},t) , \\ \Psi_{CL}(\mathbf{x},t) &\to -C(\Psi^c_{CL})^*(-\mathbf{x},t) = -C(\Psi_{RC})^\dagger(-\mathbf{x},t) , \\ \Psi_{RC}(\mathbf{x},t) &\to -C(\Psi^c_{RC})^*(-\mathbf{x},t) = -C(\Psi_{CL})^\dagger(-\mathbf{x},t) . \end{aligned} \tag{4.20b}$$

Finally, the scalars transform under P as

$$\begin{aligned} &\Phi_C(\mathbf{x},t) \to \Phi^\dagger_C(-\mathbf{x},t), \quad \chi_{LR}(\mathbf{x},t) \to \chi^\dagger_{LR}(-\mathbf{x},t), \quad \phi^I_{LR}(\mathbf{x},t) \to (\phi^I_{LR})^\dagger(-\mathbf{x},t) , \\ &\Phi_L(\mathbf{x},t) \to \Phi^\dagger_R(-\mathbf{x},t), \quad \chi_{CL}(\mathbf{x},t) \to \chi^\dagger_{RC}(-\mathbf{x},t) , \quad \phi^I_{CL}(\mathbf{x},t) \to (\phi^I_{RC})^\dagger(-\mathbf{x},t) , \\ &\Phi_R(\mathbf{x},t) \to \Phi^\dagger_L(-\mathbf{x},t) , \quad \chi_{RC}(\mathbf{x},t) \to \chi^\dagger_{CL}(-\mathbf{x},t), \quad \phi^I_{RC}(\mathbf{x},t) \to (\phi^I_{CL})^\dagger(-\mathbf{x},t) . \end{aligned} \tag{4.20c}$$

It is straightforward to check that the Yukawa interactions in (4.16a) and (4.16b) remain invariant under parity only if the f^I and g coupling matrices are Hermitian. The fermion mass matrices can thus be complex, but their determinants are real. So $\bar{\theta} = \theta_{\text{QCD}} + \theta_{\text{QFD}}$ vanishes at tree level, and the $SU(4)^3$ model provides a possible solution to the strong CP problem.

We next diagonalize the fermion mass matrices in (4.19a) and (4.19b) neglecting small intergenerational mixing between families. The masses of standard model quarks and charged leptons fix the diagonal elements in f^u, f^d, and g:

$$f^d \simeq \frac{\Lambda_{LR}}{v_L v_R}\begin{bmatrix} m_d=0.009 & & \\ & m_s=0.181 & \\ & & m_b=4.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0007 & & \\ & 0.0147 & \\ & & 0.3654 \end{bmatrix},$$

$$f^u \simeq \frac{\Lambda_{LR}}{v_L v_R}\begin{bmatrix} m_u=0.005 & & \\ & m_c=1.5 & \\ & & m_t=130 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0004 & & \\ & 0.1218 & \\ & & 10.556 \end{bmatrix}, \tag{4.21}$$

$$g \simeq \frac{\Lambda_{LR}}{v_L v_R}\begin{bmatrix} \frac{(m_u+m_d)^2}{m_e+m_d} & & \\ & \frac{(m_c+m_s)^2}{m_\mu+m_s} & \\ & & \frac{(m_t+m_b)^2}{m_\tau+m_b} \end{bmatrix}$$

$$= \begin{bmatrix} 0.0017 & & \\ & 0.7998 & \\ & & 233.9 \end{bmatrix}.$$

We have numerically evaluated these matrices using the indicated GeV quark masses and the scale values in (4.10). The resulting Yukawa couplings for the first and second families are reasonable in size. Unfortunately, the results for the third family are corrupted by the huge top quark mass. The large value for m_t can of course be offset by adjusting the inverted seesaw prefactor in (4.21). But then we are left with very small Yukawa couplings for the lightest quarks and leptons as in the standard model. So to avoid a nonperturbative top quark Yukawa coupling, we must either diminish the hierarchy between the v_L, v_R, and Λ_{LR} scales or else introduce additional scalar fields to provide greater flexibility in the scalar sector. Neither of these options is attractive. Further study is clearly needed to determine whether a more elegant explanation for the fermion mass hierarchy can be developed in this model.[3]

Finally, we investigate proton decay in the $SU(4)^3$ theory. Recall that left-handed standard model fermions and antifermions appear in separate multiplets in (4.8). Therefore, gauge boson exchange cannot mediate fermion number violating transitions such as $P \to \pi^0 e^+$. Proton decay only proceeds through χ scalar exchange graphs like the one illustrated in Fig. 2. We expect the mass of the $\chi^{4/3}$ scalar shown in the figure to be on the order of

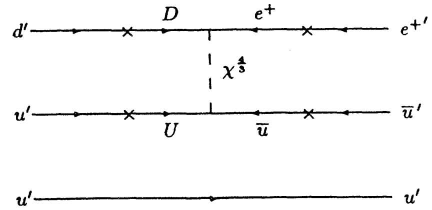

FIG. 2. Dominant contribution to proton decay from χ scalar exchange in the $SU(4)^3$ model. Primed and unprimed fields denote mass and gauge eigenstates, respectively.

the unification scale $M_{\rm GUT}=6.47\times10^{11}$ GeV. This mass seems much too light to yield a proton lifetime consistent with the experimental lower limit [10]

$$\tau_P > 5\times10^{32}\ \text{yr}. \tag{4.22}$$

However, the diagram in Fig. 2 is further suppressed by $O(v_R/\Lambda_{LR})^2$ as a result of seesaw mixing between fermion gauge and mass eigenstates. Such seesaw suppression of proton decay is generic in all unified seesaw models. Naive dimensional analysis yields the proton lifetime estimate.

$$\tau_P \simeq \frac{16\pi}{(g_{11})^4}\left[\frac{\Lambda_{LR}}{v_R}\right]^4 \frac{M_\chi^4}{m_P^5}, \tag{4.23}$$

where g_{11} is the Yukawa coupling for the first family in (4.16b) while 16π represents a two-body phase-space factor. Inserting numerical values, we find $\tau_P \simeq 4.6\times10^{33}$ yr which is consistent with the bound in (4.22).

V. CONCLUSION

The $SU(5)\times SU(5)$, $SO(10)\times SO(10)$, and $SU(4)\times SU(4)\times SU(4)$ models that we have investigated in this paper illustrate the basic features of all unified universal seesaw theories. They generalize several well-known GUT models that have been studied in the past. In addition, they provide a more natural basis for previously proposed un-unified seesaw models and offer a possible resolution to the strong *CP* problem.

Many possible extensions of this work would be interesting to pursue. Gauge boson mixing, neutrino masses, and loop contributions to $\bar{\theta}$ should all be further analyzed in these models. Moreover, a number of alternatives to the symmetry-breaking patterns that we have considered here and which may well be phenomenologically viable remain to be examined in the $SO(10)\times SO(10)$ and $SU(4)^3$ theories. Generalizations to $E_6\times E_6$ and $SU(5)^3$ that maintain left-right symmetry down to the standard model subgroup could also be constructed. In short, unified universal seesaw models represent a new class of grand unified theories in which there is much room for further exploration.

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[3]Previous attempts to understand quark and lepton mass hierarchies in the context of un-unified universal seesaw models have been somewhat more successful. See Refs. [24–26].

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