CP violation in the bosonic sector of the SM with two Higgs doublets

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(Received 14 May 1993)

We investigate CP-violation effects in the bosonic sector of the standard model (SM) with two Higgs doublets. First we calculate the mass eigenstates of the physical neutral Higgs bosons for the small but nonzero CP-violation parameter ξ_*, and then a "forward-backward" asymmetry $\mathcal{A}_{\rm FB}$ for the decay $H \to W^+W^-Z$ that would be a signal of CP violation. Although the effects are in general small ($\mathcal{A}_{\rm FB} = \Gamma_{\rm FB}/\Gamma \sim 10^{-3}$), $\mathcal{A}_{\rm FB}$ turns out to be a rather clean signal of CP violation, since neither the CP-conserving final state interactions nor the direct production background events contribute to $\Gamma_{\rm FB}$. The CKM-type CP-violation effects that could in principle also contribute to $\mathcal{A}_{\rm FB}$ are negligible. The nonzero $\mathcal{A}_{\rm FB}$, or related quantities, could possibly be detected at some later stage at the CERN LHC or SSC.

PACS number(s): 11.30.Er, 12.15.Cc, 14.80.Gt

The standard model (SM) with two Higgs doublets and CP violation in the bosonic sector has recently drawn a lot of attention [1], especially because it could give an explanation of baryogenesis [2, 3], unlike the CP violation originating from the Cabibbo-Kobayashi-Maskawa (CKM) matrix of the SM.

The most general gauge invariant potential for two Higgs doublets Φ_i ($i = 1,2$) with $Y = 1$ which induces only suppressed flavor-changing neutral currents (FCNC's) is [4]

$$\begin{aligned}V(\Phi_1,\Phi_2) = {}& \lambda_1(\Phi_1^\dagger\Phi_1 - v_1^2)^2 + \lambda_2(\Phi_2^\dagger\Phi_2 - v_2^2)^2 \\ & +\lambda_3[(\Phi_1^\dagger\Phi_1 - v_1^2) + (\Phi_2^\dagger\Phi_2 - v_2^2)]^2 + \lambda_4[(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) - (\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)] \\ & +\lambda_5[\mathrm{Re}(\Phi_1^\dagger\Phi_2) - v_1v_2\cos\xi]^2 + \lambda_6[\mathrm{Im}(\Phi_1^\dagger\Phi_2) - v_1v_2\sin\xi]^2 \; . \end{aligned} \tag{1}$$

The potential spontaneously breaks $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ down to $\mathrm{U}(1)_{\rm EM}$. The fact that the discrete symmetry $\Phi_1 \mapsto -\Phi_1$ is only softly violated (by terms of dimension two) guarantees that the FCNC's are not too large. The six parameters λ_i are in general of the order $(M_{\rm scalar}/v)^2 \sim 1$. The minimum of the potential is at

$$\langle\Phi_1\rangle_0 = \begin{pmatrix}0\\ v_1\end{pmatrix}, \qquad \langle\Phi_2\rangle_0 = e^{i\xi}\begin{pmatrix}0\\ v_2\end{pmatrix},$$
$$\left(v_1^2+v_2^2 = \frac{1}{2}v^2 = \frac{1}{2}246^2 \text{ GeV}^2\right) . \tag{2}$$

For ξ_* $[= (\lambda_5-\lambda_6)\xi] = 0$, we have no CP violation, and the neutral physical scalars H_+^0,h_+^0,A_-^0 have well-defined CP (equal to $+1, +1, -1$, respectively). These scalars and their masses are known (we use the notations of [4]):

$$\begin{pmatrix}H_+^0\\ h_+^0\end{pmatrix} = \sqrt{2}\begin{pmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}\mathrm{Re}\Phi_1^0 - v_1\\ \mathrm{Re}\Phi_2^0 - v_2\end{pmatrix},$$
$$A_-^0 = \sqrt{2}(-\mathrm{Im}\Phi_1^0\sin\beta + \mathrm{Im}\Phi_2^0\cos\beta) \; , \tag{3}$$

where

$$\beta = \arctan\left(\frac{v_2}{v_1}\right), \qquad \alpha = \frac{1}{2}\arctan\frac{2\mathcal{M}_{12}}{\mathcal{M}_{11}-\mathcal{M}_{22}} \; ,$$
$$\mathrm{sgn}(\sin 2\alpha) = \mathrm{sgn}(\mathcal{M}_{12}) \; ,$$
$$\mathcal{M} = \begin{pmatrix}4v_1^2(\lambda_1+\lambda_3)+v_2^2\lambda_5 & (4\lambda_3+\lambda_5)v_1v_2\\ (4\lambda_3+\lambda_5)v_1v_2 & 4v_2^2(\lambda_2+\lambda_3)+v_1^2\lambda_5\end{pmatrix},$$
$$\begin{pmatrix}M_{H^0}^2\\ M_{h^0}^2\end{pmatrix} = \frac{1}{2}[\mathcal{M}_{11}+\mathcal{M}_{22} \pm \sqrt{(\mathcal{M}_{11}-\mathcal{M}_{22})^2+4\mathcal{M}_{12}^2}] \; ,$$
$$M_{A^0} = \frac{1}{2}\lambda_6 v^2 \; . \tag{4}$$

On the other hand, in the case of $\xi_* = (\lambda_5-\lambda_6)\xi \neq 0$ we do have CP violation. It is possible to find the physical scalar mass eigenstates in this case, if we make the expansion in powers of ξ in the potential (1). Denoting the mass eigenstates (H_+^0, h_+^0, A_-^0) of the $\xi_* = 0$ case as $v_{(1)}(0)$, $v_{(2)}(0)$, $v_{(3)}(0)$, respectively, we obtain the three mass eigenstates $v_{(j)}(\xi_*)$ after some lengthy algebra:

$$v_{(j)}(\xi_*) = U_{jk}(\xi_*)v_{(k)}(0) \; , \tag{5}$$

where $U(\xi_*)$ is a 3×3 orthogonal matrix,

$$U_{11}(\xi_*) = 1 - \frac{1}{2}\xi_*^2 \frac{\sin^2\delta}{(x_3-x_1)^2} + O((\lambda_5-\lambda_6)^2\xi^4) \ ,$$
$$U_{22}(\xi_*) = 1 - \frac{1}{2}\xi_*^2 \frac{\cos^2\delta}{(x_3-x_2)^2} + O((\lambda_5-\lambda_6)^2\xi^4) \ ,$$
$$U_{33}(\xi_*) = 1 - \frac{1}{2}\xi_*^2 \left[\frac{\sin^2\delta}{(x_3-x_1)^2} + \frac{\cos^2\delta}{(x_3-x_2)^2}\right] + O((\lambda_5-\lambda_6)^2\xi^4) \ ,$$
$$U_{12}(\xi_*) = \xi^2(\lambda_5-\lambda_6)\frac{(\lambda_5-x_1)\sin(2\delta)}{2(x_1-x_2)(x_1-x_3)} + O((\lambda_5-\lambda_6)\xi^3) \ ,$$
$$U_{21}(\xi_*) = \xi^2(\lambda_5-\lambda_6)\frac{(\lambda_5-x_2)\sin(2\delta)}{2(x_2-x_1)(x_2-x_3)} + O((\lambda_5-\lambda_6)\xi^3) \ ,$$
$$U_{31}(\xi_*) = \xi_*\frac{\sin\delta}{x_1-x_3} + O((\lambda_5-\lambda_6)\xi^3) = -U_{13}(\xi_*) \ ,$$
$$U_{32}(\xi_*) = \xi_*\frac{\cos\delta}{x_2-x_3} + O((\lambda_5-\lambda_6)\xi^3) = -U_{23}(\xi_*) \ , \tag{6}$$

where

$$\xi_* = (\lambda_5-\lambda_6)\xi \ , \qquad \delta = \beta+\alpha \ , \qquad x_j = 2M_j^2(0)/v^2 \quad (j=1,2,3) \ . \tag{7}$$

Here, $M_j(0)$ $(j=1,2,3)$ are masses of $v_{(j)}(0)$ [of Eq. (4)]. The masses of $M_j(\xi_*)$ of $v_{(j)}(\xi_*)$ differ from those of the $\xi_*=0$ case only slightly (for $\xi \ll 1$):

$$M_j^2(\xi_*) = M_j^2(0) + \xi^2(\lambda_5-\lambda_6)\frac{v^2}{2}Y_j + O((\lambda_5-\lambda_6)\xi^4) \ , \tag{8}$$

where

$$Y_1 = \frac{x_1-\lambda_5}{\lambda_6-x_1}\sin^2\delta \ , \qquad Y_2 = \frac{x_2-\lambda_5}{\lambda_6-x_2}\cos^2\delta \ ,$$
$$Y_3 = 1-(\lambda_5-\lambda_6)\left[\frac{\cos^2\delta}{x_2-\lambda_6} + \frac{\sin^2\delta}{x_1-\lambda_6}\right] \ . \tag{9}$$

The quantities δ and x_j are dimensionless, in general of order 1. Note that the charged scalar sector remains unaffected by the introduction of $\xi_* \neq 0$.

Equation (5) tells us that the mass eigenstates of the neutral physical scalars are in general linear combinations of $CP=+1$ and $CP=-1$ components. This feature of CP violation could be tested experimentally by looking at the decays of the (heavy) Higgs boson H^0 [$= v_{(1)}(\xi_*)$] to W^+W^-Z, as proposed within a more general context in Ref. [5]. As argued there, in the unitary gauge, the decay amplitude T_+ mediated by $W^{*\pm}$ and Z^* exchange [Figs. 1(a,b)] would yield final states with $CP=+1$ (just like in the minimal SM), while the decay amplitude T_- mediated by (neutral) physical scalars (Fig. 2) would yield final states with[1] $CP=-1$. On the other hand, the two-body decays $H^0\to W^+W^-, ZZ$ at the tree level would not test the mixed CP structure of H^0, because the final state is an S wave due to the coupling without derivatives [$L_{\rm fin}=0 \Rightarrow S_{\rm fin}=0 \Rightarrow CP(W^+W^-) = (-1)^{S_{\rm fin}} = +1 = CP(ZZ)$].

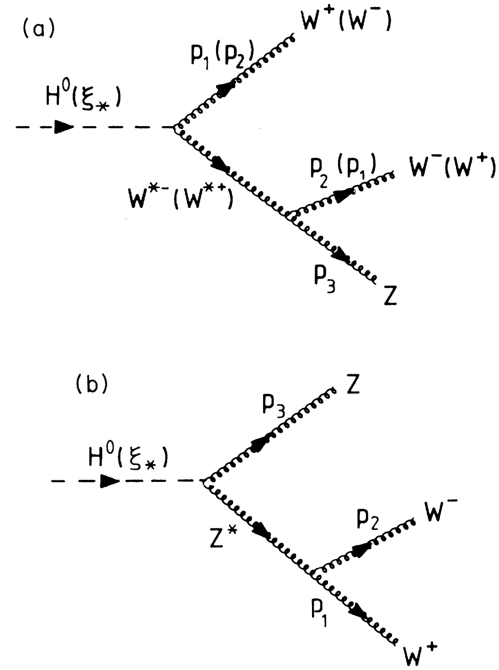

FIG. 1. Decay diagrams yielding final states with $CP=+1$.

For the decay $H^0\to W^+W^-Z$, we can construct the following experimentally relevant "forward-backward" asymmetry width parameter $\Gamma_{\rm FB}$ which would be a signal of this CP violation:

$$\Gamma_{\rm FB}(H^0\to W^+W^-Z) = \left[\int_0^{+1} d(\cos\theta)\frac{d\Gamma(H^0\to W^+W^-Z)}{d(\cos\theta)} - \int_{-1}^{0} d(\cos\theta)\frac{d\Gamma(H^0\to W^+W^-Z)}{d(\cos\theta)}\right] \ , \tag{10}$$

where θ is the angle between $\mathbf{p}_{W^+}$ and $-\mathbf{p}_Z$ in the c.m. system of (W^+W^-) (Fig. 3), and the sum over the helicities of the final particles is implicitly assumed.[2]

[1] The latter amplitude is zero if $\xi_*=0$, as expected.

[2] Note that $\Gamma_{\rm FB}$ is in principle obtained by measuring the corresponding "forward-backward" difference $N_{\rm FB}$ of the number of these decays: $\Gamma_{\rm FB} = N_{\rm FB}\frac{\Gamma(H^0)}{L\sigma}$, where $\Gamma(H^0)$ is the total decay width, L is the integrated luminosity, and σ is the production cross section for the (heavy) Higgs boson.

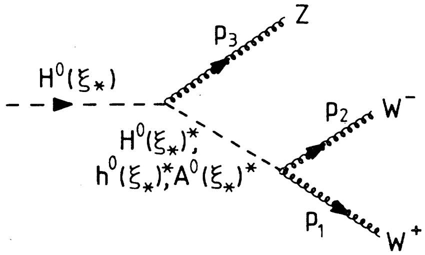

FIG. 2. Decay diagram yielding final states with $CP=-1$.

Note that $\theta \mapsto \pi-\theta$ under the CP transformation of the final state W^+W^-Z, and hence $\Gamma_{\rm FB} \mapsto -\Gamma_{\rm FB}$ if H^0 were a pure $CP=+1$ or $CP=-1$ state. Therefore, in the case of CP conservation we must have $\Gamma_{\rm FB}=0$. Hence, $\Gamma_{\rm FB}\neq 0$ is a signal of CP violation. We can show in general, using the formalism of partial wave expansions of decay amplitudes [6], that $\Gamma_{\rm FB}$ is an expression proportional to the interference terms of the $CP=\pm 1$ decay amplitudes

$$\Gamma_{\rm FB}(H^0 \to W^+W^-Z) \propto \int d(\text{configuration space})(T_-^*T_+ + T_-T_+^*) \times \text{sgn}(\cos\theta) \ . \tag{11}$$

In the specific model at hand, we can check this by explicit calculation. The tree-level transition amplitudes[3] $T_\pm$ in this case turn out to be

$$T_\pm = \epsilon^{\mu_1}(p_1h_1)\epsilon^{\mu_2}(p_2h_2)\epsilon^{\mu_3}(p_3h_3)\mathcal{F}^{(\pm)}_{\mu_1\mu_2\mu_3}(p_1,p_2,p_3) \ , \tag{12}$$

where p_j and h_j ($j=1,2,3$) denote momenta and helicities of W^+, W^-, and Z, respectively, and

$$\mathcal{F}^{(+)} = \mathcal{F}^{(+,1)} + \mathcal{F}^{(+,2)} + \mathcal{F}^{(+,3)} \ ,$$

$$\mathcal{F}^{(+,1)}_{\mu_1\mu_2\mu_3}(p_1p_2p_3) = \frac{g^2\cos^2\theta_W M_Z(U_{11}\cos\eta + U_{12}\sin\eta)}{(p_2+p_3)^2 - M_W^2 + i\Gamma_W M_W} \times \left[2(p_{2\mu_3}g_{\mu_1\mu_2} - p_{3\mu_2}g_{\mu_1\mu_3}) - (p_2-p_3)_{\mu_1}g_{\mu_2\mu_3} - \frac{\sin^2\theta_W}{\cos^2\theta_W}(p_1+p_2+p_3)_{\mu_1}g_{\mu_2\mu_3}\right] \ ,$$

$$\mathcal{F}^{(+,2)}_{\mu_1\mu_2\mu_3}(p_1p_2p_3) = -\mathcal{F}^{(+,1)}_{\mu_2\mu_1\mu_3}(p_2p_1p_3) \ ,$$

$$\mathcal{F}^{(+,3)}_{\mu_1\mu_2\mu_3}(p_1p_2p_3) = \frac{g^2 M_Z(U_{11}\cos\eta + U_{12}\sin\eta)}{(p_1+p_2)^2 - M_Z^2 + i\Gamma_Z M_Z}\left[2(p_{1\mu_2}g_{\mu_1\mu_3} - p_{2\mu_1}g_{\mu_2\mu_3}) - (p_1-p_2)_{\mu_3}g_{\mu_1\mu_2}\right] \ , \tag{13}$$

$$\mathcal{F}^{(-)}_{\mu_1\mu_2\mu_3}(p_1p_2p_3) = \frac{ig^2M_W}{\cos\theta_W}g_{\mu_1\mu_2}(p_1+p_2+p_3)_{\mu_3}\mathcal{A}(p_1\cdot p_2) \ , \tag{14}$$

$$\mathcal{A}(p_1\cdot p_2) = \sum_{j=1}^{3} A_j[(p_1+p_2)^2 - M_j^2 + i\Gamma_j M_j]^{-1} \ ,$$

$$A_j = U_{13}[\cos\eta\sin\eta(U_{j1}^2 - U_{j2}^2) - \cos(2\eta)U_{j1}U_{j2}] + U_{12}[\cos^2\eta U_{j1}U_{j2} + \cos\eta\sin\eta U_{j2}U_{j3}] - U_{11}[\cos\eta\sin\eta U_{j1}U_{j3} + \sin^2\eta U_{j2}U_{j3}] \ . \tag{15}$$

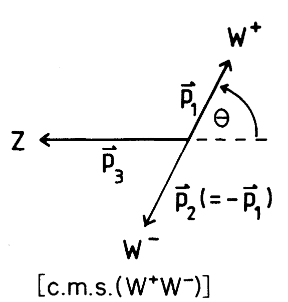

FIG. 3. Angle θ between $\mathbf{p}_{W^+}$ and $(-\mathbf{p}_Z)$ in the center-of-mass system of W^+W^-.

M_j are the masses of the three physical scalars [$M_j = M_j(\xi_*) \simeq M_j(0)$], Γ_j are the corresponding widths, and $\eta=(\beta-\alpha)$. In this particular case, we explicitly see that $|\,T_+\,|^2$ and $|\,T_-\,|^2$ are symmetric under $p_1 \leftrightarrow p_2$,

[3]Strictly speaking these are not tree-level amplitudes, because the dominant final-state interactions in the propagator are taken into account by nonzero widths of the mediating bosons. The amplitudes can be calculated in any R_ζ gauge, do not depend on the logitudinal parts of the propagators, and have no ζ dependence.

while $(T_+^* T_- + T_+ T_-^*)$ is antisymmetric (summation over the final helicities h_j is always assumed). Therefore, $| T_+ |^2$ and $| T_- |^2$ contribute to Γ and not to Γ_{FB}, while $(T_+^* T_- + T_+ T_-^*)$ contributes to Γ_{FB} and not to Γ (for the decay $H^0 \to W^+W^-Z$). Hence, we see explicitly that relation (11) holds in the specific case discussed.

In the further calculation, we will assume that M_1 $(= M_{H^0}) > (2M_W + M_Z)$ and $(M_2,\ M_3) < 2M_W$, and that Γ_2 and Γ_3 are consequently negligible ($\Gamma_2, \Gamma_3 \ll \Gamma_Z \simeq 2.5$ GeV). The asymmetry signal Γ_{FB} would then be proportional to Γ_W and Γ_Z. Furthermore, we will assume $\xi < 1$ and $\xi_* = (\lambda_5 - \lambda_6)\xi < 1$, in order to use the expressions (6). Then it follows

$$\mathcal{A}(p_1 \cdot p_2) \simeq \frac{(A_{l=2})(M_2^2 - M_3^2)}{[(p_1+p_2)^2 - M_2^2][(p_1+p_2)^2 - M_3^2]} \times [1 + O(\xi\xi_*)] \ ,$$

$$\Gamma_{FB}(H^0 \to W^+W^-Z) \simeq \Delta(\cos\delta \cos\eta \sin^2\eta)\xi_* \left[1 + O(\xi\xi_*)\right] \ . \quad (16)$$

The width Δ in the above formula[4] is

$$\Delta = \frac{M_W^2 v^2 g^4}{M_{H^0}^3 2^8 \pi^3 \cos^2\theta_W} I\left(\left\{\frac{M_j}{M_W}\right\}\right) \simeq \frac{2700\ \text{GeV}^4}{M_{H^0}^3} I \ , \quad (17)$$

where $I(\{M_j/M_W\})$ is a specific "forward-backward" asymmetry integral on the corresponding Dalitz plot, containing suppression factors Γ_W/M_W and Γ_Z/M_Z. Numerical calculations yield, for M_1 $(= M_{H^0}) = 400 -$ 800 GeV and $M_2, M_3 = 0 - 100$ GeV, $0.23 \lesssim I \lesssim 90$. The corresponding values for Δ are given in Table I. If assuming $0.2 < \xi_*(< 1)$ and $\cos\delta \cos\eta \sin^2\eta \gtrsim 0.5$, (16) gives

$$\Gamma_{FB}(H^0 \to W^+W^-Z) \gtrsim (0.1)\Delta \ .$$

TABLE I. Δ widths (in keV) and ρ numbers [Eqs. (17),(18)] for various values (in GeV) of the heavy Higgs boson mass M_{H^0} $(= M_1)$. The superscripts (1), (2), (3) denote these values for the cases when the masses (M_2, M_3) of the other two physical scalars (in GeV) are (0, 0), (100, 0) or (0, 100), and (100, 100), respectively.

M_{H^0}	$\Delta^{(1)}$	$\Delta^{(2)}$	$\Delta^{(3)}$	$10^3\rho^{(1)}$	$10^3\rho^{(2)}$	$10^3\rho^{(3)}$
300	0.26	0.38	0.56	0.98	1.44	2.10
400	9.8	13.0	17.2	1.49	2.00	2.65
500	40.5	52.0	67.0	1.16	1.50	1.92
600	101.0	124.0	155.0	0.91	1.12	1.40
700	196.0	236.0	300.0	0.73	0.88	1.10
800	330.0	400.0	492.0	0.61	0.74	0.91

We may also construct the dimensionless asymmetry parameter

$$\mathcal{A}_{FB} = \frac{\Gamma_{FB}(H^0 \to W^+W^-Z)}{\Gamma(H^0 \to W^+W^-Z)} = \frac{N_{FB}}{N} \simeq \rho \frac{\cos\delta \sin^2\eta}{\cos\eta} \xi_* [1 + O(\xi\xi_*)] \ . \quad (18)$$

The dimensionless parameter ρ is small ($\sim 10^{-3}$), due to the suppression factors Γ_W/M_W and Γ_Z/M_Z, and its values are also included in Table I.

Here we have to mention that the relation (18) is not valid in the limiting case of $\cos\eta \to 0$, because in such a case $\Gamma(H^0 \to W^+W^-Z) \simeq 0 + O(\xi_*^2)$. In such a case, $\mathcal{A}_{FB}$ could be large ($\lesssim 1$).

For completeness, we also write the decay width $\Gamma(H^0 \to W^+W^-Z)$ in the theory discussed here

$$\Gamma(H^0 \to W^+W^-Z) = \Gamma_+ + \Gamma_- ,$$

where

$$\Gamma_+ = \Gamma(H^0 \to (W^+W^-Z)_{CP=+1}) = (U_{11}\cos\eta + U_{12}\sin\eta)^2\, \Gamma^{MSM}(H^0 \to W^+W^-Z) \simeq \cos^2\eta\, \Gamma^{MSM}(H^0 \to W^+W^-Z)[1 + O(\xi\xi_*)] \ , \quad (19)$$

$$\Gamma_- = \Gamma(H^0 \to (W^+W^-Z)_{CP=-1}) \simeq (\cos^2\delta \sin^4\eta)\xi_*^2 G \left[1 + O(\xi^2)\right] \ , \quad (20)$$

where the widths G, as well as $\Gamma^{MSM}(H^0 \to W^+W^-Z)$ of the minimal SM, are given in Table II for various values of the scalar masses.[5] Note that Γ_+ is the contribution from diagrams of Figs. 1(a,b) ($\propto | T_+ |^2$), and Γ_- from

[4]Strictly speaking,

$$\Gamma_{FB} \simeq \Delta(\cos\delta\cos\eta\sin^2\eta)\left[1 + \frac{M_2^2 - M_3^2}{M_1^2 - M_3^2}\tan\delta\cot\eta\right]\xi_* + O(\xi\xi_*^2\Delta)$$

$(\delta = \beta + \alpha,\ \eta = \beta - \alpha)$, which reduces to the above form for $M_2^2, M_3^2 \ll M_1^2$, or for $M_2 = M_3$.

[5]Note that the angle $\eta = \beta - \alpha$ may be obtained from experiments measuring the $W^+ - W^-$-Higgs couplings. The angle β may be restricted indirectly by experiments whose results depend on the ratio of the vacuum expectation values, within the considered theory.

TABLE II. G widths [Eq. (20)], and the decay width $\Gamma^{\rm MSM}$ of the minimal SM, for various values of the Higgs boson mass. All values are in GeV. The superscripts of G have the same meaning as those in Table I.

M_{H^0}	$G^{(1)}$	$G^{(2)}$	$G^{(3)}$	$\Gamma^{\rm MSM}$ $\times(H^0 \to WWZ)$
300	2.3×10^{-4}	5.0×10^{-4}	1.12×10^{-3}	1.16×10^{-3}
400	5.0×10^{-3}	9.4×10^{-3}	1.84×10^{-2}	2.88×10^{-2}
500	2.04×10^{-2}	3.6×10^{-2}	6.5×10^{-2}	1.54×10^{-1}
600	5.15×10^{-2}	8.6×10^{-2}	0.152	0.490
700	0.103	0.167	0.287	1.18
800	0.179	0.285	0.480	2.39

the diagram of Fig. 2 ($\propto| T_- |^2$). Here we see that the width $\Gamma(H^0 \to W^+W^-Z)$ is not substantially affected by the small ξ_* parameter. Anyway, this width does not provide an experimental signature for detecting CP violation. $\Gamma^{\rm MSM}$ for this decay have also been calculated by other authors ([7,8], and references therein).

We find that the parameter $\Gamma_{\rm FB}$ [Eq. (10)] and the related $\mathcal{A}_{\rm FB}$ [Eq. (18)] are possibly relevant *in general* for experimental investigations of purely bosonic CP-violation effects. We calculated this quantity within the minimal extension of the SM (two Higgs doublets), and found that $\Gamma_{\rm FB}$ may be appreciable, although in general much smaller than $\Gamma(H^0 \to W^+W^-Z)$. Interestingly enough, the final-state interactions do not represent any problem; i.e., they do not give any spurious (CP-conserving) contributions to $\Gamma_{\rm FB}$. On the other hand, the final-state interactions (dominated by the W and Z width) are in fact crucial, together with $\xi_* \neq 0$, for the nonzero CP-violation signal $\Gamma_{\rm FB} \neq 0$. Furthermore, the CP violation coming from the CKM matrix would not give any contribution to CP-violating effects for the considered decay at the tree or one-loop level, but possibly only at the two-loop level, and can be therefore safely ignored.

The CERN Large Hadron Collider (LHC) and Superconducting Super Collider (SSC) should be able to produce the Higgs boson with mass of several hundred GeV (if such a Higgs exists), mostly through the gluon fusion mechanism [9] and the intermediate boson fusion mechanism [10]. Taking the yearly estimated event rate at LHC for the integrated luminosity to be 10^{41} cm^{-2} (10^{40} cm^{-2} at SSC), we expect roughly 10^3 events $H^0 \to W^+W^-Z$ per year.[6]

Several sources of background would pose a problem for identifying such events—particularly the direct production of W^+W^-Z ($p\bar{p} \to W^+W^-Z$) and the QCD continuum ($p\bar{p} \to Z + 4$ jets). It has been argued [7] that the background effects of the direct production would not be a major problem for measuring $\Gamma(H^0 \to W^+W^-Z)$ at SSC for $M_H \approx 500 - 600$ GeV.

However, $\Gamma_{\rm FB}(H^0 \to W^+W^-Z)$ is such a difference of the widths for which any background effects of the direct production that do not violate CP symmetry are canceled out. To see this, we must recall that in the "forward-backward" difference of events $N_{\rm FB}(H^0 \to W^+W^-Z)$ ($\propto \Gamma_{\rm FB}$) we make the sum (average) over the polarizations of the incoming constituent particles (unpolarized). For the case of direct production we choose the spin basis $| S, S_z\rangle$ for polarizations of the initial $q\bar{q}$ (or gg) states.[7] These initial states have well-defined CP [$CP(q\bar{q}) = (-1)^{S_{qq}+1}$, $CP(gg) = (-1)^{S_{gg}}$], and hence also the resulting directly produced W^+W^-Z states would have the same well-defined CP, provided the direct production processes themselves do not contain appreciable CP-violating vertices. Therefore, such events would contribute zero to $N_{\rm FB}(W^+W^-Z)$ [$\propto \Gamma_{\rm FB}(W^+W^-Z)$]. This argument also holds if the initial $q\bar{q}$ have opposite polarizations ($S = 1$). The CKM-type CP-violation effects in the direct production could in principle contribute to $N_{\rm FB}$, but their effects are very small for $q\bar{q}$ ($q = u, d$) and gg initial states (CP-violating asymmetries $\lesssim 10^{-6}$) [11].

Most of the QCD continuum background may be eliminated with on-mass-shell constraints and certain additional cuts [7]. However, several aspects of this problem remain open, and this background may pose a problem for determining $\Gamma_{\rm FB}$.

It is possible that the CP-violating phase ξ_* also enters the general Yukawa couplings, specifically the coupling between H^0 and the top quark. Within the latter scenario, Chang and Keung [12] have recently investigated CP-violating asymmetries for the decays $H^0 \to W^+W^-, t\bar{t}$, where the source of the asymmetries is the imaginary part ($\propto \xi_*$) of the Yukawa coupling of H^0 to the top quark. They concluded that such asymmetries can be measurable in future colliders such as the SSC or LHC. These asymmetries were

$$\mathcal{A}(H^0 \to W^+W^-) = [N(W_L^+W_L^-) - N(W_R^+W_R^-)]/N(W^+W^-)$$

and

$$\mathcal{A}(H^0 \to t\bar{t}) = [N(t_L\bar{t}_L) - N(t_R\bar{t}_R)]/N(t\bar{t}),$$

[6] In Ref. [7], the numbers N of decays are given for the minimal SM, but can be used also in the present model as order-of-magnitude estimates.

[7] We are allowed to take any convenient polarization basis, since at the end we sum over all initial polarizations.

where L and R stand for helicities -1, $+1$, respectively. They found out that $\mathcal{A}(H^0 \to W^+W^-) \lesssim 10^{-3}$ and $\mathcal{A}(H^0 \to t\bar{t}) \lesssim 10^{-1}$, for $m_t \approx 150$ GeV. Furthermore, the branching ratios for these decays are larger by one order of magnitude than those for $H^0 \to W^+W^-Z$. For the latter decay, $\mathcal{A}_{\rm FB}$ in most cases does not exceed $\sim 10^{-3}$, according to Eq. (18) and Table I. On the other hand, the helicity asymmetries $\mathcal{A}(H^0 \to W^+W^-)$, $\mathcal{A}(H^0 \to t\bar{t})$ cannot be measured directly, but have to be decoded from the asymmetry of the energy distributions of the resulting final leptons. It appears that roughly one order of magnitude is lost due to this decoding, i.e., $\mathcal{A}_{l^+l^-} = (\langle E_{l^-}\rangle - \langle E_{l^+}\rangle)/\langle E_{l^+}\rangle$ are about one order of magnitude smaller than the corresponding $\mathcal{A}(H^0 \to W^+W^-)$, $\mathcal{A}(H^0 \to t\bar{t})$ (the decoding for $H^0 \to ZZ$ is even harder). Hence, comparing the results of Ref. [12] with the results of the present paper, we conclude that the measurability of $\mathcal{A}(H^0 \to W^+W^-)$ (and the problems connected with it) is comparable to the measurability of $\mathcal{A}_{\rm FB}(H^0 \to W^+W^-Z)$, while the measurement of $\mathcal{A}(H^0 \to t\bar{t})$ (for $m_t \approx 150$ GeV) clearly appears to be more promising.

One major problem in measuring $\Gamma_{\rm FB}$ (or $\mathcal{A}_{\rm FB}$) would be a somewhat low production rate of heavy Higgs bosons at the SSC and LHC ($N \sim 10^3$ decays $H^0 \to W^+W^-Z$ per year). Since $\mathcal{A}_{\rm FB}(H^0 \to W^+W^-Z)$ [$= (N_{\rm forward} - N_{\rm backward})/N$], in the framework discussed here, is in most cases not exceeding $\sim 10^{-3}$ [Eq. (18) and Table I], many years of measurements would be needed to obtain possibly statistically significant effects. Nonetheless, we believe that the proposed quantity $\Gamma_{\rm FB}$ (or $\mathcal{A}_{\rm FB}$), or related quantities, may eventually become relevant for experimental tests of CP violation of the purely bosonic sector. Furthermore, the $\Gamma_{\rm FB}$ and $\mathcal{A}_{\rm FB}$ parameters should be investigated numerically also for the case of larger ξ_* parameter of CP violation, and they may be substantially larger in this case.

Note added. An analysis similar to that of Ref. [12], but involving the entire scattering process $e^+e^- \to t\bar{t}\nu\bar{\nu}$, has been recently made by Pilaftsis and Nowakowski [13]. The neutral Higgs boson arises from W^+W^- fusion and decays into $t\bar{t}$. Particular care is taken to ensure the gauge and CPT invariance. The numerical results are similar to those of Ref. [12].

The author would like to thank K.J. Abraham, M. Nowakowski, E.A. Paschos, A. Pilaftsis, and Y.-L. Wu for helpful discussions, particularly concerning the possible CKM-type of CP-violation effects. The author wishes to thank the Deutsche Forschungsgemeinschaft (DFG) and the CEC Science Project No. SC1-CT91-0729 for financial support during the progress of this work.

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