Neutrinoless double-β decay in an $SU(3)_L \otimes U(1)_N$ model

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We consider a model for the electroweak interactions with the $SU(3)_L \otimes U(1)_N$ gauge symmetry. We show that the conservation of the quantum number $F = L + B$ forbids the appearance of massive neutrinos and the neutrinoless double-β decay $(\beta\beta)_{0\nu}$. Explicit or/and spontaneous breaking of F implies that the neutrinos have an arbitrary mass. In addition the $(\beta\beta)_{0\nu}$ decay also has some channels that do not depend explicitly on the neutrino mass.

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I. INTRODUCTION

Recently extensions of the standard electroweak model based on the $SU(3)_L \otimes U(1)_N$ gauge symmetry have been proposed [1–6]. This sort of model allows us to relate the number of families with the number of colors obtaining an anomaly-free model. Another interesting feature of these models is that the weak mixing angle of the standard model has an upper limit. For instance, in the model of Refs. [1, 2] $\sin^2\theta_W$ has to be smaller than 1/4. Therefore, it is possible to compute an upper limit to the mass scale of the SU(3) breaking of about 1.7 TeV [7]. In this work we consider how the neutrinoless double-β decay $(\beta\beta)_{0\nu}$ can occur in an $SU(3)_L \otimes U(1)_N$ model with double charged vector bosons [1, 2].

It is well known that the observation of the neutrinoless double-β decay would imply a new physics beyond the standard model. Usually, two kinds of mechanisms for this decay were independently considered: massive Majorana neutrinos and right-handed currents [8]. In the latter case, the neutrino is not required to have a mass. However, if the right-handed currents are part of a gauge theory, it has been argued that at least some neutrinos must have a nonzero mass [9]. It is also well known that whatever mechanism generates the neutrinoless double-β decay, it also generates a Majorana mass term [10, 11]. In this sense, the fundamental requirement underlying that decay is the existence of massive neutrinos. An important point is to investigate the mechanism associated with the $(\beta\beta)_{0\nu}$ process. In fact in these models, we will see that there are contributions to the no-neutrino $\beta\beta$ decay that do not depend explicitly on the neutrino mass.

We will see that it is possible to have $(\beta\beta)_{0\nu}$ decay assumming neutrinos with arbitrarily small mass in the context of an $SU(3)_L \otimes U(1)_N$ model. In order to emphasize this fact, in most of this paper the neutrinos will be considered as being massless. Notwithstanding, we will show at the end that in order to maintain the naturalness of the model, the neutrinos must be massive. Here, the word *natural* is used in the technical sense [12]. It means that the masses in a theory are *finite* and *calculable* if there is a zeroth-order mass relation which is invariant under arbitrary changes of parameters presented in the theory. In fact, in renormalizable theories any particle mass is either zero, due to some unbroken symmetry or, arbitrary, due to the counterterm which is necessary in order to implement the renormalization program. Hence, if a mass is zero at the tree level, and there is no respective counterterm, loop corrections cannot be divergent. This is so because there would be no counterterm available to cancel the infinities.

However, the main point is that in this sort of model $(\beta\beta)_{0\nu}$ decay requires less neutrino mass than it does in most extensions of the standard electroweak model. Our requirement of massive neutrinos has no relation with the bad high energy behavior of processes such as $W^-V^- \to e^-e^-$ [9], since in these models the doubly charged gauge boson U^{--} cancels out the divergent part of such a process.

If one wants the lepton number to be conserved, one must assume that the $V^\pm, U^{\pm\pm}$ gauge bosons carry lepton number. Notice that lepton number conservation can be maintained in the Yukawa sector by assigning an appropriate lepton number to the scalar fields as well.

This paper is organized as follows: A new quantum number, i.e., the *leptobaryon* number $F = L + B$, is defined in Sec. II. The conservation of this quantum number forbids the existence of massive neutrinos and $(\beta\beta)_{0\nu}$. We add trilinear terms to the Higgs potential, which explicitly violate F, in Sec. III. In Sec. IV we consider the $(\beta\beta_{0\nu})$ decay. Section V is devoted to showing that explicit F violation also implies the spontaneous breaking of this quantum number, and the raising of neutrinos with arbitrary mass.

II. $SU(3)_L \otimes U(1)_N$ MODEL

Let us first recall some points of the $SU(3) \otimes U(1)$ model [1]. The model of Ref. [2] has a slightly different representation content. However, the main points concerning $(\beta\beta)_{0\nu}$ decay do not depend on this.

The representation content is the following: The leptons transform as triplets,

$$\psi_{aL} = \begin{pmatrix} \nu_a \\ l_a \\ l_a^c \end{pmatrix}_L \sim (\mathbf{3}, 0), \tag{2.1}$$

with $a = e, \mu, \tau$. In the quark sector we have the triplet

$$Q_{1L} = \begin{pmatrix} u_1 \\ d_1 \\ J_1 \end{pmatrix}_L \sim (\mathbf{3}, +\tfrac{2}{3}), \tag{2.2}$$

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for the left-handed fields, and singlets

$$u_{1R} \sim (\mathbf{1},+\tfrac{2}{3})\ ,d_{1R} \sim (\mathbf{1},-\tfrac{1}{3})\ ,J_{1R} \sim (\mathbf{1},+\tfrac{5}{3}), \qquad (2.3)$$

for the respective right-handed fields (we have not introduced right-handed neutrinos).

The second and third families of quarks are antitriplets $(\mathbf{3}^*,-\frac{1}{3})$

$$Q_{2L} = \begin{pmatrix} J_2 \\ u_2 \\ d_2 \end{pmatrix}_L , \quad Q_{3L} = \begin{pmatrix} J_3 \\ u_3 \\ d_3 \end{pmatrix}_L . \qquad (2.4)$$

The respective right-handed quarks are also singlets. In fact, every two of the three quark generations transform identically in contrast with the third one. The model is anomaly-free if we have an equal number of triplets and antitriplets, by considering the color of $SU(3)_c$. Furthermore, we require the sum of all fermion charges to vanish. The anomaly cancellation occurs for the three generations together, and not generation by generation. In Eqs. (2.2) and (2.4) all the quarks are linear combinations of the mass eigenstates except the one with charge $+\frac{5}{3}$.

For the first generation of quarks we have the charged current interactions

$$\mathcal{L}^{CC}_{Q_1W} = -\frac{g}{\sqrt{2}}(\ \bar{u}_L\gamma^\mu d_{\theta L}W^+_\mu + \bar{J}_{1L}\gamma^\mu u_L V^+_\mu + \bar{d}_{\theta L}\gamma^\mu J_{1L}U^{--} + \text{H.c.}), \qquad (2.5)$$

and for the second generation of quarks we have

$$\mathcal{L}^{CC}_{Q_2W} = -\frac{g}{\sqrt{2}}(\ \bar{c}_L\gamma^\mu d_{\theta L}W^+_\mu - \bar{s}_{\theta L}\gamma^\mu J_{2\phi L}V^+_\mu + \bar{c}_L\gamma^\mu J_{2\phi L}U^{++} + \text{H.c.}). \qquad (2.6)$$

The charge-changing interactions for the third generation of quarks are obtained from those of the second generation by making $c \to t$, $s \to b$, and $J_2 \to J_3$. We have mixing only in the $Q=-\frac{1}{3}$ and $Q=-\frac{4}{3}$ sectors. Thus in Eqs. (2.5) and (2.6) d_θ, s_θ, and $J_{2\phi}$ are the Cabibbo-Kobayashi-Maskawa states in the three- and two-dimensional flavor spaces d,s,b and J_2,J_3, respectively. In the leptonic sector we have the charged currents

$$\mathcal{L}^{CC}_l = -\frac{g}{\sqrt{2}}\sum_l(\ \bar{\nu}_{lL}\gamma^\mu l_L W^+_\mu + \bar{l}^c_L\gamma^\mu \nu_{lL}V^+_\mu + \bar{l}^c_L\gamma^\mu l_L U^{++}_\mu + \text{H.c.}). \qquad (2.7)$$

In order to generate the quark masses, it is necessary to introduce the Higgs scalars

$$\eta = \begin{pmatrix} \eta^0 \\ \eta_1^- \\ \eta_2^+ \end{pmatrix}, \quad \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{++} \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi^- \\ \chi^{--} \\ \chi^0 \end{pmatrix}, \qquad (2.8)$$

which transform, under $SU(3)\otimes U(1)$, as $(\mathbf{3},0)$, $(\mathbf{3},1)$, and $(\mathbf{3},-1)$, respectively.

The lepton mass term transforms as $(\mathbf{3}\otimes\mathbf{3}) = \mathbf{3}^*\oplus\mathbf{6}_S$. Thus we can introduce a triplet, such as η, or a symmetric antisextet $S = (\mathbf{6}^*_S,0)$. In the former case one of the charged leptons remains massless, and the other two are mass degenerate. Hence, we choose the latter one [4] in order to obtain an arbitrary mass for leptons.

The charge assignment for $(\mathbf{6}^*,0)$ is

$$S = \begin{pmatrix} \sigma_1^0 & h_2^+ & h_1^- \\ h_2^+ & H_1^{++} & \sigma_2^0 \\ h_1^- & \sigma_2^0 & H_2^{--} \end{pmatrix}. \qquad (2.9)$$

The quark–Higgs-boson interaction is

$$\mathcal{L}_Y = \bar{Q}_{1L}(G^u_{1\alpha}U_{\alpha R}\eta + G^d_{1\alpha}D_{\alpha R}\rho + G^j J_{1R}\chi) + \bar{Q}_{iL}(F^u_{i\alpha}U_{\alpha R}\rho^* + F^d_{i\alpha}D_{\alpha R}\eta^* + F^j_{ik}J_{kR}\chi^*) + \text{H.c.}, \qquad (2.10)$$

where $\alpha = 1,2,3$, $i,k = 2,3$, $U_{\alpha R} = u_{1R},u_{2R},u_{3R}$, $D_{\alpha R} = d_{1R},d_{2R},d_{3R}$, and all fields are still symmetry eigenstates. Explicitly from Eq. (2.10) one has

$$\begin{aligned} -\mathcal{L}_{QY} = {} & G^u_{1\alpha}(\bar{u}_{1L}\eta^0 + \bar{d}_{1L}\eta_1^- + \bar{J}_{1L}\eta_2^+)U_{\alpha R} + G^d_{1\alpha}(\bar{u}_{1L}\rho^+ + \bar{d}_{1L}\rho^0 + \bar{J}_{1L}\rho^{++})D_{\alpha R} \\ & + G^j(\bar{u}_{1L}\chi^- + \bar{d}_{1L}\chi^{--} + \bar{J}_{1L}\chi^0)J_{1R} + F^u_{i\alpha}(\bar{J}_{iL}\rho^{--} + \bar{u}_{iL}\rho^{0*} + \bar{d}_{iL}\rho^-)u_{\alpha R} \\ & + F^d_{iL}(\bar{J}_{iL}\eta_2^- + \bar{u}_{iL}\eta_1^+ + \bar{d}_{iL}\eta^{0*})D_{\alpha R} + F^j_{iL}(\bar{J}_{iL}\chi^{0*} + \bar{u}_{iL}\chi^{++} + \bar{d}_{iL}\chi^+)J_{kR} + \text{H.c.} \end{aligned} \qquad (2.11)$$

In the leptonic sector we have the interaction

$$\mathcal{L}_{lS} = -\frac{1}{2}\sum_{ab} G_{ab}\bar{\psi}^c_{iaL}\psi_{jbL}S^{ij}\ , \qquad (2.12)$$

with $\psi^c = C\bar{\psi}^T$. Here C is the charge conjugation matrix. Explicitly we have

$$\begin{aligned} 2\mathcal{L}_S = {} & -\sum_{ab} G_{ab}[\bar{\nu}^c_{aR}\nu_{bL}\sigma_1^0 + \bar{l}^c_{aR}l_{bL}H_1^{++} + \bar{l}_{aR}l^c_{bL}H_2^{--} + (\bar{\nu}^c_{aR}l_{bL} + \bar{l}^c_{aR}\nu_{bL})h_2^+ \\ & + (\bar{\nu}^c_{aR}l^c_{bL} + \bar{l}_{aR}\nu_{bL})h_1^- + (\bar{l}^c_{aR}l^c_{bL} + \bar{l}_{aR}l_{bL})\sigma_2^0] + \text{H.c.} \end{aligned} \qquad (2.13)$$

If we impose that $\langle\sigma_1^0\rangle = 0$, then the neutrinos remain massless, at least at the tree level.

In addition to (2.13) there is the additional Yukawa coupling between leptons and the scalar triplet η:

$$\mathcal{L}_{l\eta} = -\frac{1}{2}\sum_{ab} f_{ab}\bar{\psi}^c_{aiR}\psi_{bjL}\epsilon^{ijk}\eta_k + \text{H.c.}, \qquad (2.14)$$

where a,b denote family indices, i,j denote SU(3) indices, and ϵ^{ijk} is the totally antisymmetric symbol. The Yukawa coupling f_{ab} must be antisymmetric, i.e., $f_{ab} = -f_{ba}$, due to Fermi statistics, and the antisymmetry of the charge conjugation matrix C. Thus, Eq. (2.14) connects leptons of different families. Typical terms of Eq. (2.14) read

$$(\bar{\nu}^c_{eR}\mu^-_L - \bar{e}^c_R\nu_{\mu L})\eta^+_2, \quad (\bar{\nu}^c_{eR}\mu^+_L - \bar{e}_R\nu_{\mu L})\eta^-_1. \tag{2.15}$$

As we have said in the last section, let us define the *lepto-baryon* number, which is additively conserved as follows:

$$F = L + B, \tag{2.16}$$

where L is the total lepton number, i.e., $L = \sum_a L_a$, $a = e, \mu, \tau$, and B is the baryon number. As usual, $B(l) = 0$ for any lepton l, $L(q) = 0$ for any quark q,

$$F(l) = F(\nu_l) = +1, \tag{2.17}$$

and

$$F(u_\alpha) = F(d_\alpha) = \tfrac{1}{3}, \quad F(J_1) = -\tfrac{5}{3}, \quad F(J_i) = \tfrac{7}{3}, \tag{2.18}$$

where $\alpha = 1,2,3$ and $i = 2,3$. From Eqs. (2.17) and (2.18) we see that if

$$F(V^-) = F(U^{--}) = +2, \tag{2.19}$$

the interactions (2.5), (2.6), and (2.7) conserve F.

In order to make F a conserved quantum number in the Yukawa sector [see Eqs. (2.11) and (2.13)], we also assign to the scalar fields the values

$$-F(H^{--}_2) = -F(\rho^{++}) = F(\chi^-) = F(\chi^{--}) = +2,$$

$$-F(H^{--}_2) = F(H^{++}_1) = F(h^+_2) = F(\sigma^0_1) = -2, \tag{2.20}$$

and with all the other scalar fields carrying $F = 0$.

Although the process $W^-V^- \to e^-e^-$ occurs in this kind of model with the exchange of massless neutrinos, it does not imply the $(\beta\beta)_{0\nu}$ decay, since the vertex $\bar{d}_L\gamma^\mu u_L V^+_\mu$ is forbidden by F conservation. This symmetry also forbids a mixing between W^- and V^-. Hence, we see that the F symmetry must be broken in order to allow the $(\beta\beta)_{0\nu}$ to occur.

III. SCALAR SECTOR

Let

$$\begin{aligned} V(\eta,\rho,\chi,S) = {} & \mu_1^2\eta^\dagger\eta + \mu_2^2\rho^\dagger\rho + \mu_3^2\chi^\dagger\chi + \mu_4^2\mathrm{Tr}(S^\dagger S) + \lambda_1(\eta^\dagger\eta)^2 + \lambda_2(\rho^\dagger\rho)^2 \\ & + \lambda_3(\chi^\dagger\chi)^2 + (\eta^\dagger\eta)\left[\lambda_4(\rho^\dagger\rho) + \lambda_5(\chi^\dagger\chi)\right] + \lambda_6(\rho^\dagger\rho)(\chi^\dagger\chi) \\ & + \lambda_7\left[\mathrm{Tr}(S^\dagger S)\right]^2 + \lambda_8\mathrm{Tr}(S^\dagger S S^\dagger S) + \mathrm{Tr}(S^\dagger S)[\lambda_9(\eta^\dagger\eta) + \lambda_{10}(\rho^\dagger\rho) + \lambda_{11}(\chi^\dagger\chi)] \\ & + \lambda_{12}(\rho^\dagger\eta)(\eta^\dagger\rho) + \lambda_{13}(\chi^\dagger\eta)(\eta^\dagger\chi) + \lambda_{14}(\rho^\dagger\chi)(\chi^\dagger\rho) \\ & + \left(f_1\epsilon^{ijk}\eta_i\rho_j\chi_k + \frac{1}{2}f_2\rho_i\chi_j S^{\dagger ij} + f_3\eta_i\eta_j S^{\dagger ij} + \frac{1}{3!}f_4\epsilon^{ijk}\epsilon^{lmn}S_{il}S_{jm}S_{kn} + \text{H.c.}\right). \end{aligned} \tag{3.1}$$

This is the most general SU(3)⊗U(1) gauge invariant renormalizable Higgs potential for the three triplets and the sextet. The constants f_i, $i = 1,2,3,4$ have dimension of mass. It is possible to show that the potential (3.1) has a local minimum at the following vacuum expectation values (VEV's) for the scalar neutral fields [13]

$$\langle\eta\rangle = \begin{pmatrix} v_\eta \\ 0 \\ 0 \end{pmatrix}, \quad \langle\rho\rangle = \begin{pmatrix} 0 \\ v_\rho \\ 0 \end{pmatrix}, \quad \langle\chi\rangle = \begin{pmatrix} 0 \\ 0 \\ v_\chi \end{pmatrix}, \tag{3.2}$$

and

$$\langle S\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & v_H \\ 0 & v_H & 0 \end{pmatrix}. \tag{3.3}$$

Since we have chosen $\langle\sigma^0_1\rangle = 0$, the neutrinos do not gain mass at the tree level. However, we can verify the *naturalness* of this choice. The situation is similar when a triplet is added to the standard model [14]. We will return to this point in Sec. V.

Redefining all neutral scalars as $\varphi = v_\varphi + \varphi_1 + i\varphi_2$, except for the case of σ^0_1, we can analyze the scalar spectrum. For simplicity we will not consider relative phases in the vacuum expectation values. Requiring that the shifted potential have no linear terms in any of the $\varphi_{1,2}$ components of all neutral scalars we obtain in the tree approximation the constraint equations:

$$\begin{aligned} & \mu_1^2 + 2\lambda_1 v_\eta^2 + \lambda_4 v_\rho^2 + \lambda_5 v_\chi^2 + 2\lambda_9 v_H^2 + f_1 v_\eta^{-1} v_\rho v_\chi = 0, \\ & \mu_2^2 + 2\lambda_2 v_\rho^2 + \lambda_4 v_\eta^2 + \lambda_6 v_\chi^2 + 2\lambda_{10} v_H^2 + f_1 v_\eta v_\rho^{-1} v_\chi + f_2 v_\chi v_H v_\rho^{-1} = 0, \\ & \mu_3^2 + 2\lambda_3 v_\chi^2 + \lambda_5 v_\eta^2 + \lambda_6 v_\rho^2 + 2\lambda_{11} v_H^2 + f_1 v_\eta v_\rho v_\chi^{-1} + f_2 v_\rho v_H v_\chi^{-1} = 0, \\ & \mu_4^2 + 4\lambda_7 v_H^2 + 2\lambda_8 v_H^2 + \lambda_9 v_\eta^2 + \lambda_{10} v_\rho^2 + \lambda_{11} v_\chi^2 + \frac{f_2}{2} v_\rho v_\chi v_H^{-1} = 0, \\ & f_3 v_\eta^2 - f_4 v_H^2 = 0, \\ & \mathrm{Im}\, f_i = 0, \quad i = 1,2,3,4, \end{aligned} \tag{3.4}$$

and the mass matrix in the $\eta^-_1, \rho^-, \eta^-_2, \chi^-, h^-_1, h^-_2$ basis is

$$v_\chi^2 \begin{pmatrix} A_1 & A_2 & -F_3 c & 0 & 0 & -F_3 a \\ A_2 & A_3 & 0 & 0 & -F_2 & 0 \\ -F_3 c & 0 & B_1 & B_2 & -F_3 a & 0 \\ 0 & 0 & B_2 & B_3 & 0 & -F_2 b \\ 0 & -F_2 & -F_3 a & 0 & C_1 & C_2 \\ -F_3 a & 0 & 0 & -F_2 b & C_2 & C_1 \end{pmatrix}, \quad (3.5)$$

where

$$A_1 = F_1 b a^{-1} - \lambda_{12} b^2, \quad A_2 = F_1 - \lambda_{12} ab,$$
$$A_3 = (F_1 a + F_2 c) b^{-1} - \lambda_{12} a^2, \quad (3.6)$$

$$B_1 = F_1 b a^{-1} - \lambda_{13}, \quad B_2 = F_1 b - \lambda_{13} a,$$
$$B_3 = (F_1 a + F_2 c) b - \lambda_{13} a^2, \quad (3.7)$$

$$C_1 = F_2 b c^{-1}, \quad C_2 = -F_3 a^2 c^{-1}. \quad (3.8)$$

Here we have defined the dimensionless constants $F_i = f_i/v_\chi$, $a = v_\eta/v_\chi$, $b = v_\rho/v_\chi$, and $c = v_H/v_\chi$. The mass matrix in Eq. (3.5) has just two Goldstone bosons. It implies a mixing among all charged scalars. Thus the physical charged scalars are linear combinations of $\eta_i^-, h_i^- (i = 1,2)$, ρ^-, and χ^- which have no well defined value of F. Since quarks u, d interact according Eq. (2.11) with η_1^-, ρ^-, and the last fields are linear combinations of mass eigenstates, we see that the diagram in Fig. 1 is possible even if the neutrinos are massless.

IV. $(\beta\beta)_{0\nu}$ DECAY

The F symmetry is softly broken by the $f_{3,4}$ terms in the scalar potential (3.1). As we have said in the last section, the singly charged scalars are not eigenstates of F. Then, if Φ_1^- in Fig. 1 is one of the scalar mass eigenstates, we have, in general,

$$\phi_i^- = \sum_{ij} a_{ij} \Phi_j^- , \quad (4.1)$$

with $\phi_i^- = \eta_1^-,\ \eta_2^-,\ \rho^-,\ \chi^-,\ h_1^-,\ h_2^-$, and a_{ij} the mixing parameters.

We can estimate a lower bound on the mass of ϕ_1^- by assuming that its contribution to $(\beta\beta)_{0\nu}$ is less than the amplitude due to the massive Majorana neutrinos and vector boson W^- exchange. The latter amplitude is characterized by a strength which is proportional to

$$\frac{g^4 m_\nu^{\rm eff}}{m_W^4 \langle p^2 \rangle}, \quad (4.2)$$

where $m_\nu^{\rm eff} = |\sum_j U_{lj}^2 m_j|$ is the "effective neutrino mass" [8]. Here, $\langle p^2 \rangle$ is an average square four-momentum carried by the virtual neutrino. Its value is usually (10 MeV)^2 [10]. The experimental limit on $(\beta\beta)_{0\nu}$ decay rate implies that $m_\nu^{\rm eff} < M_\nu = (1\text{–}2)$ eV [9]. On the other hand, the amplitude of the process in Fig. 1 is proportional to

$$\frac{(a_{11} a_{21})^2 G_{ud}^4 G_{ee}^4}{m_\phi^4 \langle p^2 \rangle^{\frac{1}{2}}}. \quad (4.3)$$

Next, assuming that Eq. (4.3) is less than Eq. (4.2) when $m_\nu^{\rm eff} = M_\nu$, we have

$$m_{\phi_1}^4 > \frac{(a_{11} a_{21})^2 G_{ud}^4 G_{ee}^4 \sqrt{2} \langle p^2 \rangle^{\frac{1}{2}}}{32 G_F^2 M_\nu}$$
$$\simeq (6.9 \text{ TeV})^4 (a_{11} a_{21})^2 G_{ud}^4 G_{ee}^4. \quad (4.4)$$

The factors in Eq. (4.3) arise as follows. In Eq. (2.11) the fields are symmetry eigenstates. It is possible to redefine the quark fields as

$$q_L' = V_L^Q q_L, \quad q_R' = V_R^Q q_R , \quad (4.5)$$

with $V_{L,R}^Q$ being unitary matrices in the flavor space, and the unprimed fields denoting mass eigenstates for the respective charge-Q sector. Equation (2.11) implies interactions such as $G_{ud} \bar{d}_L u_R \eta_1^-$ with $G_{ud} = (V_L^{(-\frac{1}{3})} G^u V_R^{(\frac{2}{3})})_{ud}$ and d, u are mass eigenstates. The coefficients G_{ee} appear in Eq. (2.13). Since these mixing parameters in Eq. (4.4) can be very small, this does not imply a strong lower bound on the mass of the scalar fields.

There are not contributions to $(\beta\beta)_{0\nu}$ from trilinear Higgs interactions such as $\eta_1^- h_2^- H^{++}$. In models in which these contributions exist, they are negligible [10] unless a neighboring mass scale ($\sim 10^4$ GeV) exists [15].

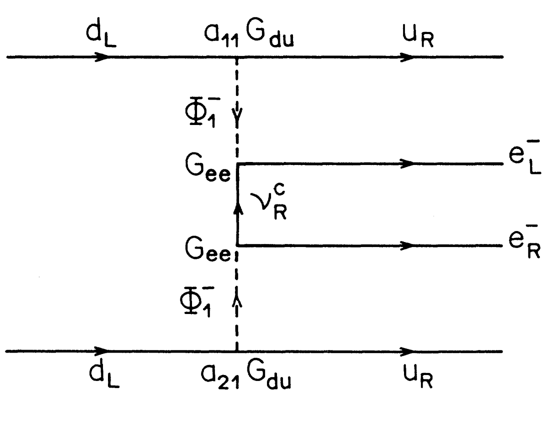

FIG. 1. Scalar contribution to the $(\beta\beta)_{0\nu}$. $G_{ud,ee}$ are Yukawa couplings and $a_{11,21}$ mixing parameters in the scalar sector.

V. CONCLUSIONS

We now consider the question of the neutrino masses. First of all, notice that if we forbid the trilinear terms in $f_{3,4}$, say, by assuming a discrete symmetry [4], the mixing that arises from Eq. (3.5) is among η_1^-, ρ^-, h_1^-, and separately among η_2^-, χ^-, h_2^-. Hence F is conserved and $(\beta\beta)_{0\nu}$ is forbidden. In this context it is important that the F symmetry is softly broken by the trilinear terms $f_{3,4}$ in the scalar potential (3.1). Since we have made $\langle \sigma_1^0 \rangle = 0$, we can think that the neutrino masses vanish at the tree level, but that they are finite and calculable, in the sense of Sec. I. Let us consider this issue more in detail.

From Eqs. (2.13) and (2.15), it is easy to convince ourselves that the neutrinos gain finite masses through loop diagrams as in Figs. 2(a) and 2(b). Now, joining the neutrino lines in Fig. 2 by σ_1^0, we obtain the divergent contribution to $\langle\sigma_1^0\rangle$ appearing in Figs. 3(a,b). This implies a counterterm and makes it impossible to maintain $\langle\sigma_1^0\rangle = 0$, at least in a *natural* way. Hence, neutrinos gain an arbitrarily small mass, since we can always assume that $\langle\sigma_1^0\rangle \simeq 0$.

If $\langle\sigma_1^0\rangle \neq 0$, there is a mixing in the charged vector sector W^+–V^+:

$$W^{\pm} = \alpha X_1^{\pm} + \beta X_2^{\pm}, \quad V^{\pm} = -\beta X_1^{\pm} + \alpha X_2^{\pm},$$
$$\alpha^2 + \beta^2 = 1, \tag{5.1}$$

where $X_{1,2}^{\pm}$ are mass eigenstates. Hence the $(\beta\beta)_{0\nu}$ proceeds also as in Fig. 4, without a direct dependence on the neutrino mass, but this contribution is suppressed by the large mass of the vector boson X_2^-, or by the mixing parameters since $\beta \simeq 0$. If $\langle\sigma_1^0\rangle \neq 0$, and assuming discrete symmetries to forbid the explicit violations in Eq. (3.1), we have spontaneous breaking of the F symmetry, since σ_1^0 carries $F = -2$ [see Eq. (2.20)], implying a Majoron-Goldstone-like boson. This is so because σ_1^0 belongs to a triplet under SU(2) together with h_2^- and H_1^{--}. The phenomenology of this Goldstone boson may or may not be similar to that of the Majoron [16], and it deserves a more detailed study. As $\langle\sigma_1^0\rangle \neq 0$ we expect

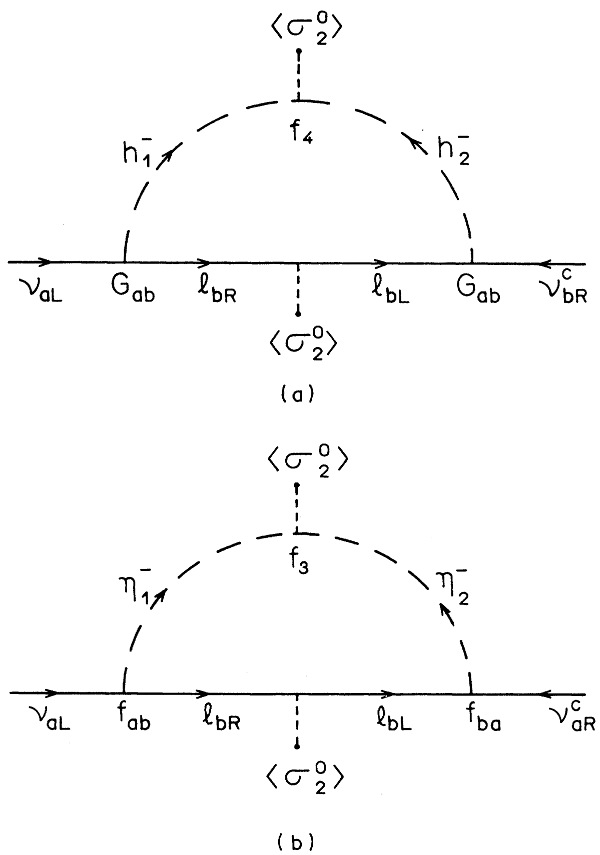

FIG. 2. Finite contribution to the Majorana neutrino mass due to the Yukawa couplings in Eqs. (2.13) and (2.15) and the trilinear terms f_3 and f_4 in the scalar potential (3.1).

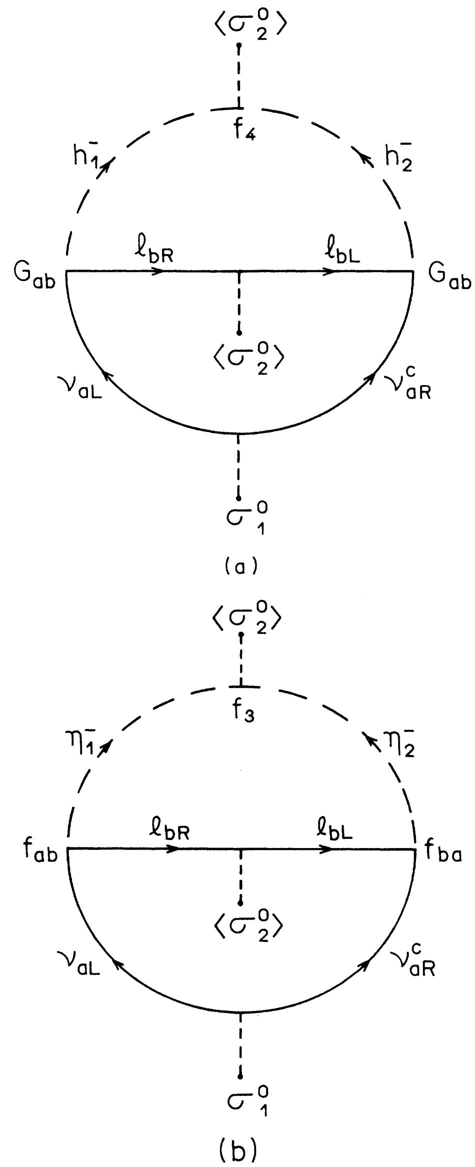

FIG. 3. Joining the neutrino lines by σ_1^0 in Fig. 2 we obtain divergent contributions to $\langle\sigma_1^0\rangle$.

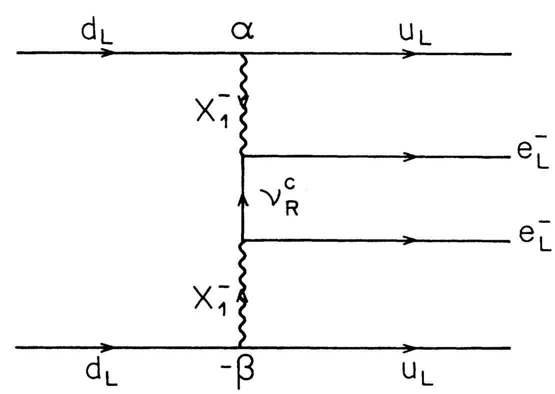

FIG. 4. Contribution to the $(\beta\beta)_{0\nu}$ due the charged vector boson exchange if $\langle\sigma_1^0\rangle \neq 0$. X_1^+ is a linear combination of W^+ and V^+. See Eq. (5.1).

a deviation from the $\rho = 1$ value ($\rho = \cos^2\theta_W M_Z^2/M_W^2$). From the experimental value $\rho = 0.995 \pm 0.013$ [17] we obtain $\langle\sigma_1^0\rangle < 10$ MeV which is the same obtained for the VEV of the Gelmini-Roncadelli triplet coming from astrophysical constraints [18].

Summarizing, we see that $(\beta\beta)_{0\nu}$ proceeds in this model also as a Higgs boson effect with almost massless neutrinos. Recall that if we had forbidden all trilinears in Eq. (3.1), except those with f_1, f_2, neutrinos could remain massless but the mixing in the charged scalar sector would be only among η_1^-, ρ^-, h_1^- and η_2^-, χ^-, h_2^- separately. Hence the $(\beta\beta)_{0\nu}$ cannot occur as was shown in Ref. [1].

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