Probing *CP* violation via Higgs boson decays to four leptons

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Since decays to four leptons is widely considered a promising way to search for the Higgs particle, we show how the same final state can also be used to search for signals of *CP* nonconservation. Energy asymmetries and triple correlations are related to parameters in the underlying *CP*-violating effective interaction at the H^0-W-W and H^0-Z-Z vertices. It is argued that the expected size of the effects is extremely small in the standard model, and it is shown that the result also seems small to be observable in an extension of the standard model with an extra Higgs doublet.

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The search for the Higgs particle is clearly one of the top priorities in particle physics. In view of the remarkable successes of the standard model (SM), some manifestation of the Higgs boson should exist, responsible for the spontaneously breaking of the gauge group $SU(2)_L \times U(1)$ down to U(1). Another important problem in particle physics is the origin of *CP* violation. In the SM, *CP* violation is accounted for by the Kobayashi-Maskawa (KM) phase [1]. However, there is considerable interest in searching for sources of *CP* violation other than the KM phase. For example, it is generally believed that the KM mechanism alone cannot produce sufficient baryon asymmetry in the Universe [2]. A number of extensions of the SM have *CP* violation other than the KM phase. Tests of *CP* nonconservation will therefore test theories beyond the SM.

Once the Higgs particle is discovered, its properties will have to be investigated vigorously. In particular, its role in *CP* nonconservation will undoubtedly get a close scrutiny. Since a very promising way to search for the Higgs boson is through its decay to four leptons, in this paper we show how that final state can also, simultaneously, be used to study the *CP* properties of the Higgs particle. We will thus consider the following processes: (I) $H^0 \to Z^*Z^* \to l^+l^-\bar{\nu}\nu$; (II) $H^0 \to W^*W^* \to l^+\nu l'^-\bar{\nu}'$; (III) $H^0 \to Z^*Z^* \to l^+l^-l'^+l'^-$ [3]. Here l and l' stand for either the electron or the muon, and ν and ν' stand for any species of neutrino allowed by lepton flavor conservation and W^* and Z^* can be either on or off shell. Even though we will keep our formalism general, we are primarily interested in the heavy Higgs boson mass region ($m_H \geq 2m_W$) in this paper since the corresponding branching fractions are larger. Also these processes are expected to have considerable background problems in the intermediate mass region ($m_Z \leq m_H \leq 2m_W$) [4]. In the heavy mass region the branching fractions are approximately 8×10^{-3}, 2×10^{-2}, and 10^{-3} [5], respectively, for processes (I)–(III) where we have summed over electron and muon final states.

CP-odd observables that we will discuss are the energy asymmetries and the *CP*-odd angular correlations of the charged leptons. Indeed these are the only *CP*-odd observables that can be constructed for the above processes if we assume that polarizations of the final-state leptons are not observed. There are several different energy asymmetries that can be defined. The first one is

$$\delta_1 = \frac{\langle E_+ - E_- \rangle}{\langle E_+ + E_- \rangle} \ . \tag{1}$$

This is suitable for process (I), where $E_\pm$ stands for the energy of positively and negatively charged leptons in this paper. Strictly speaking, the energy asymmetry for process (II) is meaningful only when l^+ is the antiparticle of l'^-. However, assuming lepton universality and setting all lepton masses to zero, we can also define a kind of "flavor-blind" energy asymmetry. Thus δ_2, the energy asymmetry for process (II), is unambiguously defined, in analogy to Eq. (1). Two independent energy asymmetries can also be defined for process (III): i.e.,

$$\delta_3 = \frac{\langle E_+ - E_- \rangle}{\langle E_+ + E_- \rangle}$$

and (2)

$$\delta_3' = \frac{\langle E_+ + E_+' - E_- - E_-' \rangle}{\langle E_+ + E_+' + E_- + E_-' \rangle} \ .$$

We recall that for any of the energy asymmetries to receive a nonvanishing contribution the amplitude for the process must have an absorptive part as required by *CPT* invariance since the energy asymmetry is a *CP* odd but naive *T* even object [6].

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CP noninvariance can also show up in the angular correlation of the two decay planes defined by the momenta of the final-state leptons. This is a straightforward generalization of Yang's parity test [7]. In the case of only four particles in the final state, all their momenta need to be tracked down to determine the angular correlations. Thus the *CP*-odd angular correlation is only useful in the process (III), i.e., $H^0 \to l'^+ l'^- l^+ l^-$. The angular correlation between the decay planes can be parametrized as [8]

$$\frac{d\Gamma}{d\phi} = \frac{\Gamma}{2\pi}[1+\lambda_1\cos\phi+\lambda_2\cos2\phi+\lambda_3\sin\phi+\lambda_4\sin2\phi] , \tag{3}$$

where Γ is the partial decay width for process (III) and ϕ is the angle between the two decay planes. A nonvanishing λ_1 indicates parity is violated, whereas nonvanishing λ_3 and/or λ_4 are indications of *CP* nonconservation. *CP*-violating angular correlation effects will tend to be washed out for two identical lepton pairs in the final state, since there is ambiguity in identifying ϕ and $-\phi$. However, both parity and *CP*-violating effects can be picked up if two lepton pairs are different, thus we will only consider nonidentical lepton pairs in process (III) when we consider angular correlations.

It is easy to show that λ_3 is related to the usual *CP*-odd triple product correlation. Let $\mathbf{p}_1$ and $\mathbf{p}_2$ be momenta of one pair of leptons coupled to one Z boson and $\mathbf{k}_1$ and $\mathbf{k}_2$ be momenta of the other pair of leptons. A *CP*-odd triple product Ω can be defined as

$$\Omega \equiv (\mathbf{p}_1 - \mathbf{p}_2)\cdot(\mathbf{k}_1 \times \mathbf{k}_2) . \tag{4}$$

In the rest frame of Higgs boson, Ω is the only independent *CP*-odd triple product that can be formed by using lepton momenta only. Then a measure of *CP* asymmetry A_{CP} defined as

$$A_{CP} \equiv \frac{\Gamma(\Omega>0)-\Gamma(\Omega<0)}{\Gamma(\Omega>0)+\Gamma(\Omega<0)} \tag{5}$$

is just $2\lambda_3/\pi$.

To simplify the calculations we will consistently neglect lepton masses. Then the most general tensor structure of the H^0VV (V stands for either the Z or the W) vertex relevant for decays to massless leptons assumes the form

$$m_V[\rho_V g_{\mu\nu} + \sqrt{2}G_F\zeta_V(q_1\cdot q_2 g_{\mu\nu} - q_{1\nu}q_{2\mu}) + \sqrt{2}G_F\vartheta_V\epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta] . \tag{6}$$

Here m_V stands for the mass of Z or W; G_F is the Fermi constant, q_1 is the momentum of one of the vector bosons coupled to the lepton current j_1^μ; and q_2 is the momentum of the other vector boson coupled to the lepton current j_2^ν. We have inserted m_V and $\sqrt{2}G_F$ in Eq. (6) to make ρ_V, ζ_V, and ϑ_V dimensionless. Coefficients ρ_V, ζ_V, and ϑ_V are functions of q_1^2 and q_2^2, in general. The origins of different terms in Eq. (6) can be identified as follows. The first term is the familiar H^0VV tree-level coupling accompanying the Higgs mechanism for giving masses to the gauge bosons. The second term is from the dimension-5 operator H^0FF (F being the field strength of the vector filed V) which can be generated by "integrating out" heavier particles in the theory. The last term is from another dimension-5 operator $H^0F\tilde{F}$, $\tilde{F}$ being the conjugate of F. Note that *CP* is violated if both ζ_V and ϑ_V are simultaneously present since FF and $F\tilde{F}$ have opposite transformation properties under *CP*. ϑ_V and ρ_V cannot coexist either if *CP* were a good symmetry [9].

In all the calculations in this paper, only the first term in Eq. (6), i.e., the tree-level coupling, and its interference with the third term in Eq. (6) will be kept. The reason for this is that we are interested in retaining only numerically the most significant terms that reflect *CP* violation. It is not difficult to see that this procedure is justified for our purpose because both ζ_V and ϑ_V in Eq. (6) are radiatively induced. For example, interference between the second and the third term in Eq. (6) is also *CP* violating, but it is of higher order. Furthermore, the *CP*-conserving interference between the first and the second term in Eq. (6) will also contribute to the total decay width, but for our purpose, it can be neglected as compared to the pure tree-level contribution.

First we discuss the energy asymmetries. It is straightforward to show that

$$\delta_1 = \delta_3 = \delta_3' = -\frac{8c_Vc_A}{c_V^2+Z_A^2}\delta_Z \quad \text{and} \quad \delta_2 = 4\delta_W ,$$

where $c_V = -1+4\sin^2\theta_W$ and $c_A = 1$ are, respectively, the vector and the axial-vector coupling constants of the Z boson to the charged leptons; δ_Z and δ_W are defined as [10]

$$\delta_V = \sqrt{2}G_F \frac{\int dq_1^2 dq_2^2 \rho_V \mathrm{Im}\vartheta_V |\mathbf{q}|^3 q_1^2 q_2^2 \Delta_V}{\int dq_1^2 dq_2^2 \rho_V^2 |\mathbf{q}|(3q_1^2q_2^2 + m_H^2|\mathbf{q}|^2)\Delta_V} . \tag{7}$$

In Eq. (7), $|\mathbf{q}|$ is the magnitude of the spatial momentum of either of the gauge bosons in the rest frame of the Higgs boson:

$$|\mathbf{q}| = \frac{\sqrt{m_H^4+q_1^4+q_2^4-2(m_H^2q_1^2+m_H^2q_2^2+q_1^2q_2^2)}}{2m_H} , \tag{8}$$

where m_H is the Higgs boson mass; Δ_V is from the propagators of the two gauge bosons,

$$\Delta_V = \frac{1}{[(q_1^2-m_V^2)^2+m_V^2\Gamma_V^2][(q_2^2-m_V^2)^2+m_V^2\Gamma_V^2]} ,$$

Γ_V being the total width of the gauge boson V. The integrations in both the numerator and the denominator of Eq. (7) are over the region $\sqrt{q_1^2}+\sqrt{q_2^2} < m_H$.

In general, ϑ_V is a function of both q_1^2 and q_2^2. However, we will assume ϑ_V to be a constant here. This approximation is justified if ϑ_V is a slowly varying function of q_1^2 and q_2^2. Furthermore, the integrands in Eq. (7) are peaked in the region where either of the propagators can be on shell. As we have indicated earlier, we are mostly interested in the case when both the vector bosons are on-shell since the branching ratios are larger. Thus it is not a bad approximation to replace the function ϑ_V by its value at q_1^2 and/or q_2^2 set equal to m_V^2. Using the narrow width approximation for both vector bosons, Eq. (7) becomes

$$\delta_V=\frac{\mathrm{Im}\vartheta_V}{\rho_V}\frac{\sqrt{2}G_F(m_H^2-4m_V^2)}{12+(m_H^2/m_V^2)(m_H^2/m_V^2-4)} . \tag{9}$$

Both the exact result Eq. (7) and the on-shell approximation Eq. (9) for δ_1 and δ_2 are plotted in Fig. 1. From the figure we see that δ_1 is about $10^{-3}(\mathrm{Im}\vartheta_Z/\rho_Z)$ and δ_2 is about $10^{-2}(\mathrm{Im}\vartheta_W/\rho_W)$.

Now we discuss the CP-odd angular correlations. It is straightforward to show that the differential partial decay rate is

$$\frac{d^3\Gamma}{dq_1^2dq_2^2d\phi}=\frac{(c_V^2+c_A^2)^2m_Z^2|\mathbf{q}|\Delta Z}{128(2\pi)^6m_H^2}$$
$$\times\left[\rho_Z^2\left[\frac{8}{3}q_1^2q_2^2+\frac{8}{9}m_H^2|\mathbf{q}|^2+\frac{\pi^2(c_Vc_A)^2}{(c_V^2+c_A^2)^2}q_1\cdot q_2\sqrt{q_1^2q_2^2}\cos\phi+\frac{4}{9}q_1^2q_2^2\cos 2\phi\right]-\sqrt{2}G_F\rho_Z\mathrm{Re}(\vartheta_Z)m_H|\mathbf{q}|\right.$$
$$\left.\times\left[\frac{\pi^2(c_Vc_A)^2}{(c_V^2+c_A^2)^2}q_1\cdot q_2\sqrt{q_1^2q_2^2}\sin\phi+\frac{8}{9}q_1^2q_2^2\sin 2\phi\right]\right] . \tag{10}$$

(For definiteness, we note that ϕ is the difference in the azimuthal angles of the two same sign leptons, coming from the two Z's, with respect to the momentum of one of the Z's.) It is obvious from the above equation that the parity-violating coefficient λ_1 is suppressed by a factor of

$$\pi^2(c_Vc_A)^2/(c_V^2+c_A^2)^2\sim 10^{-2}$$

(see also Refs. [8,11]). In addition to this suppression, λ_3 is further suppressed by the ratio $\mathrm{Re}(\vartheta_Z)/\rho_Z$ rendering λ_3 extremely small. The single differential decay rate with respect to ϕ can be obtained by numerically integrating Eq. (10). λ_3 and λ_4 are plotted in Fig. 2; again, we have assumed that ρ_V and ϑ_V are constants. The units in Fig. 2 are taken to be $\mathrm{Re}\vartheta_Z/\rho_Z$. From the figure we see that λ_3 stays approximately constant for $m_H>200$ GeV at 6×10^{-4}; λ_4 peaks around $m_H=190$ GeV, it varies between 8×10^{-4} and 8×10^{-3}.

In the SM, H^0 is a scalar particle. It couples to Z and W bosons at the tree level with $\rho_Z=g/\cos\theta_W$ and $\rho_W=g$. HFF is induced at one loop level, whereas CP-violating interaction $HF\tilde{F}$ does not arise till two-loop order for W bosons and at three loops for Z bosons. In addition to the suppression by powers of 4π associated with these loops, CP violation in the SM will necessarily involve product of small mixing angles and also perhaps small ratio of masses. While an explicit calculation is lacking, a simple dimensional analysis indicates

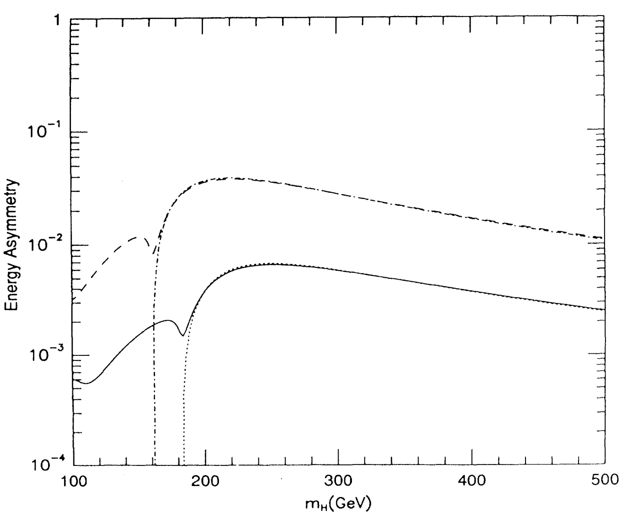

FIG. 1. δ_1 from Eq. (7) (solid line) and from the on-shell approximation (dotted line) in the units of $\mathrm{Im}\vartheta_Z/\rho_Z$. δ_2 from Eq. (7) (dashed line) and the on-shell result (dash-dotted line) in the units of $\mathrm{Im}\vartheta_W/\rho_W$.

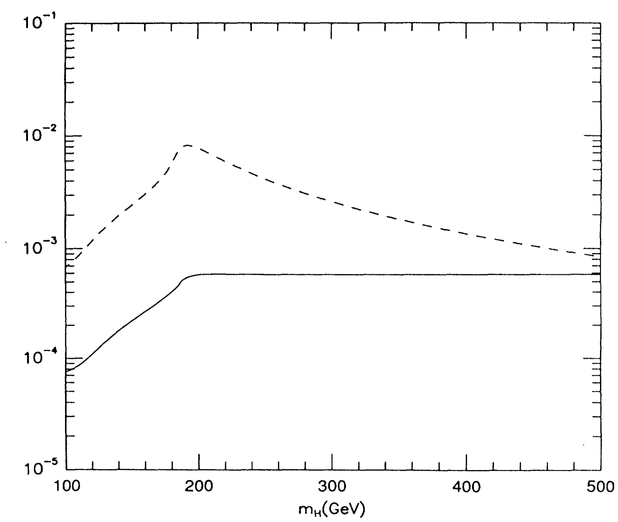

FIG. 2. λ_3 (solid line) and λ_4 (dashed line) in the units of $\mathrm{Re}\vartheta_Z/\rho_Z$.

$$\vartheta_V \sim J \frac{(m_t - m_c)(m_c - m_u)(m_t - m_u)(m_b - m_s)(m_s - m_d)(m_b - m_d)}{m_W^6}$$
$$\sim 10^{-7} J (m_t/m_W)^2 ,$$

where J is the *CP*-nonconserving invariant combination of KM angles [12]. Taking $J \sim 10^{-4}$ and $m_t \sim 130$ GeV, then ϑ_V is around 10^{-11}. It is clear that this is such an extremely small effect that it is well outside the range of experimental sensitivity.

On the other hand, many extensions of the SM have other sources of *CP* violation in addition to the KM phase which may enhance the rate of *CP*-violating decay of the Higgs particle significantly. As an illustration we will consider a two-Higgs-doublet model with softly symmetry-breaking term [13]. Unlike the SM, this model will generate a *CP*-violating effective interaction in Eq. (6) at one-loop level. The relevant Yukawa coupling in such a model is taken to be

$$-\mathcal{L}_Y = \lambda_{ij} \bar{Q}_L^i \tilde{\phi}_2 U_R^j + g_{ij} \bar{Q}_L^i \phi_1 D_R^j + \text{H.c.} \tag{11}$$

There are three spin-0 neutral bosons φ_1^0, φ_2^0, and φ_3^0 in the theory. If there is no *CP* violation from the scalar sector, two of them will be *CP*-even and the other one is *CP* odd. In the presence of *CP* violation, they mix through their mass matrix. Let us call the three neutral mass eigenstates H_1^0, H_2^0, and H_3^0, then

$$H_i^0 = O_{ij} \varphi_j , \tag{12}$$

where O_{ij} are the matrix elements of an orthogonal matrix that diagonalizes the mass matrix. ρ_Z and ρ_W arise from the tree-level coupling of H_i^0 to two Z and W bosons, while coefficients ϑ_Z and ϑ_W arise through the top quark loop shown in Fig. 3. It is easy to show that

$$\left[\frac{\vartheta_Z}{\rho_Z}\right]_i = \frac{3m_t^2 \kappa_i}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \frac{2\tilde{c}_A^2 (x+y) + \tilde{c}_V^2 - \tilde{c}_A^2}{q_1^2(x^2 - x) + q_2^2(y^2 - y) - 2xy q_1 \cdot q_2 + m_t^2} , \tag{13}$$

$$\left[\frac{\vartheta_W}{\rho_W}\right]_i = \frac{12 m_t^2 \kappa_i}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \frac{(x+y)}{q_1^2(x^2 - x) + q_2^2(y^2 - y) - 2xy q_1 \cdot q_2 + m_t^2} , \tag{14}$$

where $\tilde{c}_A = -1$ and $\tilde{c}_V = 1 - \frac{8}{3} \sin^2\theta_W$ are axial-vector and vector coupling constants of Z to the top quark; $\kappa_i = \cot\beta O_{i3}/O_{i1}$, where $\cot\beta = |v_1|/|v_2|$, $|v_2|$ and $|v_1|$ are vacuum expectation values of ϕ_2 and ϕ_1, respectively. In Fig. 4 we show both the real and the imaginary parts of ϑ_Z/ρ_Z and ϑ_W/ρ_W for on-shell W and Z, with $m_t = 130$ GeV. Since we are interested in the kinematic region $m_H > m_W(m_Z)$, one of the vector bosons can be well approximated by the on-shell approximation. Then the q^2 dependence of Eqs. (13) and (14) should be constant to a very good approximation. This was explicitly checked numerically for one case as indicated in Fig. 5. In that figure we show the q^2 dependence by fixing either q_1^2 or q_2^2 to m_Z^2 and m_W^2 for Eqs. (13) and (14), respectively. Figure 5 clearly shows our previous assumption of ϑ_V being slowly varying function of q^2 is a valid one, at least in the region where the Higgs boson mass is heavier than m_Z.

It is clear that both Eqs. (13) and (14) will develop an imaginary part only when $m_H > 2m_t$. Thus, within the

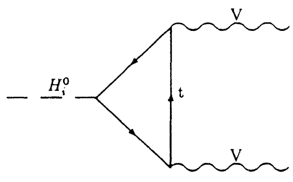

FIG. 3. One-loop Feynman diagram contributing to ϑ_V in the two-Higgs-doublet model.

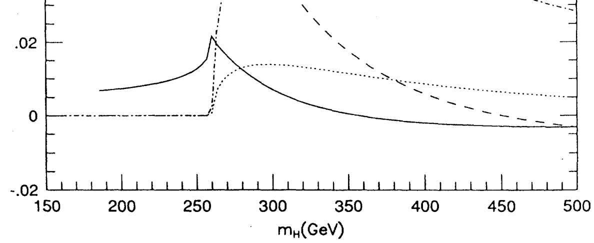

FIG. 4. Real (solid line) and imaginary (dotted line) part of $(\vartheta_Z/\rho_Z)_i$, and real (dashed line) and imaginary (dash-dotted line) part of $(\vartheta_W/\rho_W)_i$. They are all in the units of κ_i and m_t is taken to be 130 GeV.

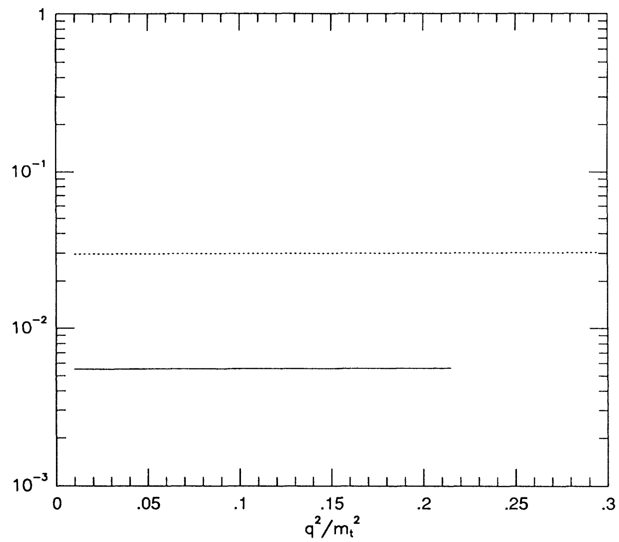

FIG. 5. q^2 dependence of $(\vartheta_Z/\rho_Z)_i$ (solid line), $(\vartheta_W/\rho_W)_i$ (dotted line). They are plotted in units of κ_i, m_t is taken to be 130 GeV, and $m_H = 150$ GeV. One of the gauge boson has been set on-shell in both cases.

context of the two-Higgs-doublet model with soft symmetry breaking, energy asymmetry is a useful observable only when $m_H > 2m_t$. Nevertheless, the angular correlation is useful both for $m_H > 2m_t$ and also for $m_H < 2m_t$. In this model, κ_i is completely undetermined; no constraints similar to the ones derived by Weinberg [13] exist for κ_i, although one may invoke some naturalness argument that it should not be too different from order one. Assuming κ_i is of order one, then we see that typically $\delta_1 \sim 10^{-5}$ and $\delta_2 \sim 10^{-4}$; $\lambda_3 \sim 10^{-6}$ and $\lambda_4 \sim 10^{-5}$. Thus in this model process (II) appears the most promising. Indeed, from Figs. 1 and 4 we see that δ_2 can be as large as 1.5×10^{-3} for m_H about 300 GeV. Assuming that the branching ratio (B) for this process is about 3×10^{-2}, as it is in the SM, we see that the number of Higgs bosons needed to see such an asymmetry, given roughly by $(\delta_2^2 \times B)^{-1}$, is about 10^7. This number is about a factor of 40 larger than the expected number of Higgs bosons at the hadron supercolliders, based again on the SM [5]. Consequently, at least in this extension of the standard model the resulting asymmetries appear too small to be observable. On the other hand, there is a large uncertainty in these estimates as many of the relevant parameters in these models have not been pinned down. Furthermore, our analysis is completely general so it may be useful to study what other extensions of the SM will yield for these asymmetries. The virtue of these tests of CP nonconservation is that they can be done at little or no extra cost.

Note added. After the completion of this paper, we learned of the existence of a related work, Ref. [14]. We thank Darwin Chang for informing us about the work. We were also made aware of another related, recent work, Ref. [15].

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