Cabibbo-angle-favored two-body hadronic decays of D_s^+ in the factorization scheme

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In this paper we study two-body Cabibbo-angle-favored decays of D_s^+ in the factorization scheme using two quark models: that of Bauer, Stech, and Wirbel and that of Isgur, Scora, Grinstein, and Wise. We discuss $D_s^+\to(\eta,\eta')\pi^+$, $(\eta,\eta')\rho^+$, $\phi\pi^+$, $\phi\rho^+$, $\bar{K}^0K^+$, $\bar{K}^0K^{*+}$, $\bar{K}^{*0}K^+$, and $\bar{K}^{*0}K^{*+}$ decays. We point out that the experimental observation $B(D_s^+\to\phi\rho^+)\approx B(D_s^+\to\bar{K}^{*0}K^{*+})$ is rather puzzling.

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I. INTRODUCTION

The CLEO II Collaboration has recently published their results on the branching ratios for D_s^+ decays into $\eta\pi^+$, $\eta'\pi^+$ [1] and $\eta\rho^+$, $\eta'\rho^+$, and $\phi\rho^+$ relative to $B(D_s^+\to\phi\pi^+)$ [2]. Prior to CLEO II publications there were other published results [3–7], often contradictory, on $B(D_s^+\to\eta\pi^+)$ and $B(D_s^+\to\eta'\pi^+)$. We have listed all known results on $B(D_s^+\to\eta\pi^+)$ and $B(D_s^+\to\eta'\pi^+)$ in Table I along with Particle Data Group listing [8]. In addition to the listing of Table I, the Mark II Collaboration has published [4] $\sigma(e^+e^-\to D_s^+)B(D_s^+\to\eta\pi^+)=(5.2\pm2.2)$ pb and $\sigma(e^+e^-\to D_s^+)B(D_s^+\to\eta'\pi^+)=(8.4\pm3.7)$ pb. They interpreted these numbers to imply $B(D_s^+\to\eta\pi^+)\approx12\%$ and $B(D_s^+\to\eta'\pi^+)\approx19\%$. In Ref. [5] the same numbers are interpreted as $B(D_s^+\to\eta\pi^+)/B(D_s^+\to\phi\pi^+)=3.0\pm1.1$ and $B(D_s^+\to\eta'\pi^+)/B(D_s^+\to\phi\pi^+)=4.8\pm2.1$. Obviously there is a wide spread in the data. Note, however, that Mark II [4] ratio for $B(D_s^+\to\eta\pi^+)/B(D_s^+\to\eta'\pi^+)$ is not in disagreement with the latest CLEO II determination [1].

The problem with the early data on $D_s^+\to\eta\pi^+$ and $\eta'\pi^+$ was noted by the authors of Ref. [9] who showed that the popular theoretical models for weak hadronic decays of charmed mesons had difficulty in explaining the high branching ratios measured by Mark II. There was, in addition, the problem of understanding the ratio $B(D_s^+\to\eta\pi^+)/B(D_s^+\to\eta'\pi^+)$. We will return to a discussion of theoretical developments since then at a later stage in this paper.

TABLE I. Measurements of $B(D_s^+\to\eta\pi^+)$ and $B(D_s^+\to\eta'\pi^+)$.

Expt.	$\dfrac{B(D_s^+\to\eta\pi^+)}{B(D_s^+\to\phi\pi^+)}$	$\dfrac{B(D_s^+\to\eta'\pi^+)}{B(D_s^+\to\phi\pi^+)}$
CLEO II [1]	$0.54\pm0.09\pm0.06$	$1.20\pm0.15\pm0.11$
E691 [3]	<1.5 at 90% C.L.	<1.3 at 90% C.L.
Mark II [5]	<2.5 at 90% C.L.	<1.9 at 90% C.L.
NA 14/2 [6]		$2.5\pm1.0\pm^{1.5}_{0.4}$
ARGUS [7]		$2.5\pm0.5\pm0.8$
PDG [8]	$(1.5\pm0.4)\%$[a]	$(3.7\pm1.2)\%$[a]

[a]This is the absolute branching ratio listed in [8].

The experimental situation regarding $D_s^+\to\eta\rho^+$, $\eta'\rho^+$, and $\phi\rho^+$ is summarized in Table II. Here the data are entirely from CLEO II [2].

In this paper we study Cabibbo-favored two-body decays of D_s^+ in the factorization scheme using two models: that of Bauer, Stech, and Wirbel [11–14] and that of Isgur, Scora, Grinstein, and Wise [15]. We use these models to calculate the relevant form factors.

The layout of the paper is as follows. Section II deals with preliminaries and the definition of the model. Section III deals with $D_s^+\to\eta\pi^+$ and $\eta'\pi^+$ problem. $D_s^+\to\eta\rho^+$, $\eta'\rho^+$, and $\phi\rho^+$ are described in Sec. IV. Section V deals with other decays of D_s^+, such as, $D_s^+\to\phi\pi^+$, $\bar{K}^0K^+$, $\bar{K}^0K^{*+}$, $\bar{K}^{0*}K^+$, and $\bar{K}^{*0}K^{*+}$. The results obtained are discussed in some length in Sec. VI. The details of some of the calculations are in the Appendix.

II. MODELS AND PRELIMINARIES

We use two different models to calculate the form factors, that of Bauer, Stech, and Wirbel [11–14] (BSW) and Isgur, Scora, Grinstein, and Wise [15] (ISGW).

We start with some preliminaries. The decay rate for $D_s^+\to PP$ (P=pseudoscalar meson) is given by

$$\Gamma(D_s^+\to PP)=\frac{|A|^2}{8\pi m_{D_s}^2}|\mathbf{p}| \tag{1}$$

where A is the weak decay amplitude we shall calculate and $|\mathbf{p}|$ the center-of-mass momentum in the final state.

If the weak decay amplitude for $D_s^+\to VP$ (V=vector meson) is written in the form

$$A(D_s^+\to VP)=a\,\varepsilon^*\cdot p_{D_s}\ , \tag{2}$$

then the decay rate is given by

$$\Gamma(D_s^+\to VP)=\frac{|a|^2}{8\pi m_V^2}|\mathbf{p}|^3\ . \tag{3}$$

If the weak decay amplitude for $D_s^+\to V_1V_2$ is *approximated* by

$$A(D_s^+\to V_1V_2)=b\,\varepsilon_1^*\cdot\varepsilon_2^* \tag{4}$$

then the decay rate is given by

TABLE II. Measurements of $B(D_s^+\to\eta\rho^+)$, $B(D_s^+\to\eta'\rho^+)$, and $B(D_s^+\to\phi\rho^+)$.

Expt.	$\frac{B(D_s^+\to\eta\rho^+)}{B(D_s^+\to\phi\pi^+)}$	$\frac{B(D_s^+\to\eta'\rho^+)}{B(D_s^+\to\phi\pi^+)}$	$\frac{B(D_s^+\to\phi\rho^+)}{B(D_s^+\to\phi\pi^+)}$
CLEO [2]	$2.86\pm0.38\pm^{0.36}_{0.38}$	$3.44\pm0.62\pm^{0.44}_{0.46}$	$1.86\pm0.26\pm^{0.29}_{0.40}$
ACCMOR [10]			<1.8
PDG [8]	$(7.9\pm2.1)\%$[a]	$(9.5\pm2.7)\%$[a]	$(5.2^{+1.4}_{-1.6})\%$[a]

[a]This is the absolute branching ratio listed in [8].

$$\Gamma(D_s^+\to V_1V_2)=\frac{|b|^2}{8\pi m_{D_s}^2}\left[2+\frac{(m_{D_s}^2-m_{V_1}^2-m_{V_2}^2)^2}{4m_{V_1}^2m_{V_2}^2}\right]|\mathbf{p}| \ . \tag{5}$$

The decay amplitude in (4) assumes that S waves dominate the decay. We argue in Sec. IV, in the discussion of $D_s^+\to\phi\rho^+$, that this is an adequate approximation for our purposes.

The effective weak Hamiltonian for Cabibbo-favored charm decays is (we are using the notation of Refs. [11–14])

$$H_W^{\text{eff}}=\frac{G_F}{\sqrt{2}}\cos^2\theta_C\{a_1(\bar{s}c)_H(\bar{u}d)_H+a_2(\bar{s}d)_H(\bar{u}c)_H\} \tag{6}$$

where $(\bar{s}c)$ stands for the color-singlet Dirac bilinear $\bar{s}\gamma_\mu(1-\gamma_5)c$ and the subscript H instructs us to treat this bilinear as the interpolating hadron field, that is, no further Fierz reordering in flavor and color need be done. a_1 and a_2 are the Wilson coefficients which we take to be given [11–14]: $a_1=1.2$ and $a_2=-0.5$ to -0.6. The Cabibbo angle θ_C is taken to be given by $\sin\theta_C=0.22$.

In considering decays involving η and η', we define

$$|\eta\rangle=|8\rangle\cos\theta_p-|0\rangle\sin\theta_p \ ,$$
$$|\eta'\rangle=|8\rangle\sin\theta_p+|0\rangle\cos\theta_p \ , \tag{7}$$

where the flavor singlet $|0\rangle$ and octet $|8\rangle$ are defined by

$$|0\rangle=\frac{1}{\sqrt{3}}|u\bar{u}+d\bar{d}+s\bar{s}\rangle \ ,$$
$$|0\rangle=\frac{1}{\sqrt{6}}|u\bar{u}+d\bar{d}-2s\bar{s}\rangle \ . \tag{8}$$

The mixing angle θ_p is now believed to be $\approx-19°$. For decays under consideration we also need the definitions

$$\langle\pi^+|(\bar{u}d)_H|0\rangle=f_\pi p_\pi^\mu$$

and

$$\langle\rho^+|(\bar{u}d)_H|0\rangle=\varepsilon^{*\mu}f_\rho m_\rho \ , \tag{9}$$

where $f_\pi=133$ MeV and $f_\rho=221$ MeV. The hadronic form factors entering our calculations are (we are following the convention of Refs. [11–14])

$$\langle P(p)|V_\mu|D_s^+(k)\rangle=\left\{(k+p)_\mu-\frac{(m_{D_s}^2-m_P^2)}{t}q_\mu\right\}F_1(t)+\frac{(m_{D_s}^2-m_p^2)}{t}q_\mu F_0(t),$$
$$\langle V(p)|V_\mu|D_s^+(k)\rangle=\varepsilon_{\mu\nu\rho\sigma}k^\nu p^\rho\varepsilon^{*\sigma}V(t) \ , \tag{10}$$
$$\langle V(p)|A_\mu|D_s^+(k)\rangle$$
$$=\varepsilon_\mu^*(m_{D_s}+m_V)A_1(t)-\frac{\varepsilon^*\cdot q}{(m_{D_s}+m_V)}(k+p)_\mu A_2(t)$$
$$-\frac{\varepsilon^*\cdot q}{t}(2m_V)q_\mu A_3(t)+\frac{\varepsilon^*\cdot q}{t}(2m_V)q_\mu A_0(t),$$

where $q_\mu=(k-p)_\mu$, $t=q^2$, and $A_3(0)=A_0(0)$. Of the four form factors A_i $(i=0,\ldots,3)$ only three are independent due to the relation

$$2m_VA_3(t)=(m_{D_s}+m_V)A_1(t)-(m_{D_s}-m_V)A_2(t) \ . \tag{11}$$

There are other ways (see, for example, [15]) of parametrizing the matrix elements shown in (10).

III. $D_s^+\to\eta\pi^+$ AND $\eta'\pi^+$

A. Factorization model calculation

In this section we have discussed the calculation of the decay amplitudes for $D_s^+\to\eta\pi^+$ and $\eta'\pi^+$ in the factorization scheme. We emphasize, as was noted in [9], that due to the conserved-vector-current (CVC) hypothesis applied to the current $\bar{u}\gamma_\mu d$, the W^+ annihilation amplitude vanishes. In the factorization approximation one gets (see, for example, [9])

$$A(D_s^+\to\eta\pi^+)$$
$$=\frac{G_F}{\sqrt{2}}\cos^2\theta_Ca_1\langle\pi^+|(\bar{u}d)_H|0\rangle\langle\eta|(\bar{s}c)_H|D_s^+\rangle$$
$$=\frac{G_F}{\sqrt{2}}\cos^2\theta_Ca_1f_\pi(m_{m_{D_s}}^2-m_\eta^2)$$
$$\times\left[\tfrac{2}{3}\right]^{1/2}\left[\cos\theta_p+\frac{1}{\sqrt{2}}\sin\theta_p\right]F_0^{D_s\eta}(m_\pi^2) \ . \tag{12}$$

In writing down (12) we have used the definitions introduced in (6)–(10) and we have assumed nonet symmetry, i.e., $s\bar{s}$ combinations are treated the same way independent of whether they belong to the flavor singlet $|0\rangle$ or the flavor octet $|8\rangle$.

Similarly,

$$A(D_s^+\to\eta'\pi^+)=\frac{G_F}{\sqrt{2}}\cos^2\theta_C a_1 f_\pi(m_{m_{D_s}}^2-m_{\eta'}^2)\times\frac{1}{\sqrt{3}}(-\sqrt{2}\sin\theta_p+\cos\theta_p)F_0^{D_s\eta'}(m_\pi^2)\ . \tag{13}$$

We draw attention to two obvious features of (12) and (13): First, increasing θ_p in magnitude, keeping it negative, has the effect of lowering $A(D_s^+\to\eta\pi^+)$ and raising $A(D_s^+\to\eta'\pi^+)$ and, second, the lower mass of η favors $A(D_s^+\to\eta\pi^+)$ over $A(D_s^+\to\eta'\pi^+)$ due to the kinematic factor $(m_{m_{D_s}}^2-m_\eta^2)$.

Since all factors in (12) and (13) are known, or calculable, we can calculate the absolute rates and branching ratios for $D_s^+\to\eta\pi^+$ and $\eta'\pi^+$ using (1). In the following section we have carried out the calculation of the form factor F_0 in two models, that of BSW [11–14] and ISGW [15].

B. Predictions for $B(D_s^+\to\eta\pi^+)$ and $B(D_s^+\to\eta'\pi^+)$ in the BSW and ISGW models

We begin with a discussion of the form factors $F_0^{D_s\eta}(t)$ and $F_0^{D_s\eta'}(t)$. The BSW model [11–14] calculates them at $t=0$ in an infinite momentum frame. As the calculation already appears in the references cited we simply quote the results using a monopole structure to extrapolate the form factors. However, since the form factors are needed at $t=m_\pi^2$, the exact formula used for extrapolation is immaterial. One finds [11] for the BSW model

$$\begin{aligned}F_0^{D_s\eta}(m_\pi^2)&\simeq F_0^{D_s\eta}(0)=0.72,\\ F_0^{D_s\eta'}(m_\pi^2)&\simeq F_0^{D_s\eta'}(0)=0.70\ .\end{aligned} \tag{14}$$

We are using two-figure accuracy.

The detailed calculation of $F_0(t_m)$ (t_m=maximum t) for $D_s^+\to\eta$ and $D_s^+\to\eta'$ transitions in the ISGW model [15] is given in Appendix A.

With $F_0(t_m)$ given in (A8) we extrapolate it down to $t=0$. The ISGW model [15], which uses oscillator wave functions, generates an exponential dependence of the form factors in t. We follow the practice of other authors [16] by assuming that the t dependence is governed by the nearest singularity; i.e., we assume a monopole form to extrapolate $F_0(t_m)$ down to any desired value of t, in particular, to $t=0$ by

$$F_0(0)=\left[1-\frac{t_m}{m_{0^+}^2}\right]F_0(t_m) \tag{15}$$

with $m_{0^+}=2.6$ GeV, the mass of a scalar meson with flavor content $(c\bar{s})$. We find for the ISGW model

$$F_0^{D_s\eta}(0)=0.62,\quad F_0^{D_s\eta'}(0)=0.75\ . \tag{16}$$

We emphasize that, throughout this paper, by "ISGW model" we mean that the calculation of the form factor at maximum momentum transfer is done according to Ref. [15], but the extrapolation in t is done by using a monopole formula. Using (14) and (16) in the rate formula (1) we have calculated the branching ratios $B(D_s^+\to\eta\pi^+)$ and $B(D_s^+\to\eta'\pi^+)$ and displayed them in Table III. Under the column marked "Experiment" we have used data from Ref. [1] along with the reference branching ratio $B(D_s^+\to\phi\pi^+)=(2.8\pm0.5)\%$ from Ref. [8]. In estimating $B(D_s^+\to\eta\pi^+)$ and $B(D_s^+\to\eta'\pi^+)$ from the ratios $B(D_s^+\to\eta\pi^+)/B(D_s^+\to\phi\pi^+)$ and $B(D_s^+\to\eta'\pi^+)/B(D_s^+\to\phi\pi^+)$ we have ignored the systematic errors.

TABLE III. Branching ratios for $D_s^+\to\eta\pi^+$ and $\eta'\pi^+$ in the BSW and ISGW models. Branching ratios are in percents.

Branching ratios	BSW	ISGW	Experiment
$B(D_s^+\to\eta\pi^+)$	2.22	1.65	$(1.51\pm0.37)^{a}$
$B(D_s^+\to\eta'\pi^+)$	2.29	2.63	$(3.36\pm0.73)^{a}$
$\dfrac{B(D_s^+\to\eta'\pi^+)}{B(D_s^+\to\eta\pi^+)}$	1.03	1.60	$(2.22\pm0.5)^{b}$

[a]Our estimate, using CLEO II data [1] and $B(D_s^+\to\phi\pi^+)=(2.8\pm0.5)\%$, ignoring systematic errors in $B(D_s^+\to\eta\pi^+)/B(D_s^+\to\phi\pi^+)$, etc.

[b]Our estimate, ignoring systematic errors and using CLEO II data [1].

Though the predictions of the BSW and ISGW models are within 30–40 % of each other, certain trends appear to be significant: the ISGW model produces a lower rate for $D_s^+\to\eta\pi^+$ than the BSW model and a higher one for $D_s^+\to\eta'\pi^+$. Consequently the ratio $B(D_s^+\to\eta'\pi^+)/(D_s^+\to\eta\pi^+)$ is more favorable in the ISGW model.

IV. $D_s^+\to\eta\rho^+,\ \eta'\rho^+,\ \phi\rho^+$

A. Factorization model calculation

Using the definitions introduced in (6)–(10) we obtain the following decay amplitudes in the factorization model:

$$A(D_s^+\to\eta\rho^+)=\frac{G_F}{\sqrt{2}}\cos^2\theta_C a_1(\varepsilon^*\cdot p_{D_s})f_\rho(2m_\rho)F_1^{D_s\eta}(m_\rho^2)\times\left[\frac{2}{3}\right]^{1/2}\left[\cos\theta_p+\frac{1}{\sqrt{2}}\sin\theta_p\right]\ , \tag{17}$$

$$A(D_s^+\to\eta'\rho^+)=\frac{G_F}{\sqrt{2}}\cos^2\theta_C a_1(\varepsilon^*\cdot p_{D_s})f_\rho(2m_\rho)F_1^{D_s\eta'}(m_\rho^2)\times\frac{1}{\sqrt{3}}(-\sqrt{2}\sin\theta_p+\cos\theta_p)\ , \tag{18}$$

$$A(D_s^+\to\phi\rho^+)=\frac{G_F}{\sqrt{2}}\cos^2\theta_C a_1 f_\rho m_\rho(m_{D_s}+m_\phi)\times(\varepsilon_\phi^*\cdot\varepsilon_\rho^*)A_1^{D_s\phi}(m_\rho^2)\ . \tag{19}$$

While in writing (17) and (18) only the factorization approximation has been used, we have used an additional approximation in writing (19): *We have retained only the S-wave part of the decay amplitude* that arises from the matrix element of the axial-vector current proportional to the form factor $A_1(t)$ of (10). The reason for neglecting the P-wave part [coming from the vector current and proportional to $V(t)$] and the D-wave part, proportional to $A_2(t)$, is that the coefficient of the $|A_1|^2$ term in the rate formula [17] is by far the largest. The only significant correction would come from the S-D interference. However, if $A_2(t)$ is indeed small as the experiments [18] suggest, then the interference term would also be small. The approximation of retaining only the S waves *overestimates* the rate for same sign A_1 and A_2 due to the fact that the S-D interference term has a negative sign [19]. However, for $A_2 \approx 0.5 A_1$ the approximation of retaining only the S-wave contribution is better than 10% and yet better if A_2 is still smaller.

B. Predictions for $B(D_s^+ \to \eta\rho^+)$, $B(D_s^+ \to \eta'\rho^+)$, and $B(D_s^+ \to \phi\rho^+)$ in the BSW model

Use of (17) and (18) in (2) and (3) gives us the rates for $D_s^+ \to \eta\rho^+$ and $\eta'\rho^+$. We need to know the form factor $F_1(m_\rho^2)$ which, if extrapolated with a single pole, is (the values of the form factors at $t=0$ are taken from [11])

$$F_1^{D_s\eta}(m_\rho^2) = \frac{0.72}{1 - m_\rho^2/m_{1^-}^2} = 0.83 \,, \tag{20}$$

where we have retained only a two-figure accuracy. The 1^- pole in this case is at 2.11 GeV, the D_s^+ mass:

$$F_1^{D_s\eta'}(m_\rho^2) = \frac{0.70}{1 - m_\rho^2/m_{1^-}^2} = 0.81 \,, \tag{21}$$

$$F_1^{D_s\phi}(m_\rho^2) = \frac{0.82}{1 - m_\rho^2/m_{1^+}^2} = 0.90 \,, \tag{22}$$

where the 1^+ pole is at 2.53 GeV, the D_s mass. Using $f_\rho = 221$ MeV and $\theta_p = -19°$, we get the branching ratios shown in Table IV.

The following comments on the BSW predictions are in order: (i) $B(D_s^+ \to \eta\rho^+)$ at 4.23% is lower than the CLEO II result shown in Table II if we use $B(D_s^+ \to \phi\pi^+) = (2.8 \pm 0.5)\%$; (ii) $B(D_s^+ \to \eta'\rho^+)$ at 2.12% is well below the CLEO II result perhaps by a factor of 5; (iii) $B(D_s^+ \to \phi\rho^+)$ at 21.4% is too high by a factor of about 3 to 4. The problem with the high rate prediction by the BSW model for $D_s^+ \to \phi\rho^+$ is well known and of long standing (see, for example, Ref. [17]). If we use $A_1^{D_s\phi}(0) = 0.45$ [18], the prediction for $B(D_s^+ \to \phi\rho^+)$ is scaled down to 6.44% in accord with CLEO II data. This is a supporting evidence for a smaller value of $A_1^{D_s\phi}(0)$ than that predicted by BSW model. We will return to an extensive discussion of $D_s^+ \to \phi\rho^+$ in Sec. VI.

TABLE IV. Branching ratios for $D_s^+ \to \eta\rho^+$ and $\eta'\rho^+$ and $\phi\rho^+$ in the BSW and ISGW models. Branching ratios are in percents.

Branching ratios	BSW	ISGW	Experiment
$B(D_s^+ \to \eta\rho^+)$	4.23	3.55	(8.00 ± 1.8)[a]
$B(D_s^+ \to \eta'\rho^+)$	2.12	3.71	(9.63 ± 2.44)[a]
$B(D_s^+ \to \phi\rho^+)$	21.4	19.9	(5.21 ± 1.18)[a]
$\dfrac{B(D_s^+ \to \eta'\rho^+)}{B(D_s^+ \to \eta\rho^+)}$	0.50	1.05	(1.20 ± 0.27)[b]

[a]Our estimate, using CLEO II data [2] and $B(D_s^+ \to \phi\pi^+) = (2.8 \pm 0.5)\%$ and ignoring systematic errors in $B(D_s^+ \to \eta\rho)/B(D_s^+ \to \phi\pi^+)$, etc.

[b]Our estimate, ignoring systematic errors and using CLEO II data [2].

C. Predictions for $B(D_s^+ \to \eta\rho^+)$, $B(D_s^+ \to \eta'\rho^+)$, and $B(D_s^+ \to \phi\rho^+)$ in the ISGW model

The form factor $f_+(t)$ of ISGW [15] is identical to $F_1(t)$ of BSW. In describing $D_s^+ \to \eta\rho^+$ and $\eta'\rho^+$ we encounter the mock form factor $\tilde{f}_+(\tilde{t}_m)$ [15] which we assume to be identical to the physical form factor $f_+(t_m)$ which, in turn, is identical to $F_1(t_m)$. Following [15] we obtain $\tilde{f}_+(\tilde{t}_m)$ for $D_s^+ \to \eta\rho^+$ *and* $D_s^+ \to \eta'\rho^+$ (since the mock η mass is the same as the mock η' mass): $\tilde{f}_+(\tilde{t}_m) = f_+(t_m) = 1.21$. This value is then continued to an arbitrary t using a monopole form. In particular, at $t=0$,

$$F_1(0) = (1 - t_m/m_{1^-}^2) F_1(t_m) \tag{23}$$

yields for the ISGW model

$$\begin{aligned} &F_1^{D_s\eta}(0) = 0.66 \quad (\text{cf. BSW: } 0.72) \,, \\ &F_1^{D_s\eta'}(0) = 0.93 \quad (\text{cf. BSW: } 0.70) \,. \end{aligned} \tag{24}$$

The difference between $F_1^{D_s\eta}(0)$ and $F_1^{D_s\eta'}(0)$ is due to the difference in t_m in (23). Contrast the values of the form factors in (24) with those of the BSW model. A consequence is that the use of the ISGW form factor results in a $\Gamma(D_s^+ \to \eta\rho^+)$ *lower* than that in the BSW model which, in turn, was already *lower* than CLEO II data. On the other hand, the ISGW form factor yields a higher (by 75%) rate for $D_s^+ \to \eta'\rho^+$ than the BSW model. However, the prediction is still a factor of ≈ 2.5 *below* CLEO II data. Though the absolute rates for *both* $D_s^+ \to \eta\rho^+$ and $D_s^+ \to \eta'\rho^+$ are predicted too low in the ISGW model, the ratio $B(D_s^+ \to \eta'\rho^+)/B(D_s^+ \to \eta\rho^+)$ is consistent with CLEO II data. The results are displayed in Table IV.

We now discuss $D_s^+ \to \phi\rho^+$ in the ISGW model. The ISGW model calculates a mock form factor $\tilde{f}(\tilde{t}_m)$ (see [15]) at the mock maximum momentum transfer $\tilde{t}_m$. The form factor $A_1(t)$ needed here [see (19)] is related to $\tilde{f}(\tilde{t}_m)$ by

$$\tilde{f}(\tilde{t}_m) = (\tilde{m}_{D_s} + \tilde{m}_\phi) \tilde{A}_1(\tilde{t}_m) \,, \tag{25}$$

$$A_1(t_m) = \tilde{A}_1(\tilde{t}_m) \,, \tag{26}$$

where $\tilde{m}_\phi$, etc. are the mock masses, i.e., the sum of the constituent quark masses, and $A_1(t_m)$ is the physical form factor at the physical maximum momentum transfer. The ISGW model yields [15]

$$\tilde{f}^{D_s\phi}(\tilde{t}_m)=2\tilde{m}_{D_s}\tilde{F}_3^{D_s\phi}(\tilde{t}_m)=2(\tilde{m}_{D_s}\tilde{m}_\phi)^{1/2}=3.23 \text{ GeV} , \tag{27}$$

where $\tilde{F}_3^{D_s\phi}(\tilde{t}_m)$ has been defined in (A13) and use made of universal β's. The resulting value of $A_1(t_m)$ is

$$A_1^{D_s\phi}(t_m)=0.92 . \tag{28}$$

Extrapolated to $t=0$ using a monopole formula we get ($m_{1^+}=2.53$ GeV)

$$A_1^{D_s\phi}(0)=(1-t_m/m_{1^+}^2)A_1^{D_s\phi}(t_m)=0.79 . \tag{29}$$

Contrasting this to the BSW model value of 0.82 we conclude that the ISGW and BSW models predict the same rate for $D_s^+\to\phi\rho^+$. The results are summarized in Table IV.

The prediction of the form factor $A_1^{D_s\phi}(0)$ in both models is too high by almost a factor of 2. A word of caution: The prediction of $B(D_s^+\to\phi\rho^+)$ is approximate due to the fact that we have kept only the S-wave contribution in the final state. Our estimate, as we commented at the end of Sec. IV A, is an *overestimate,* but has an accuracy of better than 10% for $A_2(0)\le 0.5A_1(0)$.

V. OTHER D_s^+ DECAYS

A. $D_s^+\to\phi\pi^+$

Using the factorization model with the definitions in (1) and (10), we find

$$A(D_s^+\to\phi\pi^+)=\frac{G_F}{\sqrt{2}}\cos^2\theta_C a_1 f_\pi(2m_\phi)\varepsilon^*\cdot p_{D_s}A_0^{D_s\phi}(m_\pi^2)$$
$$\equiv a(D_s^+\to\phi\pi^+)\varepsilon^*\cdot p_{D_s} . \tag{30}$$

The appearance of $A_0(t)$ is due to the fact that the divergence of the axial-vector current, rather than the current itself, enters our considerations. The rate is then calculated using (2) and (3).

In the BSW model $A_0^{D_s\phi}(m_\pi^2)$ is given by [11]

$$A_0^{D_s\phi}(m_\pi^2)\simeq A_0^{D_s\phi}(0)=0.70 . \tag{31}$$

The calculated branching ratio is

$$B(D_s^+\to\phi\pi^+)=3.05\% , \tag{32}$$

in excellent agreement with experiment: $B(D_s^+\to\phi\pi^+)=(2.8\pm0.5)\%$ [8].

In the Appendix we have discussed the evaluation of $A_0(t_m)$ in the ISGW model in some detail. We use $A_0(t_m)=0.99$, as given in (A14), and extrapolate it to $t=0$ using a monopole formula (we use $m_{1^+}=2.53$ GeV),

$$A_0^{D_s\phi}(m_\pi^2)\simeq A_0^{D_s\phi}(0)=(1-t_m/m_{1^+}^2)A_0^{D_s\phi}(t_m)=0.85 , \tag{33}$$

which yields

$$B(D_s^+\to\phi\pi^+)=4.5\% . \tag{34}$$

This is somewhat higher than the experimental branching ratio: $B(D_s^+\to\phi\pi^+)=(2.8\pm0.5)\%$ [8].

B. $D_s^+\to\bar{K}^0K^+$

The decay amplitude for this process is proportional to the Wilson coefficient a_2. In the spectator model the decay amplitude is

$$A(D_s^+\to\bar{K}^0K^+)=\frac{G_F}{\sqrt{2}}\cos^2\theta_C a_2 f_K(m_{m_{D_s}}^2-m_K^2)F_0(m_K^2) , \tag{35}$$

where we use $f_K=161$ MeV.

In the BSW model with $m_{0^+}=2.47$ GeV and $F_0^{D_sK}(0)=0.64$ [11] we get

$$F_0^{D_sK}(m_K^2)=\frac{0.64}{1-m_K^2/m_{0^+}^2}=0.67 , \tag{36}$$

which leads to a BSW model prediction of

$$B(D_s^+\to\bar{K}^0K^+)=\begin{cases}1.41\% & (a_2=-0.5),\\ 2.03\% & (a_2=-0.6) .\end{cases} \tag{37}$$

The value with $a_2=-0.6$ is close to the experimental value of $(2.8\pm0.7)\%$ [8].

Following the procedure outlined in the Appendix we can calculate $F_0^{D_sK}(t_m)$ in the ISGW model through Eq. (A4) with the result

$$F_0^{D_sK}(t_m)=1.12 , \tag{38}$$

which, when extrapolated down to $t=0$ using a monopole formula with a pole at $m_{0^+}=2.47$ GeV, yields

$$F_0^{D_sK}(0)=0.72 . \tag{39}$$

As this value is somewhat higher than the BSW prediction of 0.64, the ISGW model predicts a little larger branching ratio:

$$B(D_s^+\to\bar{K}^0K^+)=\begin{cases}1.79\% & (a_2=-0.5),\\ 2.57\% & (a_2=-0.6) .\end{cases} \tag{40}$$

The prediction with $a_2=-0.6$ is in agreement with data and the difference between BSW and ISGW model predictions is hardly significant.

C. $D_s^+\to\bar{K}^0K^{*+}$

The decay amplitude for this mode is also proportional to a_2. In the spectator model the decay amplitude is given by

$$A(D_s^+\to\bar{K}^0K^{*+})=\frac{G_F}{\sqrt{2}}\cos^2\theta_C a_2 f_K(2m_{K^*})$$
$$\times A_0^{D_sK^*}(m_K^2)\varepsilon^*\cdot p_{D_s}$$
$$\equiv a(D_s^+\to\bar{K}^0K^{*+})\varepsilon^*\cdot p_{D_s} . \tag{41}$$

The rate is then generated by (2) and (3).

In the BSW model, $A_0^{D_sK^*}(0)=0.63$ [11] and a monopole extrapolation to $t=m_K^2$ gives

$$A_0^{D_sK^*}(m_K^2)=\frac{0.63}{1-m_K^2/m_D^2}=0.68\ . \tag{42}$$

The resulting branching ratio in the BSW model is

$$B(D_s^+\to\bar{K}^0K^{*+})=\begin{cases}0.64\% & (a_2=-0.5),\\ 0.92\% & (a_2=-0.6)\ .\end{cases} \tag{43}$$

Both of these values are considerably *lower* than the experimental value [8]: $B(D_s^+\to\bar{K}^0K^{*+})=(3.3\pm0.9)\%$.

In the ISGW model the calculation of $A_0^{D_sK^*}(t_m)$ parallels that of $A_0^{D_s\phi}(t_m)$ given in the Appendix. Using (A14) with appropriate changes in masses, and using universal β's, we get

$$A_0^{D_sK^*}(t_m)=1.38\ . \tag{44}$$

On extrapolating down to $t=0$ using a monopole formula with a D-meson pole, we get

$$A_0^{D_sK^*}(0)=0.92\ . \tag{45}$$

This is considerably larger than the BSW model of 0.63. The ISGW model, therefore, predicts

$$B(D_s^+\to\bar{K}^0K^{*+})=\begin{cases}1.35\% & (a_2=-0.5),\\ 1.95\% & (a_2=-0.6)\ .\end{cases} \tag{46}$$

Though both of these values are also smaller than the experimental value, $(3.3\pm0.9)\%$, they are significantly higher than the corresponding predictions in the BSW model.

D. $D_s^+\to\bar{K}^{*0}K^+$

The decay amplitude in the spectator model is

$$\begin{aligned}A(D_s^+\to\bar{K}^{*0}K^+)&=\frac{G_F}{\sqrt{2}}\cos^2\theta_C a_2(2m_{K^*})\\ &\quad\times f_{K^*}F_1^{D_sK}(m_{K^*}^2)\varepsilon^*\cdot p_{D_s}\\ &\equiv a(D_s^+\to\bar{K}^{*0}K^+)\varepsilon^*\cdot p_{D_s}\ .\end{aligned} \tag{47}$$

We use $f_{K^*}=f_\rho=221$ MeV. In the BSW model $F_1^{D_sK}(0)=0.64$ which continued to $t=m_K^2$, gives

$$F_1^{D_sK}(m_{K^*}^2)=\frac{0.64}{1-m_{K^*}^2/m_{D^*}^2}=0.80\ . \tag{48}$$

The resulting branching ratio in the BSW model is

$$B(D_s^+\to\bar{K}^{*0}K^+)=\begin{cases}1.65\% & (a_2=-0.5),\\ 2.37\% & (a_2=-0.6)\ .\end{cases} \tag{49}$$

The experimental value is [8] $B(D_s^+\to\bar{K}^{*0}K^+)=(2.6\pm0.5)\%$ which is in agreement with the BSW prediction with $a_2=-0.6$.

The formula for $F_1^{D_sK}(t_m)$ in the ISGW model appears in Ref. [15]. With universal β's, one obtains

$$F_1^{D_sK}(t_m)=\tilde{F}_1^{D_sK}(\tilde{t}_m)=1.48\ , \tag{50}$$

which when extrapolated down to $t=0$ using a monopole formula yields

$$F_1^{D_sK}(0)=(1-t_m/m_{D^*}^2)F_1^{D_sK}(t_m)=0.68\ . \tag{51}$$

This is only slightly larger than the BSW model value of 0.64 [11]. The resulting branching ratio in the ISGW model is

$$B(D_s^+\to\bar{K}^{*0}K^+)=\begin{cases}1.87\% & (a_2=-0.5),\\ 2.68\% & (a_2=-0.6)\ .\end{cases} \tag{52}$$

Both the BSW and ISGW models do equally well in predicting $B(D_s^+\to\bar{K}^{*0}K^+)$, and the agreement with the experimental value [8] of $(2.6\pm0.5)\%$ is particularly good with $a_2=-0.6$.

E. $D_s^+\to\bar{K}^{*0}K^{*+}$

The decay amplitude in spectator model is given by

$$\begin{aligned}&A(D_s^+\to\bar{K}^{*0}K^{*+})\\ &\quad=\frac{G_F}{\sqrt{2}}\cos^2\theta_C m_{K^*}f_{K^*}(m_{D_s}+m_{K^*})\\ &\qquad\times a_2A_1^{D_sK^*}(m_{K^*}^2)(\varepsilon_{K^*}^*\cdot\varepsilon_{K^*}^*)\\ &\quad\equiv b(D_s^+\to\bar{K}^{*0}K^{*+})(\varepsilon_{K^*}^*\cdot\varepsilon_{K^*}^*)\ ,\end{aligned} \tag{53}$$

where we have kept only S waves in the final state. The rate is generated by Eq. (5). The BSW model calculates [11] $A_1^{D_sK^*}(0)=0.72$ which when extrapolated to $t=m_{K^*}^2$ using a monopole formula with an axial-vector pole with flavor content $c\bar{u}$, $m_{1^+}=2.42$ GeV, yields

$$A_1^{D_sK^*}(m_{K^*}^2)=\frac{0.72}{1-m_{K^*}^2/m_{1^+}^2}=0.83\ . \tag{54}$$

The resulting branching ratio is

$$B(D_s^+\to\bar{K}^{*0}K^{*+})=\begin{cases}4.05\% & (a_2=-0.5),\\ 5.83\% & (a_2=-0.6)\ .\end{cases} \tag{55}$$

The Particle Data Group lists [8] $B(D_s^+\to\bar{K}^{*0}K^{*+})=(5.0\pm1.7)\%$, in good agreement with the BSW model prediction.

In the ISGW model the form factor $f(t)$ (see Ref. [15]) is related to $A_1(t)$ by the analogue of Eq. (25). We find [see (27) for its analogue in $D_s^+\to\phi$ transition]

$$\tilde{f}^{D_sK^*}(\tilde{t}_m)=2.89\ \text{GeV}\ . \tag{56}$$

Using the analogues of (25) and (26), we obtain

$$A_1^{D_sK^*}(t_m)=0.89\ . \tag{57}$$

An extrapolation down to $t=0$ using a monopole formula with a pole mass $m_{1^+}=2.42$ GeV with flavor content $c\bar{u}$ yields

$$A_1^{D_sK^*}(0)=0.71 \ . \tag{58}$$

This agrees with the BSW model evaluation of 0.72 and the resulting branching ratios in the ISGW model are the same as in the BSW model.

In the following section we discuss the results we have obtained in this paper.

VI. DISCUSSION

We preface the following discussion by making our assumptions clear. We have used the factorization scheme and, within that scheme, the BSW and ISGW models to calculate the form factors. The BSW model calculates these form factors at $t=0$ while the ISGW model calculates them at $t_{\max}$. We then make an extra assumption of pole dominance of these form factors to extrapolate them to the required value of t.

For ready reference we have tabulated all of the predictions discussed in this paper in Table V. In the following we discuss the theoretical predictions by classifying the processes by the form factors that enter their description.

A. $D_s^+ \to \eta\pi^+$, $\eta'\pi^+$, and $\bar{K}^0K^+$

All these decays involve the form factor $F_0(t)$. From Table V it is evident that the ISGW model does fairly well in predicting all these branching ratios. The key to its success lies in the correct prediction of the *scale* of $F_0(t)$. Intuitively, since η' is heavier than η, we expect a larger overlap of the η' wave function with D_s^+ wave function than in the corresponding case with the η wave function. Thus, intuitively, we expect $F_0^{D_s\eta'}(0) > F_0^{D_s\eta}(0)$ and the ISGW model gives that result.

In the BSW model $B(D_s^+ \to \bar{K}^0K^+)$ is predicted quite well. However, $B(D_s^+ \to \eta\pi^+)$ is predicted too high and $B(D_s^+ \to \eta'\pi^+)$ too low. By making some extra assumptions, which we discuss below, it is possible to bring the BSW model prediction into agreement with CLEO II data [1]. However, we will soon discredit these assumptions when we discuss $D_s^+ \to \eta\rho^+$ and $\eta'\rho^+$.

There are at least two ways in which one can lower the rate for $D_s^+ \to \eta\pi^+$ and raise that for $D_s^+ \to \eta'\pi^+$. One involves nonet symmetry breaking [20] and the other, introduction of an annihilation amplitude [21]. In the scheme where nonet symmetry is relaxed, $|s\bar{s}\rangle$ in the flavor-singlet case is treated differently compared to $|s\bar{s}\rangle$ in the flavor-octet case. Thus Eqs. (12) and (13) modify to

$$A(D_s^+ \to \eta\pi^+) \propto \left[g_0\cos\theta_p + \frac{g_8}{\sqrt{2}}\sin\theta_p \right] , \tag{59}$$

$$A(D_s^+ \to \eta'\pi^+) \propto (-\sqrt{2}g_0\sin\theta_p + g_8\cos\theta_p) , \tag{60}$$

with $g_0^2+g_8^2=2$. If $g_0=g_8=1$, we recover the nonet-symmetry result of (12) and (13). By allowing $g_8 > g_0$ we can lower $A(D_s^+ \to \eta\pi^+)$ and raise $A(D_s^+ \to \eta'\pi^+)$.

In the scheme discussed by Lipkin [21], an annihilation term is introduced in the following way: First, express η-η' mixing in a different basis as

$$\begin{aligned} |\eta\rangle &= \sin\alpha|\sigma\rangle - \cos\alpha|s\bar{s}\rangle , \\ |\eta'\rangle &= \cos\alpha|\sigma\rangle + \sin\alpha|s\bar{s}\rangle , \end{aligned} \tag{61}$$

where $|\sigma\rangle = (1/\sqrt{2})|u\bar{u}+d\bar{d}\rangle$ and $\alpha = (35.3° - \theta_p)$. While the $s\bar{s}$ component of η and η' participates in the spectator $c \to s$ transitions, the σ component contributes to the annihilation amplitude. Thus one can write, in the notation of Ref. [21],

$$\begin{aligned} A(D_s^+ \to \eta\pi^+) &\propto (T\cos\alpha - A\sin\alpha) , \\ A(D_s^+ \to \eta'\pi^+) &\propto (T\sin\alpha + A\cos\alpha) , \end{aligned} \tag{62}$$

with $|T|^2+|A|^2=$const. Clearly, by following $T\neq 0$ one can decrease $A(D_s^+ \to \eta\pi^+)$ and raise $A(D_s^+ \to \eta'\pi^+)$.

Later in discussing $D_s^+ \to \eta\rho^+$ and $\eta'\rho^+$ we will argue to discredit these two scenarios to make BSW model predictions for $D_s^+ \to \eta\pi^+$ and $\eta'\pi^+$ to agree with experiments. We believe that the problem is that of *scale*, that is, $F_0^{D_s\eta'}(0)$ is predicted too low in the BSW model.

TABLE V. Branching ratios in percent calculated in the BSW and ISGW models. Predictions use $a_1=1.2$ and $a_2=-0.6$.

Branching ratios	Relevant form factor and Wilson coeff.	BSW	ISGW	Experiment
$B(D_s^+ \to \phi\pi^+)$	a_1A_0	3.05	4.5	(2.8 ± 0.5)[a]
$B(D_s^+ \to \eta\pi^+)$	a_1F_0	2.22	1.65	(1.51 ± 0.37)[b]
$B(D_s^+ \to \eta'\pi^+)$	a_1F_0	2.29	2.63	(3.36 ± 0.73)[b]
$B(D_s^+ \to \eta\rho^+)$	a_1F_1	4.23	3.55	(8.00 ± 1.8)[b]
$B(D_s^+ \to \eta'\rho^+)$	a_1F_1	2.12	3.71	(9.63 ± 2.44)[b]
$B(D_s^+ \to \phi\rho^+)$	a_1A_1	21.4	19.9	(5.21 ± 1.18)[a]
$B(D_s^+ \to \bar{K}^0K^+)$	a_2F_0	2.03	2.57	(2.8 ± 0.7)[a]
$B(D_s^+ \to \bar{K}^0K^{*+})$	a_2A_0	0.92	1.95	(3.3 ± 0.9)[a]
$B(D_s^+ \to \bar{K}^{*0}K^+)$	a_2F_1	2.37	2.68	(2.6 ± 0.5)[a]
$B(D_s^+ \to \bar{K}^{*0}K^{*+})$	a_2A_1	5.83	5.76	(5.0 ± 1.7)[a]

[a]Source: PDG [8].
[b]Source: Our estimate, as explained in footnotes to Tables III and IV.

B. $D_s^+ \to \eta\rho^+$, $\eta'\rho^+$, and $\bar{K}^{*0}K^+$

All these decays involve the form factor $F_1(t)$. At the very outset it is evident from Table V that both BSW and ISGW models predict $B(D_s^+ \to \bar{K}^{*0}K^+)$ well. It is also evident that the ISGW model does well in predicting the ratio $B(D_s^+ \to \eta'\rho^+)/B(D_s^+ \to \eta\rho^+)$ while the BSW model does not. An important feature to note is that *both* the BSW and ISGW models produce too low a branching ratio for *both* $D_s^+ \to \eta\rho^+$ and $\eta'\rho^+$.

The success of the ISGW model in predicting $B(D_s^+ \to \eta'\rho^+)/B(D_s^+ \to \eta\rho^+)$ correctly is due to the fact that it predicts $F_0^{D_s\eta'}(0) > F_0^{D_s\eta}(0)$, as one intuitively expects, while in the BSW model one gets $F_0^{D_s\eta'}(0) \approx F_0^{D_s\eta}(0)$.

Let us now return to the discussion of the two mechanisms, nonet symmetry breaking [20] and the annihilation mechanism [21], which patched up the difference between the BSW model prediction and CLEO II data for $D_s^+ \to \eta\pi^+$ and $\eta'\pi^+$. When applied to $D_s^+ \to \eta\rho^+$ and $\eta'\rho^+$, these mechanisms fail because *both* these rates are predicted to be lower than the data in *both* BSW and ISGW models. The mechanisms introduced in [20] and [21] tend to raise one rate and lower the other. Applied to $D_s^+ \to \eta\rho^+$ and $\eta'\rho^+$, since both rates are predicted lower than the data, these mechanisms will make one of the rates, which is already too low, even lower. Thus, we do not believe that the introduction of nonet-symmetry breaking or of the annihilation mechanism is the answer to the $D_s^+ \to \eta\pi^+$, $\eta'\pi^+$, $\eta\rho^+$, and $\eta'\rho^+$ problem.

C. $D_s^+ \to \phi\pi$ and $\bar{K}^0K^{*+}$

Both these decays depend on $A_0(t)$. From Table V it is seen that the BSW model predicts $B(D_s^+ \to \phi\pi^+)$ very well indeed. The ISGW model prediction is 50% higher. However, both these models predict $B(D_s^+ \to \bar{K}^0K^{*+})$ too low, especially the BSW model. The fact that the BSW model prediction for $D_s^+ \to \bar{K}^0K^{*+}$ is too low comes as a bit of a surprise to us considering that it generates $A_0^{D_s\phi}(0) \approx A_0^{D_sK^*}(0)$ as one might expect. The ISGW model generates $A_0^{D_s\phi}(0)$ larger than that in the BSW model, but it also generates $A_0^{D_sK^*}(0) > A_0^{D_s\phi}(0)$, leading to a larger rate for $D_s^+ \to \bar{K}^0K^{*+}$ than the BSW model.

D. $D_s^+ \to \phi\rho^+$ and $\bar{K}^{*0}K^{*+}$

Both these decays depend on $A_1(t)$. These decays are very puzzling. The decay amplitude for $D_s^+ \to \phi\rho^+$ is proportional to a_1 while that for $D_s^+ \to \bar{K}^{*0}K^{*+}$ is proportional to a_2. As the masses involved in the final state are comparable, one does not expect $A_1^{D_s\phi}(m_\rho^2)$ to be drastically different from $A_1^{D_sK^*}(m_{K^*}^2)$. The final-state momenta are almost the same. Hence theory would suggest $B(D_s^+ \to \phi\rho^+) \gg B(D_s^+ \to \bar{K}^{*0}K^{*+})$ simply from $|a_1| > |a_2|$. Yet experimentally [8] $B(D_s^+ \to \phi\rho^+) \approx B(D_s^+ \to \bar{K}^{*0}K^{*+})$. From Table V we see that both BSW and ISGW models predict $B(D_s^+ \to \phi\rho^+) \approx 20\%$ and $B(D_s^+ \to \bar{K}^{*0}K^{*+}) \approx 6\%$, the latter in agreement with experiment. This implies that both BSW and ISGW models calculate $A_1^{D_sK^*}(m_{K^*}^2)$ quite reliably, yet overestimate $A_1^{D_s\phi}(m_\rho^2)$.

Evidence that the experimental value of $A_1^{D_s\phi}(0)$ is much lower than the theoretical predictions comes from the CLEO [22] and ARGUS [23] measurement of $B(D_s^+ \to \phi e^+\nu)$. The corresponding evidence for a lower $A_1^{DK^*}(0)$ comes from the E691 [18] measurement of $B(D^+ \to \bar{K}^{*0}e^+\nu)$. E691 measurement of $A_1^{DK^*}(0)$ is $0.46 \pm 0.05 \pm 0.05$, while the BSW model predicts it to be ≈ 0.9 and the ISGW model ≈ 0.8 [18]. For further comparison of experiment with theory, see Ref. [24]. Indeed, if we were to scale down $A_1^{D_s\phi}(0)$ to ≈ 0.45 we would reproduce the experimental value of $B(D_s^+ \to \phi\rho^+)$, but a similar down-scaling of $A_1^{D_sK^*}(0)$ results in too low a prediction for $B(D_s^+ \to \bar{K}^{*0}K^{*+})$ [17].

Supporting evidence for $A_1^{D_s\phi}(0) \approx A_1^{DK^*}(0) \approx 0.45$ comes from $B(D_s^+ \to \phi e^+\nu)$ as argued below. If $A_1^{D_s\phi}(0) \approx A_1^{DK^*}(0)$, then one can show that [25]

$$\frac{B(D_s^+ \to \phi e^+\nu)}{B(D_s^+ \to \bar{K}^{*0}e^+\nu)} \approx 0.9\frac{\tau_{D_s^*}}{\tau_{D^+}} = 0.38\ . \tag{63}$$

On using $B(D_s^+ \to \bar{K}^{*0}e^+\nu) = (4.1 \pm 0.6)\%$ [8], (63) leads to $B(D_s^+ \to \phi e^+\nu) \simeq 1.6\%$ in agreement with CLEO [22] and ARGUS [23] data.

Similar conclusions about the size of $A_1^{D_s\phi}(0)$ have also been reached by Körner *et al.* [26].

The decays of D_s^+ we have discussed here have also been discussed at length in a recent paper by Pham [25] where factorization and flavor symmetry is used to relate D_s^+ decays to D decays. Pham [25] concludes that the decays $D_s^+ \to \eta'\pi^+$, $\eta\rho^+$, and $\eta'\rho^+$ are problematic, in that within the scheme of [25], theory cannot accommodate CLEO II data [1,2]. Further, semileptonic $D_s^+ \to \phi e^+\nu$ and $D \to K^*e^+\nu$ are consistent with $A_1(0) \approx 0.45$, much lower than the theoretical prediction of BSW and ISGW models. The same $A_1(0)$ is also consistent with $B(D_s^+ \to \phi\rho^+)$. We have found that the observed branching ratios for $D_s^+ \to \eta\rho^+$ and $\eta'\rho^+$ pose a problem for *both* BSW and ISGW models. $D_s^+ \to \eta\pi^+$ and $\eta'\pi^+$ are reasonably well explained in the ISGW model though they pose a problem for the BSW model.

As we discuss below, the evidence for the factorization assumption in D decays appears to be sound [27,28] albeit with some exceptions, notably $D^0 \to K^-a_1^+$, and a scaled-down form factor works well for $D^+ \to K^{*0}e^+\nu$ and $D^0 \to K^{*0}\rho^+$ (see Table X in [17]) and appears to work well for $D_s^+ \to \phi e^+\nu$ and $D_s^+ \to \phi\rho^+$ (see Table X in [17]) but *not* for $D_s^+ \to \bar{K}^{*0}K^{*+}$, which does not appear to need a scaled-down $A_1^{D_sK^*}(0)$.

E. Role of final-state interactions (FSI)

Now a word about the role of final-state interactions (FSI's) in D_s^+ decays: Could FSI's help us understand the rates for $D_s^+ \to (\eta,\eta')\pi^+$, $(\eta,\eta')\rho^+$, and $\phi\rho^+$ within the factorization model with the form factors calculated in BSW or ISGW models?

First, how good is factorization? In B two-body hadronic decays factorization has successfully been tested [29] at the 30% level. We also emphasize that for kinematic reasons FSI's do not play an important role in B hadronic decays—the final state hadrons, being energetic, escape the strong interaction region very quickly, thereby minimizing the effects of FSI's.

More recently [27,28] factorization has been tested in hadronic D decays. In [28] we have tested the factorization assumption in $D^0 \to K^-\rho^+$, $K^-a_1^+$, and $K^-\pi^+$ decays by comparing their branching ratios with $B(D^0 \to K^-e^+\nu)$, and in $D^0 \to \pi^+\pi^-$ by comparing its branching ratio with $B(D^0 \to K^-e^+\nu)$. We found that with the exception of $D^0 \to K^-a_1^+$ (see also [27]) the factorization scheme works satisfactorily but only once it is supplemented by FSI phases. These phases are known experimentally in $D \to K\rho$ and $K\pi$ decays [30] and are reasonably well bounded by experiments in $D \to \pi\pi$ decays [31]. In all the cases the final state can be in two isospin states, leading to an interference between the two amplitudes.

In Cabibbo-favored D_s^+ hadronic decays the final state involves only a single isospin, $I=1$. Thus interference between two isospin amplitudes which was important in D^0 decays does not play a role in D_s^+ decays. This does not mean that inelastic FSI's may not play a role. In principle $\eta\pi^+$, $\eta'\pi^+$, $\bar{K}^{*0}K^{*+}$ (S wave), and $\phi\rho^+$ (S wave), all having odd G parity, could couple; and similarly $\eta\rho^+$, $\eta'\rho^+$, and $\phi\pi^+$ with even G parity could couple among themselves. Though it is possible [32] to set up a coupled-channel scheme within the factorization model in such a manner that much of $\phi\rho^+$ branching fraction is channeled to $\bar{K}^{*0}K^{*+}$ and $\eta'\pi^+$ modes, such calculations are not reliable as there is no guidance as to what FSI parameters one ought to use.

Returning now to the reliability of the factorization model (by this, we emphasize, we do not mean the reliability of BSW or ISGW models, which are models to calculate form factors within the factorization scheme), we believe that strong evidence in favor of the factorization scheme in D_s^+ decays comes from comparing $D_s^+ \to \phi e^+\nu$ with $D_s^+ \to \phi\rho^+$ and $D^0 \to K^{*-}e^+\nu$ with $D^0 \to K^{*-}\rho^+$. One finds that the same scale for $A_1(0)$ fits the hadronic and semileptonic data. The problem, it appears, is not that the factorization model fails in these cases but rather that the two models, those of BSW and ISGW, fail in estimating the scale of $A_1(0)$ correctly.

Lastly, we find it puzzling that $D_s^+ \to \phi\rho^+$ is observed at the same rate as $D_s^+ \to \bar{K}^{*0}K^{*+}$ despite the fact that the latter is a "color-suppressed" mode (proportional to the Wilson coefficient a_2).

Finally, we refer the reader to calculations which claim to reproduce the experimentally measured form factors [33,34,35]. We do not claim sufficient familiarity with these calculations to make critical comments. Further references, particularly to lattice gauge calculations, can be found in [24].

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APPENDIX

Reference [15] provides calculation of the various form factors appearing in the matrix elements of the vector and axial-vector currents. Here we present a direct calculation of $F_0(t)$ and $A_0(t)$ which appear in the matrix element of the divergence of the currents.

1. A direct calculation of $F_0(t)$ in the ISGW model

We shall work out $F_0(t)$ directly by evaluating out the matrix element of the divergence of the current. From (10),

$$q^\mu \langle P(p)|V_\mu|D(k)\rangle = (m_D^2 - m_P^2)F_0(t) \; . \tag{A1}$$

We evaluate the matrix element in (A1) for mock mesons ("$\sim$" will indicate a mock quantity) at maximum mock momentum transfer $\tilde{t}_m$. Thus for mock mesons at maximum momentum transfer where $\tilde{q}_\mu = ((\tilde{m}_D - \tilde{m}_P),0)$ we get

$$\begin{aligned}\tilde{q}^{\,\mu}\langle \tilde{P}(\tilde{p})|V_\mu|\tilde{D}(\tilde{k})\rangle &= (\tilde{m}_D - \tilde{m}_P)\langle \tilde{P}(\tilde{p})|V_0|\tilde{D}(\tilde{k})\rangle \\ &= (\tilde{m}_D^2 - \tilde{m}_P^2)\tilde{F}_0(\tilde{t}_m) \; .\end{aligned} \tag{A2}$$

Thus

$$\tilde{F}_0(\tilde{t}_m) = \langle \tilde{P}(\tilde{p})|V_0|\tilde{D}(\tilde{k})\rangle / (\tilde{m}_D + \tilde{m}_P) \; . \tag{A3}$$

Using the oscillator wave functions given in Ref. [15] it is easy to show that, at maximum mock momentum transfer,

$$\tilde{F}_0(\tilde{t}_m) = \frac{2\tilde{m}_D \tilde{F}_3(\tilde{t}_m)}{(\tilde{m}_D + \tilde{m}_P)} \; , \tag{A4}$$

where

$$\tilde{F}_3(t_m) = \left[\frac{\tilde{m}_P}{\tilde{m}_D}\right]^{1/2} \left[\frac{\tilde{\beta}_D\tilde{\beta}_P}{\beta_{DP}^2}\right]^{3/2} \; , \tag{A5}$$

where β's are the parameters in the oscillator wave functions and $\tilde{\beta}_{DP}^2 = \frac{1}{2}(\tilde{\beta}_D^2 + \tilde{\beta}_P^2)$. Assuming universal β's and mock meson mass equal to the sum of the constituent quark mass with [15]

$$\begin{aligned} &m_c = 1.82 \text{ GeV}, \quad m_s = 0.55 \text{ GeV}, \\ &m_u = m_d = 0.33 \text{ GeV} \; , \end{aligned} \tag{A6}$$

we can calculate $\tilde{F}_0(\tilde{t}_m)$. The physical form factor is then obtained by hypothesizing that "the mock form factor at maximum mock momentum transfer is equal to the physical form factor at maximum physical momentum transfer." Thus

$$\tilde{F}_0(\tilde{t}_m)=F_0(t_m) . \tag{A7}$$

This form can then be continued to arbitrary values of t. For $D_s^+ \to \eta$ *and* $D_s^+ \to \eta'$ transitions, using universal β's, we obtain

$$\tilde{F}_0^{D_s\eta}(\tilde{t}_m)=\frac{2(\tilde{m}_{D_s}\tilde{m}_\eta)^{1/2}}{(\tilde{m}_{D_s}+\tilde{m}_n)}=0.89=F_0^{D_s\eta}(t_m)=F_0^{D_s\eta'}(t_m) . \tag{A8}$$

The reason for the equality of $F_0^{D_s\eta}(t_m)$ and $F_0^{D_s\eta'}(t_m)$ is that the mock mass for η is the same as that for η'; however, it must be borne in mind that t_m for $D_s^+ \to \eta$ transition is different from t_m for $D_s^+ \to \eta'$ transition. Hence, as discussed in the text, $F_0^{D_s\eta}(t)$ and $F_0^{D_s\eta'}(t)$ have different extrapolations.

2. A direct calculation of $A_0(t)$ in the ISGW model

Here we perform a direct calculation of the matrix element of the divergence of the axial current which is related directly to $A_0(t)$. From (10) we get

$$q^\mu\langle V|A_\mu|D_s^+\rangle=\varepsilon^*\cdot q(2m_V)A_0(t) . \tag{A9}$$

We use (A9) for mock mesons and evaluate it at maximum mock momentum transfer t_m:

$$\tilde{q}^\mu\langle \tilde{V}|A_\mu|\tilde{D}_s^+\rangle=\tilde{\varepsilon}^*\cdot\tilde{q}(2\tilde{m}_V)\tilde{A}_0(\tilde{t}) , \tag{A10}$$

which at $\tilde{q}^\mu=((\tilde{m}_{D_s}-\tilde{m}_V),0)$ yields (specializing to $D_s^+ \to \phi$ transition)

$$\langle\tilde{\phi}|A_0|\tilde{D}_s^+\rangle=2\tilde{p}_\phi\tilde{A}_0(\tilde{t}_m) , \tag{A11}$$

where we have allowed $\tilde{\phi}$ to have an infinitesimal three-momentum $\tilde{p}_\phi$ and the only contribution to the right-hand side of (A10) has come from the timelike component of the longitudinal polarization vector $\tilde{\varepsilon}_\mu$. The left-hand side of (A11) is easily calculated, following the method of Ref. [15], to yield,

$$\langle\tilde{\phi}|A_0|\tilde{D}_s^+\rangle=\tilde{m}_{D_s}\tilde{F}_3^{D_s\phi}(\tilde{t}_m)\tilde{p}_\phi\left[\frac{1}{m_s}-\frac{m_s}{2\tilde{m}_\phi}\frac{\tilde{\beta}_{D_s}^2}{\tilde{\beta}_{D_s\phi}^2}\frac{1}{\mu_+}\right] , \tag{A12}$$

where

$$\tilde{F}_3^{D_s\phi}(\tilde{t}_m)=\left[\frac{\tilde{m}_\phi}{\tilde{m}_{D_s}}\right]^{1/2}\left[\frac{\tilde{\beta}_{D_s}\tilde{\beta}_\phi}{\tilde{\beta}_{D_s\phi}^2}\right]^{3/2}$$

and

$$\frac{1}{\mu_+}=\left[\frac{1}{m_s}+\frac{1}{m_c}\right] . \tag{A13}$$

Using universal β's and constituent quark masses of (A6), we obtain

$$\tilde{A}_0(\tilde{t}_m)=\frac{(\tilde{m}_{D_s}\tilde{m}_\phi)^{1/2}}{2}\left[\frac{1}{m_s}-\frac{m_s}{2\tilde{m}_\phi}\frac{1}{\mu_+}\right]=0.99 . \tag{A14}$$

This is finally identified with the physical $A_0(t_m)$ which in turn is extrapolated to an arbitrary t.

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