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Perturbative QCD predictions for the fragmentation functions of the P-wave mesons with two heavy quarks

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The fragmentation functions for the heavy mesons of S and P waves are calculated in the framework of perturbative QCD and the nonrelativistic quark model. The results are manifestly process independent. As applications, the production ratios for P-wave $\bar{b}c$ mesons to the $b\bar{b}$ production at high energy colliders are predicted. The spin dependence of the fragmentation functions for various states is discussed and the predicted results for the D (D^*) mesons are consistent with the recent experimental data.

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I. INTRODUCTION

The fragmentation functions for heavy flavored mesons have been intensively studied theoretically [1–13]. One believes that kinematic factors govern the heavy meson fragmentation [1,3,4]. One of them is the PSSZ (Peterson, Schlatter, Schmitt, and Zerwas) model [4] parametrized by accounting for some kinematic factors appearing in perturbative calculations, and it is proven to describe well the fragmentation functions and is widely adopted in experimental analysis. However, this model cannot predict the spin dependence of the fragmentation functions and the relative production ratios for various states to that of $b\bar{b}$ production. The study of them is very important to understand the hadron fragmentation mechanism. In order to extract this kind of information, one needs to build specific models to calculate the dynamical details for the strong interactions. It is interesting that for the fragmentation of mesons which consist of two heavy quarks such as $c\bar{c}$, $c\bar{b}$, and $b\bar{b}$, the dynamics is considerably simplified, and can be easily calculated by using perturbative QCD and the nonrelativistic wave function of the meson. As a result, the spin dependence of the fragmentation function and the relative production ratios for various states can be predicted well.

The simple perturbative picture adopted here has been depicted by some authors [5–11]. The process was first calculated by the author of Ref. [5]. In Refs. [6] and [7] the authors extract the fragmentation function by retaining only the so-called "lowest twist" term, for a pseudoscalar meson and vector meson, respectively. The authors of Ref. [8] found that some higher twist terms have the same order contributions. In their derivation, the factorization property, i.e., the process independence of the fragmentation function, is not manifest. The authors of Ref. [10] calculated them in the axial gauge for the S-wave $b\bar{c}$ mesons and confirmed the results of Ref. [8]. In their calculation, the factorization property is manifest. The unpolarization fragmentation function for $c \to J/\psi$ was examined by the authors of Ref. [10]. The polarization of it was considered by the authors of Ref. [11]. In Ref. [13], the authors did a full calculation of the hadronic production of the B_c meson.

In addition to the S-wave states, some higher excited states such as the P wave do not have tiny production ratios as the experimental study indicated [14,15]. To search and study these states is of special interest both in theory and in experiment. For instance, discovering higher excited $b\bar{c}$ mesons and measuring their spectra is critical to testing heavy quark potential models. So it is also important to calculate the fragmentation functions of these states.

In this paper, based on a general consideration of the singularities in the amplitude, we calculate the fragmentation functions for the S- and P-wave mesons and their spin-dependent properties and give an interpretation on the process independence of the fragmentation functions in the perturbative QCD framework in detail.

The higher order collinear gluon emission processes contain terms such as $[\alpha_s(\sqrt{s})\ln(\sqrt{s}/m_Q)]^n$, where $\sqrt{s}$ and m_Q are the total energy and the heavy quark mass, which violate the scaling behavior of the fragmentation function. The contributions from these terms relate the evolution of the fragmentation function and can be easily summed up by solving the Altarelli-Parisi (AP) evolution equation [16]. In this paper, we also calculate the evolution of the fragmentation.

The rest of this paper is organized as follows. In Sec. II, we show that the amplitude can be factorized and the fragmentation functions are process independent by taking a special gauge in detail. The fragmentation functions for the mesons with various spins are derived. In Sec. III, we discuss the evolution of the fragmentation functions by solving the AP equation [16]. In Sec. VI, we apply these fragmentation functions to calculate the production rates and the energy distributions for various states.

II. DERIVATION OF THE FRAGMENTATION FUNCTIONS

In this section, we derive the fragmentation functions for both P- and S-wave mesons. According to the factorization theorem, the heavy meson fragmentation functions are independent of the hard processes by which the heavy quark is created. However, if one wants to extract them in the perturbative picture, one needs to invoke a specific process. Here we calculate a simple process $Z^0 \to H + q_1 + \bar{q}_2$, where H represents an S-wave ($^1S_0$, $^3S_1$), or a P-wave ($^1P_1$ and ^3P_J, $J = 0, 1, 2$) meson.

The lowest order QCD perturbative diagrams are shown in Fig. 1. The process contains two steps. First, a heavy-quark–antiquark pair is created from a virtue gluon emitted by the heavy quark. Then, the heavy quark and the heavy antiquark in the pair with almost the same velocity combine to form a heavy meson which is described by the meson wave function. The contribution from the interference between the first two diagrams and the last two is small and can be ignored. From now on, we only focus on the first two diagrams while the others can be easily obtained by symmetry. The amplitudes corresponding to these two diagrams read

$$A_{ij}^{1,2} = \frac{4g_s^2}{3\sqrt{3}} \bar{u}_i(\mathbf{q}_1)\, \lambda^{1,2}\, v_j(\mathbf{q}_2), \tag{1}$$

where

$$\lambda^1 = \int \frac{d^4q}{(2\pi)^4} \gamma_\mu \chi_P(q) \gamma^\mu \frac{1}{l^2} \frac{\not{k} - \not{q}_2 + m_2}{(k-q)^2 - m_2^2} \not{\epsilon}_z (g_v + \gamma_5 g_a), \tag{2}$$

$$\lambda^2 = \int \frac{d^4q}{(2\pi)^4} \gamma_\mu \chi_P(q) \not{\epsilon}_z (g_v + \gamma_5 g_a) \frac{\not{p}_2 - \not{k} + m_2}{(k-p_2)^2 - m_2^2} \gamma^\mu \frac{1}{l^2}, \tag{3}$$

where k_1, q_1, q_2 are the four-momenta of the Z^0, quark 1 with mass m_1, and quark 2 with mass m_2, respectively;

$$p_1 = \alpha_1 p + q, p_2 = \alpha_2 p - q \quad \left(\alpha_{1,2} \equiv \frac{m_{1,2}}{m_1 + m_2}\right)$$

are the momenta of two constituents of the bound state with the total and the relative momenta p and q, respectively; $l = p_1 + q_1$; $\chi_P(q)$ is the BS (Bethe-Salpeter) wave function of the produced bound state. In this process, we have several energy scales: m_z, m_1, m_2, M (the bound state mass), and $|\mathbf{q}|$. In the weak binding limit, they satisfy the inequality

$$m_z \gg M, m_1, m_2 \gg |\mathbf{q}|. \tag{4}$$

In the rest frame of the bound state, the wave function $\chi_P(q)$ can be written as

$$\chi_P(q) = \frac{1}{q_0 - a_+ - i\epsilon} \frac{1}{q_0 - a_- + i\epsilon} \phi(\mathbf{q}) \times v_m(\mathbf{q}) \bar{u}_{m'}(-\mathbf{q}) \chi_{mm'}^{ss_z} \langle ss_z; ll_z | JJ_z \rangle, \tag{5}$$

where

$$a_+ = \alpha_1 M - \sqrt{m_1^2 + \mathbf{q}^2} \simeq -\frac{|\mathbf{q}|^2}{2m_1},$$

$$a_- = -\alpha_2 M + \sqrt{m_2^2 + \mathbf{q}^2} \simeq \frac{|\mathbf{q}|^2}{2m_2};$$

$v_m(\mathbf{q})$ are the Dirac spinors of quark 1 and antiquark 2 which satisfy the free Dirac equation. $\chi_{mm'}^{ss_z}$ and $\langle ss_z; ll_z | JJ_z \rangle$ are the Clebsch-Gordon coefficients for the S-S coupling and L-S coupling; the order of $|q^0|$ is the same with that of $|a_+ - a_-|$; $\phi(\mathbf{q})$ is the scalar part of the wave function.

Equation (4) implies that in the rest frame of the bound state $\phi(q)$ drops fast when the relative momentum is compatible with the quark mass. Therefore, if we calculate the creation of S states, we can neglect the q dependence in λ^i ($i = 1, 2$) except $\phi(q)$. Integration over the relative momentum q yields that the amplitude is proportional to the wave function at the origin of the bound state. However, for the creation of the P wave, since the wave function at the origin vanishes, we need to expand the λ^i ($i = 1, 2$) [except $\phi(q)$] to linear terms of q. Consequently, the amplitude is proportional to the derivative of the wave function at the origin after integrating over the relative momentum. In both cases, the q^2 dependence or higher order terms can be ignored. The largest contribution of the process comes from the kinematic region in which the quark 1 and the formed meson are almost collinear. To give a further interpretation, let us introduce some kinematic variables:

$$z \equiv \frac{2p \cdot k}{m_z^2}, \quad x \equiv \frac{2q_2 \cdot k}{m_z^2}, \quad y \equiv \frac{2q_1 \cdot k}{m_z^2}, \quad r \equiv \frac{M^2}{m_z^2}.$$

They are constrained by $x + y + z = 2$ and $0 < x, y, z < 1$.

The denominators of the propagators of the gluon and the fermion in Figs. 1(a) and 1(b) are $(1-x)^2$ and $\alpha_1^2(1-x)(1-\alpha_2 z)$, respectively. The x variable is constrained by the upper and lower integrals bound for a fixed z [to $O(r)$]:

$$(1-x)_{\min} = \frac{(1-\alpha_2 z)^2 r}{z(1-z)}, \tag{6}$$

$$(1-x)_{\max} = z - \frac{(1-\alpha_1 z)^2 r}{z(1-z)}. \tag{7}$$

Since $(1-x)_{\min} \sim r$, when performing the integration for the absolute squared amplitude over the phase space or, equivalently, x, y, it is easy to see that the total width

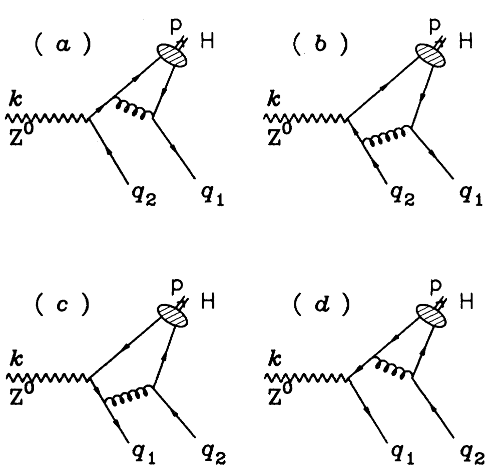

FIG. 1. Feynman diagrams responsible for $Z^0 \to H + \bar{q}_2 + q_1$.

is dominated by the region of $x \to 1$, in which the quarks and the meson are collinear. Calculating the squared amplitude of $\bar{A}^2$, one finds that the singularities of the three terms $|A_1|^2$, $|A_1 A_2|$, and $|A_2|^2$ are of the same order. The reason can be attributed to the use of a Feynman gauge for the virtual gluon propagator in A_1 and A_2. In fact, in this gauge, A_1 and A_2 can be rewritten as

$$A^{1,2}_{i,j} = \frac{4g_s^2}{3\sqrt{3}} \int \frac{d^4q}{(2\pi)^4} j^{im}_\mu(l,q) L^\mu_{mj\ 1,2}, \tag{8}$$

where

$$j^{im}_\mu(l,q) = \bar{u}_i(\mathbf{q}_1)\gamma_\mu v_m(\mathbf{p}_1) \tag{9}$$

is the colored current,

$$L^\mu_{1\ mj}(\mathbf{p}_2) = \frac{1}{l^2}\bar{u}'_m(\mathbf{p}_2)\gamma^\mu \frac{\not{k} - \not{q}_2 + m_2}{(1-x)m_z^2} \not{\epsilon}_z(g_v + \gamma_5 g_a) v_j(\mathbf{q}_2)\chi^{ss_z}_{mm'}\phi(q), \tag{10}$$

and

$$L^\mu_{2\ mj}(\mathbf{p}_2) = \frac{1}{l^2}\bar{u}'_m(\mathbf{p}_2)\not{\epsilon}_z(g_v + \gamma_5 g_a) \frac{\not{p}_2 - \not{k} + m_2}{1 - \alpha_2 z} \gamma^\mu v_j(\mathbf{q}_2)\chi^{ss_z}_{mm'}\phi(q). \tag{11}$$

When quark 1 and antiquark 2 are collinear, $j^{im}_\mu(l,q) \sim l_\mu$. However, because of the current conservation, one has

$$\int \frac{d^4q}{(2\pi)^4} l_\mu (L^\mu_{1\ mj} + L^\mu_{2\ mj}) = 0. \tag{12}$$

It means that there is a large cancellation between these two diagrams. Therefore, if we subtract these terms from $j^{im}_\mu(l,q)$, the contribution in Fig. 1(b) may be suppressed. To this end, we decompose it generally into four terms:

$$j^{im}_\mu(l,q) = a^{im} l_\mu + b^{im} k_\mu + c^{im}_1 t_{1\mu} + c^{im}_2 t_{2\mu}, \tag{13}$$

where t_1, t_2 are two linearly independent unit spacelike transverse momenta which are chosen to satisfy

$$t^\mu_{1,2} l_\mu = t^\mu_{1,2} k_\mu = 0. \tag{14}$$

Multipling Eq. (13) by l^μ and k^μ, we have two equations:

$$a^{im} l^2 + b^{im} k \cdot l = l^\mu \cdot j^{im}_\mu(l,q), \tag{15}$$

$$a^{im} l \cdot k + b^{im} k^2 = k^\mu \cdot j^{im}_\mu(l,q), \tag{16}$$

where $l^\mu \cdot j^{im}_\mu(l,q) = \bar{u}_i(\mathbf{q}_1)\left[(q_1 - m_1) + (p_1 + m_1)\right] v_m(\mathbf{p}_1) = O(q^2)$. It follows that

$$a^{im} \approx \frac{k^\mu j^{im}_\mu(l,q)}{k \cdot l}, \tag{17}$$

$$b^{im} \approx -\frac{l^2\ k^\mu j^{im}_\mu(l,q)}{(k \cdot l)^2}. \tag{18}$$

Here a^{im}, b^{im} are insensitive to the choice of vector k, as long as $k \cdot l >> l^2$. The c^{im}_1 and c^{im}_2 related to the transverse parts of the current are suppressed by factors of $\sqrt{1 - x + \delta}$ or $\frac{m_1}{m_z}$ ($\delta = \frac{2q \cdot l}{m_z^2}$). It originates from the common requirements for the angular momentum conservation and the vector coupling of the quark and gluon. Consequently, if we make a replacement

$$j^{im}_\mu(l,q) \longrightarrow j'^{im}_\mu(l,q) = j^{im}_\mu(l,q) - \frac{j^{im}(l,q) \cdot k}{k \cdot l} l_\mu, \tag{19}$$

the total amplitude will not change, but the second diagram is suppressed by a factor of $\sqrt{1 - x + \delta}$ or $\frac{m_1}{m_z}$. This replacement of $j^{im}_\mu(l,q)$ as Eq. (19) is equivalent to the replacement of the gluon propagator in the amplitude

$$\frac{g_{\mu\nu}}{l^2} \to D_{\mu\nu}(l) = \frac{1}{l^2}\left(g_{\mu\nu} - \frac{k_\mu l_\nu}{k \cdot l}\right) \tag{20}$$

or in a symmetric form

$$\frac{g_{\mu\nu}}{l^2} \to D_{\mu\nu}(l) = \frac{1}{l^2}\left(g_{\mu\nu} - \frac{k_\mu l_\nu + l_\mu k_\nu}{k \cdot l}\right). \tag{21}$$

It means that in the calculation, we can obtain the leading order contribution if we choose a gauge as Eq. (20) or Eq. (21) for the virtual gluon. In this gauge, the amplitude of the Fig. 1(a) can be rewritten as

$$A'^{ij}_1 = \frac{4g_s^2}{3\sqrt{3}}\bar{u}_i(\mathrm{q}_1)\left[\int \frac{d^4q}{(2\pi)^4}\gamma_\mu \chi_p(q)\gamma_\nu D_{\mu\nu}(l)\frac{\not{k} - \not{q}_2 + m_2}{(k-q)^2 - m_2^2}\right]\not{\epsilon}_z(g_v + \gamma_5 g_a)v_j(\mathbf{q}_2). \tag{22}$$

Now let us divide the fermion propagator into two parts (in the Z^0 rest frame):

$$S = \frac{\not{k} - \not{q}_2 + m_2}{(k-q)^2 - m_2^2} = \frac{\gamma_0(k_0 - q_{20}) + \mathbf{q}_2 \cdot \gamma + m_2}{2\omega\ (m_z - 2\omega)} + \frac{\gamma_0(k_0 - q_{20}) - \mathbf{q}_2 \cdot \gamma - m_2}{2\omega\ m_z}, \tag{23}$$

with $\omega = \sqrt{\mathbf{q}_2^2 + m_2^2} = \frac{1}{2}x m_z$, $q_{20} = (1 - \frac{1}{2}x)m_z$. The first term corresponds to the positive energy part with a singularity of $\frac{1}{1-x}$, while the second one corresponds to the negative energy part and can be ignored because it has no singularity. Thus the propagator can be approximately expressed as

$$S = \frac{\gamma_0 \omega + \mathbf{q} \cdot \gamma + m_2}{x m_z^2 (1-x)} \approx \frac{u_i(\tilde{\mathbf{q}}_2)\bar{u}_i(\tilde{\mathbf{q}}_2)}{x m_z^2 (1-x)}, \tag{24}$$

with $\tilde{q}_2 = xk - q_2$ satisfying $\tilde{q}_2^2 = m_2^2$. The factor $1-x$ in the denominator means that the virtual fermion is almost on shell when x approachs to 1. Putting Eq. (24) into Eq. (22), A'_1 can be factorized as

$$A'_{1\,ij} = \frac{4g_s^2}{3\sqrt{3}} T_{im} F_{mj}, \tag{25}$$

where

$$T_{im} = \bar{u}_i(\mathbf{q}_1)\left[\int \frac{d^4q}{(2\pi)^4}\gamma^\mu \chi_p(q)\gamma_\nu D_{\mu\nu}(l)\right] \times \frac{1}{x m_z^2(1-x)} u_m(\tilde{\mathbf{q}}_2), \tag{26}$$

$$F_{mj} = \bar{u}_m(\tilde{\mathbf{q}}_2)\not{\epsilon}_z(g_v + \gamma_5 g_a)v_j(\mathbf{q}_2). \tag{27}$$

Therefore, the squared amplitude can be written as

$$|A'_1|^2 = \bar{A}'_{1\,ji}\cdot A'_{1\,ij} = \left(\frac{4g_s^2}{3\sqrt{3}}\right)^2 \bar{F}_{jm}\bar{T}_{mi}T_{in}F_{nj}, \tag{28}$$

where $\bar{F}_{jm}F_{nj}$ is proportional to the spin-density matrix of the heavy quark before emitting the gluon which is proportional to a unit matrix [12], i.e., $\bar{F}_{jm}F_{nj} = |F|^2\delta_{mn}$. Thus, the squared amplitude can be rewritten as

$$|A'_1|^2 = \left(\frac{4g_s^2}{3\sqrt{3}}\right)^2 \bar{T}_{mi}|F|^2\delta_{mj}T_{ij}\tfrac{1}{2}\bar{F}_{km'}F_{m'k}\bar{T}_{in}T_{ni}. \tag{29}$$

Factorizing out the phase space factor, the fragmentation function reads

$$D(z) = \frac{1}{2}\left(\frac{4g_s^2}{3\sqrt{3}}\right)^2 \frac{m_z^2}{16\pi^2}\int dx|T|^2, \tag{30}$$

where the matrix T_{im} defined in Eq. (26) has a form

$$T_{ij} = \bar{u}_i(\mathbf{q}_1)\left[\int \frac{d^4q}{(2\pi)^4}\gamma_\mu \chi_p^{ss_z}(q)\phi_l(q)\cdot D_{\mu\nu}(l)\gamma_\nu\right] \times u_j(\tilde{\mathbf{q}}'_2)\,\frac{\langle l l_z; s s_z|J J_z\rangle}{m_z^2 x(1-x)}. \tag{31}$$

In the nonrelativistic approximation, $\chi_p^{ss_z}(q)$ can be written as

$$\chi_p^{ss_z}(q) = \frac{-\sqrt{2M}}{4m_1m_2}(\not{p}_1 - m_1)\frac{1}{\sqrt{2}}(\alpha\gamma_5 + \beta\not{\epsilon})(p_2 + m_2), \tag{32}$$

where $\alpha = 1\ (0), \beta = 0\ (1)$ for an $S = 0\ (1)$ meson. This form of the spinor wave function is correct up to $O(q)$. It is sufficiently accurate for our cases of S- and P-wave creation.

In the rest frame of the bound state, the scale part of the wave function $\phi_l(q)$ can be expressed as

$$\phi_l(q) = \frac{1}{q_0 - a_+ - i\epsilon}\,\frac{1}{q_0 - a_- + i\epsilon}\psi_l(\mathbf{q}). \tag{33}$$

They satisfy the well-known intergal formula

$$\int \frac{d^4q}{(2\pi)^4}\phi_0(q) = \psi_0(0), \tag{34}$$

$$\int \frac{d^4q}{(2\pi)^4} q^\alpha \phi_1(q) = i\epsilon^\alpha(l_z)\psi'_1(0), \tag{35}$$

where $\epsilon^\alpha(l_z)$ is the polarization vector related to the orbit angular momentum. Thus, integrating over the relative momentum q, for the S wave, approximately, one finds

$$T_{ij} = \bar{u}_i(\mathbf{q}_1)\gamma_\mu \chi_p^{s,s_z}(0)D_{\mu\nu}(l_0)\gamma_\nu u_j(\mathbf{q}_2)\psi_0(0)\frac{1}{m_z^2 x(1-x)}, \tag{36}$$

where $l_0 = \alpha_1 P + q_1$. For the P wave, expanding q and retaining linear terms of q in $\chi_p^{s,s_z}(q)$ and $D_{\mu\nu}(l)$, one obtains

$$T_{ij} = \bar{u}_i(\mathbf{q}_1)\gamma_\mu\left[\left.\frac{\partial\chi_p^{s,s_z}(q)}{\partial q^\alpha}\right|_{q=0} D_{\mu\nu}(l_0) + \chi_p^{s,s_z}(0)\left.\frac{\partial D_{\mu\nu}(l)}{\partial q^\alpha}\right|_{q=0}\right]\epsilon^\alpha(l_z)\gamma_\nu u_j(\mathbf{q}'_2)\psi'_1(0)\langle 1 l_z; s s_z|J J_z\rangle\frac{1}{m_z^2 x(1-x)}. \tag{37}$$

Here, the spinor part of the wave function and its derivative can be easily calculated from Eq. (32):

$$\chi_p^{s,s_z}(0) = \frac{1}{2\sqrt{M}}(\alpha\gamma_5 + \beta\not{\epsilon})(\not{p} + M), \tag{38}$$

$$\left.\frac{\partial\chi_p^{0,\ 0}(q)}{\partial q^\alpha}\right|_{q=0} = \frac{\sqrt{M}}{4m_1m_2}\gamma_5\gamma^\alpha\left[\not{p} + m_2 - m_1\right], \tag{39}$$

$$\left.\frac{\partial\chi_p^{1,s_z}(q)}{\partial q^\alpha}\right|_{q=0} = \frac{-\sqrt{M}}{4m_1m_2}\left[\gamma^\alpha\not{\epsilon}(s_z)(\not{p} + m_2 - m_1) - 2(p_1 - m_1)\epsilon^\alpha(s_z)\right], \tag{40}$$

while they were obtained in Ref. [17] for the equal mass case. Here the terms proportional to p^α have been ignored due to the factor $p\cdot q$ which is of order $O(q^2)$. The derivative of the gluon propagator can be obtained from Eq. (20):

$$\left.\frac{\partial D_{\mu\nu}(l)}{\partial q^\alpha}\right|_{q=0} = -\frac{2l_{0\alpha}}{l_0^4}\left(g_{\mu\nu} - \frac{k_\mu l_{0\nu}}{k\cdot l_0}\right) + \frac{1}{l_0^2}\frac{k_\alpha k_\mu l_{0\nu}}{(k\cdot l_0)^2} - \frac{k_\mu g_{\nu\alpha}}{l_0^2 k\cdot l_0}. \tag{41}$$

For the ^3P_J states, one has the relations [17]

$$\sum_{l_z,s_z}\langle 1l_z;1s_z|00\rangle\epsilon^\alpha(l_z)\epsilon^\beta(s_z)=\sqrt{\frac{1}{3}}\left(\frac{p^\alpha p^\beta}{m^2}-g^{\alpha\beta}\right), \tag{42}$$

$$\sum_{l_z,s_z}\langle 1l_z;1s_z|1J_z\rangle\epsilon^\alpha(l_z)\epsilon^\beta(s_z)=i\sqrt{\frac{1}{2}}\epsilon^{\alpha\beta\gamma\delta}\frac{p_\gamma}{m}\epsilon_\delta(J_z), \tag{43}$$

$$\sum_{l_z,s_z}\langle 1l_z;1s_z|2J_z\rangle\epsilon^\alpha(l_z)\epsilon^\beta(s_z)=\epsilon^{\alpha\beta}(J_z). \tag{44}$$

Here the polarization vector $\epsilon^\mu(J_z)$ and tensor $\epsilon^{\mu\nu}(J_z)$ obey the projection relations

$$\sum_{J_z}\epsilon_\mu(J_z)\epsilon_\nu(J_z)=\left(\frac{p_\mu p_\nu}{m^2}-g_{\mu\nu}\right)\equiv P_{\mu\nu}, \tag{45}$$

$$\sum_{J_z}\epsilon_{\mu v}(J_z)\epsilon_{\alpha\beta}(J_z)=\frac{1}{2}\left[P_{\mu\alpha}P_{\nu\beta}+P_{\nu\alpha}P_{\mu\beta}\right]-\frac{1}{3}P_{\mu\nu}P_{\alpha\beta}. \tag{46}$$

Using these relations, $|T|^2$ can be calculated. (In our calculation, the REDUCE computer program was employed.) From the general arguments given in the previous section, the terms with the highest order singularity in $|T|^2$ when x approaches 1 can be expanded in terms of the powers of the heavy meson mass, i.e., for the S wave,

$$|T|^2=\left[\frac{W_0(z)}{(1-x)^2}+\frac{r\,W_1(z)}{(1-x)^3}+\frac{r^2\,W_2(z)}{(1-x)^4}\right]\times\frac{|\psi_0(0)|^2}{(1-\alpha_2 z)^2\alpha_1^2 M m_z^4}. \tag{47}$$

Inserting it into Eq. (30) and integrating over x by using the integration bounds in Eq. (7), approximately, one obtains the fragmentation functions for the S wave:

$$D(z)=\frac{8\alpha_s^2|\psi_0(0)|^2}{27Mm_1^2(1-\alpha_2 z)^2}\times\left[\delta_x W_0(z)+\frac{\delta_x^2}{2}W_1(z)+\frac{\delta_x^3}{3}W_2(z)\right], \tag{48}$$

with $\delta_x\equiv\dfrac{z(1-z)}{(1-\alpha_2 z)^2}$. The W_i's are obtained from straightforward calculations on $|T|^2$ in Eq. (36).

For the S-wave pseudoscalar meson, they read

$$\begin{aligned}W_0&=2(1+z\alpha_1)^2(1-z),\\ W_1&=-2(2z^2\alpha_2^2-z^2\alpha_2-4z\alpha_2^2+4z\alpha_2\\ &\quad-3z+4\alpha_2-2)(1-z\alpha_2),\\ W_2&=-8(1-z\alpha_2)^2\alpha_1\alpha_2.\end{aligned} \tag{49}$$

For the S-wave vector meson, they read

$$\begin{aligned}W_0&=2((1+z\alpha_1)^2+2z^2)(1-z),\\ W_1&=-2(2z^2\alpha_2^2-3z^2\alpha_2+4z\alpha_2^2+4z\alpha_2\\ &\quad-9z-4\alpha_2+6)(1-z\alpha_2),\\ W_2&=-24\alpha_1\alpha_2(1-z\alpha_2)^2.\end{aligned} \tag{50}$$

The expressions listed here confirm the results of Refs. [8,10].

In addition, the longitudinal W_i^L's for the vector meson also can be calculated by using the polarization vector $\epsilon^{L\mu}=p^\mu/m-mk^\mu/(p\cdot k)$,

$$\begin{aligned}W_0^L&=2(1+z\alpha_1)^2(1-z),\\ W_1^L&=-2(2z^3\alpha_2^2-z^3\alpha_2+4z^2\alpha_2^2-3z^2\\ &\quad-8z\alpha_2+2z+4)(1-z\alpha_2)/z,\\ W_2^L&=8(2z^2\alpha_2^2-z^2\alpha_2-2z\alpha_2+1)(1-z\alpha_2)^2/z^2.\end{aligned} \tag{51}$$

The expressions listed here are consistent with the results of Ref. [11] for the equal mass case.

The transverse $W_i^T=(W_i-W_i^L)/2$ $(i=1,2,3)$ can be obtained from the above formula.

Similarly, for the P wave, $|T|^2$ can be generally written as

$$|T|^2=\left[\frac{W_0(z)}{(1-x)^2}+\frac{r\,W_1(z)}{(1-x)^3}+\frac{r^2\,W_2(z)}{(1-x)^4}+\frac{r^3\,W_3(z)}{(1-x)^5}+\frac{r^4\,W_4(z)}{(1-x)^6}\right]\frac{M|\psi_1'(0)|^2}{m_1^4 m_z^4(1-\alpha_2 z)^4}, \tag{52}$$

where the last two terms in the square brackets originate from the derivative of the gluon propagator. In the same way, substituting it into Eq. (37) and integrating over x, the fragmentation function of the P wave can be written as

$$D(z)=\frac{8\alpha_s^2|\psi_1'(0)|^2}{27Mm_1^4(1-\alpha_2 z)^4}\left[\delta_x W_0(z)+\frac{\delta_x^2}{2}W_1(z)+\frac{\delta_x^3}{3}W_2+\frac{\delta_x^4}{4}W_3+\frac{\delta_x^5}{5}W_4\right]. \tag{53}$$

We list each W_i for all P states as following.

For the $^1P_1$ (h) state,

$$\begin{aligned}W_0&=(1-3z\alpha_2+z-z^2\alpha_2^2+z^2\alpha_2+3z^2-5z^3\alpha_2^3+13z^3\alpha_2^2-9z^3\alpha_2-3z^3\\ &\quad+z^4\alpha_2^3-2z^4\alpha_2^2+3z^4\alpha_2)(1-z\alpha_2)/(32\alpha_2^2),\\ W_1&=-(8\alpha_2^2-12\alpha_2+6-16z\alpha_2^3-16z\alpha_2^2+38z\alpha_2-9z-8z^2\alpha_2^4+52z^2\alpha_2^3\\ &\quad-34z^2\alpha_2^2-10z^2\alpha_2-24z^3\alpha_2^4+38z^3\alpha_2^3-13z^3\alpha_2^2)(1-z\alpha_2)^2/(32\alpha_2^2),\\ W_2&=-\{2[(2\alpha_2-5)z-\alpha_2]\alpha_2+6\alpha_2-1+(2\alpha_2-1)z^2\alpha_2^2+2z\alpha_2^3\}(1-z\alpha_2)^3\alpha_1/(4\alpha_2),\\ W_3&=2(1-z\alpha_2)^4\alpha_2\alpha_1^2,\\ W_4&=0.\end{aligned} \tag{54}$$

For the longitudinal polarization of the $^1P_1$ state,

$$
\begin{aligned}
W_0^L &= (1 - 3z\alpha_2 + z - z^2\alpha_2^2 + z^2\alpha_2 + z^2 - 5z^3\alpha_2^3 + 13z^3\alpha_2^2 - 7z^3\alpha_2 \\
&\quad -z^3 + z^4\alpha_2^3 - 2z^4\alpha_2^2 + z^4\alpha_2)(1 - z\alpha_2)/(32\alpha_2^2), \\
W_1^L &= -(4 + 8z\alpha_2^2 - 12z\alpha_2 + 2z - 16z^2\alpha_2^3 - 4z^2\alpha_2^2 + 6z^2\alpha_2 - 3z^2 - 8z^3\alpha_2^4 + 44z^3\alpha_2^3 \\
&\quad -6z^3\alpha_2^2 - 6z^3\alpha_2 - 16z^4\alpha_2^4 + 14z^4\alpha_2^3 - 7z^4\alpha_2^2)(1 - z\alpha_2)^2/(32z\alpha_2^2), \\
W_2^L &= [4(\alpha_2^2 - 2z)z^2\alpha_2^2 + 2(4\alpha_2 - z)z\alpha_2 + (1 - 5z\alpha_2) - (4\alpha_2^2 - 4\alpha_2 - 25)z^3\alpha_2^3 \\
&\quad -2(2\alpha_2^2 + 2\alpha_2 - 1)z^4\alpha_2^3 - (16\alpha_2 + 1)z^2\alpha_2^2](1 - z\alpha_2)^3/(8z^2\alpha_2^2), \\
W_3^L &= -[2(2\alpha_2^2 - 2\alpha_2 - 1)z\alpha_2 + (2\alpha_2 - 1) + (2\alpha_2 - 1)z^2\alpha_2^2](1 - z\alpha_2)^5/(2z^2\alpha_2), \\
W_4^L &= 2(1 - z\alpha_2)^6\alpha_2\alpha_1/z^2.
\end{aligned}
\tag{55}
$$

For the $^3P_0$ (χ_0) state,

$$
\begin{aligned}
W_0 &= (4\alpha_2 - 3 - 8z\alpha_2^2 + 10z\alpha_2 - 3z + 4z^2\alpha_2^3 - 5z^2\alpha_2^2 + z^2\alpha_2)^2(1 - z)/(96\alpha_2^2), \\
W_1 &= (32\alpha_2^3 - 40\alpha_2^2 - 12\alpha_2 + 18 - 64z\alpha_2^4 + 176z\alpha_2^3 - 152z\alpha_2^2 + 18z\alpha_2 \\
&\quad +27z + 32z^2\alpha_2^5 - 184z^2\alpha_2^4 + 324z^2\alpha_2^3 - 206z^2\alpha_2^2 + 30z^2\alpha_2 \\
&\quad +32z^3\alpha_2^5 - 80z^3\alpha_2^4 + 58z^3\alpha_2^3 - 9z^3\alpha_2^2)(1 - z\alpha_2)^2/(96\alpha_2^2), \\
W_2 &= -[2(7\alpha_2 - 6z)\alpha_2 - (10\alpha_2 + 3) - 2(7\alpha_2 - 12)z\alpha_2^2 - (2\alpha_2 - 3)z^2\alpha_2^2](1 - z\alpha_2)^3\alpha_1/(12\alpha_2), \\
W_3 &= 2(1 - z\alpha_2)^4\alpha_2\alpha_1^2/3, \\
W_4 &= 0.
\end{aligned}
\tag{56}
$$

For the $^3P_1$ (χ_1) state,

$$
\begin{aligned}
W_0 &= (1 - 4z\alpha_2 + 2z + 8z^2\alpha_2^2 - 10z^2\alpha_2 + 3z^2 - 4z^3\alpha_2^3 + 6z^3\alpha_2^2 \\
&\quad -2z^3\alpha_2 + z^4\alpha_2^4 - 2z^4\alpha_2^3 + z^4\alpha_2^2)(1 - z)/(16\alpha_2^2), \\
W_1 &= (4\alpha_2 - 6 + 6z\alpha_2^2 - 10z\alpha_2 + 9z + 8z^2\alpha_2^3 - 22z^2\alpha_2^2 + 10z^2\alpha_2 \\
&\quad +2z^3\alpha_2^4 - 2z^3\alpha_2^3 + z^3\alpha_2^2)(1 - z\alpha_2)^2/(16\alpha_2^2), \\
W_2 &= (2\alpha_2^2 - 6\alpha_2 + 2 - 2z\alpha_2^3 - 6z\alpha_2^2 + 11z\alpha_2 - z^2\alpha_2^2)(1 - z\alpha_2)^3\alpha_1/(4\alpha_2), \\
W_3 &= 2(1 - z\alpha_2)^4\alpha_2\alpha_1^2, \\
W_4 &= 0.
\end{aligned}
\tag{57}
$$

For the longitudinal polarization of the $^3P_1$ state,

$$
\begin{aligned}
W_0^L &= (1 + \alpha_1 z)^2(1 - z\alpha_2)^2(1 - z)/(16\alpha_2^2), \\
W_1^L &= -(4 - 8z\alpha_2 + 2z + 2z^2\alpha_2 - 3z^2 + 6z^3\alpha_2^2 - 2z^3\alpha_2 - 2z^4\alpha_2^3 + z^4\alpha_2^2)(1 - z\alpha_2)^2/(16z\alpha_2^2), \\
W_2^L &= [(1 - 4z^2\alpha_2^3) + 4(2\alpha_2 - 1)z^3\alpha_2^2 - z^4\alpha_2^4](1 - z\alpha_2)^3/(4z^2\alpha_2^2), \\
W_3^L &= -[(2 - z\alpha_2) + 4(2\alpha_2 - 1)z^2\alpha_2^2 - 4z\alpha_2^3 - z^3\alpha_2^3](1 - z\alpha_2)^4/(4z^2\alpha_2), \\
W_4^L &= (1 - z\alpha_2)^6\alpha_1\alpha_2/z^2.
\end{aligned}
\tag{58}
$$

For the $^1P_1$ and $^3P_1$ state, we can also obtain the transverse polarization W_i^T $(i = 0 - 4)$ by

$$
W_i^T = \frac{W_i - W_i^L}{2}. \tag{59}
$$

For the $^3P_2$ (χ_2) state,

$$
\begin{aligned}
W_0 &= (2 - 8z\alpha_2 + 4z + 12z^2\alpha_2^2 - 16z^2\alpha_2 + 9z^2 - 8z^3\alpha_2^3 + 20z^3\alpha_2^2 \\
&\quad -28z^3\alpha_2 + 10z^3 + 2z^4\alpha_2^4 - 8z^4\alpha_2^3 + 24z^4\alpha_2^2 - 20z^4\alpha_2 + 5z^4)(1 - z)/24, \\
W_1 &= (16\alpha_2 - 20 - 32z\alpha_2^2 + 52z\alpha_2 - 13z + 16z^2\alpha_2^3 + 4z^2\alpha_2^2 - 30z^2\alpha_2 \\
&\quad +8z^2 + 4z^3\alpha_2^3 - 7z^3\alpha_2^2 - 28z^3\alpha_2 + 30z^3)(1 - z\alpha_2)^2/24, \\
W_2 &= (26\alpha_2 - 34 - 26z\alpha_2^2 - 18z\alpha_2 + 45z - 8z^2\alpha_2^2 + 15z^2\alpha_2)(1 - z\alpha_2)^3\alpha_1/12, \\
W_3 &= 10(1 - z\alpha_2)^4\alpha_2\alpha_1^2/3, \\
W_4 &= 0.
\end{aligned}
\tag{60}
$$

The symmetric traceless polarization tensor has five independent W_i with three different, which correspond to the helicity $0, \pm 1$, and ± 2 contributions. Denoting them by W_i^0, W_i^1, and W_i^2 $(i = 0, 1, \ldots, 4)$, we get

$$\begin{aligned}
W_0^0 &= [(1+z\alpha_1)^2 - z\alpha_1]^2(1-z)/12,\\
W_1^0 &= -(12 - 32z\alpha_2 + 4z + 28z^2\alpha_2^2 - 14z^2\alpha_2 + 2z^2 - 8z^3\alpha_2^3 + 4z^3\alpha_2^2\\
&\quad +18z^3\alpha_2 - 10z^3 - 2z^4\alpha_2^3 - 7z^4\alpha_2^2 + 8z^4\alpha_2 - 3z^4)(1-z\alpha_2)^2/(12z),\\
W_2^0 &= (24 - 60z\alpha_2 - 3z + 52z^2\alpha_2^2 - 3z^2\alpha_2 - z^2 - 16z^3\alpha_2^3 + 7z^3\alpha_2^2\\
&\quad +12z^3\alpha_2 - 6z^3 - 7z^4\alpha_2^3 + z^4\alpha_2^2)(1-z\alpha_2)^3/(6z^2),\\
W_3^0 &= -[2(90\alpha_2 + 6 - 32z\alpha_2^2 + 10z\alpha_2 + z)z^2\alpha_2 + 12(6-z)\\
&\quad -3(4\alpha_2 - 1)z^4\alpha_2^2 - 192z\alpha_2 - 9z^2](1-z\alpha_2)^4/(12z^3),\\
W_4^0 &= [(4\alpha_2 - 1)z^2\alpha_2 + 3 - 6z\alpha_2](1-z\alpha_2)^6/z^4.
\end{aligned} \tag{61}$$

For helicity=1 polarization of a $^3P_2$ state,

$$\begin{aligned}
W_0^1 &= [(2\alpha_2 - 1)z - 1]^2(1-z)z^2/16,\\
W_1^1 &= (8 - 16z\alpha_2 - 4z + 8z^2\alpha_2^2 + 8z^2\alpha_2 - 7z^2 + 4z^3\alpha_2^2 + 2z^3\alpha_2 + 4z^3 - 7z^4\alpha_2^2 - 4z^4\alpha_2 + 4z^4)(1-z\alpha_2)^2/(16z),\\
W_2^1 &= -(20 - 44z\alpha_2 - 10z + 26z^2\alpha_2^2 + 18z^2\alpha_2 - 4z^2 - 2z^3\alpha_2^3\\
&\quad +2z^3\alpha_2^2 - 5z^3\alpha_2 + 7z^3 - 2z^4\alpha_2^3 - 7z^4\alpha_2^2 + z^4\alpha_2)(1-z\alpha_2)^3/(8z^2),\\
W_3^1 &= (32 - 80z\alpha_2 - 12z + 60z^2\alpha_2^2 + 20z^2\alpha_2 + 3z^2 - 8z^3\alpha_2^3\\
&\quad -12z^3\alpha_2^2 + 2z^3\alpha_2 - 4z^4\alpha_2^3 - z^4\alpha_2^2)(1-z\alpha_2)^4/(8z^3),\\
W_4^1 &= [(218\alpha_2 + 6 - 15z\alpha_2)z^3\alpha_2^2 + (204\alpha_2 + 20 - 15z\alpha_2)z^5\alpha_2^4\\
&\quad +(32-z)z\alpha_2 + 2(13\alpha_2 + 3)z^7\alpha_2^6 - (3\alpha_2 + 1)z^8\alpha_2^7 - 4 - 111z^2\alpha_2^2 - 265z^4\alpha_2^4 - 97z^6\alpha_2^6]/(2z^4),
\end{aligned} \tag{62}$$

while the W_i^2's can be obtained by

$$W_i^2 = \frac{W_i - W_i^0 - 2W_i^1}{2}. \tag{63}$$

Thus, the fragmentation functions for each of the S and P waves are determined. It is easy to see that the shapes of the fragmentation functions depend on the variable z and the quark mass ratios, i.e., α_1 or α_2. This scaling behavior is the lowest order result and may be violated as involving the higher order effects. The production ratio of the S wave is proportional to a factor $|\psi_0(0)|^2/m_1^2M$ while that of the P wave is proportional to a factor $|\psi_1'(0)|^2/m_1^4M$. It means that the smaller m_1 is, the larger the production ratios are, as expected.

It is interesting to compare the results with the PSSZ model. It is easy to see that the kinematics factor $x(1-x)^2$ is contained in this model and the PSSZ model, which was first derived by Bjorken [1].

Comparisons of the predictions for the fragmentation functions with data for the D meson cases and the B mesons are shown in Fig. 2. Here we take our parameters α_1 to be 0.25 and 0.08 for the D and B mesons, respectively; in the PSSZ model the parameters ε_c and ε_b defined in Ref. [4] are determinted by updated experimental data as $\varepsilon_c = 0.06$ and $\varepsilon_b = 0.006$ in the PSSZ model [18]. From Fig. 2, one can see that the results of these two models are close.

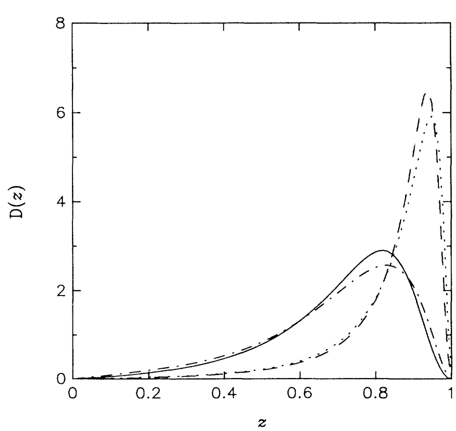

FIG. 2. The normalized fragmentation functions predicted by this model for the D meson with the parameter $\alpha_1 = 0.25$ (dot-dashed line) and the B meson with $\alpha_1 = 0.08$ (dotted line) compared with the PSSZ [4] model for the D meson with the parameter $\varepsilon_c = 0.06$ (solid line) and the B meson with $\varepsilon_b = 0.006$ (dashed line), respectively.

III. EVOLUTION OF THE FRAGMENTATION FUNCTIONS

The fragmentation functions derived in the previous section are independent of the energy scale as the lowest order perturbative results. If the energy scale of the process is very large, the off-shell heavy quark produced by a hard process may emit gluons. The collinear emission will contribute large correction terms to the fragmentation function, such as $[\alpha_s(S)\ln(S/m_Q^2)]^n$. Here the heavy quark mass m_Q provides a natural cutoff for the collinear singularity. These terms can be summed up by employing the parton shower [19] Monte Carlo simulation or by solving the AP equation [16]. Here we adopt the latter approach. It is well known that the fragmentation function $\bar{D}(z,t)$ obeys the AP evolution equation [16]

$$\frac{d\bar{D}(z,t)}{dt} = \int \frac{dy}{y}\bar{D}(y,t)P_{qq}\left(\frac{z}{y}\right), \tag{64}$$

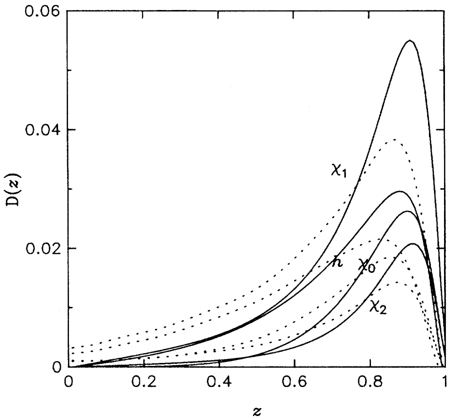

FIG. 3. Evolution of the fragmentation functions (all times 10^{-3}) for the P-wave $\bar{b}c$ mesons at a scale $Q = Q_0 = 13$ GeV (solid line), $Q = 90$ GeV (dotted line) for the ${}^1P_1$, ${}^3P_0$, ${}^3P_1$, and ${}^3P_2$ states, respectively.

where

$$t = \frac{6}{33 - 2N_f} \ln \frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)},$$

N_f is the number of flavors, and Λ is the QCD scale parameter. In our calculation, we take $\Lambda = 250$ MeV, $Q^2 = s$ is the total energy square of the process, and Q_0 is the energy scale of the lowest order fragmentation function which is obtained in last section. Thus $D(z)$ is related to $\bar{D}(z,t)$ as $D(z) = \bar{D}(z,0)$. Solving Eq. (64), one will obtain the fragmentation function evolution from the energy scale Q_0 to $\sqrt{s}$. There is some arbitrariness for the choice of the parameter Q_0. Here we choose it based on the following consideration. If we calculate the fragmentation function to the next order in perturbative QCD, a term such as $\alpha_s(Q_0)\ln(cM/Q_0)$ with a large contribution will emerge. To minimize this term's contribution, we may roughly take $Q_0 = 2M \simeq M + m_1 + m_2$, which corresponds to the threshold total energy of the process at which the bound state can be produced. The Altarelli-Parisi splitting function $P_{qq}(z)$ reads [16]

$$P_{qq}(z) = \frac{4}{3}\left[\frac{1+z^2}{(1+z)_+} + \frac{3}{2}\delta(1-z)\right]. \qquad (65)$$

AP equation (64) can be conveniently solved by taking the Mellin transformation. The evolution of the normalized fragmentation functions of P-wave $\bar{b}c$ mesons from $Q_0 = 13$ to 90 GeV is shown in Fig. 3.

IV. APPLICATIONS

As the first application, the formula can be used to estimate the production rates of $b\bar{c}$ $c\bar{c}$ $b\bar{b}$ mesons at high energy e^+e^- colliders. $b\bar{c}$ is another heavy-quark–antiquark system in addition to $c\bar{c}$ and $b\bar{b}$. The mass for various states can be predicted by flavor-independent heavy quark potential models which fit the $c\bar{c}$ and $b\bar{b}$ spectra well. So the measurements for its spectrum can be used to test the potential models. However, there are some difficulties in creating these states in practical experiments. One realistic production mechanism is that $b\bar{c}$ created by the heavy quark fragmentation mechanism as discussed in Refs. [6,8] and this paper. The S-wave production rates have been estimated in Refs. [6,8,10]. The P-wave production rates can be estimated by Eq. (48) and Eqs. (54)–(62). The derivative of the wave function at the origin is calculated by solving the Schrödinger equation for the bound state. Using the derivative of the wave function at the origin predicted by the potential I in Ref. [20], the production ratio which is defined by

$$R \equiv \frac{B(e^+e^- \to (\bar{b}c) + b + \bar{c})}{B(e^+e^- \to b\bar{b})}$$

can be obtained and the results are listed in Table I. The listed result here about the J/ψ confirmed that in the Ref. [11].

The fragmentation functions for the P-wave $\bar{b}c$ mesons are shown in Fig. 4.

Here we point out that the predicted R here is not very reliable for the B_c production at hadronic colliders in which the total cross sections are dominated by the lower subprocess total energy. In the latter case, a full QCD perturbative calculation is more appropriate [13].

Second, we can try to use the formula to predict the spin dependence of the production of the D or D^* mesons. Following Ref. [12], we define

$$P_V \equiv \frac{R_V}{R_V + R_P}$$

and

$$\alpha(z) \equiv \frac{D_L(z) - D_T(z)}{D_T(z)}$$

where R_P and R_V are the production rates of the

TABLE I. The production ratios (all times 10^{-5}) and the avarge z of the fragmentation functions for $c\bar{b}$, $c\bar{c}$, and $b\bar{b}$ mesons with unpolarized and polarized states. The wave functions at the origin (in $\text{GeV}^{3/2}$) and the derivatives (in $\text{GeV}^{5/2}$) are obtained from the potential I [20], with $\Lambda_{\overline{\text{MS}}} = 150$ MeV. α_s is taken as $\alpha_s = 0.15$. h is the helicity of the final hadrons.

	Ratio			Average z	
	$c\bar{b}$	$c\bar{c}$	$b\bar{b}$	$c\bar{b}$	$c\bar{c}$ and $b\bar{b}$
$\lvert\psi_0(0)\rvert$	0.35	0.25	0.66		
$\lvert\psi_1'(0)\rvert$	0.21	0.13	0.57		
${}^1S_0$	12	11	2.6	0.68	0.57
${}^3S_1$	16	11	2.6	0.72	0.62
${}^3S_1$ $(h=0)$	5.0	3.4	0.83	0.72	0.61
${}^1P_1$	1.1	1.4	0.1	0.71	0.62
${}^1P_1$ $(h=0)$	0.64	0.8	0.058	0.75	0.67
${}^3P_0$	0.69	1.8	0.13	0.80	0.65
${}^3P_1$	1.4	2.0	0.14	0.73	0.62
${}^3P_1$ $(h=0)$	0.36	0.56	0.040	0.66	0.58
${}^3P_2$	1.6	0.78	0.055	0.76	0.66
${}^3P_2$ $(h=0)$	0.53	0.025	0.018	0.79	0.69
${}^3P_2$ $(h=1)$	0.44	0.021	0.015	0.78	0.68

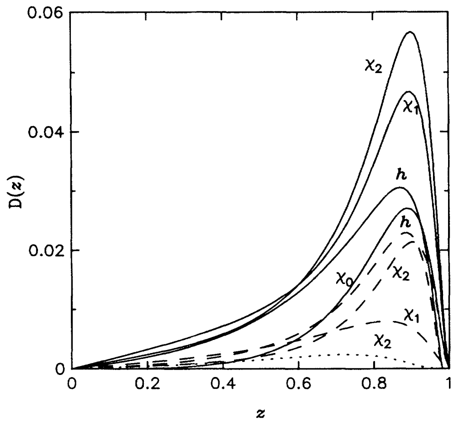

FIG. 4. Prediction for the fragmentation functions (all times 10^{-3}) of the unpolarized (solid line) P-wave $\bar{b}c$ mesons and various polarization ones such as helicity $h = 0$ (dashed line) and $h = 2$ (dotted line), respectively.

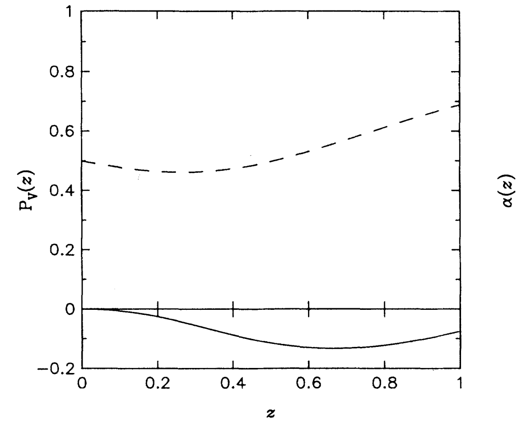

FIG. 5. Prediction for the $P_V(z)$ (dashed line) and $\alpha(z)$ (solid line) of the D and D^* mesons defined in the text.

pseudoscalar meson and the vector meson, respectively; $D_L(z)$, $D_T(z)$ are the longitudinal part and transverse part of the fragmentation function, respectively. The predicted fragmentation functions with $\alpha_1 = 0.25$ are shown in Fig. 5. The average P_V is about 0.6 which is consistent with the ALEPH Collaboration experimental data $P_V = 0.60 \pm 0.08 \pm 0.05$ [21]. $\alpha(z)$ is close to zero (nearly unpolarized), which is consistent with the CLEO results [22]. The relative production ratios of the P- to the S-wave charmed mesons can also be predicted and the results are close to that of the $\bar{b}c$ mesons because the values of α_1 are very close for these two kinds of mesons. These results need to be examined by future experiments.

V. DISCUSSIONS AND CONCLUSIONS

In this paper, the fragmentation functions for heavy mesons of the P wave are calculated by using the lowest order perturbative QCD. Starting from this simple model, the spin dependence of the fragmentation functions for both the S and P waves is discussed. $P_V \equiv R_V/(R_P + R_V) \approx 0.6$ and $\alpha(z) \sim 0$ are obtained. Comparing with the experimental data, it is found that this simple model well describes the fragmentation functions of the D, D^* and B, B^* mesons. It may indicate that the heavy meson fragmentation function is dominated by perturbative effects. To improve these results, one needs to consider higher order perturbative QCD corrections, nonperturbative QCD correction such as the multigluon exchange, quark and gluon condensates, and a better treatment of the integral over the relative momentum in the amplitude.

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