# Photon neutrino scattering

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The cross section for photon neutrino scattering is calculated in the standard model assuming that the neutrino is massless and that the center-of-mass energy is small compared to any charged lepton mass. Although the scattered photons can acquire a (parity-violating) circular polarization of order unity, the cross section in this limit is highly suppressed.

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## I. INTRODUCTION

Low energy neutrino photon scattering, e.g., $\gamma\gamma \to \nu\bar{\nu}$ could be of interest in astrophysics. Unfortunately this interaction is known to be highly suppressed. In the four-Fermi limit of the standard model it is exactly zero [1] since a vector (or axial vector) cannot couple to two massless vectors [2]. This situation persists in the full gauge theory, where the $Z$-boson $\gamma\gamma$ triangle loop contribution vanishes [2], and the leading-order contribution from $W$-boson loops is also zero [3]. Thus the interaction is smaller than a naive counting of couplings would suggest by factors of $\omega/M_W$ or $m_\nu/M_W$ where $\omega$ is the photon energy, $m_\nu$ is a neutrino mass, and $M_W$ is the $W$-boson mass.

Estimates [4] have been made as to how much this suppression in the interaction would reduce the cross sections for neutrino photon scattering but a precise calculation in the standard model seems not to have been carried out. The one-loop diagrams for neutrino photon scattering are shown in Fig. 1. In this paper, we perform the calculation in the zero neutrino mass limit, assuming that the energy is much smaller than the mass of the charged lepton.

## II. CALCULATIONS

A recent paper by Liu [5] considers nonstandard interactions which could give a nonzero neutrino-two photon interaction at $O(\alpha G_F)$ where $\alpha$ is the fine structure constant and $G_F$ is the Fermi coupling. We consider only standard model interactions and calculate to order $O(G_F^2)$.

The calculation of the diagrams in Fig. 1 is simplified by the following observations.

(1) The matrix element for each diagram is of the form

$$\bar{u}(p)(1+\gamma_5)\Gamma\,(1-\gamma_5)\,v(p'),$$

where $\bar{u}$ and $v$ are the spinors of the massless neutrinos and $\Gamma$ is the appropriate combination of $\gamma$ matrices for the particular diagram. This can be Fierz rearranged into

$$\frac{1}{2}\,\bar{u}(p)\gamma^\lambda(1-\gamma_5)v(p')\,\mathrm{Tr}\,[\Gamma\,\gamma_\lambda(1+\gamma_5)]$$

and the trace can then be carried out using some algebraic manipulation software. (We used FORM [6] and SCHOONSCHIP [7].)

(2) A nonlinear $R_\xi$ gauge condition can be chosen such that the $\gamma W \phi$ coupling is zero [8], where $\phi$ is the Goldstone boson part of the $W$. This gauge condition also modifies the $\gamma\gamma WW$ and $\gamma WW$ couplings but the rules are especially simple in the 't Hooft–Feynman gauge $\xi = 1$. We need calculate only the diagrams in Fig. 1 plus those with every $W$ replaced by $\phi$. Further, the two contact diagrams [Fig. 1(d)] are zero. This choice of gauge also makes it possible to check gauge invariance of

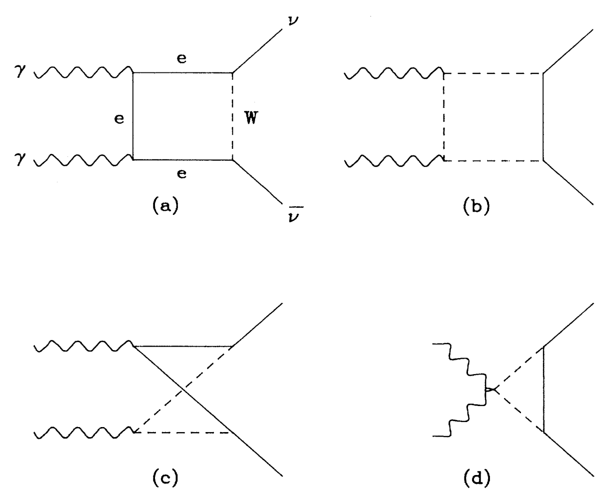


FIG. 1. Diagrams for the process $\gamma\gamma \to \nu_e\bar{\nu}_e$. For each of (a), (b), (c) there are also the diagrams with the photons interchanged.

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the photon couplings analytically rather than relying on a numerical check. For the case of zero neutrino mass, the $W$-exchange and $\phi$-exchange diagrams are separately gauge invariant.

After performing the loop integrations associated with each of the diagrams in Fig. 1, the remaining parameter integrals were expanded in powers of the square of the center of mass energy $s$ divided by a polynomial in the charged lepton mass squared $m_\ell^2$ and $M_W^2$. All parameter integrals are easily evaluated and the $W$-exchange amplitude $(\mathcal{M}_W)_{\lambda\lambda'}$, where $\lambda$ and $\lambda'$ are the photon helicities, is

$$(\mathcal{M}_W)_{\lambda\lambda'} = \frac{\sin\theta}{M_W^4}\left[1+\frac{4}{3}\ln\left(\frac{M_W^2}{m_\ell^2}\right)\right] \times\left(s\,t(1-\lambda\lambda')+\frac{s^2}{2}(1-\lambda)(1+\lambda')\right). \quad (1)$$

Here, $\theta$ is the scattering angle in the center of mass and only the photon helicities need be specified since the neutrino helicities are fixed. Notice that only $(\mathcal{M}_W)_{+-}$ and $(\mathcal{M}_W)_{-+}$ are nonvanishing. The $W$-exchange amplitudes contain no inverse powers of $m_\ell^2$ and are therefore of order $G_F^2$, as mentioned in the Introduction. Explicit calculation shows that the $\phi$-exchange amplitudes share this property. Since the $\phi$-lepton couplings introduce a factor $m_\ell^2/M_W^2$, these amplitudes can be neglected to leading order.

The cross section for $\gamma\gamma \to \nu\bar{\nu}$ is given by

$$\frac{d\sigma}{dz} = \frac{\sin^4\theta_W}{2^{13}\pi^5}\frac{(M_W^2 G_F)^4}{\omega^2}\sum |(\mathcal{M}_W)_{\lambda\lambda'}|^2, \quad (2)$$

where $z$ is the cosine of the scattering angle, $\theta_W$ is the weak mixing angle, and $\omega$ is the center of mass energy of the photon. The sum of the squared matrix elements is given by

$$\sum |(\mathcal{M}_W)_{\lambda\lambda'}|^2 = \frac{2^4}{M_W^8}\left[1+\frac{4}{3}\ln\left(\frac{M_W^2}{m_\ell^2}\right)\right]^2 \times\left[-t^3(s+t)-t(s+t)^3\right], \quad (3)$$

with $s = 4\omega^2$ and $t = -2\omega^2(1-z)$. $m_\ell$ is the mass of the charged lepton ($e, \mu$, or $\tau$). The presence of the logarithm in the matrix element makes the cross section dependent on the flavor of neutrino.

The amplitudes for $\gamma\nu \to \gamma\nu$ and $\gamma\bar{\nu} \to \gamma\bar{\nu}$ can be obtained from Eq. (1) with the appropriate interchange of $s, t$, and $u$. For the channel $\gamma\nu \to \gamma\nu$, the resulting amplitude $(\tilde{\mathcal{M}}_W)_{\lambda\lambda'}$ is

$$(\tilde{\mathcal{M}}_W)_{\lambda\lambda'} = -2i\frac{\cos(\theta/2)}{M_W^4}\left[1+\frac{4}{3}\ln\left(\frac{M_W^2}{m_\ell^2}\right)\right] \times\left(s^2(1+\lambda\lambda')+\frac{s\,t}{2}(1+\lambda)(1+\lambda')\right), \quad (4)$$

with $\theta$ denoting the scattering angle in the $\gamma\nu$ center of mass. In this case, $(\tilde{\mathcal{M}}_W)_{++}$ and $(\tilde{\mathcal{M}}_W)_{--}$ are nonvanishing. The sum of the squared matrix elements is given by Eq. (3) with $s$ and $t$ interchanged and an overall minus sign. Because interaction is parity violating, the scattered photons can acquire circular polarization. From Eq. (4), the polarization $\mathcal{P}(\theta)$ is

$$\mathcal{P}(\theta) = \frac{|(\tilde{\mathcal{M}}_W)_{++}|^2 - |(\tilde{\mathcal{M}}_W)_{--}|^2}{|(\tilde{\mathcal{M}}_W)_{++}|^2 + |(\tilde{\mathcal{M}}_W)_{--}|^2} = \frac{\cos^4(\theta/2)-1}{\cos^4(\theta/2)+1}. \quad (5)$$

Since there is no scattering in the Born approximation, polarization is of order unity. Furthermore, the absence of any photon helicity flip amplitude means that the scattering cannot produce linearly polarized photons. $\mathcal{P}(\theta)$ is plotted in Fig. 2.

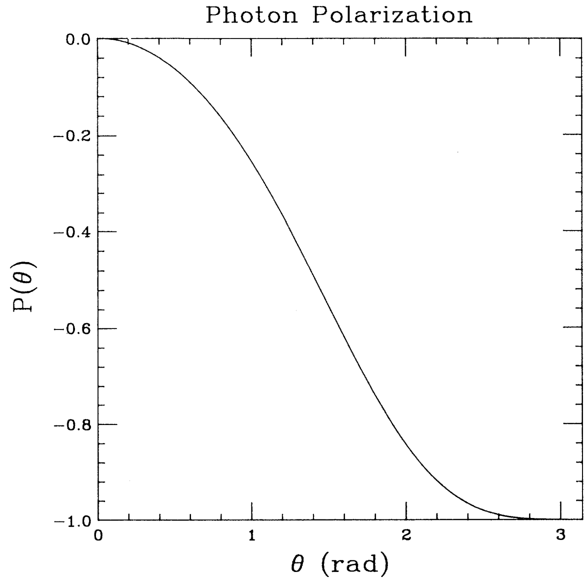


FIG. 2. Photon polarization for the process $\gamma\nu \to \gamma\nu$.

## III. DISCUSSION

The total cross sections for $\gamma\gamma \to \nu\bar{\nu}$ are obtained from Eq. (2) using Eq. (1), and those for $\gamma\nu$ elastic scattering are obtained using Eq. (4) together with the appropriate spin average. For electron neutrinos, the results are

$$\sigma_{\gamma\gamma\to\nu\bar{\nu}} = 7.38\times10^{-50}\left(\frac{\omega}{m_p}\right)^6 \text{cm}^2$$

$$\sigma_{\gamma\nu\to\gamma\nu} = 1.11\times10^{-48}\left(\frac{\omega}{m_p}\right)^6 \text{cm}^2,$$

where $m_p$ is the proton mass. In each case $\sigma_{\gamma\bar{\nu}\to\gamma\bar{\nu}}$ equals $\sigma_{\gamma\nu\to\gamma\nu}$. The cross sections for all lepton species

TABLE I. $\gamma\nu$ cross sections in units of $(\omega/m_p)^6$ cm$^2$.

| Charged lepton | $\sigma_{\gamma\gamma\to\nu\bar{\nu}}$ | $\sigma_{\gamma\nu\to\gamma\nu}$ |
|---|---|---|
| e | $7.38\times10^{-50}$ | $1.11\times10^{-48}$ |
| $\mu$ | $2.38\times10^{-50}$ | $3.56\times10^{-49}$ |
| $\tau$ | $8.47\times10^{-51}$ | $1.27\times10^{-49}$ |

in units of $(\omega/m_p)^6$ cm$^2$ are summarized in Table I. For comparison the cross section for $\nu_e\nu_\mu \to \nu_e\nu_\mu$ is $7 \times 10^{-39}$ $(\omega/m_p)^2$ cm$^2$.

The above expressions are valid only for energies $\omega \ll m_\ell$ as well as $\omega \ll M_W$. These limits cover most astrophysical applications and mean that the standard model cross sections are negligibly small. Since nonstandard model amplitudes can be of order $\alpha G_F$ [5], it is possible that the cross sections are larger in supersymmetric (SUSY) models. The existence of a characteristic signature such as circular polarization which could be attributed to the interaction of light with the SUSY candidates for dark matter is worth further investigation.

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