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CP violation and leptogenesis

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Recently a model of chaotic inflation was proposed in which the right-handed sneutrinos drive the baryogenesis. We study some of the details of the model, particularly the aspect of CP violation, and determine the number of right-handed sneutrinos required for the viability of such models.

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Recently a model has been proposed [1] in which the inflation is driven by the superpartner of the right-handed neutrino. Although the inflationary scenario is quite appealing for the solution of the horizon and flatness problems [2], it is difficult to formulate a particle physics model consistently. In addition to solving this problem, this model generates $(B-L)$ asymmetry at lower temperature in the post inflationary universe through violation of lepton number. This model requires the right-handed neutrinos which are singlets under all groups for one of these fields to be the inflaton. The Majorana mass terms for these fields have been introduced in this model by hand to explain the Cosmic Background Explorer (COBE) result [3].

The main objective of this article is to study CP violation in these models, determine the number of singlet fields required for CP violation, and count the number of CP-violating phases which can be present in the combination of the Yukawa couplings appearing in the processes under consideration [4]. We consider the particle content to be exactly the same as that of Ref. [1], with the exception of the number of right-handed neutrinos (N_α's). We start with $\bar{n}$ number of these scalars ($N_\alpha,\ \alpha=1,...,\bar{n}$) all of which carry a $(B-L)$ number −1. While any of the N_α can serve as the inflaton, for the time being let us consider the scalar partner of N_1 to be the inflaton.

The Yukawa part of the superpotential is given by

$$W=\tfrac{1}{2}M_{\alpha\beta}N_\alpha^cN_\beta^c+\mu H_uH_d+h^{\alpha j}N_\alpha^cl_jH_u+k^{ij}e_i^cl_jH_d\ , \tag{1}$$

where $\alpha,\beta=1,...,\bar{n}$ are the indices for the fields N_α and $i,j=1,...,n$ are the indices for the left-handed leptons. Although finally we are interested in $n=3$, for our analysis we shall take n to be arbitrary.

Until the time $H\sim M_{\alpha\beta}$ the $\tilde{N}_1$ field will roll down the potential and after that oscillate around its minimum, $\tilde{N}_1=0$, with frequency M, where M is the mass of $\tilde{N}_1$ and of order 10^{13} GeV to explain the COBE data [1,3,5]. Once the Universe cools down to about 10^4 GeV, the coherent oscillation starts to decay into lighter particles, thus reheating the Universe again to about 10^{10} GeV. During this period of reheating the decay of $\tilde{N}_1$ field produces $(B-L)$ number asymmetry through lepton number violation due to CP violation. This $(B-L)$ asymmetry remains as a baryon number asymmetry after the electroweak anomalous processes [6].

In the generation of the $(B-L)$ asymmetry through the lepton number asymmetry [1,6–8], the CP violation plays a crucial role [9], and we now study systematically this question of CP violation and the decay of the heavy neutrinos in a general way, trying to determine the number of phases present in the Yukawa-type couplings. The CP-violating parameter ϵ, which is the asymmetry in the $\tilde{N}_\alpha$ decay between that into leptons $l_i\tilde{H}_u$ and that into antisleptons $\tilde{l}_i^*H_u^*$, is defined as

$$\epsilon=\frac{|A(\tilde{N}_\alpha\to l_i\tilde{H}_u)|^2-|A(\tilde{N}_\alpha\to\tilde{l}_i^*H_u^*)|^2}{|A|^2} \tag{2}$$

and is given by (see Fig. 1 of Ref. [1])

$$\epsilon=\frac{\ln 2}{8\pi}\frac{\mathrm{Im}(h_{\alpha i}h_{\alpha j}h_{\beta i}^*h_{\beta j}^*)}{h_{\alpha i}h_{\alpha i}^*}=\frac{\ln 2}{8\pi}\frac{T_{\alpha\beta ij}}{|h_{\alpha i}|^2}\ , \tag{3}$$

with $t_{\alpha\beta ij}=(h_{\alpha i}h_{\alpha j}h_{\beta i}^*h_{\beta j}^*)$ and $T_{\alpha\beta ij}=\mathrm{Im}(t_{\alpha\beta ij})$. While this expression is clearly invariant under rephasing of the fields l_i [8], the invariance of ϵ under the transformation $N_\alpha\to e^{i\delta_\alpha}N_\alpha$ is not obvious. Indeed, under such a transformation the quantity $t_{\alpha\beta ij}\to e^{2i(\delta_\alpha-\delta_\beta)}t_{\alpha\beta ij}$. It would appear that a suitable choice of the phases $\delta_\alpha-\delta_\beta$ can make $t_{\alpha\beta ij}$ real, and thus give $\epsilon=0$. However, rephasing of the fields N_α introduces new phases in the propagator and wave function through complex phases in the mass parameter which exactly cancel the extra phase picked up by $t_{\alpha\beta ij}$, leaving ϵ invariant. In the leptogenesis models we need only consider the effect of CP violation in the decay of heavy neutrinos at temperatures well above the electroweak symmetry breaking scale, where the Higgs field has not acquired a vacuum expectation value (VEV), and the left-handed neutrinos are massless. We thus choose to work in the basis where N_α are the real, positive mass eigenstates of the right-handed neutrinos, and note that this choice exhausts our freedom to make phase transformations on the fields N_α.

The CP-violating parameter describing the asymmetry in the decay of Majorana neutrinos N_α into leptons vs antileptons is proportional to the same combination of Higgs-Yukawa couplings $T_{\alpha\beta ij}$ appearing in Eq. (3) (see Ref. [8]). We wish to determine under what conditions T is nonzero, and in general, find the number of independent T's which can appear in the asymmetry parameter. From the definition of the T's, one can immediately get the relations

$$T_{\alpha\beta ij} = T_{\alpha\beta ji} = -T_{\beta\alpha ij} = -T_{\beta\alpha ji}; \tag{4}$$

i.e., T is antisymmetric under the interchange of the heavy singlet indices, while it is symmetric under the interchange of the ν_i's. Thus, for $\bar{n} = 1$, i.e., only one generation of heavy singlet field N_α, there is no CP violation. This relation, however, does not determine the minimum number of neutrinos required for CP violation.

We now proceed to calculate the number of phases that can appear in $T_{\alpha\beta ij}$ when n, the number of left-handed neutrinos, and $\bar{n}$, the number of right-handed neutrinos, are arbitrary. We find that the maximum number of phases which can occur is equal to the number of independent phases in the $\bar{n} \times \bar{n}$ Hermitian matrix H:

$$H_{\alpha\beta} = h_{\alpha j} h^*_{\beta j} = (hh^\dagger)_{\alpha\beta}, \tag{5}$$

where $h_{\alpha j}$ is the Yukawa coupling between N_α and l_j. To see this recall that the left-handed neutrinos are massless, and hence degenerate. Therefore, there are no observable consequences to performing a unitary transformation on the fields l_i; in particular, the magnitude of CP violation in the decay of N_α must be invariant under such transformations. Inspection of Eq. (3) reveals that $H_{\alpha\beta}$ is the minimum set of Yukawa couplings appearing in the CP violation parameter which is invariant under this transformation. It should be noted that unitary transformations of the l_i leave the structure of the standard model Lagrangian unchanged.

If $n \geq \bar{n}$, H is an unconstrained $\bar{n} \times \bar{n}$ Hermitian matrix, whereas if $n < \bar{n}$, there are additional constraints on the number of independent elements in H. We find that the number of independent complex phases in H is given by[1]

$$N_{\text{phases}} = \begin{cases} \frac{\bar{n}(\bar{n}-1)}{2}, & n \geq \bar{n}, \\ n\bar{n} - \frac{n(n+1)}{2}, & n < \bar{n} . \end{cases} \tag{6}$$

For the case of $n \geq \bar{n}$ it is possible to choose a basis where the matrix of Higgs Yukawa couplings $h_{\alpha i}$ can itself be written as an $\bar{n} \times \bar{n}$ Hermitian matrix. To see this explicitly note that for $n \geq \bar{n}$ we can construct a unitary matrix V_{jk} such that

$$h^{\alpha j} V_{jk} = 0 \quad \text{for } k > \bar{n}. \tag{7}$$

By performing the unitary transformation on the lepton doublet l_j, $l_k = V^{-1}_{kj} l_j$ it is clear that only the l_k with $k \leq \bar{n}$ couple to the right-handed singlet fields N_α. In this basis the couplings are completely described by an $\bar{n} \times \bar{n}$ square matrix h'. Finally, recall that any square matrix h' can be written as the product $h' = hW$, where W is unitary and h is Hermitian. Thus by again performing a unitary transformation on the lepton doublets l_k, the Higgs Yukawa coupling becomes

$$h^{\alpha j} N^c_\alpha l_j H_u , \tag{8}$$

where, as advertised, h is an $\bar{n} \times \bar{n}$ Hermitian matrix.

We may use the fact that h is Hermitian to show how the complex phases manifest themselves in the quantity $T_{\alpha\beta ij}$, and do this explicitly for $\bar{n} = 2$ and 3. (If $\bar{n} = 1$, h is real and T vanishes identically; hence, as discussed above, there is no CP violation.) For $\bar{n} = 2$, h contains one phase, and there are three distinct T's: T_{1211}, T_{1212}, and T_{1222}. These are related to each other by

$$\frac{T_{1211}}{|t_{1211}|} = \frac{T_{1212}}{|t_{1212}|} = \frac{T_{1222}}{|t_{1222}|} = \sin(2\delta_{12}) , \tag{9}$$

where $\delta_{\alpha j}$ is defined by $h^{\alpha j} = |h^{\alpha j}| e^{i\delta_{\alpha j}}$. For $\bar{n} = 3$ there are 18 distinct T's; however, there are only 3 independent phases, appearing in 9 possible combinations. These are summarized by the three relations

$$\frac{T_{\alpha\beta\alpha\alpha}}{|t_{\alpha\beta\alpha\alpha}|} = \frac{T_{\alpha\beta\beta\beta}}{|t_{\alpha\beta\beta\beta}|} = \frac{T_{\alpha\beta\alpha\beta}}{|t_{\alpha\beta\alpha\beta}|} = \sin(2\delta_{\alpha\beta}) , \tag{10}$$

$$\frac{T_{\alpha\beta\alpha\gamma}}{|t_{\alpha\beta\alpha\gamma}|} = \frac{T_{\alpha\beta\beta\gamma}}{|t_{\alpha\beta\beta\gamma}|} = \sin(\delta_{\alpha\beta} + \delta_{\alpha\gamma} - \delta_{\beta\gamma}) , \tag{11}$$

$$\frac{T_{\alpha\beta\gamma\gamma}}{|t_{\alpha\beta\gamma\gamma}|} = \sin(2\delta_{\alpha\gamma} - 2\delta_{\beta\gamma}) , \tag{12}$$

where $\alpha < \beta$ and $\gamma \neq \alpha \neq \beta$. In this notation the three independent phases appearing in $T_{\alpha\beta ij}$ are δ_{12}, δ_{13}, and δ_{23}.

In conclusion we have examined leptogenesis models where the initial lepton number asymmetry is generated by a CP asymmetry in the decay of a heavy neutrino (sneutrino) into leptons with respect to the decay into antileptons (sleptons). The Higgs Yukawa couplings $h_{\alpha i}$ that give rise to these decays must contain at least one complex phase. We have calculated the number of complex phases that can occur in this coupling for an arbitrary number of heavy right-handed neutrinos and mass-

[1] This is the same number of phases that can occur in an $\bar{n} \times \bar{n}$ Majorana mass matrix [4]; however, the physical context here is quite different.

less left-handed neutrinos. We find that, for $\bar{n}$ heavy right-handed neutrinos and n left-handed neutrinos, the number of independent phases appearing in the asymmetry parameter ϵ is given by Eq. (6). Thus a minimum of two right-handed neutrinos (sneutrinos) and one left-handed neutrino are required for CP violation to occur in right-handed neutrino decay. For the three generation case ($n = \bar{n} = 3$) there are three independent phases in the Higgs Yukawa couplings. We note that this result is qualitatively different from CP violation in charged-current weak interactions arising from a complex phase in the Kobayashi-Maskawa matrix.

Finally, we would like to add one minor comment. One may seek the origin of the Majorana mass term in a cooperative effect of gravity-induced effective terms and the grand unified theory (GUT) symmetry breaking: An interesting coincidence $M \sim M_G^2/M_P \sim 10^{13}$ GeV (M_G and M_P are the GUT and Planck scales, respectively) inspires us to use one of such gravity-induced terms,

$$\frac{1}{M_P} C_{\alpha\beta} N_\alpha N_\beta \Sigma\Sigma, \tag{13}$$

where Σ is a 24-plet of SU(5) GUT; when Σ acquires a VEV, this term will induce the Majorana mass with the right magnitude. Note, however, that the term is necessarily accompanied by a quartic coupling

$$\frac{1}{M_P^2} \Sigma^2 (C_{\alpha\beta} \tilde{N}_\alpha \tilde{N}_\beta)^2 \sim \lambda \tilde{N}^4 \tag{14}$$

because of the supersymmetry. The coupling $\lambda \sim 10^{-6}$ is too big for this term to be negligible compared to the mass term $\sim M^2 \tilde{N}^2$ during the inflation. Although it is an attractive idea, the explanation of the Majorana mass along the above line does not work.

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