Light-front QCD. II. Two-component theory

Wei-Min Zhang* and Avaroth Harindranath
Department of Physics, The Ohio State University, Columbus, Ohio 43210
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The light-front gauge $A_a^+ = 0$ is known to be a convenient gauge in practical QCD calculations for short-distance behavior, but there are persistent concerns about its use because of its "singular" nature. The study of nonperturbative field theory *quantizing on a light-front plane* for hadronic bound states requires one to gain *a priori* systematic control of such gauge singularities. In the second paper of this series we study the two-component old-fashioned perturbation theory and various severe infrared divergences occurring in old-fashioned light-front Hamiltonian calculations for QCD. We also analyze the ultraviolet divergences associated with a large transverse momentum and examine three currently used regulators: an explicit transverse cutoff, transverse dimensional regularization, and a global cutoff. We discuss possible difficulties caused by the light-front gauge singularity in the applications of light-front QCD to both old-fashioned perturbative calculations for short-distance physics and upcoming nonperturbative investigations for hadronic bound states.

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I. INTRODUCTION

An intuitive physical picture of high-energy processes in QCD is provided by the partonic interpretation [1]. It is well known that such a picture emerges most naturally in the light-front canonical quantization with light-front gauge $A_a^+ = A_a^0 + A_a^3 = 0$. Based on factorization theorems [2], the hadronic structure functions which one extracts from measured cross sections can be separated into hard and soft parts for large momentum transfer Q^2. The hard part is the so-called hard scattering coefficient, which is the short-distance contribution of quarks and gluons to the structure functions. The soft part measures the low-energy (nonperturbative) properties of quarks and gluons in the parent hadron. When QCD is quantized on a light-front plane with light-front gauge, the soft contribution can be identified with parton distribution functions, i.e., the number density of partons as a function of the fraction of the light-front longitudinal momentum of the parent hadron. In the last two decades, QCD with either covariant gauge or light-front gauge has been used extensively to calculate the hard scattering coefficients that are relevant for the scale evolution of hadronic structure functions. However, the QCD based exploration of the nonperturbative part, the parton distribution functions, is still in a very preliminary stage.

Several practical calculations for short-distance QCD have been performed in the standard Feynman perturbation theory with the light-front gauge [3,4]. In this scheme, Feynman rules are derived by the use of the path integral approach with a (light-front) gauge-fixing term [5], and various hard scattering coefficients are calculated via either the operator product expansion [3] or a Feynman diagrammatic approach [4]. Despite the successful perturbative calculations, it is still not clear whether one can extend the Feynman approach to the nonperturbative studies, namely, the QCD calculation of parton distribution functions, which requires the full information of hadronic bound states.

Current attempts to explore nonperturbative QCD in LFFT are based on the Hamiltonian theory, which is defined by quantizing the theory on a light-front plane via equal-x^+ (the light-front time) commutation relations, as was first developed by Kogut and Soper for QED [6]. In the light-front QCD Hamiltonian theory (LFQCD), the hadronic bound states may be obtained by diagonalizing a light-front QCD Hamiltonian in a truncated quark-gluon Fock space [7], based on the old ideas of Tamm and Dancoff [8]. In this formalism, the low-energy hadronic structure, such as parton distribution functions and various hadronic form factors, can be addressed directly from QCD. Meanwhile, the short-distance behavior can also be studied in the same framework, i.e., in old-fashioned perturbation theory, or explicitly in the x^+-ordered perturbative QCD Hamiltonian theory based on equal-x^+ commutation relations [9]. Indeed, the interpretation of high-energy processes via the parton picture had led to extensive investigations of perturbative field theory in the infinite momentum frame in the early 1970s [10,11]. At that time, Drell, Levy, and Yan [10] and Bjorken, Kogut, and Soper [12] had already pointed out that in the old-fashioned theory the physical picture for various real physical processes becomes much clearer. The parton picture is just a typical example.

Yet, beyond many interesting applications for exclusive processes given by Lepage and Brodsky in the early 1980s [9], x^+-ordered Hamiltonian QCD has not been explored extensively in the last decade. Only very recently, loop calculations in the x^+-ordered perturbative QED and QCD theory have been performed [13–15]. It is seen that there are severe divergences in LFQCD which are associated with light-front gauge singularities. However, a systematic computational method, which involves vari-

*Address after 1 January 1994: Institute of Physics, Academia Sinica, Taipei, Taiwan.

ous regularization and renormalization schemes for light-front singularities and ultraviolet transverse divergences, is not yet well established.

In this series of papers, we study various problems of QCD in the equal-x^+ light-front canonical Hamiltonian theory that have not been addressed in detail before and that might be faced when we attempt to solve hadronic structures in this framework. In the first paper [16] (called paper I hereafter), we have explored phase space canonical quantization and light-front infrared singularities in LFQCD. The latter originates from the light-front longitudinal boundary integrals when we eliminate the unphysical longitudinal gauge potential in $A_a^+ = 0$ gauge. In this second paper, we focus on problems of various light-front divergences in the x^+-ordered Hamiltonian theory and some regularization schemes, which must be handled properly in both the old-fashioned perturbative calculations for short-distance behavior and the upcoming nonperturbative study for hadronic bound states in LFQCD. In the following paper [17], we will present a calculation of the ultraviolet divergent part of the quark-gluon vertex up to one loop in x^+-ordered Hamiltonian theory.

Because of the two requirements of field theory, namely, covariance and unitarity, quantization of non-Abelian gauge theory is not simple. In the usual covariant path integral quantization, the unphysical gauge components violate unitarity and therefore one has to introduce ghost fields to cancel the unphysical states in the theory. A covariant formalism has several advantages in many aspects. Yet, the physical picture becomes obscure once unphysical particles are introduced. On the other hand, in the canonical quantization, one has to choose a physical gauge, such as axial gauge or light-front gauge, where unitarity is automatically satisfied but covariance is no longer manifest. Although most QCD investigations for short-distance physics are based on physical gauges, particularly on light-front gauge, one still prefers to use the covariant formulation. The main complaint against light-front gauge is that the principal value prescription for the gauge singularity in this gauge prohibits any continuation to Euclidean space (Wick rotation) and hence the power counting for Feynman loop integrals. As a result, nonlocal counterterms have to be introduced, which break the multiplicative renormalizability [18,19].

However, even in covariant perturbation theory, formal multiplicative renormalizability may not be significantly useful in the calculations of hard scattering coefficients. This is because gauge invariant composite operators in the operator product expansion are not multiplicatively renormalizable [20], and the gauge variant operators will induce gauge invariant counterterms that do contribute to gauge invariant matrix elements. It has recently been shown that by taking into account this mixing in the covariant theory the long-standing discrepancy between the covariant and light-front gauge calculations for the second-order anomalous dimension of the gluon composite operators can be resolved [21]. In fact, the mixing of gauge invariant operators is an intrinsic property that exists in any covariant formulation of QCD. The above mixing problem of gauge invariant operators apparently disappears in physical gauges, and perturbative calculations become straightforward. However, a new spurious mixing of ultraviolet and infrared divergences associated with the gauge singularity does occur in light-front gauge using the principal value prescription. Here we have to be careful to see whether or not the spurious mixing affects physical quantities. In light-front gauge Feynman approach, it has been shown that the most severe infrared divergences caused by the choice of $A_a^+ = 0$ gauge with the principal value prescription are indeed canceled in gauge invariant sectors [22]. However, in Hamiltonian theory it has not been explored how this severe spurious mixing behaves and whether the resultant divergences canceled for gauge invariant quantities. These are essential problems in both perturbative and nonperturbative investigations, which we shall address in this and the following paper [17].

Since gauge fixing of $A_a^+ = 0$ removes the unphysical degrees of freedom, unitarity is automatically satisfied and no ghost field is needed, with the price to pay being the loss of manifest covariance. Indeed, the choice of $A_a^+ = 0$ gauge promises that QCD involves only two-component gauge fields and two-component quark fields in light-front coordinates. In the early stage of the development of light-front QED, Bjorken, Kogut, and Soper used the two-component formulation to discuss various physical QED processes [12]. In the middle 1980s, a two-component Feynman perturbation theory with Mandelstam-Leibbrandt prescription [23] was developed by Capper *et al.* [19] and Lee *et al.* [24] for pure Yang-Mills theory. It has been shown that there are many advantages in such a two-component light-front field theory. However, the extension of the two-component theory to the study of light-front canonical quantization (i.e., x^+-ordered theory) of QCD has not been explored in the literature. In the old-fashioned light-front perturbative QCD formulation of Lepage and Brodsky [9], the unphysical degrees of freedom were eliminated for the quantization, but in the final step they recombined the vertices together to express the Hamiltonian in terms of four-component quark and gauge fields. In this formulation, the three point vertices look the same as the covariant form and therefore it becomes useful for comparing with the covariant expressions. Lepage and Brodsky have made an extensive application of their theory to QCD exclusive processes at the tree level. For loop diagrams, the calculations become quite complicated. It is pleasantly surprising to see that the two-component formulation not only provides a transparent physical picture but also simplifies very much the calculation scheme of Lepage and Brodsky. In order to understand clearly the origin of light-front singularities in various physical processes and simplify practical calculations, we derive the two-component LFQCD in this paper and then study light-front regularization and renormalization based on the two-component formulation.

The paper is organized as follows. In Sec. II, following paper I, we further formulate LFQCD in terms of physical degrees of freedom so that both the gauge and quark fields become two-component. The equal-x^+ canonical quantization conditions are obtained by use of the one-form structure of physical phase space, as shown in paper I. The origin of the gauge singularity and associated divergences are also discussed. In Sec. III, we develop the two-component x^+-ordered (old-fashioned) perturbation theory for LFQCD in detail. We construct the corresponding diagrammatic rules for the x^+-ordered LFQCD calculations. To understand the basic light-front singularity problem and the associated divergences in loop integrals in the x^+-ordered two-component LFQCD theory, we present a calculation of the quark mass and wave function renormalization constants to the second order in Sec. IV. We also discuss the counterterms required for gluon mass and wave function renormalization in this section. Finally, we include an appendix for the construction of the two-component Feynman theory in light-front coordinates and a comparison with the old-fashioned Hamiltonian theory.

II. TWO-COMPONENT FORMALISM

The QCD Lagrangian is

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu}) + \bar{\psi}(i\gamma_\mu D^\mu - m)\psi, \tag{2.1}$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu]$ are the gluon field strength tensors and $A^\mu = \sum_a A_a^\mu T^a$ are the 3×3 gluon field matrices with T^a the Gell-Mann SU(3) matrices: $[T^a, T^b] = if^{abc}T^c$ and $\mathrm{Tr}(T^aT^b) = \frac{1}{2}\delta_{ab}$. The field variable ψ describes quarks with three colors and N_f flavors, and $D^\mu = \partial^\mu - igA^\mu$ are the covariant derivatives, while m is an $N_f \times N_f$ diagonal quark mass matrix.

In light-front coordinates, we choose

$$x^\pm = x^0 \pm x^3, \qquad x_\perp^i = x^i \quad (i = 1, 2), \tag{2.2}$$

as the local coordinates which lead to a space-time metric tensor $g^{+-} = g^{-+} = \frac{1}{2}, g^{ij} = -\delta_{ij}$, with the other components equal to zero. We also choose x^+ as the "time" parameter so that x^- and x^i become the longitudinal and transverse coordinates.

A. Two-component gauge fields

In (Abelian or non-Abelian) gauge theory, only two-components, i.e., the transverse components, of the vector gauge potentials are physically independent degrees of freedom. The other two components are unphysical and may be eliminated in the canonical Hamiltonian formulation by a suitable choice of gauge condition. In light-front coordinates, such a gauge choice is $A_a^+ = A_a^0 + A_a^3 = 0$ (light-front gauge). With the light-front gauge, as we have shown in paper I, the Lagrangian can be *rewritten* as [16]

$$\mathcal{L} = \frac{1}{2}F_a^{+i}(\partial^- A_a^i) + i\psi_+^\dagger(\partial^-\psi_+) - \mathcal{H} - \left\{A_a^-\mathcal{C}_a + \frac{1}{2}(\psi_-^\dagger\mathcal{C} + \mathcal{C}^\dagger\psi_-)\right\}, \tag{2.3}$$

where

$$\mathcal{H} = \frac{1}{2}\left\{E_a^{-2} + B_a^{-2}\right\} + \left\{\psi_+^\dagger\{\alpha_\perp\cdot(i\partial_\perp + gA_\perp) + \beta m\}\psi_-\right\} + \left\{\frac{1}{2}\partial^+(E_a^- A_a^-) - \partial^i(E_a^i A_a^-)\right\} \tag{2.4}$$

is the Hamiltonian density and

$$\mathcal{C}_a = \frac{1}{2}\partial^+ E_a^- - (\partial^i E_a^i + gf^{abc}A_b^iE_c^i) + g\psi_+^\dagger T^a\psi_+, \tag{2.5}$$

$$\mathcal{C} = i\partial^+\psi_- - (i\alpha_\perp\cdot\partial_\perp + g\alpha_\perp\cdot A_\perp + \beta m)\psi_+. \tag{2.6}$$

Here, $E_a^{-,i} = -\frac{1}{2}\partial^+ A_a^{-,i}$ and $B_a^- = F_a^{12}$ are components of the light-front color electric field and the longitudinal component of the light-front color magnetic field, and ψ_+ and ψ_- are the light-front up and down components of the quark field [6]: $\psi = \psi_+ + \psi_-$, $\psi_\pm = \Lambda_\pm\psi = \frac{1}{2}\gamma^0\gamma^\pm\psi$, where $\Lambda_+ + \Lambda_- = I$, $\Lambda_\pm{}^2 = \Lambda_\pm$, and $\Lambda_+\Lambda_- = 0$.

From Eq. (2.3) it becomes clear that the independent dynamical degrees of freedom in LFQCD are the transverse gauge fields A_a^i and the up-component quark field ψ_+. The Lagrangian equations of motion show that $\mathcal{C}_a = 0$ and $\mathcal{C} = 0$, which implies that the longitudinal gauge fields A_a^- and the down-component quark field ψ_- are Lagrange multipliers. By solving these constraints, $\mathcal{C}_a = 0$ and $\mathcal{C} = 0$, which determine the Lagrange multipliers, the LFQCD Hamiltonian can be expressed formally as

$$H = \int dx^- d^2x_\perp \left\{ \frac{1}{2}(\partial^i A_a^j)^2 + gf^{abc}A_a^iA_b^j\partial^iA_c^j + \frac{g^2}{4}f^{abc}f^{ade}A_b^iA_c^jA_d^iA_e^j + \left[\psi_+^\dagger\{\sigma_\perp\cdot(i\partial_\perp + gA_\perp) - im\}\right.\right.$$
$$\left. \times\left(\frac{1}{i\partial^+}\right)\{\sigma_\perp\cdot(i\partial_\perp + gA_\perp) + im\}\psi_+\right] + g\partial^iA_a^i\left(\frac{1}{\partial^+}\right)(f^{abc}A_b^j\partial^+A_c^j + 2\psi_+^\dagger T^a\psi_+)$$
$$\left. + \frac{g^2}{2}\left(\frac{1}{\partial^+}\right)(f^{abc}A_b^i\partial^+A_c^i + 2\psi_+^\dagger T^a\psi_+)\left(\frac{1}{\partial^+}\right)(f^{ade}A_d^j\partial^+A_e^j + 2\psi_+^\dagger T^a\psi_+)\right\} + \text{ surface terms}, \tag{2.7}$$

where the surface terms are given by the last term in Eq. (2.4).

B. Two-component quark spinors

Equation (2.7) indicates that the canonical Hamiltonian theory of QCD can be formulated purely in terms of the transverse (physical) gauge fields, as we expect after gauge fixing [16]. Furthermore, it is interesting to see here that in light-front coordinates the four-component fermion field can also be reduced to a two-component field. This is indeed one of the main advantages of light-front field theory that simplifies the relativistic fermion structure.

The two-component quark field can be explicitly formulated in a light-front representation of the γ matrix defined by

$$\gamma^+ = \begin{bmatrix} 0 & 0 \\ 2i & 0 \end{bmatrix}, \quad \gamma^- = \begin{bmatrix} 0 & -2i \\ 0 & 0 \end{bmatrix},$$
$$\gamma^i = \begin{bmatrix} -i\sigma^i & 0 \\ 0 & i\sigma^i \end{bmatrix}, \quad \gamma^5 = \begin{bmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{bmatrix}. \tag{2.8}$$

Then the projection operators $\Lambda_\pm$ become

$$\Lambda_+ = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \Lambda_- = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \tag{2.9}$$

and

$$\psi_+ = \begin{bmatrix} \xi \\ 0 \end{bmatrix}, \quad \psi_- = \begin{bmatrix} 0 \\ \left(\frac{1}{i\partial^+}\right)[\sigma^i(i\partial^i + gA^i) + im]\xi \end{bmatrix}. \tag{2.10}$$

This immediately shows that ψ_+ and ψ_- are two-component fermion fields. Hereafter, we shall simply let ξ represent the light-front quark field.

It must be emphasized that the above two-component light-front field still describes relativistic spin-1/2 particles, which are intrinsically different from spin-1/2 nonrelativistic particles. In other words, it contains quarks and antiquarks. To see how antiquarks can be described in the two-component representation, we need to construct the charge conjugate for ξ. By using the condition $C\gamma^{\mu T}C^{-1} = -\gamma^\mu$, it is easy to find that the charge conjugation operator in the γ representation of Eq. (2.8) is

$$C = i\gamma^3\gamma^1 = \begin{bmatrix} 0 & -i\sigma^i \\ i\sigma^i & 0 \end{bmatrix}. \tag{2.11}$$

Hence,

$$\psi^c = \eta C\gamma^0\psi^* \longrightarrow \xi^c = \eta\sigma^1\xi^*, \tag{2.12}$$

where η is a phase factor which will be set to unity in this paper. Equation (2.12) indicates that quark and antiquark can be described in a two-component formalism in light-front coordinates. An explicit solution for free quarks and antiquarks will be given in the next section.

In the two-component representation, the Hamiltonian (apart from total transverse spatial derivatives) becomes

$$H = \int dx^- d^2x_\perp \Big\{ \frac{1}{2}(\partial^i A_a^j)^2 + gf^{abc}A_a^i A_b^j \partial^i A_c^j + \frac{g^2}{4} f^{abc}f^{ade}A_b^i A_c^j A_d^i A_e^j + \Big[\xi^\dagger\{\sigma_\perp\cdot(i\partial_\perp + gA_\perp) - im\}$$
$$\times \left(\frac{1}{i\partial^+}\right)\{\sigma_\perp\cdot(i\partial_\perp + gA_\perp) + im\}\xi \Big] + g\partial^i A_a^i \left(\frac{1}{\partial^+}\right)(f^{abc}A_b^j\partial^+ A_c^j + 2\xi^\dagger T^a\xi)$$
$$+\frac{g^2}{2}\left(\frac{1}{\partial^+}\right)(f^{abc}A_b^i\partial^+ A_c^i + 2\xi^\dagger T^a\xi)\left(\frac{1}{\partial^+}\right)(f^{ade}A_d^j\partial^+ A_e^j + 2\xi^\dagger T^a\xi)\Big\} + \text{ surface terms.} \tag{2.13}$$

This is the canonical LFQCD Hamiltonian that is expressed only in terms of two-component gluon and two-component quark field variables.

To quantize the above two-component LFQCD, we use the canonical phase space approach, as shown in paper I. The resulting equal-x^+ commutation relations for the physical degrees of freedom are

$$[A_a^i(x), \partial^+ A_b^j(y)]_{x^+=y^+} = i\delta_{ab}\delta^{ij}\delta^3(x-y), \tag{2.14}$$
$$\{\xi(x), \xi^\dagger(y)\}_{x^+=y^+} = \delta^3(x-y), \tag{2.15}$$

where $\delta^3(x-y) = \delta(x^- - y^-)\delta^2(x_\perp - y_\perp)$. Equation (2.14) shows that the basic equal-x^+ commutation relations of the physical gauge fields are the relations between the A_a^i themselves, which is essentially different from the conventional equal time quantization. To find a commutation relation between A_a^i, we have to define the operator $1/\partial^+$ explicitly. As we have discussed in paper I, the gauge singularity of light-front gauge is hidden in this definition, so this definition will have a significant effect on the regularization and renormalization of LFQCD.

C. Light-front gauge singularity

The gauge singularities in light-front QCD that arise when one tries to eliminate the unphysical gauge degrees of freedom by solving the constraint equations can be seen clearly in momentum space. In momentum space, the constraint Eqs. (2.5) and (2.6) cannot determine the dependent fields in terms of physical fields for the single longitudinal momentum $k^+ = 0$. In coordinate space, this implies that the $A_a^+ = 0$ gauge has a singularity at longitudinal boundary. A careful treatment of the definition of $1/\partial^+$ is therefore necessary in the study of LFQCD, as we have pointed out in paper I.

Since the LFQCD Lagrangian (2.14) is only a linear function of the first-order light-front time derivative of field variables, and the equations of motion for the gauge potential has the form

$$\frac{\partial^2}{\partial^+\partial^-}A_a^i = F(A_a^i, \xi), \tag{2.16}$$

the light-front initial value problem is relevant to the boundary value at longitudinal boundary. According to the discussion in paper I, a suitable definition of $(1/\partial^+)$, which determines uniquely the initial value problem at $x^+ = 0$ [25], is

$$\left(\frac{1}{\partial^+}\right) f(x^-) = \frac{1}{4}\int_{-\infty}^{\infty} dx'^- \epsilon(x^- - x'^-) f(x'^-). \tag{2.17}$$

With the above definition, we have shown that the residual gauge freedom in $A_a^+ = 0$ gauge is completely fixed [26]. The singularity at $k^+ = 0$ is removed and the nontrivial topological properties of QCD are manifest in the nonvanishing boundary behavior of A_a^i at longitudinal boundary. Using this definition, the LFQCD Hamiltonian becomes

$$\begin{aligned} H = \int dx^- d^2x_\perp \Bigg\{ & \frac{1}{2}(\partial^i A_a^j)^2 + g f^{abc} A_a^i A_b^j \partial^i A_c^j + \frac{g^2}{4} f^{abc} f^{ade} A_a^i A_b^j A_d^i A_e^j \\ & \frac{1}{4}\int_{-\infty}^{\infty} dx'^- \left\{ 2g\partial^i A_a^i \epsilon(x^- - x'^-)\rho_a(x'^-, x) - i\xi^\dagger \{\alpha_\perp^i (i\partial_\perp^i + gA^i) + \beta m\} \epsilon(x^- - x'^-) \{\alpha_\perp^j (i\partial_\perp^j + gA^j) + \beta m\}\xi \right\} \\ & - \frac{g^2}{4}\int_{-\infty}^{\infty} dx'^- \rho_a(x^-, x)|x^- - x'^-|\rho_a(x'^-, x) \Bigg\} + \left(\lim_{\lambda\to\infty} \lambda\right) \frac{g^2}{4}\int d^2x_\perp \left\{\int_{-\infty}^{\infty} dx^- \rho_a(x^-, x)\right\}^2, \end{aligned} \tag{2.18}$$

where we have defined $\rho_a(x^-, x) = \frac{1}{2}(f^{abc} A_b^i \partial^+ A_c^i + 2\xi^\dagger T^a \xi)$ and have used the identity [27]

$$\frac{1}{2}\int_{-\lambda}^{\lambda} dx'^- \epsilon(x^- - x'^-)\epsilon(x'^- - x''^-) = |x^- - x''^-| - \lambda, \tag{2.19}$$

and the surface terms in (2.13) vanish [16]. The equal-x^+ commutation relation for A_a^i can also be found from (2.14):

$$[A_a^i(x), A_b^j(y)]_{x^+ = y^+} = -i\frac{\delta_{ab}\delta^{ij}}{4}\epsilon(x^- - y^-)\delta^2(x_\perp - y_\perp). \tag{2.20}$$

It has been verified that the above formulation is consistent with the Lagrangian equations of motion [16].

From Eq. (2.19) we see that the LFQCD Hamiltonian contains a boundary term [the last term in Eq. (2.18)] due to the linear instantaneous interaction. This term seems to be problematic since it contains an infinite factor. However, as we have examined in paper I, although the $k^+ = 0$ singularity is removed by Eq. (2.17), the infrared divergences from the small longitudinal momentum, i.e., surrounding the $k^+ = 0$ region, are still present in the above Hamiltonian. These infrared divergences associated with light-front gauge singularity are hidden in the three point quark-gluon and gluon-gluon vertices in Eq. (2.18). There are mainly two types of the light-front infrared divergences: linear and logarithmic. In paper I, we have demonstrated in a one-loop calculation that with our consistent definition of the product of two principal value prescriptions based on Eq. (2.17), the linear infrared divergences are canceled by the infinite boundary term [the last term in Eq. (2.18)]. Thus this infinite term is necessary for a singularity-free LFQCD Hamiltonian with the principal value prescription. For detailed discussion see paper I. In this paper, we will address the logarithmic infrared divergences.

III. THE X^+-ORDERED LFQCD PERTURBATION THEORY

In this section, we shall develop the x^+-ordered perturbation theory for the two-component canonical Hamiltonian LFQCD.

A. The x^+-ordered perturbation theory

The LFQCD Hamiltonian can be rewritten as a free term plus interactions:

$$H = \int dx^- d^2x_\perp (\mathcal{H}_0 + \mathcal{H}_{\rm int}). \tag{3.1}$$

It is easy to show that

$$\mathcal{H}_0 = \frac{1}{2}(\partial^i A_a^j)(\partial^i A_a^j) + \xi^\dagger \left(\frac{-\partial_\perp^2 + m^2}{i\partial^+}\right)\xi, \tag{3.2}$$

$$\mathcal{H}_{\rm int} = \mathcal{H}_{qqg} + \mathcal{H}_{ggg} + \mathcal{H}_{qqgg} + \mathcal{H}_{qqqq} + \mathcal{H}_{gggg}, \tag{3.3}$$

and

$$\mathcal{H}_{qqg} = g\xi^\dagger \left\{ -2\left(\frac{1}{\partial^+}\right)(\partial_\perp \cdot A_\perp) + \sigma \cdot A_\perp \left(\frac{1}{\partial^+}\right)(\sigma\cdot\partial_\perp + m) + \left(\frac{1}{\partial^+}\right)(\sigma\cdot\partial_\perp - m)\sigma\cdot A_\perp \right\}\xi, \tag{3.4}$$

$$\mathcal{H}_{ggg} = gf^{abc}\left\{\partial^i A_a^j A_b^i A_c^j + (\partial^i A_a^i)\left(\frac{1}{\partial^+}\right)(A_b^j\partial^+ A_c^j)\right\}, \tag{3.5}$$

$$\begin{aligned}\mathcal{H}_{qqgg} &= g^2\left\{\xi^\dagger\sigma\cdot A_\perp\left(\frac{1}{i\partial^+}\right)\sigma\cdot A_\perp\xi + 2\left(\frac{1}{\partial^+}\right)(f^{abc}A_b^i\partial^+A_c^i)\left(\frac{1}{\partial^+}\right)(\xi^\dagger T^a\xi)\right\}\\ &= \mathcal{H}_{qqgg1} + \mathcal{H}_{qqgg2},\end{aligned} \tag{3.6}$$

$$\mathcal{H}_{qqqq} = 2g^2\left\{\left(\frac{1}{\partial^+}\right)(\xi^\dagger T^a\xi)\left(\frac{1}{\partial^+}\right)(\xi^\dagger T^a\xi)\right\}, \tag{3.7}$$

$$\begin{aligned}\mathcal{H}_{gggg} &= \frac{g^2}{4}f^{abc}f^{ade}\left\{A_b^iA_c^jA_d^iA_e^j + 2\left(\frac{1}{\partial^+}\right)(A_b^i\partial^+A_c^i)\left(\frac{1}{\partial^+}\right)(A_d^j\partial^+A_e^j)\right\}\\ &= \mathcal{H}_{gggg1} + \mathcal{H}_{gggg2}.\end{aligned} \tag{3.8}$$

To derive the x^+-ordered LFQCD perturbative theory, we undress the Heisenberg operators and go to the interaction picture by using the usual U-matrix transformation,

$$U(x^+, x_0^+) = T_+ \exp\left(-\frac{i}{2}\int_{x_0^+}^{x^+} d\tau H_{\rm int}(\tau)\right), \tag{3.9}$$

where T_+ is an x^+-ordering operator. Thus, the full interacting Heisenberg operators O and state vectors $|\Psi\rangle$ are determined by

$$O(x^+) = U(0,x^+)O_I(x^+)U(x^+,0), \tag{3.10}$$

$$|\Psi\rangle = U(0,x^+)|\Psi\rangle_I. \tag{3.11}$$

In the interaction picture, the equations of motion are

$$i\partial^-|\Psi\rangle_I = H_{\rm int}|\Psi\rangle_I \ , \quad \partial^- O_I = \frac{1}{i}[O_I, H_0]. \tag{3.12}$$

The perturbation expansion is then the same as that familiar from quantum mechanics. For convenience, we will drop the subscript I in the following. By using the adiabatic assumption [28]

$$U_\epsilon(x^+, x_0^+) = \sum_{n=0}^{\infty}\left(\frac{-i}{2}\right)^n\frac{1}{n!}\int_{x_0^+}^{x^+}d\tau_1\cdots\int_{x_0^+}^{x^+}d\tau_n \times e^{-\frac{\epsilon}{2}(|\tau_1|+\cdots+|\tau_n|)}T_+[H_{\rm int}(\tau_1)\cdots H_{\rm int}(\tau_n)], \tag{3.13}$$

where ϵ is an infinitesimal number (one should let it go to zero at the final step of the procedure), we obtain the following perturbative expansion [29].

(i) The perturbative expansion of state vectors is given as

$$|\Psi\rangle = U|\Phi\rangle = \sum_{n=0}^{\infty}\left(\frac{-i}{2}\right)^n\frac{1}{n!}\int_0^\infty d\tau_1\cdots\int_0^\infty d\tau_n \times e^{-\frac{\epsilon}{2}(|\tau_1|+\cdots+|\tau_n|)}T_+[H_{\rm int}(-\tau_1)\cdots H_{\rm int}(-\tau_n)]|\Phi\rangle, \tag{3.14}$$

where $U \equiv U_\epsilon(0,-\infty)$, and $|\Phi\rangle$ is an eigenstate of H_0.

(ii) The perturbative expansion of the matrix element of the operator O between the initial state $|\Psi_i\rangle$ and the final state $|\Psi_f\rangle$ is

$$\begin{aligned}\langle\Psi_f|O(0)|\Psi_i\rangle &= \langle\Phi_f|U^{-1}O(0)U|\Phi_i\rangle\\ &= \sum_{n=0}^{\infty}\left(\frac{-i}{2}\right)^n\frac{1}{n!}\int_{-\infty}^{\infty}d\tau_1\cdots d\tau_n e^{-\frac{\epsilon}{2}(|\tau_1|+\cdots+|\tau_n|)}\langle\Phi_f|T_+[H_{\rm int}(\tau_1)\cdots H_{\rm int}(\tau_n)O(0)]|\Phi_i\rangle.\end{aligned} \tag{3.15}$$

B. Momentum space representation

In practice, it is convenient to perform perturbative calculations in momentum space. In the interaction picture, the Fourier transform of $A_a^i(x)$ and $\xi(x)$ can be obtained by solving the equation of motion in momentum space for the free Hamiltonian. The equations of motion are determined by Eq. (3.12):

$$\begin{aligned}\partial^- A_a^i(x) &= \frac{1}{i}[A_a^i(x), H_0]\\ &= \frac{1}{4}\int_{-\infty}^{\infty}dx'^-\epsilon(x^- - x'^-)\partial_\perp^2 A_a^i(x^+, x'^-, x_\perp),\end{aligned} \tag{3.16}$$

$$\begin{aligned}\partial^-\xi(x) &= \frac{1}{i}[\xi(x), H_0]\\ &= \frac{1}{4}\int_{-\infty}^{\infty}dx'^-\epsilon(x^- - x'^-)(\partial_\perp^2 - m^2) \times\xi(x^+, x'^-, x_\perp),\end{aligned} \tag{3.17}$$

and their solutions are

$$A^i(x) = \sum_\lambda\int\frac{dq^+d^2q_\perp}{2(2\pi)^3[q^+]}\left[\varepsilon_\lambda^i a(q,\lambda)e^{-iqx} + \text{H.c.}\right], \tag{3.18}$$

$$\xi(x) = \sum_\lambda \chi_\lambda \int \frac{dp^+ d^2p_\perp}{2(2\pi)^3} \left[b(p,\lambda)e^{-ipx} + d^\dagger(p,-\lambda)e^{ipx} \right], \quad (3.19)$$

with

$$q^- = \frac{q_\perp^2}{[q^+]}, \quad p^- = \frac{p_\perp^2 + m^2}{[p^+]}. \quad (3.20)$$

In Eqs. (3.19) and (3.20), λ is defined as the helicity,

$$\lambda = \begin{cases} 1 \\ -1 \end{cases} \text{for gluons}, \quad \lambda = \begin{cases} 1/2 \\ -1/2 \end{cases} \text{for quarks}. \quad (3.21)$$

The gluon polarization vectors are $\varepsilon_1^i = \frac{1}{\sqrt{2}}(1,i)$ and $\varepsilon_{-1}^i = \frac{1}{\sqrt{2}}(1,-i)$. The quark spinors are simply the eigenstates of a spin-1/2 nonrelativistic particle, $\chi_{1/2} = \binom{1}{0}$ and $\chi_{-1/2} = \binom{0}{1}$. For the definition of Eq. (2.17), the multiple principal value prescription is given as

$$\begin{aligned} \left(\frac{1}{\partial^+}\right)^n f(x^-) &= \left(\frac{1}{4}\right)^n \int_{-\infty}^{\infty} dx_1^- \cdots dx_n^- \epsilon(x^- - x_1^-) \\ &\quad \times \cdots \epsilon(x_{n-1}^- - x_n^-) f(x_n^-) \\ &\to \left[\frac{1}{2}\left(\frac{1}{k^+ + i\epsilon} + \frac{1}{k^+ - i\epsilon}\right)\right]^n f(k^+) \\ &= \frac{1}{[k^+]^n} f(k^+). \end{aligned} \quad (3.22)$$

It is this prescription that regularizes the infinite term in Eq. (2.18). The creation and annihilation operators in (3.19) and (3.20) satisfy the basic commutation relations

$$[a(q,\lambda), a^\dagger(q',\lambda')] = 2(2\pi)^3 k^+ \delta_{\lambda,\lambda'} \delta^3(q-q'), \quad (3.23)$$

$$\{b(p,\lambda), b^\dagger(p',\lambda')\} = \{d(p,\lambda), d^\dagger(p',\lambda')\} = 2(2\pi)^3 \delta_{\lambda,\lambda'} \delta^3(p-p'), \quad (3.24)$$

where $\delta^3(p-p') = \delta(p^+ - p'^+)\delta^2(p_\perp - p'_\perp)$. The charge conjugate of ξ [see Eq. (2.12)],

$$\xi^c(x) = \sum_\lambda \chi_\lambda \int \frac{dp^+ d^2p_\perp}{2(2\pi)^3} \left[-d(p,\lambda)e^{-ipx} + b^\dagger(p,-\lambda)e^{ipx} \right], \quad (3.25)$$

shows that $b(p,\lambda), b^\dagger(p,\lambda)$ and $d(p,\lambda), d^\dagger(p,\lambda)$ are the quark and antiquark creation and annihilation operators.

C. Old-fashioned renormalization theory

In momentum space, the renormalization of mass, wave function, and coupling constants in x^+-ordered Hamiltonian perturbation theory (the old-fashioned renormalization theory) can be determined from the perturbative correction to the masses, the wave functions, and the coupling constants.

(i) *Wave function renormalization*: In momentum space, the perturbative expansion of state vectors is given by

$$\begin{aligned} U|\Phi\rangle = \Bigg\{ |\Phi\rangle &+ \sum_{n_1} \frac{|n_1\rangle\langle n_1|H_{\rm int}(0)|\Phi\rangle}{p^- - p_{n_1}^- + i\epsilon} \\ &+ \sum_{n_1 n_2} \frac{|n_1\rangle\langle n_1|H_{\rm int}(0)|n_2\rangle\langle n_2|H_{\rm int}(0)|\Phi\rangle}{(p^- - p_{n_1}^- + i\epsilon)(p^- - p_{n_2}^- + i\epsilon)} \\ &+ \cdots \Bigg\}, \end{aligned} \quad (3.26)$$

where $|n_1\rangle, |n_2\rangle, \ldots$ are properly symmetrized (antisymmetrized) states with respect to identical bosons (fermions) in the states. If the initial state is a single particle state, we can extract a factor from this expansion [10,12]

$$\begin{aligned} U|\Phi\rangle = \sqrt{Z_\Phi} \Bigg\{ |\Phi\rangle &+ \sum_{n_1}{}' \frac{|n_1\rangle\langle n_1|H_{\rm int}(0)|\Phi\rangle}{p^- - p_{n_1}^- + i\epsilon} \\ &+ \sum_{n_1 n_2}{}' \frac{|n_1\rangle\langle n_1|H_{\rm int}(0)|n_2\rangle\langle n_2|H_{\rm int}(0)|\Phi\rangle}{(p^- - p_{n_1}^- + i\epsilon)(p^- - p_{n_2}^- + i\epsilon)} \\ &+ \cdots \Bigg\}, \end{aligned} \quad (3.27)$$

and the factor Z_Φ is called the wave function renormalization constant determined by the normalization condition

$$\langle \Phi'|U^{-1}U|\Phi\rangle = \delta_{\Phi'\Phi}. \quad (3.28)$$

Note that in contrast with the summation in Eq. (3.26), $\sum'$ in Eq. (3.27) sums over all intermediate states except the initial state $|\Phi\rangle$. Furthermore, Eq. (3.28) leads to

$$Z_\Phi^{-1} = 1 + \sum_{n_1}{}' \frac{|\langle n_1|H_{\rm int}(0)|\Phi\rangle|^2}{(p^- - p_{n_1}^- + i\epsilon)^2} + \cdots. \quad (3.29)$$

(ii) *Mass and coupling constant renormalization*: The perturbative calculation of matrix elements, (3.15), in momentum space becomes

$$\begin{aligned} \langle \Psi_f|O(0)|\Psi_i\rangle = \langle \Phi_f|O(0)|\Phi_i\rangle &+ \sum_{n_1} \frac{\langle \Phi_f|H_{\rm int}(0)|n_1\rangle\langle n_1|O(0)|\Phi_i\rangle}{p_f^- - p_{n_1}^- + i\epsilon} \\ &+ \sum_{n_1,n_2} \frac{\langle \Phi_f|H_{\rm int}(0)|n_1\rangle\langle n_1|H_{\rm int}(0)|n_2\rangle\langle n_2|O(0)]|\Phi_i\rangle}{(p_f^- - p_{n_1}^- + i\epsilon)(p_f^- - p_{n_2}^- + i\epsilon)} + \cdots. \end{aligned} \quad (3.30)$$

The mass correction can then be computed from "energy-level" shift, i.e., the correction to the energy of an on-mass-shell particle:

$$\begin{aligned}\delta p^- &= \langle\Phi|(H-H_0)|\Psi\rangle \\ &= \langle\Phi|H_{\rm int}(0)|\Phi\rangle + \sum_{n_1}\frac{|\langle n_1|H_{\rm int}(0)|\Phi\rangle|^2}{p^- - p^-_{n_1} + i\epsilon} \\ &\quad + \cdots . \end{aligned} \tag{3.31}$$

Using the mass-shell equation $m^2 = p^+p^- - \mathbf{p}_\perp^2$, and recalling that p^+ and $\mathbf{p}_\perp$ are the conserved light-front kinematical momenta, we obtain the mass renormalization in the light-front field theory:

$$\begin{aligned}\delta m^2 = p^+\delta p^- &= p^+\langle\Phi|H_{\rm int}(0)|\Phi\rangle \\ &\quad + p^+\sum_{n_1}\frac{|\langle n_1|H_{\rm int}(0)|\Phi\rangle|^2}{p^- - p^-_{n_1} + i\epsilon} + \cdots . \end{aligned} \tag{3.32}$$

The coupling constant renormalization is

$$\begin{aligned}\langle\Psi_f|H^i_{\rm int}(0)|\Psi_i\rangle &= \langle\Phi_f|H^i_{\rm int}(0)|\Phi_i\rangle \\ &\quad + \sum_{n_1}\frac{\langle\Phi_f|H_{\rm int}(0)|n_1\rangle\langle n_1|H^i_{\rm int}(0)|\Psi_i\rangle}{p^-_f - p^-_{n_1} + i\epsilon} + \sum_{n_1,n_2}\frac{\langle\Phi_f|H_{\rm int}(0)|n_1\rangle\langle n_1|H_{\rm int}(0)|n_2\rangle\langle n_2|H^i_{\rm int}(0)]|\Phi_i\rangle}{(p^-_f - p^-_{n_1} + i\epsilon)(p^-_f - p^-_{n_2} + i\epsilon)} + \cdots , \\ &\equiv Z_g\sqrt{Z_iZ_f}\langle\Phi_f|H^i_{\rm int}(0)|\Phi_i\rangle, \end{aligned} \tag{3.33}$$

where $H^i_{\rm int}$ represents various interaction terms in Eq. (3.3) that are proportional to the coupling constant g, Z_g is the multiplicative coupling constant renormalization, and Z_i and Z_f are the wave function renormalization constants of the initial and final states.

Next we develop the diagrammatic approach for the perturbative expansions discussed above. All matrix elements of $H_{\rm int}$, namely, the LFQCD vertices, are listed in Table I with the corresponding diagrams. These are obtained by directly calculating the matrix elements between free particle states. The rules to write the expression of perturbative expansions from diagrams are as follows.

(a) Draw all topologically distinct x^+-ordered diagrams.

(b) For each internal line, sum over helicity and integrate using $\int \frac{dk^+d^2k_\perp}{16\pi^3}\theta(k^+)$ for quarks and $\int \frac{dk^+d^2k_\perp}{16\pi^3}\theta(k^+)$ for gluons.

(c) For each vertex, include a factor of $16\pi^3\delta^3(p_f-p_i)$ and a simple matrix element listed in Table I. Each gluon line connected to the vertex contributes a factor $\frac{1}{\sqrt{k^+}}$ from the normalization of the single gluon state.

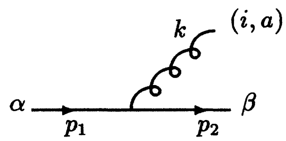
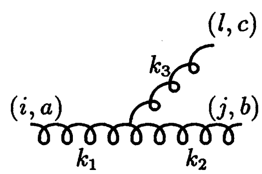
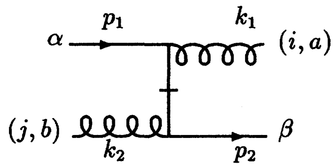
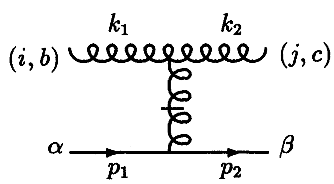
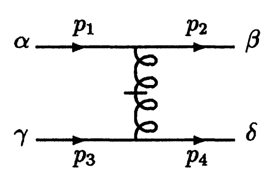
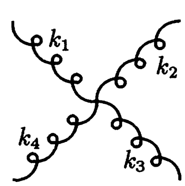
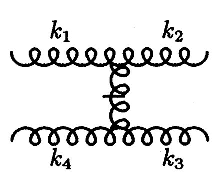
(d) Include a factor $(p^-_f - \sum_n p^-_n + i\epsilon)^{-1}$ for each intermediate state, where $\sum_n p^-_n$ sum over all on-mass-shell intermediate particle energies.

(e) Add a symmetry factor S^{-1} for each gluon loop, coming from the symmetrized boson states.

We conclude this section with a discussion of the major difference between the above two-component perturbation theory and the four-component formalism of Lepage and Brodsky (LB) [9]. In the LB formalism, the three point vertex has the same structure as in covariant theory and thereby the spinor and polarization vector of quarks and gluons contain unphysical components. The four-component theory should be useful when we compare the calculations with the covariant expressions. In the two-component theory, the vertices are complicated [see Eqs. (3.4) and (3.8)] but they have been expressed purely in terms of physical degrees of freedom, so each term corresponds to a real dynamical process. For example, the quark-gluon vertex in the two-component theory contains three terms [see Eq. (4.2)]: the first term originates from the elimination of the longitudinal component of the gauge field, which induces the main gauge singularity, the second term is a helicity-conserving quark-gluon interaction, and the last term is the helicity-flip quark-gluon interaction. Thus the two-component theory is practically useful when we study the interactions involved in real physical processes and discuss the origin of various divergences in renormalization, as we will see in the following paper [17]. Furthermore, as we will see in the next section, although formally the three point vertices in the two-component formalism look complicated, their matrix elements in momentum space can be immediately reduced to only depend on light-front relative momenta and thereby the corresponding diagrammatic calculations become much simpler.

To illustrate the above computation scheme and to discuss light-front infrared divergences, we next calculate some basic x^+-ordered diagrams.

TABLE I. The χ^+-ordered Hamiltonian diagrammatic rules. All gluon momentum directions are x^+-ordered (from left to right). The indices $\alpha, \beta, \ldots,$ correspond to quark color and run from 1 to 3; $a, b, c, \ldots,$ represent gluon color and run from 1 to 8; while $i, j, \ldots,$ are transverse space indices that run from 1 to 2.

H_{int}	Diagram	Vertex
H_{qqg}		$-gT^a_{\beta\alpha}\chi^\dagger\left\{2\frac{k^i}{[k^+]} - \frac{\sigma\cdot p_{2\perp}-im}{[p_2^+]}\sigma^i - \sigma^i\frac{\sigma\cdot p_{1\perp}+im}{[p_1^+]}\right\}\chi\epsilon^{i*}$ $\equiv T^a_{\beta\alpha}\chi^\dagger\Gamma^i_{q0}(p_1,p_2,k)\chi\epsilon^{i*}$
H_{ggg}		$-igf^{abc}\varepsilon^i\ \varepsilon^{j*}\varepsilon^{l*}\Big\{\Big[(k_2-k_3)^i - \frac{k_1^i}{[k_1^+]}(k_2^+-k_3^+)\Big]\delta_{jl} + \Big[(k_3+k_1)^j - \frac{k_2^j}{[k_2^+]}(k_3^++k_1^+)\Big]\delta_{li} + \Big[-(k_1+k_2)^l + \frac{k_3^l}{[k_3^+]}(k_1^++k_2^+)\Big]\delta_{ij}\Big\}$ $\equiv -igf^{abc}\Gamma^{ijl}_{g0}(k_1,k_2,k_3)\varepsilon^i\ \varepsilon^{j*}\varepsilon^{l*}$
H_{qqgg1}		$g^2\chi^\dagger\frac{(T^bT^a)_{\beta\alpha}\sigma^j\sigma^i}{[p_1^+-k_1^+]}\chi\epsilon^{i*}\epsilon^j$
H_{qqgg2}		$2g^2iT^a_{\beta\alpha}f^{abc}\chi^\dagger\frac{k_1^++k_2^+}{[k_1^+-k_2^+]^2}\chi\delta_{ij}\epsilon^i\ \epsilon^{j*}$
H_{qqqq}		$4g^2T^a_{\beta\alpha}T^a_{\delta\gamma}\chi^\dagger\chi\frac{1}{[p_1^+-p_2^+]^2}\chi^\dagger\chi$
H_{gggg1}		$g^2\left\{f^{a_1a_2b}f^{a_3a_4b}\left[\delta_{i_1i_3}\delta_{i_2i_4}-\delta_{i_1i_4}\delta_{i_2i_3}\right] +(2\to3)+(2\to4)\right\}$
H_{gggg2}		$g^2\left\{f^{a_1a_2b}f^{a_3a_4b}\delta_{i_1i_2}\delta_{i_3i_4}\frac{(k_1^++k_2^+)(k_3^++k_4^+)}{[k_1^+-k_2^+][k_3^+-k_4^+]} +(2\to3)+(2\to4)\cdot\right\}$

IV. EXAMPLES OF LIGHT-FRONT RENORMALIZATION

In this section, we compute mass and wave function renormalization up to one-loop for quarks and gluons in the x^+-ordered two-component theory, providing a detailed analysis for most severe light-front divergences *in old-fashioned Hamiltonian theory.*

In the x^+-ordered loop integrals, the internal momenta (k^+, k^i) can always be expressed in terms of relative momentum as

$$x = \frac{k^+}{[p^+]} \ , \quad \kappa^i = k^i - \frac{k^+}{[p^+]}p^i \ , \tag{4.1}$$

where (p^+, p^i) are total momenta. *In the two-component theory*, the three point vertex can be written as a function of the relative momenta.

(a) Quark-quark-gluon vertex [see Figs. 1(a) and 1(b)]:

$$\begin{aligned}\Gamma^i_{q0}(p, p-k, k) &\equiv 2\frac{k^i}{[k^+]} - \frac{\sigma^j(p^j - k^j) - im}{[p^+ - k^+]}\sigma^i - \sigma^i\frac{\sigma^j p^j + im}{[p^+]} \\ &= \frac{1}{[p^+ - k^+]}\left\{2\frac{p^+}{[k^+]}\left(k^i - \frac{k^+}{[p^+]}p^i\right) - \sigma^i\left(\sigma^j k^j - \frac{k^+}{[p^+]}\sigma^j p^j\right) + i\sigma^i m\frac{k^+}{[p^+]}\right\} \\ &= \frac{1}{[p^+][1-x]}\left\{\frac{2}{[x]}\kappa^i - \sigma^i \sigma\cdot\kappa_\perp + i\sigma^i m x\right\},\end{aligned} \tag{4.2}$$

$$\begin{aligned}\Gamma^i_{q0}(p-k, p, k) &\equiv 2\frac{k^i}{[k^+]} - \frac{\sigma^j p^j - im}{[p^+]}\sigma^i - \sigma^i\frac{\sigma^j(p^j - k^j) + im}{[p^+ - k^+]} \\ &= \frac{1}{[p^+ - k^+]}\left\{2\frac{p^+}{[k^+]}\left(k^i - \frac{k^+}{[p^+]}p^i\right) - \left(\sigma^j k^j - \frac{k^+}{[p^+]}\sigma^j p^j\right)\sigma^i - i\sigma^i m\frac{k^+}{[p^+]}\right\} \\ &= \frac{1}{[p^+][1-x]}\left\{\frac{2}{[x]}\kappa^i - \sigma\cdot\kappa_\perp\sigma^i - i\sigma^i m x\right\}.\end{aligned} \tag{4.3}$$

(b) Three-gluon vertex [see Fig. 1(c)].

$$\begin{aligned}\Gamma^{ijl}_{g0}(p, -k, p-k) &\equiv \left[(p-2k)^i - \frac{p^i}{[p^+]}(p^+ - 2k^+)\right]\delta_{jl} \\ &\quad + \left[(k-2p)^j - \frac{k^j}{[k^+]}(k^+ - 2p^+)\right]\delta_{li} + \left[(p+k)^l - \frac{p^l - k^l}{[p^+ - k^+]}(p^+ + k^+)\right]\delta_{ij} \\ &= 2\left\{-\left(k^i - \frac{k^+}{[p^+]}p^i\right)\delta_{jl} + \frac{p^+}{[k^+]}\left(k^j - \frac{k^+}{[p^+]}p^j\right)\delta_{li} + \frac{p^+}{[p^+ - k^+]}\left(k^l - \frac{k^+}{[p^+]}p^l\right)\delta_{ij}\right\} \\ &= 2\left\{-\kappa^i\delta_{jl} + \frac{1}{[x]}\kappa^j\delta_{li} + \frac{1}{[1-x]}\kappa^l\delta_{ij}\right\}.\end{aligned} \tag{4.4}$$

These identities in the two-component theory greatly simplify the complicated x^+-ordered diagrammatic calculations. The four-component diagrammatic rules [9] lack this simplification.

From (3.27), we also see that if we shift p^- of the initial state to off-mass-shell, the one-loop wave function renormalization in Eq. (3.29) is given by

$$Z_\Phi^{-1} = 1 - \left.\frac{\partial(\delta p^-(p))}{\partial p^-}\right|_{p^2=m^2}. \tag{4.5}$$

Thus, the mass and wave function renormalization can be found directly from $\delta p^-|_{p^2\neq m^2}$ at the one-loop level.

A. Quark mass and wave function renormalization

Based on the x^+-ordered perturbative theory, the light-front quark energy correction up to one-loop is determined by

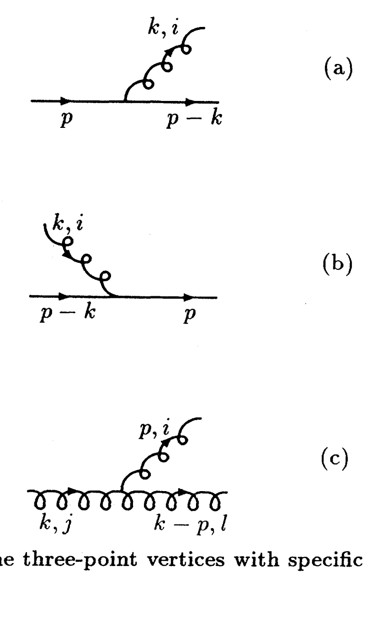


FIG. 1. The three-point vertices with specific momenta.

$$\begin{aligned}\delta p^- &= \langle p',\lambda'|H-H_0|p\lambda\rangle \\ &= \left\{\delta p_1^- + \delta p_2^- + \delta p_3^-\right\}\delta(p-p')\delta_{\lambda,\lambda'},\end{aligned} \tag{4.6}$$

where $|p,\lambda\rangle$ is a dressed single quark state. The three terms in the second equality of Eq. (4.6) correspond to the three diagrams shown in Fig. 2. Using the rules listed in Table I, we find, for Fig. 2(a),

$$\begin{aligned}\delta p_1^- = (-g)^2(T^aT^a)\int\frac{dk^+d^2k_\perp}{16\pi^3}\frac{\theta(k^+)\theta(p^+-k^+)}{[k^+]} \\ \times\Gamma^i_{q0}(p-k,-k,p)\Gamma^i_{q0}(p,k,p-k) \\ \times\frac{1}{p^- - k^- - (p-k)^-}.\end{aligned} \tag{4.7}$$

Figures 2(b) and 2(c) are the self-inertia diagrams from the normal ordering of the instantaneous interactions H_{qqgg1} and H_{qqqq}. Their contributions to the quark energy correction are

$$\delta p_2^- = g^2(T^aT^a)\int\frac{dk^+d^2k_\perp}{16\pi^3}\frac{\theta(k^+)}{[k^+]}\frac{\sigma^i\sigma^i}{[p^+-k^+]}, \tag{4.8}$$

$$\delta p_3^- = 2g^2(T^aT^a)\int\frac{dk^+d^2k_\perp}{16\pi^3}\theta(k^+)\left\{\frac{1}{[p^+-k^+]^2} - \frac{1}{[p^++k^+]^2}\right\}. \tag{4.9}$$

Without any explicit specification, the k^+ integrals are from zero to infinity, and the k^i integrals are from negative infinity to positive infinity. The last factor in Eq. (4.7) is the energy denominator, where the light-front energies for internal lines are always on-shell energies, namely, they satisfy Eq. (3.20) for quarks and gluons. We set the initial light-front energy p^- to be off mass shell, so that the mass and wave function renormalization can be determined directly from δp^-. Also, note that the boundary term in Eq. (2.18) has an explicit contribution to δp_3^- in the principal value prescription of Eq. (3.22). In Appendix B of paper I, we have demonstrated the cancellation of linear infrared divergences in the quark mass correction of Eqs. (4.7)–(4.9) which includes the boundary term contribution. Here we present the complete calculations for quark mass and wave function renormalization.

By using the identity Eqs. (4.2) and (4.3), it is easy to find that

$$\delta p_1^- = \frac{g^2C_f}{[p^+]}\int\frac{d^2\kappa_\perp}{16\pi^3}\int_0^1\frac{dx}{[1-x]} \times\frac{\left(4\frac{1}{[x]^2}-4\frac{1}{[x]}+2\right)\boldsymbol{\kappa}_\perp^2+2m^2x^2}{x(1-x)p^2-\boldsymbol{\kappa}_\perp^2-xm^2}, \tag{4.10}$$

$$\delta p_2^- = 2\frac{g^2C_f}{[p^+]}\int\frac{d^2\kappa_\perp}{16\pi^3}\int_0^\infty\frac{dx}{[x][1-x]}, \tag{4.11}$$

$$\delta p_3^- = 2\frac{g^2C_f}{[p^+]}\int\frac{d^2\kappa_\perp}{16\pi^3}\int_0^\infty dx\left\{\frac{1}{[1-x]^2}-\frac{1}{(1+x)^2}\right\}, \tag{4.12}$$

where $C_f \equiv (T^aT^a) = (N^2-1)/2N$, N is the number of colors (i.e., $N=3$).

Equations (4.10)–(4.12) involve various infrared divergences for longitudinal momentum integrals as well as ultraviolet (UV) divergence for transverse momentum integrals. The infrared divergences are regularized by the principal value prescription of Eq. (3.22). To carry out the integrations explicitly, we have to regularize the UV divergences. The regulator in Feynman theory that preserves covariance and gauge invariance is dimensional regularization. This regularization plays a crucial role in proving the renormalizability of perturbative Yang-Mills theory. Unfortunately, dimensional regularization is not available in old-fashioned perturbative theory. In fact, in the nonperturbative study of the x^+-ordered Hamiltonian theory, renormalization requires us to introduce a cutoff procedure. Currently, there are three regularization schemes for UV divergences used in x^+-ordered Hamiltonian theory: (i) an explicit cutoff for the transverse momentum [30]; (ii) transverse dimensional regularization [31]; and (iii) a global cutoff regularization [9]. These regulators break some symmetries of the theory, and renormalization requires counterterms to restore them. In Ref. [9] it is suggested that the global cutoff could be a suitable regulator for the x^+-ordered Hamiltonian QCD calculations. To compare the different regularization schemes in the old-fashioned theory, we will examine all three prescriptions.

(i) *Transverse cutoff regularization*: $|\kappa_\perp| \leq \Lambda_\perp$. For this simplest regularization scheme, Eqs. (4.10)–(4.12) turn out to be

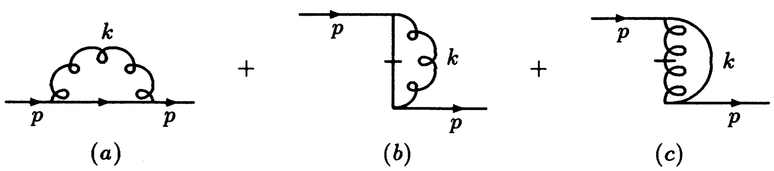


FIG. 2. The x^+-ordered graphs for the one-loop correction to the quark mass and wave function renormalization in two-component LFQCD.

$$\delta p_1^- = -\frac{g^2}{8\pi^2}C_f\left\{\frac{p^2-m^2}{[p^+]}\left[\left(2\ln\frac{p^+}{\epsilon}-\frac{3}{2}\right)\ln\Lambda_\perp^2 - \int_0^1 dx\left(\frac{2}{[x]}-2+x\right)\ln f(x)\right] + \frac{m^2}{[p^+]}\left(-2\ln\Lambda_\perp^2+2\int_0^1 dx\ln f(x)\right) + \frac{\Lambda_\perp^2}{[p^+]}\left(\frac{\pi p^+}{2\epsilon}-1+\ln\frac{p^+}{\epsilon}\right)\right\}, \quad (4.13)$$

$$\delta p_2^- = \frac{g^2}{8\pi^2}C_f\frac{\Lambda_\perp^2}{[p^+]}\ln\frac{p^+}{\epsilon}, \quad (4.14)$$

$$\delta p_3^- = \frac{g^2}{8\pi^2}C_f\frac{\Lambda_\perp^2}{[p^+]}\left(\frac{\pi p^+}{2\epsilon}-1\right), \quad (4.15)$$

where we have defined $f(x) = [xm^2 - x(1-x)p^2]$. From the above result, we see that in the one-loop quark energy correction, one-gluon exchange gives rise to both linear and logarithmic infrared divergences. The instantaneous fermion interaction contribution [see δp_2^- in Fig. 2(b)] contains only one logarithmic divergence which cancels the logarithmic divergence in δp_1^-. The instantaneous gluon interaction contribution [δp_3^- of Fig. 2(c)] has a linear infrared divergence which precisely cancels the same divergence in δp_1^-. As we have pointed out in paper I, this cancellation of linear infrared divergences is based on the use of the regularization for $k^+ \to 0$ in Eq. (3.22), which consistently includes the boundary term contribution in Eq. (2.18).

Now it is easy to find the quark mass correction (dropping the finite part)

$$\delta m^2 = p^+\delta p^-|_{p^2=m^2} = \frac{g^2}{4\pi^2}C_f m^2\ln\frac{\Lambda_\perp^2}{m^2} \quad (4.16)$$

which is longitudinal infrared divergence free. Note that the coefficient (1/4) in this mass correction is different from the covariant result (3/8) because of the different regularization schemes.

The quark wave function renormalization constant is

$$Z_2 = 1 + \left.\frac{\partial\delta p^-}{\partial p^-}\right|_{p^2=m^2} = 1 + \frac{g^2}{8\pi^2}C_f\left\{\left(\frac{3}{2}-2\ln\frac{p^+}{\epsilon}\right)\ln\frac{\Lambda_\perp^2}{m^2} + 2\ln\frac{p^+}{\epsilon}\left(1-\ln\frac{p^+}{\epsilon}\right)\right\}. \quad (4.17)$$

In wave function renormalization we see that there is an additional type of divergence, the mixing of infrared and ultraviolet divergences. This is the "spurious" mixing associated with the gauge singularity. It corresponds to the so-called light-front double pole problem in the Feynman theory with $A_a^+ = 0$ and the principal value prescription that prohibits any continuation to Euclidean space and power counting in Feynman loop integrals. In the x^+-ordered Hamiltonian perturbative theory, the power counting is different, as was recently pointed out by Wilson [32]. The above argument of power counting for Feynman loop integrals may be irrelevant. Furthermore, since the second order correction to wave functions must be negative [see Eq. (3.29)], from Eq. (4.17) we see that it is the additional infrared divergence that gives a consistent answer for wave function renormalization. As we will show in the following paper [17], this type of divergence is indeed canceled completely for gauge invariant quantities and therefore the mixing divergences in the old-fashioned Hamiltonian theory may not be a problem for LFQCD.

(ii) *Transverse dimensional regularization*: Using transverse dimensional regularization, we find that

$$\delta p_1^- = -\frac{g^2}{8\pi^2}C_f\left\{\frac{p^2-m^2}{[p^+]}\left[\left(2\ln\frac{p^+}{\epsilon}-\frac{3}{2}\right)\frac{1}{\epsilon_t} + \int_0^1 dx\left(\frac{2}{[x]}-2+x\right)\ln\frac{\mu^2}{f(x)}\right] + \frac{m^2}{[p^+]}\left(-\frac{2}{\epsilon_t}-2\int_0^1 dx\ln\frac{\mu^2}{f(x)}\right)\right\}, \quad (4.18)$$

$$\delta p_2^- = \delta p_3^- = 0, \quad (4.19)$$

where μ^2 is a scale introduced by dimensional regularization, and $\epsilon_t \equiv 1 - d_t/2$ with the dimension of the transverse space being d_t. Comparing with the explicit transverse momentum cutoff, it is obvious that dimensional regularization removes some infrared divergences associated with quadratic UV divergences, which should otherwise be canceled consistently in the theory. The mass and wave function renormalization in the transverse dimensional regularization scheme are

$$\delta m^2 = p^+\delta p^-|_{p^2=m^2} = \frac{g^2}{4\pi^2}C_f\frac{m^2}{\epsilon_t}, \quad (4.20)$$

$$Z_2 = 1 + \frac{g^2}{8\pi^2}C_f\left\{\left(\frac{3}{2}-2\ln\frac{p^+}{\epsilon}\right)\frac{1}{\epsilon_t} + 2\ln\frac{p^+}{\epsilon}\left(1-\ln\frac{p^+}{\epsilon}-\ln\frac{\mu^2}{m^2}\right)\right\}, \quad (4.21)$$

which gives the same results as the explicit cutoff for UV divergences (by replacing $\frac{1}{\epsilon_t}$ with $\ln\frac{\Lambda^2}{m^2}$). In wave function renormalization, there is an additional pure infrared divergence which is associated with the mass scale in dimensional regularization.

(iii) *Global cutoff*: Another cutoff scheme in the x^+-ordered fight-front field theory is called a global cutoff [9] and it is defined as follows. The momentum of any intermediate state is restricted by

$$\sum_i \frac{p_{\perp i}^2 + m_i^2}{[p_i^+]} \leq \frac{\Lambda^2 + p_\perp^2}{[p^+]}, \quad (4.22)$$

where $\{p^+, p_\perp\}$ is the momentum of the initial (or final) state. For one-loop quark mass and wave function corrections, the intermediate state contains one gluon and one quark so that (4.22) is reduced to

$$\frac{k_\perp^2}{[k^+]} + \frac{(p_\perp - k_\perp)^2 + m^2}{[p^+ - k^+]} \leq \frac{\Lambda^2 + p_\perp^2}{[p^+]}, \quad (4.23)$$

which can be further simplified as

$$\frac{\kappa_\perp^2}{[x]} + \frac{\kappa_\perp^2 + m^2}{[1-x]} \leq \Lambda^2, \quad \Lambda^2 >> \kappa_\perp^2, m^2. \tag{4.24}$$

The cutoff condition, Eq. (4.24), is independent of the total momentum and therefore is obviously boost invariant. With this regulator, the integrals in (4.10)–(4.12) are restricted by

$$\int dx d^2\kappa_\perp \longrightarrow \int dx d^2\kappa_\perp \Theta\left(\Lambda^2 - \frac{\kappa_\perp^2}{[x]} - \frac{\kappa_\perp^2 + m^2}{[1-x]}\right)$$
$$= \int dx \theta(\kappa_{\max}) \int_0^{\kappa_{\max}} d^2\kappa_\perp, \tag{4.25}$$

where

$$\kappa_{\max}^2 = x(1-x)\Lambda^2 - xm^2, \tag{4.26}$$

and $\theta(\kappa_{\max})$ leads to an additional condition on the longitudinal momentum:

$$0 \leq x \leq 1 - \frac{m^2}{\Lambda^2}. \tag{4.27}$$

Equation (4.27) implies that the singularity of the gluon longitudinal momentum for $x \to 1$ is regularized by the global cutoff. However, there is still an infrared singularity when $x \to 0$, which can be treated by the principal value prescription, $\frac{1}{[x]} = \frac{1}{2}\left\{\frac{1}{x+i\epsilon_x} + \frac{1}{x-i\epsilon_x}\right\}$, where ϵ_x is dimensionless and is boost invariant.

Formally, the instantaneous diagrams [Figs. 2(b) and 2(c)] do not involve intermediate states, which apparently implies that the global cutoff cannot be applied to these diagrams. However, the instantaneous diagrams arise from the normal-ordering Hamiltonian, and in principle, are accompanied by Fig. 2(a). Thus, before we replace the integrals in Eqs. (4.10)–(4.12) by Eq. (4.25), we shall first combine Eqs. (4.10)–(4.12) together. Note that

$$\int_0^\infty dx \frac{1}{[x][1-x]} = \int_0^1 \frac{dx}{[x]}, \tag{4.28}$$
$$\int_0^\infty dx \left(\frac{1}{[1-x]^2} - \frac{1}{(1+x)^2}\right) = 2\int_0^1 \frac{dx}{[1-x]^2}$$
$$\longrightarrow 2\int_0^1 \frac{dx}{[x]^2}, \tag{4.29}$$

where the second step in Eq. (4.29) is obtained by replacing the quark momentum in the integral [see Fig. 2(c)] by the gluon momentum due to momentum conservation. Using Eqs. (4.28) and (4.29) to combine Eqs. (4.10)–(4.12) together, we find

$$\delta p^- = -\frac{g^2 C_f}{8\pi^2[p^+]}\left\{(p^2 - m^2)\int_0^1 dx \int d\kappa_\perp^2 \frac{2\frac{1}{[x]} - 2 + x}{\kappa_\perp^2 + f(x)}\right.$$
$$-2m^2 \int_0^1 dx \int \kappa_\perp^2 \frac{1}{\kappa_\perp^2 + f(x)}$$
$$\left. + \int_0^1 dx \int d\kappa_\perp^2 \left(\frac{1}{[1-x]} - \frac{1}{[x]}\right)\right\}. \tag{4.30}$$

Now applying the regulator of Eq. (4.25) to the integrals in Eq. (4.30), the mass and wave function renormalization become

$$\delta m^2 = p^+ \delta p^-|_{p^2=m^2} = \frac{3g^2}{8\pi^2} C_f m^2 \ln\frac{\Lambda^2}{m^2}, \tag{4.31}$$
$$Z_2 = 1 + \frac{g^2}{8\pi^2} C_f \left\{\left(\frac{3}{2} - 2\ln\frac{1}{\epsilon_x}\right)\ln\frac{\Lambda^2}{m^2}\right.$$
$$\left. + \ln\frac{1}{\epsilon_x}\left(2 - \ln\frac{1}{\epsilon_x}\right)\right\}. \tag{4.32}$$

It is of interest to see that with the global cutoff, the second order mass correction gives the same coefficient (3/8) as in a covariant calculation. Comparing with the other two regulators, the additional mass correction originates from the first term in the last integral of Eq. (4.30), a contribution from the one-gluon exchange diagram [Fig. 2(a)]. This contribution is canceled by the fermion instantaneous interaction [Fig. 2(b)] in the explicit transverse cutoff regulator, and is "removed" in transverse dimensional regularization. Here, the fermion instantaneous interaction gives a finite contribution to the mass correction.

The wave function renormalization constant from the global cutoff is the same as from the other two regulators for UV divergences, and behaves also very similar for IR divergences. Also, the mixing of UV and IR divergences is exactly the same for all three different regulators. In x^+-ordered Hamiltonian theory, the mixing divergences cannot and should not be removed as indicated by the negativity of the second order correction of the wave function renormalization constant that must result in a physical theory.

Comparing the three different regulators, it is hard to see which regulator is better at the one-loop level. In fact, the quark mass and wave function renormalization in QCD are the same as the electron mass and wave function renormalizations in QED [13], except for a color factor (C_f). The infrared divergences associated with the gauge singularity are not surprising. The severe problem of gauge singularities (if these exist in the present formalism) should be exhibited in the second order correction to the gluon state, which we now turn to discuss.

B. Gluon mass and wave function renormalization

Similar to the x^+-ordered calculation of the quark mass and wave function renormalization, the gluon energy correction up to one-loop is given by

$$\delta(q^-)_{ij}^{ab} = \langle b, j, q', \lambda'|H - H_0|a, i, q, \lambda\rangle$$
$$= \{\delta q_1^- + \delta q_2^- + \delta q_3^- + \delta q_4^-$$
$$+ \delta q_5^-\}\delta^3(q - q')\delta_{\lambda\lambda'}\delta_{ab}\delta_{ij}, \tag{4.33}$$

where $|a, i, q, \lambda\rangle$ is a dressed single gluon state. The corresponding diagrams are shown in Fig. 3. For Figs. 3(a), 3(b), and 3(e), using the diagrammatic rules listed in Table I, we have

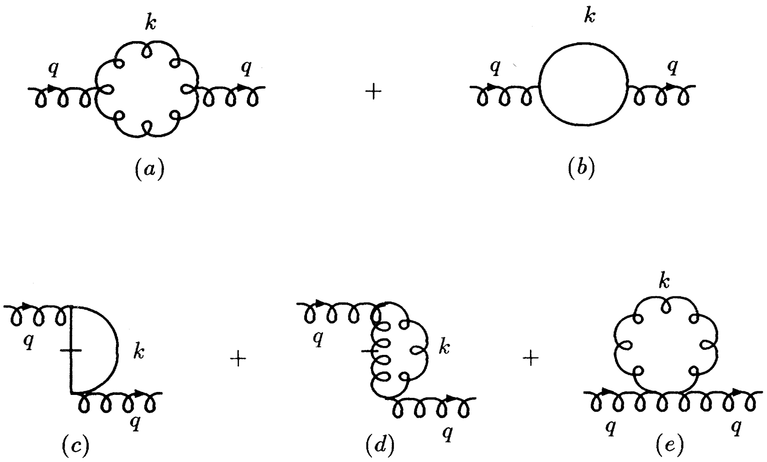


FIG. 3. The x^+-ordered graphs for the one-loop correction to the gluon mass and wave function renormalization in two-component LFQCD.

$$\delta q_1^- \delta_{ab}\delta_{ij} = \frac{1}{2}\frac{(ig)^2}{[q^+]}(f^{acd}f^{bdc})\int \frac{dk^+d^2k_\perp}{16\pi^3}\frac{\theta(k^+)}{[k^+]} \times\frac{\theta(q^+-k^+)}{[q^+-k^+]}\Gamma_{g0}^{jlm}(q,-k,k-q) \times\Gamma_{g0}^{iml}(-q,q-k,k)\frac{1}{q^- - k^- - (q-k)^-}, \tag{4.34}$$

$$\delta q_2^- \delta_{ab}\delta_{ij} = \frac{(-g)^2}{[q^+]}\mathrm{Tr}(T^aT^b)\int \frac{dk^+d^2k_\perp}{16\pi^3}\theta(k^+) \times\theta(q^+-k^+)\mathrm{Tr}[\Gamma_{q0}^{i}(k,k-q,q) \times\Gamma_{q0}^{j}(k-q,k,-q)]\frac{1}{q^- - k^- - (q-k)^-}, \tag{4.35}$$

$$\delta q_5^- \delta_{ab}\delta_{ij} = \frac{1}{2}\frac{g^2}{[q^+]}(f^{acd}f^{bcd})\int \frac{dk^+d^2k_\perp}{16\pi^3}\frac{\theta(k^+)}{k^+} \times 2\,(\delta_{ij}\delta_{ll}-\delta_{il}\delta_{jl})\,. \tag{4.36}$$

Figures 3(c) and 3(d) are the self-inertia diagrams from the normal ordering of the instantaneous interactions H_{qqgg1} and H_{gggg2}. Their contributions to the quark energy correction are

$$\delta q_3^- \delta_{ab}\delta_{ij} = \frac{g^2}{[q^+]}\mathrm{Tr}(T^aT^b)\int \frac{dk^+d^2k_\perp}{16\pi^3}\theta(k^+) \times\mathrm{Tr}(\sigma^i\sigma^j)\left(\frac{1}{[q^+-k^+]}-\frac{1}{[q^++k^+]}\right), \tag{4.37}$$

$$\delta q_4^- \delta_{ab}\delta_{ij} = \frac{1}{2}\frac{g^2}{[q^+]}(f^{acd}f^{bcd})\int \frac{dk^+d^2k_\perp}{16\pi^3}\frac{\theta(k^+)}{k^+} \times\delta_{il}\delta_{jl}\left\{\frac{(q^++k^+)^2}{[q^+-k^+]^2}+\frac{(q^+-k^+)^2}{(q^++k^+)^2}\right\}. \tag{4.38}$$

In Eqs. (4.34), (4.36), and (4.38), the factor $\frac{1}{2}$ is a symmetry factor for two-boson states. The above expression can be simplified by using Eqs. (4.2)–(4.4):

$$\delta q_1^- = 2\frac{g^2}{[q^+]}C_A\int\frac{d^2\kappa_\perp}{16\pi^3}\int_0^1 dx\frac{\boldsymbol{\kappa}_\perp^2}{x(1-x)q^2-\boldsymbol{\kappa}_\perp^2} \times\left(1+\frac{1}{[x]^2}+\frac{1}{[1-x]^2}\right), \tag{4.39}$$

$$\delta q_2^- = 2\frac{g^2}{[q^+]}T_fN_f\int\frac{d^2\kappa_\perp}{16\pi^3}\int_0^1 dx\frac{1}{[1-x]} \times\frac{\boldsymbol{\kappa}_\perp^2\left(2x-2+\frac{1}{[x]}\right)+\frac{1}{[x]}m^2}{x(1-x)q^2-\boldsymbol{\kappa}_\perp^2-m^2}, \tag{4.40}$$

$$\delta q_3^- = 2\frac{g^2}{[q^+]}T_fN_f\int\frac{d^2\kappa_\perp}{16\pi^3}\int_0^\infty dx\left(\frac{1}{[1-x]}-\frac{1}{1+x}\right), \tag{4.41}$$

$$\delta q_4^- = \frac{g^2}{2[q^+]}C_A\int\frac{d^2\kappa_\perp}{16\pi^3}\int_0^\infty\frac{dx}{[x]}\left(\frac{(1+x)^2}{[1-x]^2}+\frac{(1-x)^2}{(1+x)^2}\right), \tag{4.42}$$

$$\delta q_5^- = \frac{g^2}{[q^+]}C_A\int\frac{d^2\kappa_\perp}{16\pi^3}\int_0^\infty\frac{dx}{[x]}, \tag{4.43}$$

where $C_A\delta_{ab}=f^{acd}f^{bcd}=N\delta_{ab}$, $T_f\delta_{ab}=\mathrm{Tr}(T^aT^b)=\frac{1}{2}\delta_{ab}$, and N_f is the total flavor number for quarks. Again, we now put the gluon off mass shell so that the mass and wave function renormalization can be found simultaneously from Eqs. (4.39)–(4.43). In the previous subsection, we have shown that there are three regulators for UV divergences in x^+-ordered LFQCD. To understand these regularization schemes for the gluon sector, we compute the integrals in Eqs. (4.39)–(4.43) for all three schemes.

(i) *Transverse cutoff regularization*: Using the explicit transverse cutoff regularization for the UV divergences, we find

$$\delta q_1^- = -\frac{g^2}{8\pi^2}C_A\left\{\frac{q^2}{[q^+]}\left[\left(2\ln\frac{q^+}{\epsilon}-\frac{11}{6}\right)\ln\Lambda_\perp^2 + \int_0^1 dx\left(2-x(1-x)-\frac{2}{[x]}\right)\ln[x(1-x)q^2]\right] + \frac{\Lambda_\perp^2}{[q^+]}\frac{\pi q^+}{2\epsilon}\right\}, \quad (4.44)$$

$$\delta q_2^- = -\frac{g^2}{8\pi^2}T_f N_f\left\{\frac{q^2}{[q^+]}\left[\frac{2}{3}\ln\Lambda_\perp^2 - \int_0^1 dx(2x^2-2x+1)\ln[m^2-x(1-x)q^2]\right] + \frac{m^2}{[q^+]}\left[2\ln\Lambda_\perp^2 - 2\int_0^1 dx\ln[m^2-x(1-x)q^2]\right] + 2\frac{\Lambda_\perp^2}{[q^+]}\left(\ln\frac{q^+}{\epsilon}-1\right)\right\}, \quad (4.45)$$

$$\delta q_3^- = -\frac{g^2}{4\pi^2}\frac{\Lambda_\perp^2}{[q^+]}T_f N_f \ln\frac{k_\infty^+}{q^+}, \quad (4.46)$$

$$\delta q_4^- = \frac{g^2}{8\pi^2}C_A\frac{\Lambda_\perp^2}{[q^+]}\left\{\frac{\pi q^+}{2\epsilon} - 1 + \ln\frac{k_\infty^+}{\epsilon}\right\}, \quad (4.47)$$

$$\delta q_5^- = \frac{g^2}{16\pi^2}C_A\frac{\Lambda_\perp^2}{[q^+]}\ln\frac{k_\infty^+}{\epsilon}, \quad (4.48)$$

where k_∞^+ is the internal fermion momentum $k^+ \to \infty$. In Eq. (4.44), there is also a mass singularity, which is commonly known in gauge theory in the covariant formulation. Formally we introduce a small gluon mass ($q^2 = \mu_G^2$) in the energy denominators to regulate this singularity. The gluon mass and wave function renormalizations are

$$\delta\mu^2 = q^+\delta q^-|_{q^2=0} = -\frac{g^2}{4\pi^2}\left\{T_f N_f m^2 \ln\frac{\Lambda_\perp^2}{m^2} + \Lambda_\perp^2\left(\frac{C_A}{2}-T_f N_f\right)\left(1-\ln\frac{k_\infty^+}{\epsilon}\right)\right\}, \quad (4.49)$$

$$Z_3 = 1 + \left.\frac{\partial\delta q^-}{\partial q^-}\right|_{q^2=0} = 1 + \frac{g^2}{8\pi^2}\left\{\left(\frac{11}{6}-2\ln\frac{q^+}{\epsilon}\right)C_A\ln\frac{\Lambda_\perp^2}{\mu_G^2} - \frac{2}{3}T_f N_f\ln\frac{\Lambda_\perp^2}{m^2} - 2C_A\ln^2\frac{q^+}{\epsilon}\right\}. \quad (4.50)$$

Equations (4.44)–(4.48) show that all severe divergences appear in the gluon sector: quadratic and logarithmic UV divergences, linear and logarithmic IR divergences, gluon mass singularity, and an unusual large longitudinal momentum logarithmic divergence. Only the linear infrared divergences are canceled [see (4.44) and (4.48)] with our principal value prescription Eq. (3.22), as we anticipated in paper I. The gluon mass correction is not zero. The first term in (4.49) is a fermion loop contribution [Fig. 3(b)], which is the same as the photon mass correction in QED. In addition, the gluon mass correction also contains a severe mixing of quadratic UV divergences with logarithmic IR and UV divergences of longitudinal momentum. It is caused by the instantaneous fermion and gluon interaction contributions plus the tadpole effect of the normal four gluon interactions [Figs. 3(c)–3(e)]. It is interesting to see that this kind of divergence behaves the same way from both the fermion contribution and the gluon contribution. The nonzero gluon mass correction of Eq. (4.49) is not surprising because it has the same divergence feature as the photon mass correction in LFQED [Eq. (4.49) will be reduced to photon mass correction when we set $T_f = 1, C_A = 0$, and $N_f = 1$]. In a covariant calculation, the zero gluon mass correction is true only for dimensional regularization which "removes" or drops the mass correction. In the present calculation, maintaining zero gluon mass requires a mass counterterm, as is known in QED. The difference between QED and QCD is only manifest in the gauge boson wave function renormalization. For wave function renormalization, again there is an additional mixing of UV and IR divergences, which provides the correct sign for the wave function renormalization constant. In addition to this feature, there is a contribution from the gluon loop [Fig. 3(a)]. As we will see in the following paper, after the cancellation of the mixing divergences, it is this contribution that leads to asymptotic freedom in QCD.

(ii) *Transverse dimensional regularization*: Transverse dimensional regularization leads to the solution

$$\delta q_1^- = -\frac{g^2}{8\pi^2}C_A\frac{q^2}{[q^+]}\left\{\left(2\ln\frac{q^+}{\epsilon}-\frac{11}{6}\right)\left(\frac{1}{\epsilon_t}+\ln\frac{\mu^2}{q^2}\right) + \ln^2\frac{q^+}{\epsilon}\right\}, \quad (4.51)$$

$$\delta q_2^- = -\frac{g^2}{8\pi^2}T_f N_f\left\{\frac{q^2}{[q^+]}\left(\frac{2}{3}\frac{1}{\epsilon_t} + \int_0^1 dx(2x^2-2x+1)\ln\frac{\mu^2}{m^2-x(1-x)q^2}\right) + \frac{m^2}{[q^+]}\left(2\frac{1}{\epsilon_t} + 2\int_0^1 dx\ln\frac{\mu^2}{m^2-x(1-x)q^2}\right)\right\}, \quad (4.52)$$

$$\delta q_3^- = \delta q_{4+5}^- = 0. \quad (4.53)$$

The gluon mass and wave function renormalizations are

$$\delta\mu^2 = -\frac{g^2}{4\pi^2}T_f N_f m^2\frac{1}{\epsilon_t}, \quad (4.54)$$

$$Z_3 = 1 + \frac{g^2}{8\pi^2}\left\{\left[\left(\frac{11}{6}-2\ln\frac{q^+}{\epsilon}\right)C_A - \frac{2}{3}T_f N_f\right]\frac{1}{\epsilon_t} + C_A\left[\left(\frac{11}{6}-2\ln\frac{q^+}{\epsilon}\right)\ln\frac{\mu^2}{\mu_G^2} - 2\ln^2\frac{q^+}{\epsilon}\right]\right\}. \quad (4.55)$$

Obviously, transverse dimensional regularization removes all quadratic UV divergences. Thus if we only consider zero quark mass, as is usually assumed in practical perturbative calculations [22], the gluon mass correction in transverse dimensional regularization is zero.

(iii) *Global cutoff*: Up to the second order, the intermediate states for the gluon energy correction are different in different diagrams. For Fig. 3(a), the global cutoff is given by

$$\frac{\kappa_\perp^2}{[x]} + \frac{\kappa_\perp^2}{[1-x]} \leq \Lambda^2, \tag{4.56}$$

which leads to the condition

$$\kappa_{\max}^2 = x(1-x)\Lambda^2, \quad 0 \leq x \leq 1. \tag{4.57}$$

This shows that the global cutoff for a massless gluon does not remove the light-front infrared singularities from $x \to 0$ and $x \to 1$. The associated infrared divergences have to be regulated by the principal value prescription: $1/[x]$ [see after Eq. (4.27)]. However, for Fig. 3(b), the global cutoff is

$$\frac{\kappa_\perp^2 + m^2}{[x]} + \frac{\kappa_\perp^2 + m^2}{[1-x]} \leq \Lambda^2. \tag{4.58}$$

The transverse and longitudinal momenta are then restricted by

$$\kappa_{\max}^2 = x(1-x)\Lambda^2 - m^2, \quad \frac{m^2}{\Lambda^2} \leq x \leq 1 - \frac{m^2}{\Lambda^2}, \tag{4.59}$$

which regulate both UV and IR divergences. When normal ordering the Hamiltonian, the instantaneous interaction must be regulated in a similar way, namely, Fig. 3(c) is accompanied with Fig. 3(b) and Figs. 3(d), 3(e) with Fig. 3(a). The result (dropping the finite part) is

$$\delta q_{1+4+5}^- = -\frac{g^2 C_A}{8\pi^2 [q^+]} \left\{ q^2 \left(2 \ln \frac{q^+}{\epsilon} - \frac{11}{6} \right) \ln \frac{\Lambda^2}{q^2} - \frac{\Lambda^2}{3} \right\}, \tag{4.60}$$

$$\delta q_{2+3}^- = -\frac{g^2 N_f T_f}{8\pi^2 [q^+]} \left\{ q^2 \frac{2}{3} \ln \frac{\Lambda^2}{m^2} + \frac{2}{3}\Lambda^2 + 2m^2 \int_{m^2/\Lambda^2}^{1-m^2/\Lambda^2} dx \ln \frac{\Lambda^2}{m^2 - x(1-x)q^2} \right\}. \tag{4.61}$$

Therefore, with a global cutoff, the gluon mass and wave function renormalization are

$$\delta\mu^2 = -\frac{g^2}{4\pi^2} \left\{ T_f N_f m^2 \ln \frac{\Lambda^2}{m^2} - \frac{\Lambda^2}{3} \left(\frac{C_A}{2} - T_f N_f \right) \right\}, \tag{4.62}$$

$$Z_3 = 1 + \frac{g^2}{8\pi^2} \left\{ \left(\frac{11}{6} - 2 \ln \frac{1}{\epsilon_x} \right) C_A \ln \frac{\Lambda^2}{\mu_G^2} - \frac{2}{3} T_f N_f \ln \frac{\Lambda^2}{m^2} \right\}. \tag{4.63}$$

The results show that the global cutoff uses minimum regularization parameters (Λ^2, ϵ_x, and μ_G^2). In the mass correction, only quadratic and logarithmic UV divergences are involved. The wave function renormalization has the same UV divergence behavior as found using the explicit transverse cutoff, but no pure infrared divergences.

Comparing the above three calculations, we see that in each case we have to introduce a gluon mass counterterm to keep the gluon massless. In the gluon wave function renormalization, the divergences are also very similar from each regulator. None of these regulators can avoid the severe light-front divergences. Thus we cannot easily conclude that one is better than the others. Normally, the choice of a suitable regulator also depends on whether it can maintain a maximum number of symmetries. However, not only does the global cutoff preserve boost invariance, the other two regulators can be made to maintain boost invariance as well if we replace the ratio p^+/ϵ by ϵ_x in the explicit transverse cutoff and in the transverse dimensional regularization. This is because these regulators can be applied to the relative transverse momentum and the ratio of longitudinal momentum. In the three schemes, rotational invariance is preserved along the longitudinal direction but is obviously destroyed in the transverse directions. Thus, symmetry considerations alone do not provide a simple criterion for a choice between these regulators. In the light-front gauge Feynman theory, there is a regulator developed by Mandelstam and Leibbrandt [23] which can remove the severe $k^+ \to 0$ singularity. Unfortunately, this prescription depends on the sign of k^- and therefore we cannot apply it to the x^+-ordered Hamiltonian theory.

However, from the above calculations, we see that the wave function renormalization contains several complicated pure infrared divergences and a mass singularity from the massless gluon. This complexity can be avoided if we introduce a mass scale u for the minimum cutoff for transverse momentum,

$$\Lambda_\perp^2 \geq \kappa_\perp^2 \geq u^2, \tag{4.64}$$

and assume that u is much larger than all other masses in the theory. With this regulator, the quark and gluon mass and wave function renormalization become simple:

$$\delta m^2 = \frac{g^2}{4\pi^2} C_f m^2 \ln \frac{\Lambda_\perp^2}{u^2}, \tag{4.65}$$

$$\delta\mu^2 = -\frac{g^2}{4\pi^2} \left\{ T_f N_f m^2 \ln \frac{\Lambda_\perp^2}{u^2} + (\Lambda_\perp^2 - u^2) \left(\frac{C_A}{2} - T_f N_f \right) \left(1 - \ln \frac{k_\infty^+}{\epsilon} \right) \right\}, \tag{4.66}$$

$$Z_2 = 1 + \frac{g^2}{8\pi^2} C_f \left(\frac{3}{2} - 2 \ln \frac{p^+}{\epsilon} \right) \ln \frac{\Lambda_\perp^2}{u^2}, \tag{4.67}$$

$$Z_3 = 1 + \frac{g^2}{8\pi^2} \left\{ C_A \left(\frac{11}{6} - 2 \ln \frac{q^+}{\epsilon} \right) - \frac{2}{3} T_f N_f \right\} \ln \frac{\Lambda_\perp^2}{u^2}. \tag{4.68}$$

This shows that all pure infrared divergences in wave function renormalization are removed. The remaining unfamiliar divergences are the mixing of quadratic UV with logarithmic IR divergences in the gluon mass correction, and the mixing of UV and IR logarithmic divergences in wave function renormalization. The mixing divergences in the gluon mass correction could be removed by a gluon mass counterterm, while the mixing divergences in wave function renormalization are canceled in physical quantities, as we will show in the next paper [17]. Thus, in the case of the lack of a better regularization scheme, Eq. (4.64) with the principal value prescription of Eq. (3.22) may be the simplest regulator for practical old-fashioned perturbative calculations in LFQCD. We will use this regulator for further calculations in the next paper [17].

Finally, we calculate the anomalous dimension for quarks and gluons. The anomalous dimension of the quark field to order g^2 is

$$\gamma_F \equiv -\frac{1}{2Z_2}\frac{\partial Z_2}{\partial \ln \Lambda} = \frac{g^2}{8\pi^2}C_f\left(2\ln\frac{p^+}{\epsilon}-\frac{3}{2}\right), \tag{4.69}$$

which is the same as Lepage and Brodsky obtained [9]. The momentum-dependent term implies that the quark anomalous dimension is gauge dependent. The anomalous dimension for the gluon field is

$$\gamma_G \equiv -\frac{1}{2Z_3}\frac{\partial Z_3}{\partial \ln \Lambda} = \frac{g^2}{8\pi^2}\left\{C_f\left(2\ln\frac{q^+}{\epsilon}-\frac{11}{6}\right)+\frac{2}{3}T_f N_f\right\}, \tag{4.70}$$

which is also gauge dependent. In the case of setting $q^+ = 0$, the gauge-dependent term can be removed, and Eq. (4.70) is reduced Gross-Wilczek result in their Feynman calculation with $A_a^+ = 0$ and $q^+ = 0$ [3].

V. DISCUSSIONS AND SUMMARY

In this paper, we have not provided any new features of QCD in old-fashioned Hamiltonian theory. However, to the best of our knowledge, this is the first attempt to show in detail various light-front divergences in Hamiltonian theory. We are motivated to analyze these severe light-front divergences by the current investigations of nonperturbative light-front field theory for QCD. Systematic control of these divergences is required *a priori* before we perform any practical numerical calculation in light-front coordinates for QCD bound states. From the basic one-loop calculations presented in this paper, we learn that in the old-fashioned perturbation theory, LFQCD involves various UV and IR divergences. Some of the divergences have not even been encountered in covariant and noncovariant Feynman calculations to the same order. We have explored these divergences in the two-component theory, which simplifies greatly the calculations based on the formalism developed by Lepage and Brodsky, and in which we can also clearly see the origin of each divergence from the structure of physical quark-gluon interactions. We see that the currently available regularization schemes for Hamiltonian theory do not remove these unwanted divergences.

Among various light-front divergences, there are two severe divergences we have to deal with in the old-fashioned theory for LFQCD. The first is the mixing of UV and IR logarithmic divergences in wave function renormalization. As we have argued, the occurrence of the mixing divergences may not be a severe problem. In light-front gauge Feynman theory with principal value prescription, such a mixing divergence prohibits any continuation to Euclidean space and thereby breaks down the standard power counting used to analyze Feynman loop integrals. As a result, proving renormalizability of the theory is problematic. The Mandelstam-Leibbrandt (ML) prescription can resolve this problem. However, in old-fashioned Hamiltonian theory, this argument is no longer available because power counting in the light-front field theory is totally different, as pointed out recently by Wilson [32]. In fact, although the use of light-front gauge has always been regarded as dangerous due to the occurrence of the mixing divergences, it has been shown in light-front gauge Feynman theory that these mixing divergences are indeed canceled in gauge invariant quantities [22]. It will be seen in the following paper [17] that the mixing divergences are also canceled completely in the old-fashioned Hamiltonian theory for the coupling constant renormalization, as has also been shown recently by Perry based on the LB formalism [15]. Consequently, we believe that the problem of mixing divergences may not exist when we consider real physical processes.

The second problem is the infinite gluon mass correction. In the old-fashioned Hamiltonian theory, dimensional regularization is not available to avoid the nonzero gluon mass correction. To have a massless gluon in perturbation theory, we have to introduce a gluon mass counterterm. In the leading order (one-loop) calculation, there is no difficulty arising from a gluon mass counterterm. However, when we go to the next order, it can be found that the gluon mass counterterm leads to a noncancellation of infrared divergences. Based on the power counting for light-front QCD [32], we can see, from the light-front energy dispersion relation,

$$P^- = \frac{P_\perp^2 + m^2}{P^+}, \tag{5.1}$$

that the nonvanishing infrared divergences could introduce nonlocal counterterms in both the longitudinal and transverse directions. In equal-time quantization, such nonlocal counterterms are forbidden for a renormalizable theory. Here, these nonlocal counterterms are allowed by the light-front power counting. This is a special feature of LFQCD. One speculation from this property is that the nonlocal counterterms for infrared divergences may also provide a source for quark confinement [32].

In summary, renormalization in *light-front QCD Hamiltonian theory* is very different from conventional Feynman theory and it is an entirely new subject where investigations are still in their preliminary stage. Very recently, a renormalization group based approach has been proposed for light-front field theory [35]. It would be interesting to see the extension of this approach to LFQCD. In perturbative calculations, careful treatment could remove all severe infrared divergences for interesting physical quantities in LFQCD. For nonperturbative studies, the cancellation of severe infrared divergences may not work because certain approximations (e.g., Fock space truncation), might be used. These approximations may also break many important symmetries, such as gauge invariance and rotational invariance. It is the hope of the current investigation of light-front renormalization theory that the counterterms for the light-front infrared divergences may restore the broken symmetries and also provide an effective confining LFQCD Hamiltonian for hadronic bound states.

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APPENDIX: TWO-COMPONENT FEYNMAN PERTURBATIVE THEORY

For comparison with the old-fashioned x^+-ordered perturbation theory explored in this paper, we also present in this appendix the two-component LFQCD Feynman theory. The two-component LFQCD Lagrangian has a canonical form

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\rm int}, \tag{A1}$$

where

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu A_a^i \partial^\mu A_a^i - \xi^\dagger \left(\frac{1}{i\partial^+}\right)(\Box^2 + m^2)\xi,$$
$$\mathcal{L}_{\rm int} = -\mathcal{H}_{\rm int}. \tag{A2}$$

The Feynman rules can be derived from the path integral formalism [33]. Here we use the procedure of 't Hooft and Veltman [34]. The result is as follows. The free quark and gluon propagators in two-component LFQCD are

$$S_{0\alpha\beta}(p) = \frac{ip^+\delta_{\alpha\beta}}{p^2 - m^2} \quad , \quad D_{0ab}^{ij} = \delta_{ij}\delta_{ab}\frac{i}{k^2}. \tag{A3}$$

The diagrammatic rules for various vertices are listed in Table II. The rest of the Feynman rules are the same as in instant form for boson and fermion theories.

In this appendix, we will use the two-component Feynman perturbation theory to calculate quark and gluon self-energies and compare these to the x^+-ordered calculations in Sec. IV.

(i) *The two-component Feynman calculation of quark self-energy.* To compare the x^+-ordered Hamiltonian with Feynman perturbative calculations, we evaluate the quark self-energy by use of the Feynman rules listed in Table II.

In the two-component Feynman perturbation theory, the one-loop quark self-energy is given by

$$\Sigma = \Sigma_1 + \Sigma_2 + \Sigma_3, \tag{A4}$$

where the three terms on the right-hand side are denoted by the three Feynman diagrams shown in Fig. 4. From Table II, we find

$$-i\Sigma_1 = (ig)^2(T^aT^a)\int \frac{d^4k}{(2\pi)^4}S_0(p-k) \times\Gamma_{q0}^i(p-k,-k)D_{0ij}^{aa}(k)\Gamma_{q0}^j(p,k), \tag{A5}$$

$$-i\Sigma_2 = (-ig^2)(T^aT^a)\int \frac{d^4k}{(2\pi)^4}D_{0ii}^{aa}(k)\frac{(\sigma^i)^2}{[p^+ - k^+]}, \tag{A6}$$

$$-i\Sigma_3 = (-2ig^2)(T^aT^a)\int \frac{d^4k}{(2\pi)^4}S_0(k)\left(\frac{1}{[p^+ - k^+]^2} - \frac{1}{[p^+ + k^+]^2}\right), \tag{A7}$$

which can be reduced to

$$\Sigma_1 = ig^2C_f\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2[(p-k)^2 - m^2]}\frac{1}{[p^+ - k^+]} \times\left\{\left[4\left(\frac{p^+}{[k^+]}\right)^2 - 4\frac{p^+}{[k^+]} + 2\right]\left(\mathbf{k}_\perp - \frac{\mathbf{k}^+}{\mathbf{p}^+}\mathbf{p}_\perp\right)^2 + 2m^2\left(\frac{k^+}{p^+}\right)^2\right\}, \tag{A8}$$

$$\Sigma_2 = i2g^2C_f\int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2}\frac{1}{[p^+ - k^+]}, \tag{A9}$$

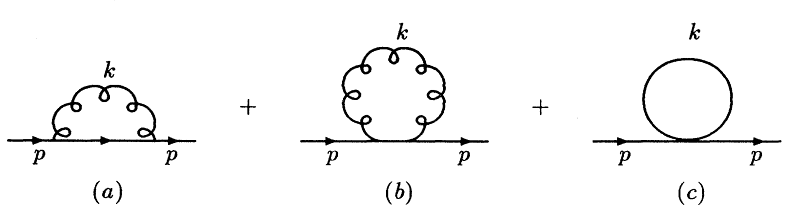
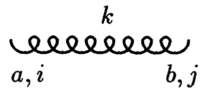
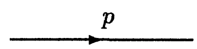
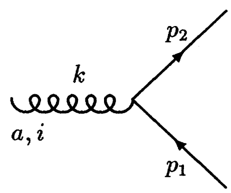
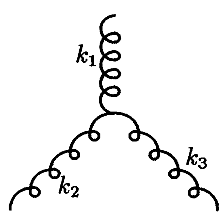
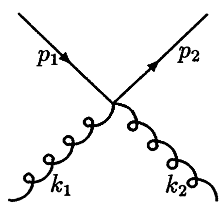
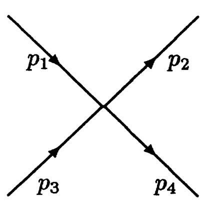
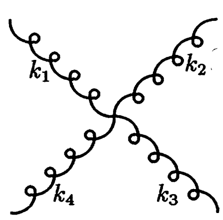


FIG. 4. The Feynman diagrams for the one-loop quark self-energy in two-component LFQCD.

TABLE II. Two-component Feynman diagrammatic rules. $\mathcal{L}_{\rm int} = -\mathcal{H}_{\rm int}$. All gluon momenta are outgoing.

$\mathcal{L}_{int}$	Diagram	Vertex
$\mathcal{L}_{0g}$		$i\,\frac{\delta_{ab}\delta_{ij}}{k^2}$
$\mathcal{L}_{0q}$		$i\,\frac{p^+}{p^2-m^2}$
$\mathcal{L}_{qqg}$		$igT^a\left\{2\frac{k^i}{[k^+]}-\frac{\sigma_\perp\cdot p_{2\perp}-im}{[p_2^+]}\sigma^i-\sigma^i\frac{\sigma_\perp\cdot p_{1\perp}+im}{[p_1^+]}\right\}$
$\mathcal{L}_{ggg}$		$-gf^{a_1a_2a_3}\left\{\left[(k_2-k_3)^{i_1}-\frac{k_1^{i_1}}{[k_1^+]}(k_2^+-k_3^+)\right]\delta_{i_2i_3}\right.$ $+\left[(k_3-k_1)^{i_2}\frac{k_2^{i_2}}{[k_2^+]}(k_3^+-k_1^+)\right]\delta_{i_3i_1}$ $\left.+\left[(k_1-k_2)^{i_3}-\frac{k_3^{i_3}}{[k_3^+]}(k_1^+-k_2^+)\right]\delta_{i_1i_2}\right\}$
$\mathcal{L}_{qqgg}$		$-ig^2\left\{\frac{(T^{a_2}T^{a_1})\sigma^{i_2}\sigma^{i_1}}{[p_1^+-k_2^+]}+\frac{(T^{a_1}T^{a_2})\sigma^{i_1}\sigma^{i_2}}{[p_1^+-k_1^+]}\right.$ $\left.-2iT^af^{aa_1a_2}\frac{k_1^+-k_2^+}{[k_1^++k_2^+]^2}\delta_{i_1i_2}\right\}$
$\mathcal{L}_{qqqq}$		$-2ig^2\left\{\frac{T^aT^a}{[p_1^+-p_2^+]^2}-\frac{T^aT^a}{[p_1^+-p_4^+]^2}\right\}$
$\mathcal{L}_{gggg}$		$-ig^2\left\{f^{a_1a_2b}f^{a_3a_4b}\left[\delta_{i_1i_3}\delta_{i_2i_4}-\delta_{i_1i_4}\delta_{i_2i_3}\right.\right.$ $\left.\left.+\delta_{i_1i_2}\delta_{i_3i_4}\frac{(k_1^+-k_2^+)[k_3^+-k_4^+)}{[k_1^++k_2^+][k_3^++k_4^+]}\right]+(2\to 3)+(2\to 4)\right\}$

$$\Sigma_3 = i2g^2C_f \int \frac{d^4k}{(2\pi)^4} \frac{k^+}{k^2-m^2} \left(\frac{1}{[p^+-k^+]^2} - \frac{1}{[p^++k^+]^2} \right). \quad \text{(A10)}$$

Using the principal value prescription, we can integrate over k^- in Eqs. (A8)–(A10) and show that $\Sigma_i = \delta p_i^-$. To do so, we consider a general integral

$$I = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2+i\epsilon)[(p-k)^2-m^2+i\epsilon]} \times f\left(\frac{1}{[k^+]}, \frac{1}{[p^+-k^+]}\right), \quad \text{(A11)}$$

where f is an arbitrary analytical function of $\frac{1}{[k^+]}$ and $\frac{1}{[p^+-k^+]}$. To integrate over k^-, we define

$$I = \int \frac{dk^+d^2k_\perp}{16\pi^3} I^-(k^+,k_\perp), \quad \text{(A12)}$$

$$I^-(k^+,k_\perp) \equiv \int_{-\infty}^{\infty} \frac{dk^-}{2\pi} f\left(\frac{1}{[k^+]}, \frac{1}{[p^+-k^+]}\right) \frac{1}{(k^+k^- - k_\perp^2 + i\epsilon)} \frac{1}{(p^+-k^+)(p^- - k^-) - (p-k)_\perp^2 - m^2 + i\epsilon}. \quad \text{(A13)}$$

Using the identity

$$\frac{1}{x+i\epsilon} = -i \int_0^{\infty} d\alpha e^{i\alpha(x+i\epsilon)}, \quad \text{(A14)}$$

to rewrite the last two factors in Eq. (A13) and then integrating over k^-, we have

$$I^-(k^+,k_\perp) = -\int_0^\infty d\alpha \int_0^\infty d\beta f\left(\frac{1}{[k^+]}, \frac{1}{[p^+-k^+]}\right) e^{-i\left\{\alpha(k_\perp^2-i\epsilon)+\beta[(p-k)_\perp^2+m^2-(p^+-k^+)p^- - i\epsilon]\right\}} \times \delta[\alpha k^+ - \beta(p^+-k^+)]. \quad \text{(A15)}$$

Consider the case $p^+ \geq 0$. Since $\alpha, \beta > 0$,

$$k^+ > p^+ : \quad I^-(k^+,k_\perp) = 0, \quad \text{(A16)}$$

$$k^+ = p^+ : \quad I^-(k^+,k_\perp) = \frac{i}{[k^+][(p-k)_\perp^2+m^2-i\epsilon]} f\left(\frac{1}{[k^+]}, 0\right), \quad \text{(A17)}$$

$0 < k^+ < p^+$:

$$I^-(k^+,k_\perp) = f\left(\frac{1}{[k^+]}, \frac{1}{[p^+-k^+]}\right) \frac{1}{[k^+][p^+-k^+]} \frac{-i}{\left(p^- - \frac{(p-k)_\perp^2+m^2}{[p^+-k^+]} - \frac{k_\perp^2}{[k^+]} + i\epsilon\right)}, \quad \text{(A18)}$$

$$k^+ = 0 : \quad I^-(k^+,k_\perp) = \frac{i}{[p^+](k_\perp^2 - i\epsilon)} f\left(0, \frac{1}{[p^+-k^+]}\right), \quad \text{(A19)}$$

$$k^+ < 0 : \quad I^-(k^+,k_\perp) = 0, \quad \text{(A20)}$$

where we have used the principal value prescription $\frac{1}{[k^+]} = 0$ for $k^+ = 0$ [see Eq. (3.22)]. Combining the above results together, we can simply write $I^-(k^+,k_\perp)$ as

$$I^-(k^+,k_\perp) = f\left(\frac{1}{[k^+]}, \frac{1}{[p^+-k^+]}\right) \frac{1}{[k^+][p^+-k^+]} \times \frac{-i}{\left(p^- - \frac{(p-k)_\perp^2+m^2}{[p^+-k^+]} - \frac{k_\perp^2}{[k^+]} + i\epsilon\right)} \quad \text{(A21)}$$

for $0 \leq k^+ \leq p^+$ and zero for others. Substituting this result into Eq. (A11) and using the relative momenta [see Eq. (4.1)], we have

$$I = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2[(p-k)^2-m^2]} f\left(\frac{1}{[k^+]}, \frac{1}{[p^+-k^+]}\right) = -i \int \frac{d^2\kappa_\perp}{16\pi^3} \int_0^1 \frac{dx}{x(1-x)p^2 - \kappa_\perp^2 - xm^2} \times f\left(\frac{1}{p^+[x]}, \frac{1}{p^+[1-x]}\right). \quad \text{(A22)}$$

The same result can be obtained for $p^+ \leq 0$. Similarly, it can also be shown that

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} = -i \int \frac{d^2\kappa_\perp}{16\pi^3} \int_0^\infty \frac{dx}{[x]}. \quad \text{(A23)}$$

Using the above two solutions, Eqs. (A8)–(A10) are immediately reduced to (4.10)–(4.12) and therefore $\Sigma_i = \delta p_i^-$.

(ii) *Gluon self-energy in the two-component Feynman calculation.* In the two-component Feynman perturbation theory, the one-loop gluon self-energy can be written as

$$\Pi_{ij}^{ab} = \Pi_{ij1}^{ab} + \Pi_{ij2}^{ab} + \Pi_{ij3}^{ab} + \Pi_{ij4}^{ab}, \qquad \text{(A24)}$$

where the terms on the right-hand side are represented by the Feynman diagrams shown in Fig. 5 in which Fig. 5(d) corresponds to Figs. 3(d) and 3(e) of the x^+-ordered diagrams. Using the rules listed in Table II, we have

$$-i\Pi_{ij1}^{ab} = \frac{1}{2} g^2 (f^{ace} f^{bfd}) \int \frac{d^4k}{(2\pi)^4} \Gamma_{g0}^{jmn}(q,-k) \times D_{0ml}^{cd} D_{0kn}^{ef} \Gamma_{g0}^{ikl}(-q, q-k), \qquad \text{(A25)}$$

$$-i\Pi_{ij2}^{ab} = -(ig)^2 (T^a T^b) \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[\Gamma_{q0}^i(q-k,k) \times S_0(k) S_0(q-k) \Gamma_{q0}^j(k,q)], \qquad \text{(A26)}$$

$$-i\Pi_{ij3}^{ab} = (-2ig^2)(T^a T^b) \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}(\sigma^i \sigma^i) S_0(k) \times \left(\frac{1}{[q^+ - k^+]} - \frac{1}{q^+ + k^+} \right), \qquad \text{(A27)}$$

$$-i\Pi_{ij4}^{ab} = \frac{1}{2}(-ig^2)(f^{acd} f^{bcd}) \int \frac{d^4k}{(2\pi)^4} D_{0ii}^{aa} \times \left\{ \left(\delta_{ij}\delta_{ll} - \delta_{il}\delta_{jl} + \delta_{il}\delta_{jl} \frac{(p^+ + k^+)^2}{(p^- - k^+)^2} \right) + \left(\delta_{ij}\delta_{ll} - \delta_{il}\delta_{jl} + \delta_{il}\delta_{jl} \frac{(p^+ - k^+)^2}{(p^+ + k^+)^2} \right) \right\}. \qquad \text{(A28)}$$

Explicitly,

$$-i\Pi_{ij1}^{ab} = -\frac{1}{2} \frac{g^2}{[q^+]} (f^{acd} f^{bdc}) \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 (q-k)^2} \times \Gamma_{g0}^{jlm}(q,-k) \Gamma_{g0}^{iml}(-q, q-k), \qquad \text{(A29)}$$

$$-i\Pi_{ij2}^{ab} = \frac{g^2}{[q^+]} \mathrm{Tr}(T^a T^b) \int \frac{d^4k}{(2\pi)^4} \frac{k^+}{k^2 - m^2} \frac{q^+ - k^+}{(q-k)^2 - m^2} \times \mathrm{Tr}[\Gamma_{q0}^i(k,q) \Gamma_{q0}^j(k-q,-q)], \qquad \text{(A30)}$$

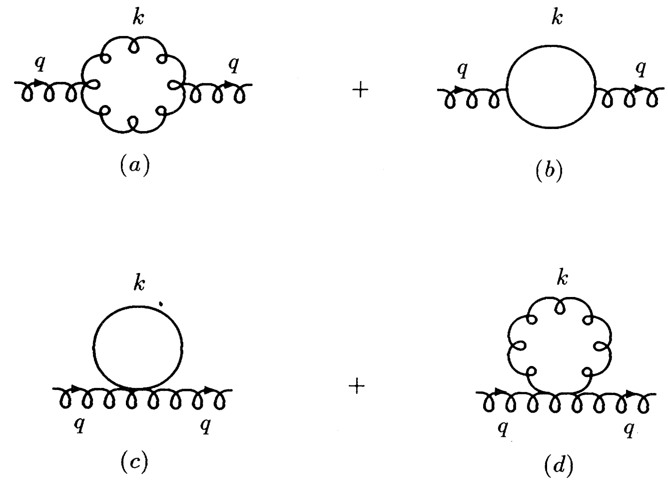


FIG. 5. The Feynman diagrams for the one-loop gluon self-energy in two-component LFQCD.

$$-i\Pi_{ij3}^{ab} = \frac{g^2}{[q^+]} \mathrm{Tr}(T^a T^b) \int \frac{d^4k}{(2\pi)^4} \frac{k^+}{k^2 - m^2} \mathrm{Tr}(\sigma^i \sigma^j) \times \left(\frac{1}{[q^+ - k^+]} - \frac{1}{[q^+ + k^+]} \right), \qquad \text{(A31)}$$

$$-i\Pi_{ij4}^{ab} = \frac{1}{2} g^2 (f^{acd} f^{bcd}) \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \delta_{ij} \left\{ 2 + \frac{(p^+ + k^+)^2}{(p^+ - k^+)^2} + \frac{(p^+ - k^+)^2}{(p^+ + k^+)^2} \right\}. \qquad \text{(A32)}$$

Again, by using (A22) and (A23), it is easy to find that $\Pi_{ij}^{ab}(q) = \delta q^-(q)\delta_{ab}\delta ij$. Therefore, in perturbation theory, the two-component Feynman theory is reduced to the x^+-ordered Hamiltonian theory if we integrate over k^- first.

One advantage of two-component Feynman perturbation theory is that we can use the ML prescription and dimensional regulator to regulate the light-front infrared divergences and ultraviolet divergences in Feynman loop integrals, and may recover the multiplicative renormalizability of the theory, at least in the one-loop approximation, as shown in Ref. [24].

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