PHYSICAL REVIEW D VOLUME 48, NUMBER 10 15 NOVEMBER 1993

Light-front QCD.
I. Role of longitudinal boundary integrals

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(Received 10 June 1993)

In this first part in a series of papers on light-front QCD, we address the global properties of light-front canonical structure. In the light-front canonical gauge theory, the elimination of unphysical gauge degrees of freedom leads to a set of boundary integrals which are associated with the light-front infrared singularity. We find that a consistent treatment of the boundary integrals leads to the cancellation of the light-front linear infrared divergences. For physical states, the requirement of finite energy density in the light-front gauge ($A_a^+ = 0$) results in equations which determine the asymptotic behavior of the transverse (physical) gauge degrees of freedom at longitudinal boundary. These asymptotic fields are generated by the boundary integrals and they involve nonlocal behavior in the transverse direction that leads to nonlocal forces which may be the source of QCD confinement.

PACS number(s): 11.10.Ef, 11.10.Gh, 12.38.Aw, 12.38.Bx

I. INTRODUCTION

Quantum chromodynamics (QCD), the theory of the strong interaction, is a theory in physics that has been accepted widely as fundamental but has not been tested by precise experiments. The lack of precision experimental tests comes from the fact that we are still unable to apply QCD to describe strong interaction processes at low and intermediate energies, although many of these processes have been measured for a long time. For a typical example, we do not know how to *accurately* calculate the proton magnetic moment which is extremely well-known experimentally. Also, we do not know how to use chiral symmetry in QCD to *precisely* address the issue of PCAC (partial conservation of axial-vector current), which underlies almost all the theoretical understanding of the low-energy strong interaction. At high energies, the asymptotic freedom behavior allows us to use perturbation theory. Even there the employment of QCD requires assumptions of factorization that separate low- and high-energy contributions to physically accessible observables, and the perturbation theory is only applicable to the high-energy contribution [1]. A currently available nonperturbative calculation scheme for low and intermediate energy QCD is lattice gauge theory [2]. Extensive work on lattice gauge theory has led to significant progress in various avenues of research; however, accurate information on the bound states of light quarks is still not available. The difficulty in solving low-energy QCD is that theorists have been unable to demonstrate a clear physical picture of the quark confinement and the dynamical mechanism of chiral symmetry breaking from the fundamental theory.

Recall that QCD was initially proposed as a strong interaction field theory *in* light-front coordinates, motivated by light-front current algebra [3]. In recent years, the search for nonperturbative solutions of QCD has led to an extensive exploration of light-front field theory (LFFT). The main attractions for studying nonperturbative QCD in light-front coordinates, called front form by Dirac [4], are that [5] (1) boost invariance in LFFT is a kinematical symmetry, which is important in the study of composite systems, particularly the hadrons in QCD, (2) LFFT is a relativistic field theory with nonrelativistic structure so that the relativistic bound-state equations are reduced to Schrödinger-type equations, from which the nonrelativistic quark model may find its justification in QCD in light-front coordinates, and (3) the positivity of the longitudinal momentum ($k^+ \geq 0$) in light-front Hamiltonian field theory implies that the light-front vacuum consists only of particles with longitudinal momentum $k^+ = 0$, which may simplify the QCD vacuum structure. These properties provide a hope to solve QCD in light-front coordinates for hadrons.

A systematic formulation of the light-front field theory for QCD [QCD in light-front coordinates with light-front gauge, (LFQCD)] was given by Casher as well as by Bardeen *et al.* about 17 years ago [6], based on the light-front quantization approach developed by Kogut and Soper [7] for QED, while the light-front pure Yang-Mills theory was studied by Tomboulis even earlier [8]. The perturbative LFQCD for various exclusive processes has also been investigated extensively by Lepage and Brodsky in the very early 1980's [9]. However, nonperturbative LFQCD has not been explored. In order to understand basic nonperturbative relativistic bound-state problems in LFFT, in the last few years many works have mainly focused on various $(1+1)$ dimensional field theory models, and some on the (3+1) dimensional Yukawa model and QED [10,11]. One main obstacle in extending the study to nonperturbative LFQCD is that a formalism to address simultaneously the major difficulties of QCD in light front coordinates is still not in place. These

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difficulties include the renormalization problem (even in perturbation theory), the confinement problem, and the problem of the QCD vacuum and dynamical chiral symmetry breaking.

The renormalization problem in the study of relativistic bound states in LFFT has several aspects. Since power counting is different on the light front [12], there are additional ultraviolet divergences in LFFT, compared to the instant form (i.e., equal-time quantization) [4]. The additional ultraviolet divergences have received some attention recently in the context of the relativistic bound-state problem in the (3+1) light-front Yukawa model [13]. A renormalization group based approach has been proposed for LFFT [14]. Ultraviolet divergences in light-front perturbation theory is also not straightforward due to the lack of covariance in the formalism [15], and noncovariant counterterms are needed to restore Lorentz covariance [16]. LFQCD also contains severe light-front infrared divergences. The resolution of the light-front infrared divergence problems is not complete even in perturbative LFQCD [17]. Issues arising from the possible mixing of the ultraviolet and infrared divergences in the relativistic bound-state problems in LFFT have not been addressed so far.

Understanding confinement is crucial for building hadronic bound states in QCD. In the canonical LFQCD formulation of Lepage and Brodsky [9], the associated Hamiltonian contains a linear potential between color charges only in the longitudinal direction, which does not provide a confinement mechanism for quarks and gluons in $3+1$ dimensions. Therefore, it may not be suitable for describing low-energy hadronic structure. Recently, Wilson proposed a formalism to construct a confining light-front quark-gluon Hamiltonian for LFQCD [12]. Wilson suggested that a starting point for analyzing the full QCD with confinement in light-front coordinates is the light-front infrared divergences. Based on light-front power counting, the counterterms for the light-front infrared divergences can involve the color charge densities and involve unknown nonlocal behavior in transverse direction that become a possible source for transverse confinement. However, the analysis is not yet complete and a scheme for practical calculation has yet to be developed.

Dynamical chiral symmetry breaking is another important issue in the study of QCD for hadrons. In instant form, dynamical chiral symmetry breaking is associated with a nontrivial vacuum through the Goldstone mechanism. In LFQCD, the vacuum is trivial when the $k^+=0$ sector is ignored. Therefore, it seems to be natural to argue that in order to obtain a nontrivial vacuum, one has to solve the $k^+=0$ modes [11]. Solving the $k^+=0$ modes may provide us with mechanism for spontaneous chiral symmetry breaking. Yet, the $k^+=0$ sector is singular and is very ambiguous. This singularity may exist even in free field theory. Thus, it is not clear whether the nontrivial structure of LFQCD *must* be associated with the $k^+=0$ modes. Furthermore, by involving the $k^+=0$ sector, the main advantage of LFQCD that simplifies nonperturbative bound states is lost, and therefore there is no strong reason why we should study nonperturbative QCD in light-front coordinates. In fact, dynamical symmetry breaking can be manifested in different ways in different frames. It may be more attractive if we could formulate LFQCD with a trivial vacuum such that the dynamical breaking of chiral symmetry is manifested explicitly via effective interactions. However, the attempt in this direction has not yet started.

All these problems mentioned above are essential and should involve non-Abelian gauge degrees of freedom in QCD. We are still unable to solve QCD at the moment. As a starting point, we shall address in this paper the problems of *light-front infrared singularity* based on a canonical quantization approach to LFQCD. We hope that these discussions will provide some insight for solving QCD in light-front coordinates in the future.

We apply the conventional canonical procedure [18] to QCD in light-front coordinates. It turns out naturally that QCD is a generalized Hamiltonian system [19] where the first-class gauge and quark constraints emerge explicitly in the Lagrangian. As is known, in the light-front gauge these first-class constraints become solvable first-order differential equations, and are used to eliminate unphysical degrees of freedom to all orders of the coupling constant. However, the gauge constraint equations contains a set of boundary integrals at longitudinal boundary for the longitudinal color electric fields [see Eq. (16) and the following discussions]. These longitudinal boundary integrals are the color charge density integrated over the longitudinal space (x^-) and are the source of the light-front infrared singularity. The resulting LFQCD Hamiltonian contains a boundary term proportional to these boundary integrals which are overlooked in previous investigations of light-front gauge theory.

We find that in perturbation theory the boundary integrals serve to remove linear infrared divergences in loop integrals. Removing the linear infrared divergences in LFQCD is a serious problem that has not been solved completely. In usual Feynman theory of perturbative LFQCD, by use of the gauge-fixing term, one can derive the gauge propagator involving $1/k^+$ singularity. Beyond the leading order calculation, this singularity leads to linear infrared divergence in the principal value prescription. In x^+-ordered Hamiltonian perturbation theory, the linear infrared divergences emerge even in tree-level and one-loop diagrams. By including the boundary term in the Hamiltonian, we obtain a consistent distribution function for the product of two principal value prescriptions, from which the linear divergences in loop integrals are removed by the same divergences from instantaneous interactions. This finding may be useful for perturbative LFQCD calculations in high-energy processes.

The relevant boundary integrals in the Hamiltonian formulation of axial gauge were indeed pointed out first by Schwinger in 1962 [20]. Because of the different structure between LFFT and the field theory in instant form, the consequences from the boundary integrals we study in this paper have not yet been realized explicitly in axial gauge. One of the important differences is related to the QCD vacuum. In instant form with axial gauge, the QCD vacuum cannot be simple. In LFQCD, generally the vacuum should also be nontrivial because of the $k^+=0$ modes. However, the choice of antisymmetric boundary conditions for field variables at longitudinal boundary excludes the $k^+ = 0$ modes. In this case, the LFQCD vacuum remains trivial as the bare vacuum, and thus *the nontrivial QCD structure must be carried purely by the light-front infrared behavior of gauge fields.*

For physical states, the requirement of finite energy density results in asymptotic equations for transverse (physical) gauge fields at longitudinal boundary. These asymptotic gauge fields are generated by the boundary integrals, and they involve not only the color charge densities in transverse space but also nonlocal behavior in the transverse direction. We find that by replacing the nontrivial boundary condition with a trivial one for the transverse gluon fields, many transverse nonlocal interactions are induced by the boundary integrals, which may lead to quark and gluon confinement. This possibility will be explored in further investigations.

The paper is organized as follows. In Sec. II, a canonical procedure for LFQCD is studied where we focus on the problem of the boundary conditions in solving the light-front constraints. In Sec. III, we discuss canonical quantization of LFQCD by use of the rigorous phase space structure [21] rather than the Dirac procedure [22]. In Sec. IV, the roles of boundary integrals are explored in detail. Some remarks are made for relevant problems in Sec. V. Finally, in the Appendix, we demonstrate the cancellation of linear infrared divergences at the tree level in $q\bar{q}$ scattering and in one-loop diagrams for the quark mass correction. The detailed perturbative calculations will be given in the following papers [23,24].

II. CANONICAL FORMULATION AND BOUNDARY CONDITION

We start from the QCD Lagrangian

$$\mathcal{L} = -\tfrac{1}{2}\mathrm{Tr}(F^{\mu\nu}F_{\mu\nu}) + \overline{\psi}(i\gamma_\mu D^\mu - m)\psi, \tag{1}$$

where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - ig[A^\mu, A^\nu]$, $A^\mu = \sum_a A_a^\mu T^a$ is a 3×3 gluon field color matrix, and the T^a are the generators of the SU(3) color group: $[T^a, T^b] = if^{abc}T^c$ and $\mathrm{Tr}(T^aT^b) = \frac{1}{2}\delta_{ab}$. The field variable ψ describes quarks with three colors and N_f flavors, $D^\mu = \frac{1}{2}\overleftrightarrow{\partial^\mu} - igA^\mu$ is the symmetric covariant derivative, and m is an $N_f \times N_f$ diagonal quark mass matrix. The Lagrange equations of motion are

$$\partial_\mu F_a^{\mu\nu} + gf^{abc}A_{b\mu}F_c^{\mu\nu} + g\overline{\psi}\gamma^\nu T^a\psi = 0, \tag{2}$$

$$(i\gamma_\mu\partial^\mu - m + g\gamma_\mu A^\mu)\psi = 0. \tag{3}$$

The light-front coordinates are defined as $x^\pm \equiv x^0 \pm x^3$, $x_\perp^i \equiv x^i$ $(i = 1, 2)$, where x^+ is chosen as the "time" direction along which the states are evolved, and x^- and $x_\perp$ become naturally the longitudinal and transverse coordinates. The inner product of any two four-vectors is then $a_\mu b^\mu = \frac{1}{2}(a^+b^- + a^-b^+) - a_\perp \cdot b_\perp$, and the time and space derivatives $(\partial^\mu = \frac{\partial}{\partial x_\mu})$ and the four-dimensional volume element are given by $\partial^+ = 2\frac{\partial}{\partial x^-}$, $\partial^- = 2\frac{\partial}{\partial x^+}$, $\partial^i = -\frac{\partial}{\partial x^i}$, and $d^4x = \frac{1}{2}dx^+dx^-d^2x_\perp$, respectively.

Naively, the canonical theory of QCD in light-front coordinates is constructed by defining the conjugate momenta of field variables $\{A_a^\mu(x), \psi(x), \overline{\psi}(x)\}$ as

$$E_a^\mu(x) = \frac{\partial\mathcal{L}}{\partial(\partial^- A_{a\mu})} = -\frac{1}{2}F_a^{+\mu}(x), \tag{4}$$

$$\pi_\psi(x) = \frac{\partial\mathcal{L}}{\partial(\partial^-\psi)} = i\frac{1}{4}\bar{\psi}\gamma^+ = \frac{i}{2}\psi_+^\dagger(x), \tag{5}$$

$$\pi_{\psi^\dagger}(x) = \frac{\partial\mathcal{L}}{\partial(\partial^-\psi^\dagger)} = -i\frac{1}{4}\gamma^0\gamma^+\psi = -\frac{i}{2}\psi_+(x), \tag{6}$$

where the fermion spinor in light-front coordinates is divided into $\psi = \psi_+ + \psi_-$, $\psi_\pm = \Lambda_\pm\psi$ with $\Lambda_\pm \equiv \frac{1}{2}\gamma^0\gamma^\pm$. Following a similar procedure in instant form described by Faddeev and Slavnov [18] for gauge theory, we separate the time derivative terms from the Lagrangian

$$\mathcal{L} = \left\{\frac{1}{2}F_a^{+i}(\partial^- A_a^i) + \frac{i}{2}\psi_+^\dagger(\partial^-\psi_+) - \frac{i}{2}(\partial^-\psi_+^\dagger)\psi_+\right\} \\ -\mathcal{H} - \left\{A_a^-\mathcal{C}_a + \frac{1}{2}(\psi_-^\dagger\mathcal{C} + \mathcal{C}^\dagger\psi_-)\right\}, \tag{7}$$

where

$$\mathcal{H} = \tfrac{1}{2}\left(E_a^{-2} + B_a^{-2}\right) \\ +\tfrac{1}{2}\left(\psi_+^\dagger\{\alpha_\perp \cdot (i\partial_\perp + gA_\perp) + \beta m\}\psi_- + \mathrm{H.c.}\right) \\ + \left[\tfrac{1}{2}\partial^+(E_a^- A_a^-) - \partial^i(E_a^i A_a^-)\right] \tag{8}$$

and

$$\mathcal{C}_a = \tfrac{1}{2}(\partial^+ E_a^- + gf^{abc}A_b^+ E_c^-) \\ -(\partial^i E_a^i + gf^{abc}A_b^i E_c^i) + g\psi_+^\dagger T^a\psi_+, \tag{9}$$

$$\mathcal{C} = (i\partial^+ + gA^+)\psi_- \\ -(i\alpha_\perp \cdot \partial_\perp + g\alpha_\perp \cdot A_\perp + \beta m)\psi_+. \tag{10}$$

In Eq. (8) we have defined $B_a^- = F_a^{12}$ as the longitudinal component of the light-front color magnetic field.

The reason for writing the Lagrangian in the above form is to make the Hamiltonian density and also the dynamical variables and constraints manifest. In Eq. (7), the first term contains all the light-front time derivative terms. From the definition of Eqs. (4)–(6), it immediately follows that only the transverse gauge fields A_a^i and the up-component quark fields ψ_+ and $\psi_+^\dagger$ are dynamical variables. The second term in Eq. (7), $\mathcal{H}$, is a Hamiltonian density. It contains three parts, the first part involves the light-front color electric and magnetic fields; the second, the usual quark Hamiltonian with coupling to the gauge field, and the last a surface term. In addition to the kinetic term and the Hamiltonian density, Eq. (7) also contains an additional term. This is a constraint term which indicates that the longitudinal gauge field A_a^- and the down-component quark fields ψ_- $(\psi_-^\dagger)$ are only the Lagrange multipliers for the constraints $\mathcal{C}_a, \mathcal{C}$ $(\mathcal{C}^\dagger) = 0$. These constraints arise from the definition of canonical momenta in the light-front coordinates and are consistent with the Lagrangian equations of motion. The gauge field constraint $\mathcal{C}_a = 0$ is in fact the light-front Gauss law which is an intrinsic property of gauge theory. The fermion constraint $\mathcal{C}$ ($\mathcal{C}^\dagger$) = 0 is purely a consequence of using the light-front coordinates.

The existence of constraint terms simply implies that QCD in the light-front coordinates is a generalized Hamiltonian system [19]. These constraints are all secondary, first-class constraints [25] in the Dirac procedure of quantization [22]. To obtain a canonical formulation of LFQCD for nonperturbative calculations, we need to explicitly solve the constraints, namely, to determine the Lagrange multipliers, to all orders of the coupling constant. Generally, it is very difficult to analytically determine the Lagrange multipliers from the constraints $\mathcal{C}_a, \mathcal{C} = 0$ since they are coupled by A_a^+. Only in the light-front gauge [7,26],

$$A_a^+(x) \equiv A_a^0(x) + A_a^3(x) = 0, \tag{11}$$

are these two constraints reduced to solvable one-dimensional differential equations:

$$\begin{aligned} &\tfrac{1}{2}\partial^+ E_a^- = \partial^i E_a^i + g(f^{abc} A_b^i E_c^i - \psi_+^\dagger T^a \psi_+) \ , \\ &i\partial^+ \psi_- = (i\alpha_\perp \cdot \partial_\perp + g\alpha_\perp \cdot A_\perp + \beta m)\psi_+ \ . \end{aligned} \tag{12}$$

In order to solve Eq. (12), we have to define the operator $1/\partial^+$. In general,

$$\begin{aligned} \left(\frac{1}{\partial^+}\right) f(x^-, x^+, x_\perp) &= \frac{1}{4}\int_{-\infty}^{\infty} dx_1^- \varepsilon(x^- - x_1^-) f(x_1^-, x^+, x_\perp) \\ &\quad + C(x^+, x_\perp) \ , \end{aligned} \tag{13}$$

where $\varepsilon(x) = 1, 0, -1$ for $x > 0, = 0, < 0$, respectively, and $C(x^+, x_\perp)$ is a x^- independent constant. However, since the canonical conjugate of transverse gauge field in LFQCD is a dependent variable [$E_a^i = -\frac{1}{2}\partial^+ A_a^i$, see Eq. (4) with $A_a^+ = 0$], one has to impose *a priori* a boundary condition for A_a^i in order to derive the canonical commutation relations for the physical field variables. It has been shown [26,27] that the suitable definition of $1/\partial^+$ which uniquely determines the initial value problem at $x^+ = 0$ for independent field variables is $C(x^+, x_\perp) = 0$. This corresponds to choosing an antisymmetric boundary condition for field variables in the longitudinal direction.

Using Eq. (13) and the definition $E_a^i = -\frac{1}{2}\partial^+ A_a^i$, we can explicitly express E_a^- in terms of transverse gauge fields A_a^i and the independent light-front quark field ψ_+ from the gauge constraint in Eq. (12):

$$\begin{aligned} E_a^-(x) = -\partial^i A_a^i(x) - \frac{g}{2}\int_{-\infty}^{\infty} dx'^- \varepsilon(x^- - x'^-)\rho_a(x'^-, x) \\ + C_a(x^+, x_\perp), \end{aligned} \tag{14}$$

where we have defined the color charge density

$$\begin{aligned} \rho(x^-, x^+, x_\perp) &\equiv \rho(x^-, x) \\ &= \tfrac{1}{2}(f^{abc} A_b^i \partial^+ A_c^i + 2\psi_+^\dagger T^a \psi_+). \end{aligned} \tag{15}$$

To uniquely determine the initial values at $x^+ = 0$, we require that the E_a^- and A_a^i satisfy antisymmetric boundary conditions at longitudinal boundary, namely, $C_a(x^+, x_\perp) = 0$. As a result,

$$E_a^-(x) \overset{x^- = \pm\infty}{=} -\partial^i A_a^i|_{x^- = \pm\infty} \mp \frac{g}{2}\int_{-\infty}^{\infty} dx^- \rho_a(x^-, x). \tag{16}$$

Since E_a^- satisfies now an antisymmetric boundary condition, its boundary values at longitudinal boundary are completely determined by Eq. (16), where the second term is boundary integrals over x^- for the color charge densities. As we will see later these integrals are the source of light-front infrared singularity. *We call them the longitudinal boundary integrals, or simply the boundary integrals.*

By using the identity [20],

$$\frac{1}{2}\int_{-\lambda/2}^{\lambda/2} dx^- \varepsilon(x^- - x'^-)\varepsilon(x^- - x''^-) = -|x'^- - x''^-| + \frac{1}{2}\lambda, \tag{17}$$

where the parameter λ denotes the distance between two boundary points in the longitudinal direction, the color electric field energy in the Hamiltonian becomes

$$\begin{aligned} H_E &= \frac{1}{2}\int_{-\infty}^{\infty} dx^- d^2x_\perp (E_a^-)^2 \\ &= \frac{1}{2}\int_{-\infty}^{\infty} dx^- d^2x_\perp \Bigg\{ (\partial^i A_a^i)^2 + g\int_{-\infty}^{\infty} dx'^- \partial^i A_a^i \varepsilon(x^- - x'^-)\rho_a(x'^-, x) \\ &\quad - \frac{g^2}{2}\int_{-\infty}^{\infty} dx'^- \rho_a(x^-, x)|x^-_{} - x'^-|\rho_a(x'^-, x)\Bigg\} + \left(\lim_{\lambda\to\infty}\lambda\right)\frac{g^2}{2}\int d^2x_\perp \left\{\int_{-\infty}^{\infty} dx^- \rho_a(x^-, x)\right\}^2 \end{aligned} \tag{18}$$

From Eq. (18), we see that after eliminating the longitudinal gauge field in the light-front gauge, color charge instantaneous interactions emerge in the Hamiltonian. In addition, Eq.(18) also contains a boundary term (the last term) due to the linear instantaneous interaction. There is no such term in Coulomb gauge because the Coulomb potential vanishes at spatial infinity. This term is proportional to the square of the boundary integrals and is associated with the infrared singularity. In Sec. IV, we will show that in perturbation theory, this term is regularized by the distribution function of the product of two principal value prescriptions and leads to the cancellation of the light-front linear infrared divergences. For physical states, the requirement of finite energy density results in the asymptotic equations for the transverse gauge fields which show that the asymptotic transverse gauge fields do not vanish at longitudinal boundary and are generated by the boundary integrals. Thus, the boundary integrals can inherently affect QCD dynamics.

Now the Lagrange multipliers in Eq. (7) can be easily determined. The Lagrange multiplier ψ_- is the solution of the quark constraint in Eq. (12):

$$\psi_-(x) = -\frac{i}{4}\int_{-\infty}^{\infty} dx'^- d^2x'_\perp \varepsilon(x^- - x'^-)\delta^2(x_\perp - x'_\perp) \times\{\alpha_\perp \cdot [i\partial'_\perp + gA_\perp(x')] + \beta m\}\psi_+(x'). \tag{19}$$

The Lagrange multiplier A_a^- is obtained from the definition $E_a^- = -\frac{1}{2}\partial^+ A_a^-$ and Eq. (16):

$$A_a^-(x) = -\frac{1}{2}\int_{-\infty}^{\infty} dx'^- \varepsilon(x^- - x'^-)E_a^-(x^+, x'^-, x_\perp) . \tag{20}$$

For this solution, the first surface term in Eq. (8) vanishes. Moreover, it is reasonable to assume that the transverse color electric fields E_a^i as well as A_a^i vanish as $O(r^{-2})$ and $O(r^{-1})$ at $r = |x_\perp| \to \infty$ because the gauge freedom is totally fixed at the transverse infinity. Thus the other surface term in Eq. (8) vanishes as well.

After the determination of the Lagrange multipliers, the LFQCD Hamiltonian is given simply by

$$\begin{aligned} H &= \int dx^- d^2x_\perp \{\tfrac{1}{2}(E_a^{-2} + B_a^{-2}) \\ &\qquad + \psi_+^\dagger[\alpha^i(i\partial^i + gA^i) + \beta m]\psi_- \} \\ &= \int dx^- d^2x_\perp \Big\{ \frac{1}{2}(\partial^i A_a^j)^2 + g f^{abc} A_a^i A_b^j \partial^i A_c^j + \frac{g^2}{4} f^{abc} f^{ade} A_a^i A_b^j A_d^i A_e^j \\ &\qquad + \frac{1}{4}\int_{-\infty}^{\infty} dx'^- [2g\partial^i A_a^i \varepsilon(x^- - x'^-)\rho_a(x'^-, x) - i\psi_+^\dagger\{\alpha^i(i\partial^i + gA^i) + \beta m\}\varepsilon(x^- - x'^-) \\ &\qquad\qquad \times\{\alpha^j(i\partial^j + gA^j) + \beta m\}\psi_+] - \frac{g^2}{4}\int_{-\infty}^{\infty} dx'^- \rho_a(x^-, x)|x^- - x'^-|\rho_a(x'^-, x) \Big\} \\ &\quad + \left(\lim_{\lambda\to\infty} \lambda\right) \frac{g^2}{4}\int d^2x_\perp \left\{\int_{-\infty}^{\infty} dx^- \rho_a(x^-, x)\right\}^2 . \end{aligned} \tag{21}$$

III. PHASE SPACE APPROACH TO LIGHT-FRONT QUANTIZATION

A self-consistent formulation of LFQCD requires that the resulting Hamiltonian must generate the correct equations of motion for the physical degrees of freedom $(A_a^i, \psi_+, \psi_+^\dagger)$. This section is devoted to the derivation of canonical quantization and a check of the consistency.

To see how to correctly reproduce the Lagrangian equations of motion, we need to find consistent commutators for physical field variables. In the light-front gauge, the Lagrangian of Eq. (7) is reduced to

$$\mathcal{L} = \frac{1}{2}\partial^+ A_a^i \partial^- A_a^i + \frac{i}{2}(\psi_+^\dagger \partial^- \psi_+ - \partial^- \psi_+^\dagger \psi_+) - \mathcal{H}. \tag{22}$$

The canonical momenta of the physical field variables $A_a^i, \psi_+, \psi_+^\dagger$ are

$$\mathcal{E}_a^i = \frac{1}{2}\partial^+ A_a^i \ , \quad \pi_{\psi_+} = \frac{i}{2}\psi_+^\dagger \ , \quad \pi_{\psi_+^\dagger} = -\frac{i}{2}\psi_+. \tag{23}$$

However, Eq. (23) shows that all the canonical momenta are functions of the independent field variables. Thus, after determining all the Lagrange multipliers, the system is still a constrained Hamiltonian system. Usually, in order to quantize such a constrained Hamiltonian system, one has to use the Dirac procedure, by imposing the so-called primary, second-class constraints $E_a^i + \frac{1}{2}\partial^+ A_a^i = 0$ (similarly for $\pi_{\psi_+, \psi_+^\dagger}$) to construct Dirac brackets. However, for these trivial primary constraints, the mathematically well-defined canonical one-form offers a rigorous phase space structure for canonical quantization [21,28]. Now, we will use such an approach for light-front quantization.

The phase space structure for the physical variables $(\mathcal{E}_a^i, A_a^i; \pi_{\psi_+}, \psi_+; \pi_{\psi_+^\dagger}, \psi_+^\dagger)$ is determined rigorously by rewriting Eq. (22) as a Lagrangian one-form $\mathcal{L}dx^+$ (apart from a total light-front time derivative):

$$\begin{aligned} \mathcal{L}dx^+ &= \tfrac{1}{2}2(\mathcal{E}_a^i dA_a^i + \pi_{\psi_+} d\psi_+ + d\psi_+^\dagger \pi_{\psi_+^\dagger} \\ &\qquad - A_a^i d\mathcal{E}_a^i - d\pi_{\psi_+}\psi_+ - \psi_+^\dagger d\pi_{\psi_+^\dagger}) - \mathcal{H}dx^+ \\ &= \tfrac{1}{2}q^\alpha \Gamma_{\alpha\beta} dq^\beta - \mathcal{H}dx^+ , \end{aligned} \tag{24}$$

where the first term on the right-hand side is called the canonical one-form of the physical phase space, and quark fields are anticommuting c numbers (Grassmann variables). Correspondingly, the symplectic structure or the Poisson brackets of the phase space is given by

$$\omega = \tfrac{1}{2}\Gamma_{\alpha\beta}dq^\alpha dq^\beta \quad \text{or} \quad [q^\beta, q^\alpha]_P = \Gamma^{-1}_{\alpha\beta}. \tag{25}$$

Canonical quantization is realized by replacing the Poisson brackets by the equal-x^+ commutation relations

$$[q^\beta, q^\alpha] = i\Gamma^{-1}_{\alpha\beta}. \tag{26}$$

Explicitly

$$[A^i_a(x), \mathcal{E}^j_b(y)]_{x^+=y^+} = i\tfrac{1}{2}\delta_{ab}\delta^{ij}\delta^3(x-y), \tag{27}$$

$$\{\psi_+(x), \pi_{\psi_+}(y)\}_{x^+=y^+} = i\frac{\Lambda_+}{2}\delta^3(x-y), \tag{28}$$

$$\{\psi^\dagger_+(x), \pi_{\psi^\dagger_+}(y)\}_{x^+=y^+} = -i\frac{\Lambda_+}{2}\delta^3(x-y) \ , \tag{29}$$

or

$$[A^i_a(x), \partial^+ A^j_b(y)]_{x^+=y^+} = i\delta_{ab}\delta^{ij}\delta^3(x-y), \tag{30}$$

$$[A^i_a(x), A^j_b(y)]_{x^+=y^+} = -i\delta_{ab}\delta^{ij}\frac{1}{4}\varepsilon(x^- - y^-)\delta^2(x_\perp - y_\perp), \tag{31}$$

$$\{\psi_+(x), \psi^\dagger_+(y)\}_{x^+=y^+} = \Lambda_+\delta^3(x-y) \ , \tag{32}$$

where $\delta^3(x-y) \equiv \delta(x^- - y^-)\delta^2(x_\perp - y_\perp)$. All other commutators between the physical degrees of freedom vanish. Note that, unlike in the instant form, the commutator $[A^i_a(x), A^j_b(y)]$ does not vanish since the canonical conjugate $\mathcal{E}^i_a$ is a function of the field variable A^i_a. Equation (31) is obtained by the use of Eq. (13) with the antisymmetric boundary condition (i.e., $C = 0$), which leads to the antisymmetric boundary condition for $A^i_a(x)$: $A^i_a(-\infty) = -A^i_a(\infty)$ [29]. Using the basic commutation relations of Eqs. (31) and (32), it is straightforward to verify that the equations of motion are consistent with Eqs. (2) and (3):

$$\begin{aligned}\partial^-\psi_+ &= \frac{1}{i}[\psi_+ \ , \ H] \\ &= \Big\{ igA^- - \{\alpha_\perp \cdot (i\partial_\perp + gA_\perp) + \beta m\} \\ &\qquad \times \frac{1}{\partial^+}\{\alpha_\perp \cdot (i\partial_\perp + gA_\perp) + \beta m\}\Big\}\psi_+ \ .\end{aligned} \tag{33}$$

$$\begin{aligned}\partial^- A^i_a &= \frac{1}{i}[A^i_a \ , \ H] \\ &= \frac{1}{\partial^+}[D^j_{ab}F^{ji}_b - D^i_{ab}E^-_b - gj^i_a - gf^{abc}A^-_b\partial^+A^i_c] \ ,\end{aligned} \tag{34}$$

where $D^i_{ab} = \delta_{ab}\partial^i - gf^{abc}A^i_c$, and A^-_a and E^-_a are given by Eqs. (20) and (16) [30].

In addition, choosing antisymmetric boundary conditions in the longitudinal direction in LFQCD has the following advantages.

(1) With any other boundary condition, Eq. (14) contains an arbitrary x^--independent function. Such an arbitrary function leads to ambiguities in formulating LFQCD. Only with an antisymmetric boundary conditions, is this arbitrary term zero and formally LFQCD can be completely defined. Furthermore, by the choice of antisymmetric boundary conditions, the residual gauge freedom in $A^+_a = 0$ is completely fixed [31].

(2) For the definition Eq. (13) with $C(x^+, x_\perp) = 0$, all field variables in LFQCD satisfy antisymmetric boundary conditions at longitudinal boundary except $\psi_+(x)$, whose boundary condition is not specified. However, the equation of motion for $\psi_+(x)$ contains $(\frac{1}{\partial^+})\psi_+$ [see Eq. (33)] which forces it to satisfy the antisymmetric boundary condition. As a result, the $k^+ = 0$ modes are completely excluded in the momentum expansion of field variables. Since the LFFT vacuum is occupied only by the $k^+ = 0$ particles, with the help of antisymmetric boundary condition the LFQCD vacuum is ensured to be trivial as the bare vacuum. The nontrivial structure of QCD (the $k^+ = 0$ mode effect) is carried purely by light-front infrared properties in field operators. We also must emphasize that the exclusion of the $k^+ = 0$ mode in field variables by the antisymmetric boundary conditions does not mean that the zero modes of composite operators are removed.

It is worth pointing out that the antisymmetric boundary conditions provide a well-defined regulator to the light-front infrared divergences in momentum space, namely, the principal value prescription. In Feynman perturbation theory, the use of the principal value prescription leads to the "spurious" poles in light-front Feynman integrals, which prohibit any continuation to Euclidean space (Wick rotation) and hence the use of standard power counting arguments for Feynman loop integrals [32]. This causes difficulties in addressing renormalization of QCD in Feynman perturbation theory with light-front gauge. In the last decade there are many investigations in attempting to solve this problem. One excellent solution is given by Mandelstam and Leibbrandt, i.e., Mandelstam-Leibbrandt (ML) prescription [33], which allow continuation to Euclidean space and hence power counting. It has been also shown that with ML prescription, the *multiplicative* renormalization in the two-component LFQCD Feynman formulation is restored [34].

In the present paper, we study QCD in light-front equal-x^+ quantization. Unfortunately, ML prescription cannot be applied to equal-x^+ quantization because ML prescription is defined on the boundary condition which involves x^+ itself [35] and are not allowed in equal-x^+ canonical theory. Yet, as is pointed out recently by Wilson [12], light-front power counting differs completely from the power counting in equal-time quantization that noncanonical counterterms are allowed in LFFT. Furthermore, the current attempts to understand nonperturbative QCD in light-front coordinates is based on the x^+-ordered (old-fashioned) diagrams in which no Feynman integral is involved [10,11]. Thus the power counting criterion for Feynman loop integrals is no longer available in LFQCD Hamiltonian calculations. In Hamiltonian perturbation theory with the principal value prescription, LFQCD contains the severe linear and logarithmic infrared divergences generated from the boundary integrals. In perturbation theory, the logarithmic infrared divergences can be completely canceled in the complete loop diagrams, as was previously shown in the calculation of QCD correction to the scale evolution of hadronic structure function up to two-loops using the principal value prescription [36]. A simple example of such a cancellation in x^+-ordered perturbative theory is also given for quark mass renormalization in the Appendix. In the following papers of this series [23,24] we will present a detailed discussion on x^+-ordered perturbative loop calculations and renormalization of LFQCD Hamiltonian theory up to one-loop, where the logarithmic infrared divergences are systematically analyzed and their cancellation is also shown in the coupling constant renormalization. However, the severe light-front linear infrared divergences that even exist in tree level have not been explored. The renormalization of light-front Hamiltonian theory is an entirely new subject where investigations are still in their preliminary stage [12–16]. Since the light-front power counting allows noncanonical counterterms, a complete understanding of renormalized LFQCD may not be worked out within perturbation theory, and new renormalization and regularization approaches are needed to be developed. In this paper we show that the boundary integrals play an important role in removing the severe linear infrared divergences.

IV. ROLE OF BOUNDARY INTEGRALS

1. *Removing linear infrared divergences.* In the past decade, applications of LFQCD are mostly restricted in perturbation theory. In the original development of Lepage and Brodsky the boundary term in Eq. (21) is ignored so that the light-front instantaneous interactions are thought to be linear potentials [7,37]. However, we find that this negligence leads to severe infrared singularities in the perturbation theory. To see this clearly, we consider the formulation in momentum space. For the prescription of $1/\partial^+$ expressed in terms of the integral of Eq. (13), the standard Fourier transform leads to the principal value prescription in momentum space as follows:

$$\left(\frac{1}{\partial^+}\right) f(x^-) = \frac{1}{4}\int_{-\infty}^{\infty} dx'^- \varepsilon(x^- - x'^-) f(x'^-) \to \frac{1}{2}\left(\frac{1}{k^+ + i\epsilon} + \frac{1}{k^+ - i\epsilon}\right) f(k^+) \equiv \frac{1}{[k^+]} f(k^+), \tag{35}$$

$$\begin{aligned}\left(\frac{1}{\partial^+}\right)^2 f(x^-) &= \frac{1}{4^2}\int_{-\infty}^{\infty} dx'^- dx''^- \varepsilon(x^- - x'^-)\varepsilon(x'^- - x''^-) f(x''^-) \\ &\to \left[\frac{1}{2}\left(\frac{1}{k^+ + i\epsilon} + \frac{1}{k^+ - i\epsilon}\right)\right]^2 f(k^+) \equiv \frac{1}{[k^+]^2} f(k^+).\end{aligned} \tag{36}$$

Equation (36) defines the product of two principal value prescriptions of Eq. (35) in terms of the distribution function. In this derivation, it follows [see Eq. (17)] that the boundary term in Eq. (21) have been regularized. It is known that Eq. (36) leads to linear infrared divergences in loop integrals. In order to avoid this divergence, the following prescription has been introduced [9]:

$$\left(\frac{1}{\partial^+}\right)^2 f(x^-) = \frac{1}{4^2}\int_{-\infty}^{\infty} dx'^- |x^- - x'^-| f(x'^-) \to \frac{1}{2}\left(\frac{1}{(k^+ + i\epsilon)^2} + \frac{1}{(k^+ - i\epsilon)^2}\right) f(k^+) \equiv \frac{1}{[k^{+2}]} f(k^+). \tag{37}$$

This corresponds to the case that the longitudinal boundary term in Eq. (17) is ignored. Equivalently, the last term in Eq. (21) is dropped. Apparently, this prescription removes the linear infrared divergence originated from the instantaneous interactions. Unfortunately in such a prescription, beyond leading order calculations in Feynman perturbation theory or even in leading order calculation in the old-fashioned Hamiltonian perturbation theory, the product of two principal value prescriptions appearing from three-point vertex either is not defined or leads to linear infrared divergences. We shall show that it is the prescription of Eq. (36) which serves for the cancellation of linear infrared divergences originated from the three-point vertex and from the instantaneous interactions. Here we only discuss the x^+-ordered (old-fashioned) perturbative calculations.

First at the tree level, for example, for $q\bar{q}$ scattering [see (A1)], only the linear potential leads to a $1/\epsilon^2$ divergence as $k^+ \to 0$. The scattering involving one-gluon exchange is finite due to the principal value prescription. Thus in the naive prescription (37), even the lowest order $q\bar{q}$ scattering amplitude is $1/\epsilon^2$ divergent. By including the boundary term, this divergence is canceled. In loop calculations, for example, for the one-loop correction to the light-front quark energy [see (A2)], the one-gluon exchange diagram (which contains an integral of $1/[k^+]^2$) leads to a $1/\epsilon$ divergence; the linear potential is, however, infrared finite in the relevant integral of $1/[k^{+2}]$. Hence, in the naive prescription, loop calculations also contain the severe $1/\epsilon$ infrared divergence. In the prescription of (36), the instantaneous interactions are the linear potentials accompanied by the boundary term, which produce a $1/\epsilon$ divergence from the integral of $1/[k^+]^2$ that cancels precisely the same divergence in the one-gluon exchange diagram. Furthermore, the cancellation in the one-loop correction of quark-gluon vertex has also been verified

[24].

The reason that the linear infrared divergences are removed by using the prescription of (36) can be understood as follows. From Eq. (16), the k^+ singularity originated from the boundary integrals. The color electric energy in LFQCD Hamiltonian contains two sources for the k^+ singularity. One is the explicit boundary term, the last term in Eq. (21), which is $1/k^{+2}$ singular. The other belongs to the gluon emission vertex. The resultant gluon emission vertex is the first term in the square bracket in Eq. (21), which is $1/k^+$ singular. Therefore, in one-gluon exchange diagrams, it produces a $1/k^{+2}$ singularity from the product of two principal value prescriptions for the definition of Eq. (35). The associated linear divergence in loop integrals is the same as that from the $1/k^{+2}$ singularity of the boundary terms in the prescription of Eq. (36), with a different sign from an energy denominator, and therefore the linear infrared divergence is canceled. Note that in Eq. (21) there is another $1/k^+$ singularity (in the second term in the square bracket), which comes from the quark constraint [see Eq. (19)]. Yet, in one-gluon exchange diagrams, it leads to a form $1/(p_1^+ p_2^+)$ $(p_1^+ = p_2^+ + k^+)$ which does not generate infrared divergences. Thus, all linear infrared divergences originate from the same source, the boundary integrals. Any negligence of boundary term in the Hamiltonian through Eq. (37) will lead to unwanted infrared divergences.

However, the cancellation of the linear infrared divergences in higher order loop integrals (beyond the one-loop diagrams) may also depend on the regularization of ultraviolet divergences. The cancellation beyond leading order should be true for gauge invariant regularization. For gauge variant regularization, such as transverse dimensional regularization [6], boost invariant cutoff regularization [38], and the explicit cutoff regularization [39] used in the x^+-ordered perturbative LFQCD, we need to introduce gluon mass counterterms. These counterterms break gauge invariance and thereby may also spoil the cancellation of linear infrared divergences in higher order diagrams. However, if we set the quark mass $m = 0$ in perturbative LFQCD, the transverse dimensional regularization results in a zero gluon mass correction. In this case the cancellation is still satisfied in two-loop diagrams. In deep inelastic scattering, one often sets $m = 0$ in calculating high-order corrections to the scale evolution of hadronic structure functions [36]. A more detailed discussion on perturbative LFQCD will be presented in the following papers [23,24]. For low-energy dynamics, the light quark mass is crucial and perturbation theory is no longer useful. Removal of infrared divergences needs to be treated in an alternative way, which we shall discuss later.

We may point out that in 1+1 LFQCD [40], the boundary integral is the color charge operator. The corresponding boundary term occurring in the Hamiltonian [41] is then proportional to the square of color charge. It is indeed this term resulting in an infinite quark mass which is regarded as evidence of quark confinement in 1 + 1 QCD. Explicitly, the linear potential does not provide an infinite mass for the quark, as shown above (also see Ref. [42]), but the boundary term adds a $1/k^{+2}$ singularity (the $1/\epsilon$ divergence) to the quark propagator. Since there are no transverse gluons in 1 + 1 QCD to cancel this divergence, the boundary term recovers 't Hooft's solution of the infinite quark mass pole [40]. In physical (zero color charge) states, the boundary term does not contribute to physical observables because it is the square of the color charge operators. Quark confinement in gauge-invariant states arises purely from the linear potential. This implies that ignoring the boundary integral in 1 + 1 QCD may not affect any observable. In 3 + 1 QCD, the existence of transverse gluons changes these consequences.

2. *Nonlocal interactions in the transverse direction.* For physical states, finite energy density requires that the longitudinal color electric field strength must vanish at longitudinal boundary (a similar requirement was used by Chodos in axial gauge [20]):

$$E_a^-|_{x^-=\pm\infty} = 0 \tag{38}$$

or explicitly

$$\partial^i A_a^i|_{x^-=\pm\infty} = \mp\frac{g}{2}\int_{-\infty}^{\infty} dx^- \rho_a(x^-, x). \tag{39}$$

Equation (39) is consistent with our choice of antisymmetric boundary condition. Moreover, this condition explicitly shows that the transverse gauge fields at longitudinal boundary are generated by the boundary integrals [43]. Clearly, Eq. (39) is satisfied only for physical states. In perturbation theory, we cannot use this condition because in perturbative QCD, we consider not only physical states but also color nonsinglet states for which Eq. (39) may not be satisfied. Therefore the main effect of Eq. (39) should be manifested in nonperturbative dynamics, i.e., bound states.

One of the nonperturbative approaches to solve bound states in LFFT is the Tamm-Dancoff approach, which truncates the Fock space to be a few-body state space [44]. Such an approach becomes practically applicable only when the vacuum is trivial. We have given a realization of a trivial vacuum in this paper. However, to address hadronic bound states, the existence of nontrivial interactions, namely, confinement potentials, is crucial. An explicit construction of such interactions from QCD is still lacking. The LFQCD Hamiltonian contains linear interactions only in the longitudinal direction [see Eq. (21)]. Quark and color confinement certainly require similar potentials in the transverse direction as well. We suggest that these nontrivial interactions in LFQCD might hide in the condition of Eq. (39).

From Eq. (39) we see that the asymptotic A_a^i fields at longitudinal boundary are proportional to the color charge density in transverse space and also that they involve nonlocal behavior in the transverse direction (induced by the transverse derivative). Intuitively, we may separate the transverse gauge potentials into a normal part plus a boundary part:

$$A_a^i = A_{aN}^i + A_{aB}^i \ , \tag{40}$$

where

$$A^i_{aN}|_{x^-=\pm\infty}=0\ \ ,\ \ \partial^i A^i_{aN}|_{x^-=\pm\infty}=0, \qquad (41)$$

$$\partial^i A^i_{aB}|_{x^-=\pm\infty}=\mp\frac{g}{2}[\rho^g_a(x_\perp)+\rho^q_a(x_\perp)]\ . \qquad (42)$$

In Eq. (42), $\rho_a(x_\perp)$ denote the color charge densities integrated over x^-. The conditions of Eqs. (41) and (42) do not uniquely determine the separation of Eq. (40). Generally, there are two types of separation for Eq. (40). One is to consider A^i_{aB} the long-distance fields generated by the boundary integrals and A^i_{aN} the short-distance fields determined by free theory. In this case, if we are only interested in the low-energy dynamics, the effect of the A^i_{aN} fields may be ignored. This separation is physically very interesting but it is practically very difficult to be realized analytically. Another possibility is to choose a simple solution for the A^i_{aB} that satisfy Eq. (42). In this case, the A^i_{aN} have the trivial boundary condition Eq. (41) but are not determined by free theory. The Hamiltonian is then expressed only in terms of the A^i_{aN}, and the boundary behavior of transverse gauge fields are replaced by the effective interactions. A convenient choice for A^i_{aB} which satisfies Eq. (42) is

$$A^i_{aB}(x)=-\frac{g}{8}\int dx'^-dx'^i\varepsilon(x^- - x'^-)\varepsilon(x^i-x'^i) \times[\rho^g_a(x')+\rho^q_a(x')]\ ,\ \ i=1,2. \qquad (43)$$

Substituting the separation of Eq. (40) with (43) into the LFQCD Hamiltonian, we obtain a new Hamiltonian in terms of A^i_{aN} that contains many effective interactions induced by Eq. (39). All these effective interactions involve the color charge densities and involve nonlocal behavior in both the longitudinal and transverse directions. One of the lowest order interactions, for example, is given by

$$H_{b1}\propto\sum_{ij}\int_{-\infty}^{\infty}dx^idx^-dx^jdx'^-dx'^j\eta^{ij} \times\{\partial^i\rho^q_a(x^-,x^i,x^j)\}|x^- - x'^-| \times|x^j-x'^j|\{\partial^i\rho^q_a(x'^-,x^i,x'^j)\}\ , \qquad (44)$$

where $\eta^{ij}\equiv 1$ (0) for $i\neq(=)\ j$. This is a linear interaction in both the longitudinal and the transverse directions. Hence, Eq. (39) leads to numerous many-body nonlocal color charge interactions which may lead to confinement.

Still the Hamiltonian contains, in principle, an infinite number of many-body interactions generated by the boundary integrals (or obtained from the counterterms of the infrared divergences). This is a consequence of the boundary integrals in a non-Abelian gauge theory due to the existence of nonlinear gluon interactions. It is also true in other gauge choices, such as Coulomb gauge [45] or axial gauge [20]. Practically, as the first step, we may only keep two-body interaction terms, such as Eq. (44), in the new Hamiltonian. Because of the trivial vacuum in the present formulation of LFQCD, using such an approximate LFQCD Hamiltonian, we can apply the light-front Tamm-Dancoff approach [10] to find hadronic bound states, where the bound states contain only a few particles, such as one quark-antiquark pair, one quark-antiquark pair with one and two gluons. This is certainly one of the most attractive approaches for low-energy QCD. A numerical investigation along this consideration is in progress.

V. DISCUSSIONS

In the previous section, we have discussed some primary properties of boundary integrals which we think to be important in understanding LFQCD. In the current investigations of LFFT, one of the most active topics is the problem of the $k^+=0$ modes and the $A^+=0$ gauge. In this section, we have some remarks to make about the relation of the $k^+=0$ modes, the $A^+_a=0$ gauge and boundary conditions at longitudinal boundary.

In previous LFFT investigations, much attention has been paid to how to construct a nontrivial vacuum from the $k^+=0$ modes. All attempts have focused on $(1+1)$ field models [11]. The motivation for these attempts, as we have mentioned in the Introduction, is to try to understand spontaneous chiral symmetry breaking in LFFT. In instant form, the vacuum is, of course, crucial for hadronic structure since we believe that axial charges Q^a_5 create pseudoscalar particles (the lowest bound states in strong interaction region) from the vacuum. However, the role of light-front axial charges in hadronic structure is totally different. The success of light-front current algebra in describing low-energy hadronic structure is based on the properties of light-front Q^a_5 with a trivial vacuum [46]. In this case, Q^a_5 annihilate the vacuum so that the vacuum in LFFT itself is not essential in understanding chiral symmetry. The importance of light-front Q^a_5 lies in their matrix elements between hadronic states. These matrix elements are proportional to hadronic decay constants involving pseudoscalar mesons and are independent to the limit of $m_\pi\to 0$. They therefore carry the basic information of hadronic structure. In instant form, Q^a_5 cannot connect hadronic states with different 4-momenta in the limit of $m_\pi\to 0$ [47]. In other words, the instant Q^a_5 itself is practically not useful for hadronic structure except for the Nambu-Goldstone picture of spontaneous symmetry breaking, where the important ingredient is the axial-vector current. These totally opposite properties of axial charges in light-front and instant forms implies that to address dynamical breaking of chiral symmetry in LFQCD, one may need to understand the relation between light-front axial charge operators and the Hamiltonian operator rather than the structure of vacuum in LFFT.

Second, we discuss briefly the difference between LFQCD and the canonical formulation of QCD in instant form with axial gauge $A^3_a=0$. The main difference is as follows. In LFQCD, the finite energy density for physical states results explicitly in the asymptotic equation for transverse gauge fields in longitudinal boundary [see Eq. (39)] due to the fact that the conjugate momenta of A^i_a are dependent variables in light-front coordinates, ($E^i_a=\frac{1}{2}\partial^+A^i_a$ is not a light-front time derivative). In axial gauge, $A^i_a, i=1,2$ and their conjugate momenta are all the dynamically independent variables. Thus, a similar condition proposed by Chodos leads to a very complicated formalism which may not be practically useful even for perturbation theory, as noted by himself [20]. The second major difference is the vacuum. In axial gauge, the QCD vacuum is still complicated regardless of the boundary condition chosen. In such a case, it is very difficult (if not impossible) to do nonperturbative calculations before knowing the vacuum structure. In LFQCD, with antisymmetric boundary conditions, the vacuum is trivial and the nontrivial behavior of QCD would be manifested directly in Hamiltonian operators induced by the boundary integrals. Thus it is straightforward to use quantum mechanical nonperturbative approaches to compute bound states. Moreover, in axial gauge, the boost invariance is not manifested kinematically so that it is not a good framework to study low-energy QCD, which deals with composite particles of quarks and gluons. In LFQCD, as we have mentioned in the Introduction, boost invariance is a kinematical symmetry which is very convenient in addressing hadronic structures.

In summary, we show that a suitable choice of boundary conditions for physical fields in LFQCD is crucial because it determines whether the nontrivial behavior of QCD can be decoupled from the vacuum so that the property of the trivial vacuum in LFFT becomes useful for solving hadrons from QCD. We have derived the canonical formulation of LFQCD with great care for boundary integrals, which have not been paid enough attention in previous investigations. We show that the boundary integrals are the source of the light-front infrared singularity and involve color charge densities and nonlocal behavior in the transverse direction that lead to nonlocal forces generated by the boundary integrals. Clearly, our understanding of the physics from the boundary integrals in LFQCD is far from complete and much work remains to be done. For example, it is very interesting to see which terms among the numerous nonlocal interactions are essential for hadronic bound states. However, before we turn to discuss the nonperturbative LFQCD, it is necessary to study in detail the perturbative LFQCD which has not be explored extensively beyond the tree levels calculation of Lepage and Brodsky [9]. This will be presented in the following two papers [23,24].

ACKNOWLEDGMENTS

We are grateful to R. J. Perry and K. G. Wilson for many valuable comments and insightful discussions. We would also like to acknowledge fruitful discussions with S. J. Brodsky, R. J. Furnstahl, K. Hornbostel, D. Mustaki, S. Pinsky, J. Shigemitsu, and T. S. Walhout. We want to especially thank R. J. Furnstahl for his careful reading of the manuscript. This work was supported by National Science Foundation of United States under Grant Nos. PHY-9102922, PHY-8858250, PHY-9203145, and PHY-9207889.

APPENDIX: CANCELLATION OF LINEAR INFRARED DIVERGENCE

In this appendix we shall use the x^+-ordered perturbative rule which we developed recently [23] for the two-component LFQCD to check the cancellation of linear infrared divergence in perturbative LFQCD. For a complete list of the x^+-ordered perturbative rules and Feynman rules for two-component LFQCD and the detailed calculations, see Refs. [23,24].

1. Tree level ($q\bar{q}$ scattering)

The lowest-order $q\bar{q}$ scattering amplitude is given by

$$M_{fi} = M_{fi}^a + M_{fi}^b + M_{fi}^c, \tag{A1}$$

which corresponds to the diagrams shown in Fig. 1. In the x^+-ordered perturbative theory, we have

$$\begin{aligned} M_{fi}^a + M_{fi}^b = & -g^2 T_{43}^a T_{21}^a \chi_4^\dagger \Gamma_{q0}^i(-p_4, -p_3, k)\chi_3 \\ & \times \chi_2^\dagger \Gamma_{q0}^i(p_2, p_1, k)\chi_1 \left\{ \frac{\theta(p_1^+ - p_2^+)}{p_i^- - p_1^- - p_4^- - k^-}\frac{1}{k^+} \right. \\ & \left. - \frac{\theta(p_2^+ - p_1^+)}{p_i^- - p_2^- - p_3^- + k^-}\frac{1}{k^+} \right\}, \end{aligned} \tag{A2}$$

$$M_{fi}^c = \begin{cases} -4g^2 T_{21}^a T_{43}^a \chi_2^\dagger \chi_1 \frac{1}{[k^{+2}]} \chi_4^\dagger \chi_3 & \text{NB}, \\ -4g^2 T_{21}^a T_{43}^a \chi_2^\dagger \chi_1 \frac{1}{[k^+]^2} \chi_4^\dagger \chi_3 & \text{WB}, \end{cases} \tag{A3}$$

where

$$\begin{aligned} \Gamma_{q0}^i(p_2, p_1, k) = \left[2\frac{k^i}{[k^+]} - \frac{\sigma \cdot p_{2\perp} - im}{[p_2^+]}\sigma^i \right. \\ \left. - \sigma^i \frac{\sigma \cdot p_{1\perp} + im}{[p_1^+]} \right], \end{aligned} \tag{A4}$$

p_i^- is the total energy of the initial state, $k^\mu = (p_1^+ - p_2^+, p_1^i - p_2^i, \frac{(p_{1\perp} - p_{2\perp})^2 + m^2}{p_1^+ - p_2^+})$. NB denotes a no boundary term and WB means including the boundary term. It follows that in the principal value prescription, $M_{fi}^a + M_{fi}^b$ is free of infrared divergences, while M_{fi}^c without

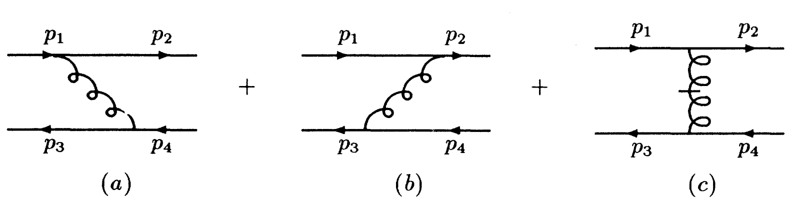

FIG. 1. The x^+-ordered graphs for the lowest-order $q\bar{q}$ scattering in perturbative LFQCD.

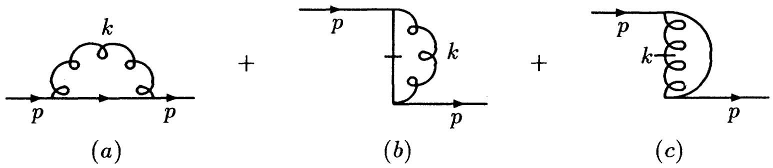

FIG. 2. The x^+-ordered graphs for the one-loop correction to the light-front quark energy in perturbative LFQCD.

the boundary term has a $1/\epsilon^2$ divergence when $k^+ \to 0$. When the boundary term is included, the $1/\epsilon^2$ can be canceled [see (A3)]. Therefore, it is necessary to include the boundary term in order to obtain a finite amplitude for the lowest-order $q\bar{q}$ scattering. A similar discussion for e^+e^- scattering in LFQED is given in Ref. [35].

2. One-loop correction to the light-front quark energy

Based on the x^+-ordered perturbative theory [23], the light-front quark energy (p^-) correction up to one-loop is determined by

$$\delta p^-(p^2 = m^2) = \delta p_a^- + \delta p_b^- + \delta p_c^- . \tag{A5}$$

The three terms on the right-hand side are denoted by the three diagrams shown in Fig. 2. Again, using the rules in Ref. [23], we find that

$$\delta p_a^- = g^2 C_f \int \frac{dk^+ d^2k_\perp}{16\pi^3} \frac{\theta(k^+)\theta(p^+ - k^+)}{[k^+]} \Gamma^i_{q0}(p-k,p,k) \times \Gamma^i_{q0}(p,p-k,-k) \frac{1}{p^- - k^- - (p-k)^-} , \tag{A6}$$

$$\delta p_b^- = 2g^2 C_f \int \frac{dk^+ d^2k_\perp}{16\pi^2} \frac{1}{[k^+][p^+ - k^+]} , \tag{A7}$$

$$\delta p_c^- = \begin{cases} 2g^2 C_f \int \frac{dk^+ d^2k_\perp}{16\pi^2} \left\{ \frac{1}{[(p^+ - k^+)^2]} - \frac{1}{[(p^+ + k^+)^2]} \right\} & \text{NB} , \\ 2g^2 C_f \int \frac{dk^+ d^2k_\perp}{16\pi^3} \left\{ \frac{1}{[p^+ - k^+]^2} - \frac{1}{[p^+ + k^+]^2} \right\} & \text{WB} , \end{cases} \tag{A8}$$

where $C_f = 4/3$. A direct calculation shows that

$$\delta p_a^- = \frac{g^2}{8\pi^2} \frac{C_f}{p^+} \int d^2k_\perp \left(1 - \frac{\pi p^+}{2\epsilon} + \ln \frac{\epsilon}{p^+} + \int_0^1 dx \frac{2m^2}{k_\perp^2 + x^2 m^2} \right) , \tag{A9}$$

$$\delta p_b^- = -\frac{g^2}{8\pi^2} \frac{C_f}{p^+} \int d^2k_\perp \left(\ln \frac{\epsilon}{p^+} \right) , \tag{A10}$$

$$\delta p_c^- = \begin{cases} \frac{g^2}{8\pi^2} \frac{C_f}{p^+} \int d^2k_\perp (-2) & \text{NB,} \\ , & \\ \frac{g^2}{8\pi^2} \frac{C_f}{p^+} \int d^2k_\perp \left(-1 + \frac{\pi p^+}{2\epsilon} \right) & \text{WB} , \end{cases} \tag{A11}$$

which tells us that in the one-loop correction to the light-front quark energy, the one-gluon exchange contains both linear and logarithmic infrared divergences. The instantaneous fermion interaction contains only one logarithmic divergence [see δp_b^- in Eq.(A10)], which cancels the logarithmic divergence in δp_a^-. The naive instantaneous-gluon interaction (namely, the linear potential in the longitudinal direction) is free of infrared divergence. Therefore, without boundary term, the quark mass correction involves a linear infrared divergence, which is an inconsistent solution. By combining the boundary term with the linear potential, we see that δp_c^- has a linear infrared divergence which precisely cancels the same divergence in δp_a^-. Thus the quark mass correction is now free of infrared divergences:

$$\delta m^2 = p^+ \delta p^- = \frac{m^2}{4\pi^2} C_f \ln \frac{\Lambda^2}{m^2} + \text{finite} , \tag{A12}$$

where Λ is the transverse momentum cutoff. In Eq. (A9), the coefficient (1/4) in the mass correction is different from the covariant result (3/8) because the regularization scheme is different. This coefficient is the same as that in the light-front calculation with dimensional regularization in the transverse direction and the explicit cutoff in the longitudinal direction [48]. Note that in their calculation, the expressions of Eqs. (A9)–(11) are different but the sum is the same as Eq. (A12) where the linear divergence is also canceled.

[1] For example, see R. D. Field, *Applications of Perturbative QCD* (Addison-Wesley, New York, 1989).
[2] See, for example, *Lattice '92*, Proceedings of the International Symposium, Amsterdam, The Netherlands, 1992, edited by J. Smit and P. van Baal [Nucl. Phys. B (Proc. Suppl.) **30** (1993)].
[3] H. Fritzsch and M. Gell-Mann, in *Proceedings of the XVI International Conference on High Energy Physics*, edited by J. D. Jackson and A. Roberts (Fermilab, Batavia, 1972), Vol. 2, p. 135; H. Fritzsch, M. Gell-Mann, and H. Leutwyler, Phys. Lett. **47B**, 365 (1973).
[4] P. A. M. Dirac, Rev. Mod. Phys. **21**, 392 (1949).
[5] For an extensive list of light-front references, see "Sources for Light-Front Physics," available via anonymous FTP from pacific.mps.ohio-state.edu in the subdirectory pub/infolight.
[6] A. Casher, Phys. Rev. D **14**, 452 (1976). Also see W. A. Bardeen and R. B. Pearson, *ibid.* **14**, 547 (1976); W. A. Bardeen, R. B. Pearson, and E. Rabinovici, *ibid.* **21**, 1037 (1980), for the LFQCD lattice formulation.

[7] J. B. Kogut and D. E. Soper, Phys. Rev. D **1**, 2901 (1970).
[8] E. Tomboulis, Phys. Rev. D **8**, 2736 (1973).
[9] G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980); S. J. Brodsky and G. P. Lepage, in *Perturbative Quantum Chromodynamics*, edited by A. H. Mueller (World Scientific, Singapore, 1989), p. 93, and references therein.
[10] H. C. Pauli and S. Brodsky, Phys. Rev. D **32**, 1993 (1985); R. J. Perry, A. Harindranath, and K. G. Wilson, Phys. Rev. Lett. **65**, 2959 (1990); S. Brodsky and H. C. Pauli, in *Recent Aspect of Quantum Field Theory*, edited by H. Mitter and H. Gausterer, Lecture Notes in Physics Vol. 396 (Springer-Verlag, Berlin, 1991), and references therein; St. Glazek, A. Harindranath, J. Shigemitsu, S. Pinsky, and K. G. Wilson, Phys. Rev. D **47**, 1599 (1993).
[11] For a summary see X. Ji, Commun. Nucl. Phys. **21**, 123 (1993); and also see S. Brodsky, G. McCartor, H. C. Pauli, and S. Pinsky, Particle World **3**, 109 (1993).
[12] K. G. Wilson, "Light-Front QCD," OSU Internal Report, 1991 (unpublished).
[13] Glazek, Harindranath, Shigemitsu, Pinsky, and Wilson [10]; P. M. Wort, Phys. Rev. D **47**, 608 (1993).
[14] B. van de Sande and S. S. Pinsky, Phys. Rev. D **46**, 5479 (1992); St. Glazek and K. G. Wilson, *ibid.* **47**, 4657 (1993); R. J. Perry and K. G. Wilson, Nucl. Phys. B (to be published); R. J. Perry, OSU report, 1993 (unpublished).
[15] A. Langnau and M. Burkardt, Phys. Rev. D **47**, 3452 (1993); A. Langnau and S. J. Brodsky (unpublished).
[16] M. Burkardt and A. Langnau, Phys. Rev. D **44**, 1187 (1991); **44**, 3857 (1991).
[17] G. Leibbrandt, Rev. Mod. Phys. **59**, 1067 (1987).
[18] L. D. Faddeev and A. A. Slavnov, *Gauge Fields* (Benjamin/Cummings, Reading, Massachusetts, 1980), Chap. 3.
[19] P. A. M. Dirac, Can. J. Math. **2**, 129 (1950).
[20] J. Schwinger, Phys. Rev. **130**, 402 (1963); Y.-P. Yao, J. Math. Phys. **5**, 1319 (1964); A. Chodos, Phys. Rev. D **17**, 2624 (1978); I. Bars and F. Green, Nucl. Phys. **B142**, 157 (1978).
[21] L. D. Faddeev and R. Jackiw, Phys. Rev. Lett. **60**, 1692 (1988); R. Jackiw, in Workshop on *Constraint Theory and Quantization Methods* (Montepulciano, Italy, 1993).
[22] P. A. M. Dirac, *Lectures on Quantum Mechanics* (Yeshiva University, New York, 1964).
[23] W. M. Zhang and A. Harindranath, following paper, Phys. Rev. D (to be published).
[24] A. Harindranath and W. M. Zhang, this issue, Phys. Rev. D (to be published), paper III.
[25] In light-front field theory, there always exist the so-called primary, second-class constraints due to the fact that the light-front Lagrangian is linear in the first-order x^+ derivative and therefore the canonical momenta are functions of field variables. These constraints are not real constraints in the generalized phase space quantization procedure and are easily handled. We will give these discussions in Sec. III.
[26] F. Rohrlich, Acta Phys. Austriaca **32**, 87 (1970); R. A. Neville and F. Rohrlich, Nuovo Cimento **A1**, 625 (1971).
[27] R. Jackiw, in *Photon-Hadron Interactions I.*, International Summer Institute in Theoretical Physics, Hamburg, Germany, 1971, Springer Tracts in Modern Physics Vol. 62 (Springer, Berlin, 1971), p. 1.
[28] If the first-class constraints cannot be solved explicitly, for example, for other gauge choices in light-front coordinates, using the Dirac procedure may be necessary to construct the canonical commutation relations for all variables.
[29] A direct check also shows that

$$\lim_{x^- \to \infty, y^- \to -\infty} \varepsilon(x^- - y^-) = 0.$$

Topologically, this requirement is identical to $\varepsilon(0) = 0$. This can easily be understood if we divide the longitudinal line into boxes and define the ε function in each box.
[30] In 1+1 QED, there is an apparent contradiction between the antisymmetric boundary conditions for field variables and equations of motion. Consider the second Maxwell equation in (1+1)-dimensional QED:

$$\partial^- E^- = gj^- = 2g\psi_-^\dagger \psi_- .$$

The right-hand side of this equation satisfies the symmetric boundary condition while the left-hand side satisfies the antisymmetric boundary condition if the longitudinal electric field E^- and hence $\partial^- E^-$ satisfy the antisymmetric boundary condition. Thus there is an apparent contradiction. Here we show that the contradiction does not exist since both $\partial^- E^-$ and j^- do vanish at the ends of the boundary as a result of charge conservation. The proof is simple: In light-front coordinates with a light-front gauge for 1+1 QED, $E^- = -g(\frac{1}{\partial^+})j^+$, where j^+ is the light-front charge density. Applying Eq. (14) with the antisymmetric boundary condition [i.e., let $C(x^+) = 0$ in Eq. (13)] to QED_{1+1} we immediately find that at the ends of the boundary, $E^- = \mp\frac{g}{4}Q$; i.e., it is proportional to the charge Q. Thus, the charge conservation ensures that the light-front time derivative of E^- is zero at the ends of the boundary. On the other hand, using the current conservation $\partial_\mu j^\mu = 0$ and using again our definition of $\frac{1}{\partial^+}$, we have $j^-(x^+, \pm\infty) = \mp\frac{1}{4}\partial^- Q = 0$.
[31] W. M. Zhang and A. Harindranath, Phys. Lett. B **314**, 223 (1993).
[32] D. M. Capper, J. J. Dulwich, and M. J. Litvak, Nucl. Phys. **B241**, 463 (1984).
[33] S. Mandelstam, Nucl. Phys. **B213**, 149 (1983); G. Leibbrandt, Phys. Rev. D **29**, 1699 (1984); Rev. Mod. Phys. **59**, 1067 (1987).
[34] H. C. Lee and M. S. Milgram, Nucl. Phys. **B268**, 543 (1986).
[35] A. C. Tang, Phys. Rev. D **37**, 3014 (1988).
[36] For an example, see G. Curci, W. Furmanski, and R. Petronzio, Nucl. Phys. **B175**, 27 (1980).
[37] T. M. Yan, Phys. Rev. D **7**, 1760 (1973).
[38] S. J. Brodsky, G. P. Lepage, T. Huang, and P. B. Mackenzie, in *Particles and Fields 2*, Proceedings of the Banff Summer Institute, Banff, Canada, 1981, edited by A. Z. Capri and A. N. Kamal (Plenum, New York, 1983).
[39] C. B. Thorn, Phys. Rev. D **20** 1934 (1979).
[40] G. 't Hooft, Nucl. Phys. **B75**, 461 (1975); For more detailed discussion see K. Hornbostel, Ph.D. thesis, Stanford University, 1988.
[41] Such a term has been noted by Bergknoff in the (1+1)-dimensional Schwinger model, see H. Bergknoff, Nucl. Phys. **B122**, 215 (1977).
[42] C. G. Callan, N. Coote, and D. Gross, Phys. Rev. D **13**

1649 (1976); M. B. Einhorn, *ibid.* **14** 3451 (1976).

[43] It may be worth pointing out that unlike the $1+1$ QCD, the boundary integrals in 3+1 LFQCD are color charge densities in the transverse space and not the color charge operators. The color charge operator is defined as

$$Q_a = \frac{1}{2}\int dx^- d^2x_\perp (f^{abc}A_b^i \partial^+ A_c^i + 2\psi_+^\dagger T^a \psi_+)$$
$$= \int dx^- d^2x_\perp [\rho_a^g(x^-, x_\perp) + \rho_a^q(x^-, x_\perp)] \ ,$$

where ρ_a^g is the charge density carried by the gluon field and ρ_a^q that by the quark field in light-front coordinates. Therefore, we cannot simply drop the boundary integrals for color singlet states. In other words, removing the boundary integrals in Eq. (21) implies ignoring the nonvanishing asymptotic A_a^i fields at the longitudinal boundary, and therefore loses the possibility to address the nontrivial properties in LFQCD.

[44] I. Tamm, J. Phys. (USSR) **9**, 449 (1949); S. M. Dancoff, Phys. Rev. **78**, 382 (1950); for the light-front Tamm-Dancoff approach, see Perry, Harindranath, and Wilson [10].

[45] N. Christ and T. D. Lee, Phys. Rev. D **22**, 939 (1980).

[46] F. Feinberg, Phys. Rev. D **7**, 540 (1973); E. Eichten, F. Feinberg, and J. F. Willemsen, *ibid.* **8**, 1204 (1973).

[47] R. Carlitz, D. Heckathorn, J. Kaur, and W.-K. Tang, Phys. Rev. D **11**, 1234 (1975).

[48] D. Mustaki, S. Pinsky, J. Shigemitsu, and K. G. Wilson, Phys. Rev. D **43**, 3411 (1991).