Twistor formulation of the nonheterotic superstring with manifest world sheet supersymmetry

A. Galperin*
Department of Physics and Astronomy, The Johns Hopkins University, Baltimore, Maryland 21218

E. Sokatchev †
Physikalisches Institut, Universität Bonn, Nussallee 12, D-5300 Bonn 1, Germany
(Received 28 April 1993)

We propose a new formulation of the $D = 3$ type II superstring which is manifestly invariant under both target-space $N = 2$ supersymmetry and world sheet $N = (1,1)$ super reparametrizations. This gives rise to a set of twistor (commuting spinor) variables, which provide a solution to the two Virasoro constraints. The world sheet supergravity fields are shown to play the role of auxiliary fields.
PACS number(s): 11.17.+y, 11.30.Pb

I. INTRODUCTION

During the last few years a new formulation of the superparticle and the heterotic superstring with a $D = 3,4,6,10$ target space has been developed [1–10]. It has $N = 1$ target supersymmetry and at the same time manifests world line (or world sheet) $N = D - 2$ local supersymmetry. The latter replaces the well-known κ symmetry [11,12] of the superparticle (string). Thus κ symmetry finds its natural explanation as an on-shell version of the off-shell local supersymmetry of the world sheet.

The key to such formulations is the use of commuting spinor ("twistor") variables, as proposed in the pioneering work of Sorokin *et al.* [1]. These variables emerge in a natural way as the world sheet supersymmetry superpartners of the target superspace Grassmann coordinates. In this context one obtains a twistorlike solution for the null momentum of the massless superparticle (or for one of the Virasoro vectors of the heterotic superstring) as a bilinear combination of the twistors. Thus, the twistor variables turn out to parametrize the sphere S^{D-2} associated with the above null vector. All of this is achieved as a consequence of one of the equations of motion of the twistorlike superparticle (string), the so-called geometrodynamical constraint. It specifies the way the world sheet superspace $\mathcal{M}$ is embedded in the target superspace $\underline{\mathcal{M}}$. Namely, one requires that the odd part of the tangent space to $\mathcal{M}$ lie entirely within the odd part of the tangent space to $\underline{\mathcal{M}}$ at any point of $\mathcal{M}$. The conditions for this particular embedding are generated dynamically from a Lagrange multiplier term in the action.

It is very natural to try to extend the above results to the nonheterotic superstring. This means solving *both* Virasoro constraints in terms of twistor variables and interpreting the κ-symmetry of the theory as a nonheterotic $N = (D-2, D-2)$ world sheet supersymmetry. However, changing from one dimension (the case of the superparticle) or essentially one dimension (the case of the heterotic superstring) to two dimensions of the world sheet is far from trivial. An attempt in this direction has recently been made by Chikalov and Pashnev [13]. There only the first half of this program was achieved. Considering an $N = 2$ target superspace, but still only $N = (1,0)$ world sheet supersymmetry, Chikalov and Pashnev obtained two twistor variables and solved both Virasoro constraints. At the same time, their world sheet possessed only one supersymmetry, which could not explain the full κ symmetry of the theory and in addition broke two-dimensional Lorentz invariance. An interesting feature of their formulation was the absence of any world sheet supergravity fields.

In the present paper we shall make a step further towards the realization of the full twistor program. We present a twistor formulation of the $D = 3$ type II (i.e., with $N = 2$ target-space supersymmetry) nonheterotic superstring. On the world sheet we have $N = (1,1)$ local supersymmetry and thus are able to completely eliminate κ symmetry. At first sight our construction closely resembles the one in the heterotic case [10]. However, significant differences appear in the analysis of the twistor constraints, which follow from the geometrodynamical embedding of $\mathcal{M}$ in $\underline{\mathcal{M}}$. If in the heterotic case it was relatively easy to show that as a result of these constraints the twistor variables parametrized the space S^{D-2}, here a careful study is needed. The solution to the twistor constraints now consists of two sectors, a regular one, which corresponds to nontrivial superstring motion, and a singular one, in which the superstring collapses into a superparticle. Another big difference is that one needs the full set of world sheet supergravity fields in order to make the superfield action super-reparametrization invariant

*On leave from the Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, Russia.

†On leave from the Institute for Nuclear Research and Nuclear Energy, Sofia, Bulgaria.

0556-2821/93/48(10)/4810(11)/$06.00

[14]. However, at the level of components one discovers that the world sheet gravitino is in fact an *auxiliary field* and can be eliminated via its algebraic field equation. The world sheet metric and the twistor variables compete for the role of reparametrization gauge fields. Algebraic elimination of the twistor variables leads to the familiar Green-Schwarz action. Finally, in the heterotic case the formalism worked equally well in all the cases $D = 3, 4, 6, 10$. However, in the nonheterotic case the attempt to go beyond $D = 3$ (thus having extended world sheet supersymmetry) causes a problem, namely, the geometrodynamical constraint starts producing equations of motion. Understanding this crucial point will probably give us some new nontrivial insight into the geometric nature of the superstring. It will also help us achieve a complete twistor formulation in all the dimesnions where the classical superstring exists.

The paper is organized as follows. In Sec. II we explain the notation and introduce the basic geometric objects of the world sheet and target superspaces. In Sec. III we present the twistor formulation of the $D = 3$ superparticle with $N = 2$ target-space and $N = (1, 1)$ world sheet supersymmetry. This is a simplified version of the superstring theory, which helps illustrate some of the new features encountered here. In Sec. IV the two terms of the superstring action, the geometrodynamical and the Wess-Zumino ones, are given and it is shown how the former allows one to establish the consistency of the latter. In Sec. V we study in detail the component structure of the action. We find out which component fields are auxiliary and by eliminating them arrive at the standard Green-Schwarz action. This analysis crucially depends on which solution of the twistor constraints we use, the regular or the singular one. In the latter case we observe the string shrinking to a particle. In the Appendix we find the general solution to the algebraic twistor constraints, which consists of a regular and a singular sector.

II. TWO- AND THREE-DIMENSIONAL SUPERGEOMETRY

In this section we shall introduce some basic concepts concerning $N = 2$ superspaces in two and three dimensions. These superspaces will serve as the world sheet and the target space of the superstring, respectively.

The world sheet of the type II $D = 3$ superstring is a $(2|2)$-dimensional superspace parametrized by two even and two odd real coordinates $Z^M = (\xi^m, \eta^\mu)$, where $m = (0, 1)$ and $\mu = (1, 2)$. We assume that it is endowed by $N = (1, 1)$ two-dimensional supergravity. The latter is described by a vierbein $E_A{}^M$ and a SO(1,1) Lorentz connection ω_A which satisfy the constraint [15]

$$\{D_\alpha, D_\beta\} = 2i(\gamma^c C)_{\alpha\beta} D_c + R_{\alpha\beta}. \tag{1}$$

Here $A = (a, \alpha)$ with $a = (0, 1)$ and $\alpha = (1, 2)$ being SO(1,1) vector and spinor indices, and the covariant derivatives are

$$D_A = E_A{}^M \partial_M + \omega_A. \tag{2}$$

Equation (1) means that we have imposed the torsion constraints

$$T_{\alpha\beta}{}^c = 2i(\gamma^c C)_{\alpha\beta}, \quad T_{\alpha\beta}{}^\gamma = 0. \tag{3}$$

The complete set of $N = (1, 1)$ supergravity constraints and their consistency have been studied in [16]. There it has been shown that two-dimensional $N = (1, 1)$ supergravity can be considered superconformally flat (ignoring moduli problems). This means that there exist superconformal transformations of the vierbeins and connections which leave (1) invariant and which can gauge away all the torsion and curvature tensors. For our purposes we shall need the infinitesimal form of the super-Weyl transformations of the vierbeins:

$$\begin{aligned} \delta E_M{}^a &= \Lambda E_M{}^a, \\ \delta E_M{}^\alpha &= \frac{1}{2}\Lambda E_M{}^\alpha - \frac{i}{2} E_M{}^a (C^{-1}\gamma_a)^{\alpha\beta} D_\beta \Lambda. \end{aligned} \tag{4}$$

In what follows we shall often use two-dimensional light-cone notation. There one employs $\gamma^{\pm\pm} = \frac{1}{2}(\gamma^0 \pm \gamma^1)$ as projection operators for the two irreducible halves of the spinor. Then the light-cone form of (1) is

$$\begin{aligned} \{D_+, D_+\} &= 2iD_{++}, \\ \{D_-, D_-\} &= 2iD_{--}, \\ \{D_+, D_-\} &= R_{+|-} \,. \end{aligned} \tag{5}$$

Our final point about two-dimensional supergravity concerns the structure of the covariant derivatives (2) taken at the point $\eta = 0$. They will be used in Sec. V for evaluating the superstring component action. From [16] we learn that in a certain gauge for the super-Weyl and tangent Lorentz groups one has

$$D_\alpha|_{\eta=0} = \partial_\alpha + \omega_\alpha, \quad D_a|_{\eta=0} = e_a{}^m(\xi)\partial_m + \psi_a{}^\mu(\xi)\partial_\mu + \omega_a, \tag{6}$$

where $e_a{}^m(\xi)$ and $\psi_a{}^\mu(\xi)$ are the two-dimensional graviton and gravitino fields.

Now we pass to the target superspace of the $D = 3$, $N = 2$ superstring. It is a $3|4$-dimensional superspace parametrized by [17] $Z^{\underline{M}} = (X^{\underline{m}}, \Theta^{\underline{\mu}}, \bar{\Theta}^{\underline{\mu}})$, where $\underline{m} = (0, 1, 2)$ and $\underline{\mu} = (1, 2)$. Note that the Grassmann variables are combined here into a *complex* doublet $\Theta^{\underline{\mu}}$ and its conjugate $\bar{\Theta}^{\underline{\mu}}$ (hence $N = 2$). Our formulation of the superstring will be of a σ model type [18]. In such a context one treats the target-space coordinates as world sheet superfields $Z^{\underline{M}}(Z^M)$. Then one can define the differential one-forms

$$E^{\underline{A}} = dZ^{\underline{M}} E_{\underline{M}}{}^{\underline{A}}(Z). \tag{7}$$

Here $E_{\underline{M}}{}^{\underline{A}}(Z)$ are the vierbeins of the target supergeometry. The flat $D = 3$ $N = 2$ superspace is characterized by the one-forms [20]

$$E^{\underline{a}} = dX^{\underline{a}} - id\Theta\gamma^{\underline{a}}\bar{\Theta} - id\bar{\Theta}\gamma^{\underline{a}}\Theta, \quad E^{\underline{\alpha}} = d\Theta^{\underline{\alpha}}. \tag{8}$$

These forms are invariant under target-space $D = 3$, $N = 2$ supersymmetry and with respect to the world sheet local symmetries. The pullbacks of these forms onto the world sheet are

$$E_A{}^{\underline{A}} = D_A Z^{\underline{M}} E_{\underline{M}}{}^{\underline{A}}(Z). \tag{9}$$

Acting on (9) with the covariant derivative D_B and performing graded antisymmetrization in A, B we obtain an important relation which involves the world sheet and target-superspace torsions:

$$D_A E_B{}^{\underline{C}} - (-)^{AB} D_B E_A{}^{\underline{C}} = T_{AB}{}^C E_C{}^{\underline{C}} - (-)^{A(B+\underline{B})} E_B{}^{\underline{B}} E_A{}^{\underline{A}} T_{\underline{AB}}{}^{\underline{C}}. \tag{10}$$

The explicit form of the flat target superspace torsion is

$$T_{\underline{\alpha\bar{\beta}}}{}^{\underline{c}} = T_{\underline{\bar{\alpha}\beta}}{}^{\underline{c}} = 2i(\gamma^{\underline{c}})_{\underline{\alpha\beta}}, \quad \text{the rest} = 0. \tag{11}$$

A characteristic feature of the superstring considered as a σ model is the presence of a Wess-Zumino term in the action. It is based on another target-superspace geometric object, the super-two-form $B_{\underline{MN}}(Z^{\underline{K}})$. In the flat case it is given by

$$B_{\underline{\mu n}} = \frac{i}{2}(\gamma_{\underline{n}}\Theta)_{\underline{\mu}}, \quad B_{\underline{\bar{\mu} n}} = \frac{i}{2}(\gamma_{\underline{n}}\bar{\Theta})_{\underline{\mu}},$$
$$B_{\underline{\mu\nu}} = -\tfrac{1}{2}(\gamma^{\underline{n}}\Theta)_{\underline{\mu}}(\gamma_{\underline{n}}\Theta)_{\underline{\nu}}, \quad B_{\underline{\bar{\mu}\bar{\nu}}} = -\tfrac{1}{2}(\gamma^{\underline{n}}\bar{\Theta})_{\underline{\mu}}(\gamma_{\underline{n}}\bar{\Theta})_{\underline{\nu}}, \tag{12}$$

the rest $= 0$.

Its field strength is a three-form:

$$H_{\underline{MNK}} = \partial_{[\underline{M}} B_{\underline{NK})}, \tag{13}$$

where $[\underline{MNK})$ means graded antisymmetrization. Using the $D = 3$ γ matrix identity

$$(\gamma^{\underline{m}})_{(\underline{\nu\lambda}}(\gamma_{\underline{m}})_{\underline{\gamma})\underline{\kappa}} = 0, \tag{14}$$

one can show that

$$H_{\underline{m\nu\lambda}} = H_{\underline{m\bar{\nu}\bar{\lambda}}} = i(\gamma_{\underline{m}})_{\underline{\nu\lambda}}, \quad \text{the rest} = 0. \tag{15}$$

In what follows we shall also need the expression for the three-form with tangent space indices,

$$H_{\underline{ABC}} = (-)^{(B+\underline{N})\underline{A}+(\underline{C}+\underline{K})(\underline{A}+\underline{B})} E_{\underline{C}}{}^{\underline{K}} E_{\underline{B}}{}^{\underline{N}} E_{\underline{A}}{}^{\underline{M}} H_{\underline{MNK}}. \tag{16}$$

Its projections are similar to those in (15):

$$H_{\underline{a\beta\gamma}} = H_{\underline{a\bar{\beta}\bar{\gamma}}} = i(\gamma_{\underline{a}})_{\underline{\beta\gamma}}, \quad \text{the rest} = 0. \tag{17}$$

Note that the part of the target-space geometry involving only the pullbacks (one-forms) and the torsion respects the automorphism group U(1) of $D = 3$, $N = 2$ supersymmetry. The latter rotates Θ by a phase factor. A peculiarity of the type II superstring is that its Wess-Zumino term violates this U(1) symmetry, as can be seen from (12). As shown in [21], this is the only way to have a closed three-form in a type II superspace. An interesting geometric interpretation of this fact has been given in [22].

III. THE $D = 3$, $N = 2$ SUPERPARTICLE

In this section we shall present a twistor formulation of the superparticle moving in $D = 3$, $N = 2$ superspace. It is the one-dimensional simplified version of the superstring. It shares some new features with the superstring and can thus serve as an introduction to the superstring. Moreover, as we shall see in Sec. V C, a specific solution to the superstring twistor constraints leads to a degenerate form of the superstring, which is just the superparticle.

In the traditional Brink-Schwarz formulation [23] of the $D = 3$, $N = 2$ superparticle one finds two κ symmetries which gauge away half of the target-space Grassmann coordinates. In a twistor formulation one expects to have two world line local supersymmetries. So we consider a super world line parametrized by τ, η^i, where $i = 1, 2$ is a doublet index of the SO(2) automorphism of the $N = 2$ supersymmetry algebra,

$$\{D_i, D_j\} = 2i\delta_{ij}\partial_\tau. \tag{18}$$

The action we propose for the $D = 3$, $N = 2$ superparticle is given by

$$S = \int d\tau d^2\eta P_{i\underline{a}}(D_i X^{\underline{a}} - iD_i\Theta\gamma^{\underline{a}}\bar{\Theta} - iD_i\bar{\Theta}\gamma^{\underline{a}}\Theta). \tag{19}$$

It contains the pullback $E_i{}^{\underline{a}}$ of the invariant one-form of target-space supersymmetry [cf. (8)] and a Lagrange multiplier. All superfields in (19) are unconstrained. As a kinematical restriction we require that the pullback $E_i{}^{\underline{\alpha}}$ defining the commuting spinor (twistor) variables be a nonvanishing matrix,

$$D_i\Theta^{\underline{\alpha}} \neq 0. \tag{20}$$

We note that this action is invariant under the $N = 2$ superconformal group. [24] Our aim is to study the component content of the above action and to show its equivalence to the Brink-Schwarz superparticle action [23]. Integrating over the world line Grassmann coordinates we get

$$\begin{aligned} S = \int d\tau [&D_1D_2P_{i\underline{a}}(D_iX^{\underline{a}} - iD_i\Theta\gamma^{\underline{a}}\bar{\Theta} - iD_i\bar{\Theta}\gamma^{\underline{a}}\Theta) + iD_2P_{1\underline{a}}(\partial_\tau X^{\underline{a}} - i\partial_\tau\Theta\gamma^{\underline{a}}\bar{\Theta} - i\partial_\tau\bar{\Theta}\gamma^{\underline{a}}\Theta - 2D_1\Theta\gamma^{\underline{a}}D_1\bar{\Theta}) \\ &-iD_1P_{2\underline{a}}(\partial_\tau X^{\underline{a}} - i\partial_\tau\Theta\gamma^{\underline{a}}\bar{\Theta} - i\partial_\tau\bar{\Theta}\gamma^{\underline{a}}\Theta - 2D_2\Theta\gamma^{\underline{a}}D_2\bar{\Theta}) \\ &+D_1P_{1\underline{a}}(D_1D_2X^{\underline{a}} - iD_1D_2\Theta\gamma^{\underline{a}}\bar{\Theta} - iD_1D_2\bar{\Theta}\gamma^{\underline{a}}\Theta + iD_1\Theta\gamma^{\underline{a}}D_2\bar{\Theta} + iD_1\bar{\Theta}\gamma^{\underline{a}}D_2\Theta) \\ &+D_2P_{2\underline{a}}(D_1D_2X^{\underline{a}} - iD_1D_2\Theta\gamma^{\underline{a}}\bar{\Theta} - iD_1D_2\bar{\Theta}\gamma^{\underline{a}}\Theta - iD_1\Theta\gamma^{\underline{a}}D_2\bar{\Theta} - iD_1\bar{\Theta}\gamma^{\underline{a}}D_2\Theta) \\ &-iP_{1\underline{a}}(\tfrac{1}{2}\partial_\tau D_2X^{\underline{a}} - i\partial_\tau D_2\Theta\gamma^{\underline{a}}\bar{\Theta} + i\partial_\tau\Theta\gamma^{\underline{a}}D_2\bar{\Theta} + 2D_1D_2\Theta\gamma^{\underline{a}}D_1\bar{\Theta} + \text{H.c.}) \\ &+iP_{2\underline{a}}(\tfrac{1}{2}\partial_\tau D_1X^{\underline{a}} - i\partial_\tau D_1\Theta\gamma^{\underline{a}}\bar{\Theta} + i\partial_\tau\Theta\gamma^{\underline{a}}D_2\bar{\Theta} - 2D_1D_2\Theta\gamma^{\underline{a}}D_2\bar{\Theta} + \text{H.c.})]_{\eta=0}. \end{aligned} \tag{21}$$

The variation with respect to the component $D_1D_2P_{i\underline{a}}$ produces the auxiliary field equations (we omit the subscript $\eta = 0$)

$$D_iX^{\underline{a}} - iD_i\Theta\gamma^{\underline{a}}\bar{\Theta} - iD_i\bar{\Theta}\gamma^{\underline{a}}\Theta = 0. \quad (22)$$

The variation with respect to the components $D_1P_{1\underline{a}}$ and $D_2P_{2\underline{a}}$ defines the auxiliary component $D_1D_2X^{\underline{a}}$ and also leads to one of the twistor constraints

$$D_1\Theta\gamma^{\underline{a}}D_2\bar{\Theta} + D_1\bar{\Theta}\gamma^{\underline{a}}D_2\Theta = 0. \quad (23)$$

Further, varying with respect to the sum $D_2P_{1\underline{a}} + D_1P_{2\underline{a}}$ we get the other twistor constraint

$$D_1\Theta\gamma^{\underline{a}}D_1\bar{\Theta} - D_2\Theta\gamma^{\underline{a}}D_2\bar{\Theta} = 0. \quad (24)$$

The difference $i(D_2P_{1\underline{a}} - D_1P_{2\underline{a}})$ is identified with the particle's momentum $p_{\underline{a}}$.

Finally, the last two terms in the component action [(21) can be simplified by using the auxiliary field equations (22)] and the resulting action take the form

$$S = \int d\tau[p_{\underline{a}}(\partial_\tau X^a - i\partial_\tau\Theta\gamma^{\underline{a}}\bar{\Theta} - i\partial_\tau\bar{\Theta}\gamma^{\underline{a}}\Theta - D_i\Theta\gamma^{\underline{a}}D_i\bar{\Theta})$$
$$+2P_{1\underline{a}}(D_2\Theta\gamma^{\underline{a}}\partial_\tau\bar{\Theta} + D_2\bar{\Theta}\gamma^{\underline{a}}\partial_\tau\Theta - iD_1\Theta\gamma^{\underline{a}}D_1D_2\bar{\Theta} - iD_1\bar{\Theta}\gamma^{\underline{a}}D_1D_2\Theta)$$
$$-2P_{2\underline{a}}(D_1\Theta\gamma^{\underline{a}}\partial_\tau\bar{\Theta} + D_1\bar{\Theta}\gamma^{\underline{a}}\partial_\tau\Theta + iD_2\Theta\gamma^{\underline{a}}D_1D_2\bar{\Theta} + iD_2\bar{\Theta}\gamma^{\underline{a}}D_1D_2\Theta)]. \quad (25)$$

Here the twistor variables $D_i\Theta^{\underline{\alpha}}$ and $D_i\bar{\Theta}^{\underline{\alpha}}$ are restricted by the constraints (23) and (24). Our next step is to solve these constraints. Using the explicit representation for the $D = 3$ γ matrices in the light-cone basis

$$\gamma^{++} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \; \gamma^{--} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \; \gamma^{+-} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (26)$$

we easily find the general solution to the twistor constraints (23) and (24) under the assumption (20):

$$D_1\Theta^{\underline{\alpha}} = \lambda^{\underline{\alpha}}, \quad D_2\Theta^{\underline{\alpha}} = is\lambda^{\underline{\alpha}}, \quad s = \pm 1, \quad (27)$$

where $\lambda^{\underline{\alpha}}$ is an arbitrary *complex nonvanishing* spinor.

Substituting this solution into the action (25) we get

$$S = \int d\tau \; [p_{\underline{a}}(\partial_\tau X^a - i\partial_\tau\Theta\gamma^{\underline{a}}\bar{\Theta} - i\partial_\tau\bar{\Theta}\gamma^{\underline{a}}\Theta - 2\lambda\gamma^{\underline{a}}\bar{\lambda})$$
$$-2P_{\underline{a}}\bar{\chi}\gamma^{\underline{a}}\lambda - 2\bar{P}_{\underline{a}}\chi\gamma^{\underline{a}}\bar{\lambda}], \quad (28)$$

where

$$P_{\underline{a}} = P_{2\underline{a}} - isP_{1\underline{a}}, \quad \chi^{\underline{\alpha}} \equiv \partial_\tau\Theta^{\underline{\alpha}} + sD_1D_2\Theta^{\underline{\alpha}}.$$

Now we remark that the trilinear term in this action is purely auxiliary. Indeed, the variation with respect to $P_{\underline{a}}$ leads to the equation

$$\bar{\chi}\gamma^{\underline{a}}\lambda = 0. \quad (29)$$

Since the commuting spinor λ is nonvanishing, the only solution to this equation is $\bar{\chi} = 0$. The variation with respect to $\bar{\chi}$ in (28) implies

$$P_{\underline{a}}(\gamma^{\underline{a}}\lambda)_{\underline{\alpha}} = 0. \quad (30)$$

The general solution of this twistor equation is

$$P_{\underline{a}} = \rho(\tau)\lambda\gamma^{\underline{a}}\lambda, \quad (31)$$

with an arbitrary complex function $\rho(\tau)$. However, the right-hand side of (31) is an obvious gauge invariance of the action (28) due to the γ matrix identity (14). Hence the two last terms in (28) vanish.

Finally, we vary with respect to the twistor variable λ:

$$p_{\underline{a}}(\gamma^{\underline{a}}\bar{\lambda})_{\underline{\alpha}} = 0 \;\Rightarrow\; p^{\underline{a}} = \bar{\mu}(\tau)\bar{\lambda}\gamma^{\underline{a}}\bar{\lambda}. \quad (32)$$

The fact that the particle momentum is real and nonvanishing then implies

$$\bar{\mu}(\tau)\bar{\lambda}\gamma^{\underline{a}}\bar{\lambda} = \mu(\tau)\lambda\gamma^{\underline{a}}\lambda. \quad (33)$$

The solution to this equation is

$$\bar{\lambda}^{\underline{\alpha}} = e^{i\phi}\lambda^{\underline{\alpha}}, \quad \bar{\mu} = e^{-2i\phi}\mu. \quad (34)$$

It implies that on shell the complex spinor $\lambda^{\underline{\alpha}}$ becomes *real* modulo a phase. The arbitrary phase ϕ corresponds to the SO(2) subgroup of the superconformal invariance of the action (19) and can be completely gauged away. Then we can replace the twistor combination $\lambda\gamma^{\underline{a}}\bar{\lambda}$ in (28) by $|\mu|^{-1}p^{\underline{a}}$ and obtain the standard Brink-Schwarz superparticle action.

The conclusion is that the action (19) is equivalent to the Brink-Schwarz action upon eliminating the auxiliary fields (including the twistor variables) and fixing certain gauges. An unusual feature compared to the twistor superparticle of [1] is the presence of two twistors (the real and imaginary parts of λ) instead of only one. As we shall see in Sec. V C, it is this form of the superparticle which appears as a degenerate case of the nonheterotic superstring.

IV. THE $D = 3$, $N = 2$ SUPERSTRING ACTION

The twistor superstring action consists of two terms

$$S = S_{\rm GD} + S_{\rm WZ}. \quad (35)$$

The first one resembles very much the superparticle action of Sec. III:

$$S_{\rm GD} = \int d^2\xi d^2\eta P_{\underline{a}}{}^{\alpha}E_{\alpha}{}^{\underline{a}}. \quad (36)$$

The variation with respect to the Lagrange multiplier $P_{\underline{a}}{}^{\alpha}$ leads to the following geometrodynamical [25] constraint on the target-space coordinates treated as world sheet superfields $Z^{\underline{M}}(Z^M)$:

$$E_{\alpha}{}^{\underline{a}} = 0. \quad (37)$$

The meaning of Eq. (37) is that the pullback of the target-superspace vector vierbein onto the spinor directions of the world sheet must vanish. In other words, if one considers the world sheet as a (2|2)-dimensional hypersurface embedded in the 3|4 target superspace, then there should be no projections of the target-space even directions onto the world sheet odd ones. Using (10), (3), and (11), we obtain the important consequence

$$2(\gamma^c C)_{\alpha\beta} E_c{}^{\underline{a}} = E_\alpha \gamma^{\underline{a}} \bar{E}_\beta + \bar{E}_\alpha \gamma^{\underline{a}} E_\beta. \quad (38)$$

In two-dimensional light-cone notation (38) reads

$$E_+ \gamma^{\underline{a}} \bar{E}_+ = E_{++}{}^{\underline{a}}, \quad (39)$$

$$E_- \gamma^{\underline{a}} \bar{E}_- = E_{--}{}^{\underline{a}}, \quad (40)$$

$$E_+ \gamma^{\underline{a}} \bar{E}_- + E_- \gamma^{\underline{a}} \bar{E}_+ = 0. \quad (41)$$

These equations are constraints on the superfields $Z^{\underline{M}}(Z^M)$. In particular, they imply algebraic restrictions on the first components in the η expansion of the spinor-spinor pullbacks:

$$\mathcal{E}_\alpha{}^{\underline{\alpha}} \equiv E_\alpha{}^{\underline{\alpha}}|_{\eta=0}. \quad (42)$$

These are commuting spinors (with respect to the two- and three-dimensional Lorentz groups), which we shall call "twistor variables." In Sec. V we shall show that as a result of these restrictions the first components in the η expansion of the vectors $E_{\pm\pm}{}^{\underline{a}}$ defined by (39) and (40) are lightlike,

$$\mathcal{E}_{\pm\pm}{}^{\underline{a}} = E_{\pm\pm}{}^{\underline{a}}|_{\eta=0}: \quad (\mathcal{E}_{++}{}^{\underline{a}})^2 = (\mathcal{E}_{--}{}^{\underline{a}})^2 = 0. \quad (43)$$

In other words, one of the main purposes of the geometrodynamical constraint (37) is to provide a solution to the Virasoro constraints of the superstring in terms of the twistor variables (42).

The presence of the geometrodynamical term (36) makes it possible to introduce the leading term in the superstring action in the form of a generalized Wess-Zumino term. The latter requires certain consistency conditions, and we shall show that they are satisfied as a consequence of the geometrodynamical constraint (37).

The Wess-Zumino term has the form

$$S_{\mathrm{WZ}} = \int d^2\xi d^2\eta\, (-)^{MN+M+N} P^{MN} \times (B_{MN} - E_M{}^a E_N{}^b \epsilon_{ab} A - \partial_M Q_N). \quad (44)$$

Here P^{MN} is a graded-antisymmetric Lagrange multiplier, Q_M is another Lagrange multiplier, $B_{MN} = (-)^{(N+\underline{N})M} E_N{}^{\underline{N}} E_M{}^{\underline{M}} B_{\underline{MN}}$ is the pullback of the target-superspace two-form, and $E_M{}^a$ are world sheet vierbeins. The quantity A is to be found from the consistency conditions below. Varying with respect to P^{MN} leads to the equation of motion

$$B_{MN} - E_M{}^a E_N{}^b \epsilon_{ab} A = \partial_{[M} Q_{N)}. \quad (45)$$

The meaning of this equation is that the pullback of the two-form becomes almost "pure gauge" on shell. The consistency condition following from (45) is that the graded curl of the left-hand side of Eq. (45) must vanish. Thus we obtain

$$\begin{aligned} H_{MNK} &= \partial_{[K} B_{MN)} \\ &= 2\partial_{[K} E_M{}^a E_{N)}{}^b \epsilon_{ab} A \\ &\quad + (-)^{K(M+N)} E_{[M}{}^a E_N{}^b \epsilon_{ab} \partial_{K)} A, \end{aligned} \quad (46)$$

where H_{MNK} is the pullback of the three-form. It is convenient to pass to tangent space indices in Eq. (46), i.e., to multiply it by two-dimensional vierbeins. Using the expression for the world sheet torsion,

$$T_{AB}{}^C = -(-)^{A(B+N)} E_B{}^N E_A{}^M \partial_M E_N{}^C + \omega_{AB}{}^C - (-)^{AB}(A \leftrightarrow B), \quad (47)$$

one can rewrite (46) as:

$$H_{ABC} = (-T_{[BA}{}^d + 2\omega_{[BA}{}^d)\delta^e_{C)}\epsilon_{de} A + \delta^d_{[B}\delta^e_C \epsilon_{de} D_{A)} A\,, \quad (48)$$

where H_{ABC} is the pullback of the three-form,

$$H_{ABC} = (-)^{(B+\underline{B})A + (C+\underline{C})(A+B)} E_C{}^{\underline{C}} E_B{}^{\underline{B}} E_A{}^{\underline{A}} H_{\underline{ABC}}\,. \quad (49)$$

Let us study the different projections of the condition (48). First we consider the projections of its left-hand side. If we take all the indices spinor and use the geometrodynamical equation (37) [26], we find

$$H_{\alpha\beta\gamma} = E_\alpha{}^{\underline{\alpha}} E_\beta{}^{\underline{\beta}} E_\gamma{}^{\underline{\gamma}} H_{\underline{\alpha\beta\gamma}} + 3 E_\alpha{}^{\underline{\alpha}} E_\beta{}^{\underline{\beta}} E_\gamma{}^{\bar{\underline{\gamma}}} H_{\underline{\alpha\beta\bar{\gamma}}} + \text{c.c.} = 0 \quad (50)$$

as a consequence of (17). In accordance with this the right-hand side of Eq. (48) vanishes identically for this choice of the indices.

Further, let us take one vector and two spinor indices in (48). Substituting (17) into (49) and using (39)–(41) and (14), we find

$$H_{++|+|+} = i(E_+\gamma_{\underline{a}}E_+)E_{++}{}^{\underline{a}} + \text{c.c.} = i(E_+\gamma_{\underline{a}}E_+)(E_+\gamma^{\underline{a}}\bar{E}_+) + \text{c.c.} = 0, \quad (51)$$

$$\begin{aligned} H_{++|+|-} &= i(E_-\gamma_{\underline{a}}E_+)E_{++}{}^{\underline{a}} + \text{c.c.} = i(E_-\gamma_{\underline{a}}E_+)(E_+\gamma^{\underline{a}}\bar{E}_+) + \text{c.c.} \\ &= -\frac{i}{2}(E_+\gamma_{\underline{a}}E_+)(E_-\gamma^{\underline{a}}\bar{E}_+) + \text{c.c.} = \frac{i}{2}(E_+\gamma_{\underline{a}}E_+)(E_+\gamma^{\underline{a}}\bar{E}_-) + \text{c.c.} = 0. \end{aligned} \quad (52)$$

Similarly, we obtain

$$H_{--|-|-} = H_{--|+|-} = 0. \quad (53)$$

The only nonvanishing pullbacks with one vector and two spinor indices are

$$H_{--|+|+} = i(E_+\gamma_{\underline{a}}E_+)E_{--}{}^{\underline{a}} + \text{c.c.} = i(E_+\gamma_{\underline{a}}E_+)(E_-\gamma^{\underline{a}}\bar{E}_-) + \text{c.c.}$$
$$= -2i(E_+\gamma_{\underline{a}}E_-)(E_+\gamma^{\underline{a}}\bar{E}_-) + \text{c.c.}, \tag{54}$$

$$H_{++|-|-} = i(E_-\gamma_{\underline{a}}E_-)E_{++}{}^{\underline{a}} + \text{c.c.} = i(E_-\gamma_{\underline{a}}E_-)(E_+\gamma^{\underline{a}}\bar{E}_+) + \text{c.c.}$$
$$= -2i(E_-\gamma_{\underline{a}}E_+)(E_-\gamma^{\underline{a}}\bar{E}_+) + \text{c.c.} = -H_{--|+|+}. \tag{55}$$

If we compare these expressions for the pullbacks of the three-form with the right-hand side of Eq. (48) and use (3), we see complete agreement, provided that the quantity A is given by

$$A = \tfrac{1}{8}\,(E\gamma_a\gamma_{\underline{a}}E + \text{c.c.})\,\epsilon^{ab}E_b{}^{\underline{a}}. \tag{56}$$

The remaining possibility in Eq. (48) is to have two vector and one spinor indexes. This does not lead to new relations, since the component $H_{ab\gamma}$ of the pullback of the three-form is determined by the Bianchi identity

$$D_{[a}H_{\alpha\beta\gamma)} + T_{[a\beta}{}^{E}H_{E\gamma\delta)} = 0 \tag{57}$$

and thus automatically agrees with the right-hand side of (48).

This concludes the verification of the consistency of our Wess-Zumino term. We have seen that the pullback of the two-form itself is not closed ($dB \neq 0$), but this can be corrected by an appropriately chosen term with A given in (56).

We note that the action term (44) is invariant under the superconformal transformations (4). Indeed, we see that the vierbein factor in front of A in (44) transforms as a density of weight +2. The twistor vector $E\gamma^a\gamma_{\underline{a}}E$ and its conjugate are densities of weight −1. As to the vectors $E_a{}^{\underline{a}}$, their transformation laws are less trivial. Take, for instance,

$$\delta E_{++}{}^{\underline{a}} = -\Lambda E_{++}{}^{\underline{a}} + iD_+\Lambda E_+{}^{\underline{a}}. \tag{58}$$

The first term in (58) provides the weight −1 needed to compensate the other two factors. The second term is proportional to $E_\alpha{}^{\underline{a}}$, which vanishes according to (37). In other words, this second term can be compensated by a suitable transformation of the Lagrange multiplier $P_{\underline{a}}{}^{\alpha}$ in the action term (36). As to the term (36) itself, its super-Weyl invariance is assured by ascribing a certain weight to the Langrange multiplier.

Another remark concerns the U(1) automorphism of $D = 3$, $N = 2$ supersymmetry. The term S_{GD} of our action respects this symmetry, whereas S_{WZ} does not.

V. COMPONENT ACTION

In this section we shall obtain the component expression of the two terms (36) and (44) of the superstring action. We shall show that the Wess-Zumino term (44) is reduced to the usual superstring action of Green-Schwarz type. The geometrodynamical term (36) will turn out to be purely auxiliary.

A. Wess-Zumino term

We begin with the Wess-Zumino term (44). The variation with respect to the Lagrange multiplier Q_M produces the equation

$$\partial_N P^{NM} = 0. \tag{59}$$

Its general solution is

$$P^{MN} = \partial_K \Sigma^{MNK} + \delta^M_k \delta^N_l \epsilon^{kl} T\,\eta^2. \tag{60}$$

It consists of two parts. The first one has the form of a pure gauge transformation with graded antisymmetric parameter Σ^{MNK}. We are sure that this is an invariance of the action because the consistency condition (48) holds [27]. It is easy to see that almost all of the components of P^{MN} can be gauged away by a Σ transformation without using any parameters with space-times derivatives. Hence, one is allowed to use such a gauge in the action. The only remaining nontrivial part of P^{MN} is the second term in the solution (60). It contains a coefficient T, which is restricted by (59) to be an arbitrary constant:

$$\partial_{++}T = \partial_{--}T = 0. \tag{61}$$

Inserting the solution (60) back into the action term (44) and doing the η integration with the help of the Grassmann δ function η^2 present in (60) we obtain

$$S_{\text{WZ}} = \int d^2\xi\; T\;\epsilon^{mn}\left(B_{mn} - \frac{1}{2}E_m{}^{a}E_n{}^{b}\epsilon_{ab}\,A\right)_{\eta=0}. \tag{62}$$

To evaluate the various objects at $\eta = 0$ we use Eq. (6). Thus, $E_m{}^a|_0 = e_m{}^a(\xi)$, and so the factor in front of A becomes simply $\det e$. The quantity A from (56) takes the form

$$A|_{\eta=0} = \tfrac{1}{8}\,(\mathcal{E}\gamma_a\gamma_{\underline{a}}\mathcal{E} + \text{c.c.})\,\epsilon^{ab}\mathcal{E}_b{}^{\underline{a}}, \tag{63}$$

where

$$\mathcal{E}_\alpha{}^{\underline{\alpha}} = E_\alpha{}^{\underline{\alpha}}|_0,$$
$$\mathcal{E}_a{}^{\underline{a}} = E_a{}^{\underline{a}}|_0 = \left[(e_a{}^m\partial_m + \psi_a{}^\mu D_\mu)Z^{\underline{M}}E_{\underline{M}}{}^{\underline{a}}\right]_0$$
$$= \left[e_a{}^m\partial_m Z^{\underline{M}}E_{\underline{M}}{}^{\underline{a}}\right]_0. \tag{64}$$

Note that the only place in (62) where the gravitino field $\psi_a{}^\mu$ could occur is in (64), but even there it dropped out as a consequence of the geometrodynamical equation (37).

At this point we are going to use the solution to the $\eta = 0$ part of the constraint (38). This is the constraint on the twistor variables $\mathcal{E}_\alpha{}^{\underline{\alpha}}$ and appears as a component of the geometrodynamical term (36) (see Sec. V B). As explained in the Appendix, the twistor constraint has two solutions: a regular and a singular one. The regular solution is obtained under the assumption that the twistor matrix $\mathcal{E}_\alpha{}^{\underline{\alpha}}$ is nondegenerate,

$$\det \| \mathcal{E}_{\alpha}{}^{\underline{\alpha}} \| \neq 0, \tag{65}$$

and has the form

$$\mathcal{E}_{\alpha}{}^{\underline{\alpha}} = e^{i\phi} \begin{pmatrix} \lambda_+{}^{\underline{\alpha}} \\ i\lambda_-{}^{\underline{\alpha}} \end{pmatrix}. \tag{66}$$

Here $\phi(\xi)$ is an arbitrary phase. The spinors λ in (66) are real,

$$\bar{\lambda}_+{}^{\underline{\alpha}} = \lambda_+{}^{\underline{\alpha}}, \quad \bar{\lambda}_-{}^{\underline{\alpha}} = \lambda_-{}^{\underline{\alpha}}, \tag{67}$$

and satisfy two further relations

$$\lambda_+\gamma^{\underline{a}}\lambda_+ = \mathcal{E}_{++}{}^{\underline{a}}, \quad \lambda_-\gamma^{\underline{a}}\lambda_- = \mathcal{E}_{--}{}^{\underline{a}}. \tag{68}$$

These equations give expressions for the vectors $\mathcal{E}_{\pm\pm}{}^{\underline{a}}$ in terms of the twistor variables $E_{\alpha}{}^{\underline{\alpha}}$. Using (14), one sees that the vectors in (68) are lightlike:

$$(\mathcal{E}_{++}{}^{\underline{a}})^2 = (\mathcal{E}_{--}{}^{\underline{a}})^2 = 0. \tag{69}$$

Thus, we see that the lowest-order component of the twistor constraint (38) has reduced the 2×2 complex twistor matrix to two independent real twistors $\lambda_{\pm}{}^{\underline{\alpha}}$, in terms of which the superstring Virasoro constraints (69) are solved.

In this subsection we shall restrict ourselves to the regular solution (66). The case of the singular one (which corresponds to the case of a string collapsed into a particle) will be treated in Sec. V C. So, putting the above expressions in (63) and then in (62), we obtain

$$S_{\rm WZ} = T \int d^2\xi \left(\epsilon^{mn} B_{mn} - \tfrac{1}{2} \det e \; \cos 2\phi \; \mathcal{E}_{++}{}^{\underline{a}} \mathcal{E}_{--\underline{a}} \right). \tag{70}$$

We see that this expression is almost identical with the usual Green-Schwarz type II superstring action, if the constant T is interpreted as the string tension [28]. The only difference is in the factor containing the auxiliary scalar field $\phi(\xi)$. In Sec. V B we shall see that ϕ does not appear in the geometrodynamical term (36) of the superspace superstring action. This is not surprising, since ϕ appears as the parameter of a U(1) transformation, and $S_{\rm GD}$ respects this symmetry. Therefore we can vary with respect to ϕ in (70) and obtain the field equation

$$\sin 2\phi \; \mathcal{E}_{++}{}^{\underline{a}} \mathcal{E}_{--\underline{a}} = 0. \tag{71}$$

Using the twistor expressions (68), it is not hard to show that

$$\mathcal{E}_{++}{}^{\underline{a}} \mathcal{E}_{--\underline{a}} = (\lambda_+\gamma^{\underline{a}}\lambda_+)(\lambda_-\gamma_{\underline{a}}\lambda_-) = (\det \| \lambda_{\alpha}{}^{\underline{\alpha}} \|)^2$$
$$= -e^{-4i\phi}(\det \| \mathcal{E}_{\alpha}{}^{\underline{\alpha}} \|)^2. \tag{72}$$

Note that the first term in (72) is in fact proportional to the determinant of the induced two-dimensional metric of the superstring. Since in this subsection we assume that the twistor matrix is nonsingular [see (65)], we conclude that the solution to (71) is

$$\phi = 0. \tag{73}$$

Putting this solution back into the action (70) we obtain

$$S_{\rm WZ} = T \int d^2\xi \left(\epsilon^{mn} B_{mn} - \tfrac{1}{2} \det e \; \mathcal{E}_{++}{}^{\underline{a}} \mathcal{E}_{--\underline{a}} \right). \tag{74}$$

This is the action of a type II $D = 3$ Green-Schwarz superstring. In it the twistor variables are not present any more; they have been eliminated through the algebraic relations (68).

We have seen that in the process of derivation of the action (74) the two-dimensional gravitino field dropped out [see (64)], although in the original superfield form (44) it was needed to maintain the invariance with respect to the local supersymmetry transformations on the world sheet. Despite the absence of the gravitino, this local supersymmetry is still present in the component term (74), but now in the nonmanifest form of κ symmetry. In order to see how local supersymmetry is transformed into κ symmetry on shell let us consider the supersymmetry variation of the target-superspace coordinates $z^{\underline{M}} = Z^{\underline{M}}|_{\eta=0}$

$$\delta z^{\underline{A}} \equiv \delta z^{\underline{M}} E_{\underline{M}}{}^{\underline{A}} = \epsilon^{\alpha} D_{\alpha} Z^{\underline{M}} E_{\underline{M}}{}^{\underline{A}}|_{\eta=0} = \epsilon^{\alpha} \mathcal{E}_{\alpha}{}^{\underline{A}}. \tag{75}$$

From the geometrodynamical equation (37) follows

$$\delta z^{\underline{a}} = 0, \quad \delta z^{\underline{\alpha}} = \epsilon^{\alpha} \mathcal{E}_{\alpha}{}^{\underline{\alpha}}. \tag{76}$$

Let us now introduce an anticommuting parameter $\kappa_{\underline{\alpha}}{}^{a}$ carrying a $D = 2$ vector and a $D = 3$ spinor index by substituting

$$\epsilon_{\alpha} = (\gamma_a)_{\alpha}{}^{\beta} \mathcal{E}_{\beta}{}^{\underline{\beta}} \kappa_{\underline{\beta}}{}^{a}. \tag{77}$$

Putting this in (76) and using the solution to the constraints on the twistor matrix, Eq. (73) and the Fierz identity for the three-dimensional γ matrices, we obtain

$$\delta z^{\underline{\alpha}} = (\gamma_{\underline{a}})^{\underline{\alpha\beta}} (\mathcal{E}_{++}{}^{\underline{a}} \kappa_{\underline{\beta}}{}^{++} - \mathcal{E}_{--}{}^{\underline{a}} \kappa_{\underline{\beta}}{}^{--}). \tag{78}$$

Equation (78) coincides with the κ symmetry transformations of the type II superstring [12].

Note an interesting feature of the transition from the action term (70) to the final form (74). The former is not invariant under the U(1) automorphism of $D = 3$, $N = 2$ supersymmetry. In the first term in (70) this is due to the U(1) noninvariant two-form. In the second term in (70) the only object which breaks U(1) is the phase ϕ. Indeed, looking at (66), one sees that ϕ is shifted by the U(1) transformation of the index $\underline{\alpha}$. However, once this field has been eliminated from the action, one obtains the peculiar mixture of a noninvariant and an invariant term in (74), characteristic for the type II superstring.

B. Geometrodynamical term

In the previous subsection we saw that the usual superstring action is essentially contained in the Wess-Zumino term. Here we shall show that the role of the geometrodynamical term (36) is purely auxiliary, i.e., that it only leads to algebraic constraints on the component fields. Among them are the twistor constraints reducing the twistor matrix to the two lightlike vectors from the Virasoro constraints. Another of these constraints will allow us to express the two-dimensional gravitino field in terms of the derivatives of the Grassmann coordinates $\theta^{\underline{\alpha}}$ of the target superspace. Other equations will put the Lagrange multipliers $P_{\underline{a}}{}^{\alpha}$ to zero on shell. We shall also show that the scalar field ϕ appearing in $S_{\rm WZ}$ is not present in $S_{\rm GD}$, and so the derivation of its field equation from $S_{\rm WZ}$ in Sec. V A was correct. Thus the superstring action (35) will be reduced to the Green-Schwarz one (74).

In two-dimensional light-cone notation the term $S_{\rm GD}$ becomes [see (8)]

$$\begin{aligned} S_{\rm GD} &= \int d^2\xi D_+D_- \left[P_{-\underline{a}}(D_+X^{\underline{a}} - iD_+\Theta\gamma^{\underline{a}}\bar\Theta) + \text{c.c.} + (+ \leftrightarrow -)\right]_{\eta=0} \\ &= \int d^2\xi \,[D_+D_-P_{-\underline{a}}(D_+X^{\underline{a}} - iD_+\Theta\gamma^{\underline{a}}\bar\Theta) - D_+P_{-\underline{a}}(D_-D_+X^{\underline{a}} - iD_-D_+\Theta\gamma^{\underline{a}}\bar\Theta - iD_+\Theta\gamma^{\underline{a}}D_-\bar\Theta) \\ &\quad + D_-P_{-\underline{a}}(iD_{++}X^{\underline{a}} + D_{++}\Theta\gamma^{\underline{a}}\bar\Theta - iD_+\Theta\gamma^{\underline{a}}D_+\bar\Theta) \\ &\quad + P_{-\underline{a}}(D_+D_-D_+X^{\underline{a}} - iD_+D_-D_+\Theta\gamma^{\underline{a}}\bar\Theta \\ &\qquad + D_{++}\Theta\gamma^{\underline{a}}D_-\bar\Theta + 2iD_-D_+\Theta\gamma^{\underline{a}}D_+\bar\Theta) + \text{c.c.} - (+ \leftrightarrow -)]_{\eta=0}. \end{aligned} \tag{79}$$

The variation with respect to the components $D_+D_-P_{-\underline{a}}$ and $D_+D_-P_{+\underline{a}}$ gives two equations for the auxiliary odd components of the superfield $X^{\underline{a}}$:

$$D_\pm X^{\underline{a}} = iD_\pm\Theta\gamma^{\underline{a}}\bar\Theta + iD_\pm\bar\Theta\gamma^{\underline{a}}\Theta \tag{80}$$

(from here on we shall drop the indication $\eta = 0$). The variation with respect to the components $D_+P_{-\underline{a}}$ and $D_-P_{+\underline{a}}$ gives equations for the auxiliary even component $D_-D_+X^{\underline{a}}$ and also leads to the twistor constraint

$$D_+\Theta\gamma^{\underline{a}}D_-\bar\Theta + D_+\bar\Theta\gamma^{\underline{a}}D_-\Theta = 0. \tag{81}$$

This is just the lowest-order component of the constraint (41). The other two constraints, the $\eta = 0$ components of (39) and (40), follow from the terms with $D_-P_{-\underline{a}}$ and $D_+P_{+\underline{a}}$. This set of constraints was discussed in Sec. V A. Postponing once again the investigation of the singular solution of the constraint until Sec. V C, we consider only the regular one (66):

$$D_+\Theta^{\underline{\alpha}} = e^{i\phi}\lambda_+{}^{\underline{\alpha}}, \quad D_-\Theta^{\underline{\alpha}} = ie^{i\phi}\lambda_-{}^{\underline{\alpha}}. \tag{82}$$

Next we shall simplify the term with $P_{-\underline{a}}$ in (79). Using (80) and the anticommutation relations (1) we find that the first two terms after $P_{-\underline{a}}$ equal the third term. Further, the covariant derivative D_{++} in this third term can be written out in detail according to (6):

$$\begin{aligned} &D_{++}\Theta\gamma^{\underline{a}}D_-\bar\Theta + D_{++}\bar\Theta\gamma^{\underline{a}}D_-\Theta \\ &\quad = ie_{++}{}^m(-e^{-i\phi}\partial_m\Theta + e^{i\phi}\partial_m\bar\Theta)\gamma^{\underline{a}}\lambda_- \\ &\quad +2\psi_{++}{}^-\lambda_-\gamma^{\underline{a}}\lambda_-, \end{aligned} \tag{83}$$

where we have used (81) and (82). Note that the covariant derivative D_{++} in the $D_-P_{-\underline{a}}$ term in (79) does not contain the gravitino field, as a consequence of (80) [see also (64)].

Now we are going to put all this back into the action term (79). The purely auxiliary terms drop out and $S_{\rm GD}$ is reduced to

$$\begin{aligned} S_{\rm GD} = \int d^2\xi \,\{&P_{--\underline{a}}(\mathcal{E}_{++}{}^{\underline{a}} - 2\lambda_+\gamma^{\underline{a}}\lambda_+) \\ &+2P_{-\underline{a}}[i\chi\gamma^{\underline{a}}\lambda_+ + ie_{++}{}^m(-e^{-i\phi}\partial_m\Theta + e^{i\phi}\partial_m\bar\Theta)\gamma^{\underline{a}}\lambda_- + 2\psi_{++}{}^-\lambda_-\gamma^{\underline{a}}\lambda_-] - (+ \leftrightarrow -)\}, \end{aligned} \tag{84}$$

where $\mathcal{E}_{++}{}^{\underline{a}}$ was defined in (64) and we have introduced the notation

$$iD_-P_{-\underline{a}} = P_{--\underline{a}}, \quad \chi = e^{-i\phi}D_-D_+\Theta + e^{i\phi}D_-D_+\bar\Theta \tag{85}$$

[and similarly in the $(+ \leftrightarrow -)$ sector]. The field equation for χ is a typical twistor equation,

$$P_{-\underline{a}}(\gamma^{\underline{a}}\lambda_+)_{\underline{\alpha}} = 0, \tag{86}$$

which has the general solution

$$P_{-\underline{a}} = P_{-3}(\lambda_+\gamma^{\underline{a}}\lambda_+) \tag{87}$$

with an arbitrary odd scalar field $P_{-3}(\xi)$. Further, the gravitino field $\psi_{++}{}^-$ appears only in (84) [see (70)], and so its field equation is

$$P_{-\underline{a}}(\lambda_-\gamma^{\underline{a}}\lambda_-) = P_{-3}(\lambda_+\gamma_{\underline{a}}\lambda_+)(\lambda_-\gamma^{\underline{a}}\lambda_-) = 0. \tag{88}$$

As explained in (72), under the current assumption of nonsingularity of the twistor matrix the twistor factor in (88) is nonvanishing, and so we conclude

$$P_{-3} = 0 \;\Rightarrow\; P_{-\underline{a}} = 0. \tag{89}$$

Before inserting (89) back into (84) and thus eliminating the $P_{-\underline{a}}$ term from the action, we have to study the field equation for $P_{-\underline{a}}$ itself:

$$\begin{aligned} &i\chi\gamma^{\underline{a}}\lambda_+ + ie_{++}{}^m(-e^{-i\phi}\partial_m\Theta + e^{i\phi}\partial_m\bar\Theta)\gamma^{\underline{a}}\lambda_- \\ &\qquad + 2\psi_{++}{}^-\lambda_-\gamma^{\underline{a}}\lambda_- = 0. \end{aligned} \tag{90}$$

These are three equations (as many as the projections of the vector index $\underline{a}$). Two of them can be used to solve for the auxiliary field $\chi^{\underline{\alpha}}$ (because here we assume that the matrix $\lambda_{\pm}{}^{\underline{\alpha}}$ is invertible). The third one enables us to solve for the gravitino field $\psi_{++}{}^{-}$ [to this end one multiplies Eq. (90) by $\lambda_+\gamma_{\underline{a}}\lambda_+$ and uses the nonsingularity of the twistor factor $(\lambda_+\gamma_{\underline{a}}\lambda_+)(\lambda_-\gamma^{\underline{a}}\lambda_-)$]. Thus we see that the gravitino field is an *auxiliary* field. It is expressed in terms of the derivative $e_{++}{}^m\partial_m\theta$ (where $\theta = \Theta|_0$). This is possible since θ transforms inhomogeneously under the world sheet local supersymmetry, $\delta\theta^{\underline{\alpha}} = \epsilon^{\alpha}\lambda_{\alpha}{}^{\underline{\alpha}}$ [see (76)].

So far we have shown that the term with $P_{-\underline{a}}$ in (84) is purely auxiliary and drops out of the action. Now we shall show that the term with $P_{--\underline{a}}$ vanishes on shell as well. First we shall vary with respect to the twistor field λ_+. It appears only once [we have already put $P_{-\underline{a}} = 0$ and in the Wess-Zumino term (84) we have eliminated the twistors in favor of the vectors $\mathcal{E}_{\pm\pm}{}^{\underline{a}}$], and so we get an equation similar to (86):

$$P_{--\underline{a}}(\gamma^{\underline{a}}\lambda_+)_{\underline{\alpha}} = 0 \;\Rightarrow\; P_{--\underline{a}} = P_{-4}(\lambda_+\gamma_{\underline{a}}\lambda_+). \tag{91}$$

Further, the variation with respect to $P_{--\underline{a}}$ gives

$$\mathcal{E}_{++}{}^{\underline{a}} = 2\lambda_+\gamma^{\underline{a}}\lambda_+ \,. \tag{92}$$

Finally, we vary with respect to the vierbein fields $e_a{}^m$. They appear both in $S_{\rm GD}$ [(84)] and in $S_{\rm WZ}$ [(74)]. The variational equation for $e_m{}^{--}$ is

$$P_{--\underline{a}}\mathcal{E}_m{}^{\underline{a}} \sim e_m{}^{++}\mathcal{E}_{++}{}^{\underline{a}}\mathcal{E}_{--\underline{a}} - \mathcal{E}_m{}^{\underline{a}}\mathcal{E}_{--\underline{a}}. \tag{93}$$

Multiplying Eq. (93) by $e_{\pm\pm}{}^m$ we find

$$P_{--\underline{a}}\mathcal{E}_{++}{}^{\underline{a}} = 0, \quad P_{--\underline{a}}\mathcal{E}_{--}{}^{\underline{a}} \sim \mathcal{E}_{--}{}^{\underline{a}}\mathcal{E}_{--\underline{a}}. \tag{94}$$

Inserting the solution (91) and the $--$ analogue of (92) into (94), we finally obtain

$$P_{-4}(\lambda_+\gamma_{\underline{a}}\lambda_+)(\lambda_-\gamma^{\underline{a}}\lambda_-) = 0 \;\Rightarrow\; P_{-4} = 0 \;\Rightarrow\; P_{--\underline{a}} = 0. \tag{95}$$

Once again, we see that the zweibeins play the role of auxiliary fields (like the gravitino above). In the standard superstring theory they produce the Virasoro constraints (69). In the twistor theory these constraints are already solved in terms of twistors. Therefore the zweibeins just give rise to auxiliary equations such as (93), which help eliminate some of the Lagrange multipliers.

This concludes our demonstration that the term $S_{\rm GD}$ [(36)] in the superstring action is purely auxiliary. It does not lead to any new equations of motion for the physical fields x and θ and thus the on-shell component action is just the Green-Schwarz one (74).

C. Case of a degenerate twistor matrix

In Secs. V A and V B we studied the component content of the twistor superstring action under the assumption that the twistor algebraic constraint (38) (taken at $\eta = 0$) has the regular solution (66). Here we shall investigate the alternative singular solution. We shall show that in this case the string collapses into a particle. For simplicity we shall only consider the bosonic fields in the action.

As explained in the Appendix, the singular solution, for which $\det \| \mathcal{E}_\alpha{}^{\underline{\alpha}} \| = 0$, has the form

$$\mathcal{E}_+{}^{\underline{\alpha}} = \lambda^{\underline{\alpha}}, \quad \mathcal{E}_-{}^{\underline{\alpha}} = ir\lambda^{\underline{\alpha}}. \tag{96}$$

Here $\lambda^{\underline{\alpha}}$ is an arbitrary *complex* spinor and r is an arbitrary *real* factor. Let us insert this solution into the Wess-Zumino term of our string action. The quantity A (63) vanishes due to the γ matrix identity (14):

$$\begin{aligned} A &\sim (\mathcal{E}_-\gamma^{\underline{a}}\mathcal{E}_+)(\mathcal{E}_-\gamma_{\underline{a}}\bar{\mathcal{E}}_+) + \text{c.c.} \\ &= r^2(\lambda\gamma^{\underline{a}}\lambda)(\lambda\gamma_{\underline{a}}\bar{\lambda}) + \text{c.c.} = 0. \end{aligned} \tag{97}$$

Further, the two-form term in (62) is proportional to θ, and so it does not contribute to the bosonic terms in the action. Thus, $S_{\rm WZ}$ vanishes in this case.

Let us now turn to the geometrodynamical term (36). Dropping the fermion fields and using the solution (96), we see that the component expansion in (79) is reduced to two terms only:

$$\begin{aligned} S_{\rm GD} = \int d^2\xi\; [&P_{\underline{a}}{}^{++}(D_{++}X^{\underline{a}} - \lambda\gamma^{\underline{a}}\bar{\lambda}) \\ &+P_{\underline{a}}{}^{--}(D_{--}X^{\underline{a}} - r^2\lambda\gamma^{\underline{a}}\bar{\lambda})]. \end{aligned} \tag{98}$$

The variation with respect to the following combination of Lagrange multipliers $\delta P_{\underline{a}}{}^{--} - r^2\delta P_{\underline{a}}{}^{++}$ shows that the two vectors $D_{++}X^{\underline{a}}$ and $D_{--}X^{\underline{a}}$ tangent to the string surface are linearly dependent:

$$r^2 D_{++}X^{\underline{a}} - D_{--}X^{\underline{a}} = 0. \tag{99}$$

This means that the dependence on the one of the world sheet coordinates drops out and the string collapses into a one-dimensional object (particle). The degeneracy of the world sheet leads to an additional gauge invariance. For instance, the action (98) has the gauge invariance

$$\begin{aligned} &\delta e_{++}{}^m = \rho(\xi)r^2 e_{--}{}^m, \quad \delta\lambda^\alpha = \tfrac{1}{2}\rho r^2\lambda^\alpha, \\ &\delta e_{--}{}^m = \rho r^2 e_{--}{}^m, \\ &\delta P_{\underline{a}}^{--} = -\rho P_{\underline{a}}^{++} - \rho r^2 P_{\underline{a}}^{--}, \quad \delta P_{\underline{a}}^{++} = 0. \end{aligned} \tag{100}$$

The appearence of new gauge invariances is observed in the fermionic part of the superstring action too. Thus, for example, the world sheet gravitino drops out from the action. This is in agreement with previous twistor formulations of the superparticle (see, e.g., [3]), where one does not need a gravitino field to achieve the local world sheet supersymmetry invariance.

Using all these gauge invariances along with world sheet reparametrizations, tangent space Lorentz and Weyl transformations, we can gauge away the zweibeins and the field r. Then with the help of (99) we find

$$S_{\rm GD} = \int d^2\xi\; P_{\underline{a}}(\partial_\tau X^{\underline{a}} - \lambda\gamma^{\underline{a}}\bar{\lambda}), \tag{101}$$

where $P_{\underline{a}}$ corresponds to an orthogonal combination of the Lagrange multipliers. Integrating out the inessential world sheet coordinate (σ), we see that this is a twistor particle action of the type described in Sec. III.

The conclusion of this subsection is that when one employs the singular solution of the twistor constraint (38), the superstring action becomes degenerate. The gauge invariance widens, leaving a number of component fields arbitrary. The remaining physical fields do not depend on σ any more, and so the superstring becomes a superparticle. Since the ordinary Green-Schwarz superstring formulation does contain the superparticle as a certain singular limit, we see that both the regular and singular solutions to the twistor constraints have to be taken into account.

VI. CONCLUSIONS

In this paper we have shown how the nonheterotic $D=3$ type II superstring can be formulated with manifest $N=(1,1)$ world sheet supersymmetry. The central point in the construction was the geometrodynamical constraint (37) and its corollary (38). In particular, they reduced the initial 2×2 complex twistor matrix $\mathcal{E}_\alpha{}^{\underline{\alpha}}$ to the two null vectors from the Virasoro constraints. The rest of (37) gave rise to purely auxiliary equations.

The geometrodynamical principle is common for the twistor formulations of the superparticle [3], the heterotic superstring [9,10], and, as we have seen here, the nonheterotic $D=3$ type II superstring. One would be tempted to extrapolate this to the nonheterotic type II superstring in higher dimensions as well. Indeed, analyzing the lowest-order component of Eq. (38), one can show that the $D=3$ situation is reproduced. For instance, in $D=10$ the 16×16 complex twistor matrix is once again reduced to the two null vectors from the Virasoro constraints. However, starting from $D=4$ [and $N=(2,2)$] there is an unexpected difficulty at the next level in the η expansion of Eq. (38). One can show (most easily in the linearized approximation) that some of the constraints are equations of motion for θ. This is inadmissible, since the geometrodynamical constraint is produced by a Lagrange multiplier, which implies that some of the components of the latter will propagate as well. One clearly sees that the case $D=3$ is the only exception, due to the trivial algebra of the transverse γ matrices in $D=3$. In fact, the same problem is also encountered in the framework of the type II superparticle discussed in Sec. III. So, the main open problem now is to find a modification of the geometrodynamical constraint such that it would not imply equations of motion in $D>3$. We hope to be able to report progress in this direction elsewhere.

Note added. After this paper was completed, we received a new paper by Pasti and Tonin [30], in which they claim that a similar construction applies to the $D=11$ supermembrane with full $N=8$, $D=3$ world sheet supersymmetry. This would be very surprising, since they impose the same type of geometrodynamical constraint. As we mentioned above, in the case of extended ($N>1$) world sheet supersymmetry this constraint is most likely to produce equations of motion and the corresponding Lagrange multiplier will contain new propagating degrees of freedom. One simple argument explaining this phenomenon has been proposed to us by P. Howe. The supermembrane theory of [30] could be truncated to a $D=11$ superparticle with $N=16$ world line supersymmetry. There the geometrodynamical constraint reduces the twistor variables (i.e., the bosonic physical fields) to the sphere S^9 (modulo gauge transformations). At the same time, the 32 components of the fermion $\theta^{\underline{\alpha}}$ are brought down to 16 after taking into account the 16 local world line supersymmetries. It is clear that 9 bosons and 16 fermions do not form an off-shell supermultiplet; therefore, the geometrodynamical constraint must involve equations of motion.

ACKNOWLEDGMENTS

E.S. is grateful to N. Berkovits, F. Delduc, P. Howe, E. Ivanov, W. Nahm, A. Perelomov, and M. Scheunert for stimulating discussions. The work of A.G. has been supported by the U.S. National Science Foundation, Grant No. PHY-90-96198.

APPENDIX: SOLUTION TO THE TWISTOR CONSTRAINTS

In Sec. IV we derived the geometrodynamical constraint (38) or, in light-cone notation, (39)-(41). The lowest-order terms in the η expansion of this constraint gives restrictions on the twistor matrix $\parallel \mathcal{E}_\alpha{}^{\underline{\alpha}} \parallel$:

$$\mathcal{E}_+\gamma^{a}\bar{\mathcal{E}}_+ = \mathcal{E}_{++}{}^{a}, \tag{A1}$$

$$\mathcal{E}_-\gamma^{a}\bar{\mathcal{E}}_- = \mathcal{E}_{--}{}^{a}, \tag{A2}$$

$$\mathcal{E}_+\gamma^{a}\bar{\mathcal{E}}_- + \mathcal{E}_-\gamma^{a}\bar{\mathcal{E}}_+ = 0. \tag{A3}$$

In fact, the first two equations define two vectors $\mathcal{E}_{\pm\pm}{}^{a}$ and only the third equation constrains the twistor variables. Here we are going to solve (A3) in a general way.

We start by writing out the components of the twistor matrix:

$$\mathcal{E}_\alpha{}^{\underline{\alpha}} \equiv \begin{pmatrix} A & B \\ C & D \end{pmatrix}. \tag{A4}$$

A basic kinematic assumption about the twistor variables is that they can never vanish identically. This means that at least one element of the matrix (A4) is nonvanishing. It is convenient to write down the constraint (A3) using the light-cone basis (26) for the γ matrices. There the three projections read

$$(++):\ A\bar{C}+C\bar{A}=0; \tag{A5}$$

$$(--):\ B\bar{D}+D\bar{B}=0; \tag{A6}$$

$$(+-):\ B\bar{C}+C\bar{B}+A\bar{D}+D\bar{A}=0\ . \tag{A7}$$

The general solution to Eqs. (A5) and (A6) is given by

$$A=ae^{i\alpha},\ \ C=ice^{i\alpha},\ \ B=be^{i\beta},\ \ D=ide^{i\beta}, \tag{A8}$$

where a, b, c, and d are *real*. Substituting this into (A7) one gets

$$(ad-bc)\sin(\alpha-\beta)=0. \tag{A9}$$

Now, there are two possibilities: The matrix $\mathcal{E}_\alpha{}^{\underline{\alpha}}$ can be either degenerate or nondegenerate. With the help of

(A8) we evaluate the determinant of this matrix:

$$\det \parallel \mathcal{E}_{\alpha}{}^{\underline{\alpha}} \parallel = i(ad-bc)e^{i(\alpha+\beta)}. \tag{A10}$$

Hence if the matrix $\mathcal{E}_{\alpha}{}^{\underline{\alpha}}$ is nondegenerate, $ad-bc \neq 0$, and (A9) implies in turn $\alpha=\beta$. In the degenerate case $ad-bc=0$ [and hence $(c,d)\sim(a,b)$] and the phases α and β are independent.

In summary, the general solution to (A3) consists of two sectors. In the first sector, the matrix $\mathcal{E}_{\alpha}{}^{\underline{\alpha}}$ is *nondegenerate* and is represented as

$$\mathcal{E}_{\alpha}{}^{\underline{\alpha}} = e^{i\phi}\begin{pmatrix} a & b \\ ic & id \end{pmatrix} \equiv e^{i\phi}\begin{pmatrix} \lambda_{+}^{\underline{\alpha}} \\ i\lambda_{-}^{\underline{\alpha}} \end{pmatrix}, \tag{A11}$$

where the spinors $\lambda_{+}^{\underline{\alpha}}$ and $\lambda_{-}^{\underline{\alpha}}$ are *real* and restricted by the condition

$$\lambda_{+}^{\underline{\alpha}}\lambda_{-\underline{\alpha}} \equiv ad-bc \neq 0. \tag{A12}$$

The second sector consists of the *degenerate* matrix $\mathcal{E}_{\alpha}{}^{\underline{\alpha}}$

$$\mathcal{E}_{\alpha}{}^{\underline{\alpha}} = \begin{pmatrix} ae^{i\alpha} & be^{i\beta} \\ irae^{i\alpha} & irbe^{i\beta} \end{pmatrix} \equiv \begin{pmatrix} \lambda^{\underline{\alpha}} \\ ir\lambda^{\underline{\alpha}} \end{pmatrix} \tag{A13}$$

where $\lambda^{\underline{\alpha}}$ is now an arbitrary *complex* spinor, and r is *real* .

---

[1] D.P. Sorokin, V.I. Tkach, and D.V. Volkov, Mod. Phys. Lett. A **4**, 901 (1989); D.P. Sorokin, V.I. Tkach, D.V. Volkov, and A.A. Zheltukhin, Phys. Lett. B **216**, 302 (1989).

[2] F. Delduc and E. Sokatchev, Class. Quantum Grav. **9**, 361 (1992).

[3] A. Galperin and E. Sokatchev, Phys. Rev. D **46**, 714 (1992).

[4] N. Berkovits, Phys. Lett. B **232**, 184 (1989).

[5] E.A. Ivanov and A.A. Kapustnikov, Phys. Lett. B **267**, 175 (1991).

[6] F. Delduc, E. Ivanov, and E. Sokatchev, Nucl. Phys. **B384**, 334 (1992).

[7] M. Tonin, Phys. Lett. B **266**, 312 (1991).

[8] N. Berkovits, Nucl. Phys. **B379**, 96 (1992).

[9] M. Tonin, Int. J. Mod. Phys. A **7**, 6013 (1992).

[10] F. Delduc, A. Galperin, P. Howe, and E. Sokatchev, Phys. Rev. D **47**, 578 (1993).

[11] W. Siegel, Phys. Lett. **128B**, 397 (1983); Class. Quantum Grav. **2**, 195 (1985).

[12] M.B. Green, J.H. Schwarz, and E. Witten, *Superstring Theory* (Cambridge University Press, Cambridge, England, 1987).

[13] V. Chikalov and A. Pashnev, Mod. Phys. Lett. A **8**, 285 (1993).

[14] In the case of the superparticle no world line supergravity fields appear in the action [3]. For the heterotic superstring only one component of the two-dimensional metric is needed to generate the second Virasoro constraint [6,10].

[15] We use the following conventions for the γ and charge conjugation matrices:

$$\gamma^0 = C = \begin{pmatrix} 0\,1 \\ 1\,0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0\ \ 1 \\ -1\,0 \end{pmatrix}, \quad \{\gamma^m,\gamma^n\} = 2\eta^{mn},$$

with $\eta^{00}=1$, $\eta^{11}=-1$.

[16] P. Howe, J. Phys. A **12**, 393 (1979); M. Brown and S.J. Gates, Ann. Phys. (N.Y.) **122**, 443 (1979).

[17] To avoid the proliferation of indices we use the same letters to denote similar types of indices on the world sheet and in the target space. The distinction is made by underlining the target-space ones.

[18] The σ model nature of the superstring was revealed in [19].

[19] M. Henneaux and L. Mezincescu, Phys. Lett. **152B**, 340 (1985).

[20] Here $\gamma^{\underline{a}}$ are the ordinary $D=3$ γ matrices times the charge conjugation matrix; we use the representation

$$\gamma^{\underline{0}} = \begin{pmatrix} 1\,0 \\ 0\,1 \end{pmatrix}, \quad \gamma^{\underline{1}} = \begin{pmatrix} 0\,1 \\ 1\,0 \end{pmatrix}, \quad \gamma^{\underline{2}} = \begin{pmatrix} 1\ \ 0 \\ 0\,-1 \end{pmatrix}.$$

[21] T. Curtright, L. Mezincescu, and C. Zachos, Phys. Lett. **161B**, 79 (1985).

[22] A. Isaev and E. Ivanov, Mod. Phys. Lett. A **4**, 351 (1989).

[23] L. Brink and J.H. Schwarz, Phys. Lett. **100B**, 310 (1981).

[24] For a discussion of the extended one-dimensional superconformal groups see [3].

[25] "Geometro" because it determines the way the world sheet is embedded as a surface in the target superspace; "dynamical" because it is derived from the superstring action.

[26] Here and in what follows we frequently use the constraint (37) or its corollary (38) in the Wess-Zumino action term. This actually means that we produce terms proportional to the constraint, which can be absorbed into a redefinition of the Lagrange multiplier in (36).

[27] Note that (48) is a consequence of the geometrodynamical constraint (37). This means that the Σ gauge invariance is achieved by a suitable transformation of the Lagrange multiplier in (36).

[28] The mechanism where the string tension appears as an integration constant was proposed in a different context in [29].

[29] J.A. De Azcárraga, J.M. Izquierdo, and P.K. Townsend, Phys. Rev. D **45**, 3321 (1992); P.K. Townsend, Phys. Lett. B **277**, 285 (1992); E. Bergshoeff, L.A.J. London, and P.K. Townsend, Class. Quantum Grav. **9**, 2545 (1992).

[30] P. Pasti and M. Tonin, "Twistor-like formulation of the supermembrane in D=11," Padova Report No. DFPD/93/TH/07 (unpublished).