Reexamination of generation of baryon and lepton number asymmetries in the early Universe by heavy particle decay

Jiang Liu and Gino Segrè
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104
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It is shown that wave-function renormalization can introduce an important contribution to the generation of baryon and lepton number asymmetries by heavy particle decay. These terms, omitted in previous analyses, are of the same order of magnitude as the standard terms.

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The three key elements for baryogenesis, namely, baryon number violation, C and CP violation and departure from thermal equilibrium were clearly identified in Sakharov's historic paper [1] in 1967. Realistic calculations of baryogenesis only became possible, however, in the late 1970s, after the introduction of grand unified theories (GUT's), which provided a clear field theoretical model in which baryon number violation occurs [2].

The early calculations followed a standard pattern, colloquially referred to as the "drift and decay mechanism." Preexisting asymmetries were presumed to be erased before the breaking of the GUT symmetry. A particle S, usually a colored Higgs boson, has a long enough lifetime so that it is out of thermal equilibrium when it finally decays. Since the decaying particle has at least two decay modes with different baryon number and its couplings violate CP, the ingredients are all in place for baryogenesis.

It was realized almost immediately that one needed to go beyond the tree approximation in calculating decay amplitudes: otherwise CPT invariance leads to a zero baryon asymmetry. Therefore, the standard calculation involves an interference between, e.g., a tree-level diagram for S decaying into fermions $S\to f_1f_2$ and a one- or more-loop diagram for the same process.

Many refinements and elaborations have taken place in the past fifteen years. Departures from the so-called "drift and decay mechanism" have been numerous: the most influential one has resulted from the observation by Kuzmin, Rubakov, and Shaposhnikov that nontrivial vacuum gauge configurations can lead to a significant baryon number violation at low temperature [3] (~ 100 GeV). In this paper, we will have nothing to say directly about low-temperature baryogenesis [4]. Our comments are most applicable to the earlier calculations and variations thereof.

It was realized recently [5] that wave-function renormalization of a heavy unstable particle can introduce important effects for CP-violating asymmetries. Baryon number asymmetry is one such particularly interesting example. We have realized that, whereas vertex corrections to the $S\to f_1f_2$ decay were treated consistently, external line insertions associated with wave-function renormalizations were not. Since, in general, these are of the same order of magnitude, the calculations change substantially. In one particular example we will, in fact, show that the vertex and external line insertions cancel: since, as we said earlier, baryon asymmetry is zero at the tree level, these corrections are the leading contributions to our process.

To be specific, consider a B- and CP-violating interaction [the standard SU(5) GUT model has two additional interactions of similar form which we have omitted for simplicity]

$$\mathcal{L}_{\mathcal{I}}=G_\xi\bar{u}_{R,\alpha}e^c_R S_{\xi,\alpha}+F_\xi\bar{u}^{\,c}_{R,\alpha}d_{R,\beta}S_{\xi,\gamma}\epsilon^{\alpha\beta\gamma}+\text{H.c.}\ , \tag{1}$$

where $S_{\xi,\alpha}$ is a heavy scalar belonging to the **5**, representation of SU(5). u, d, and e^c are the charged fermions of the first generation, α, β, and γ are the color indices, and $\xi=1,2,\ldots$ labels different species of S. Complex couplings G_ξ and F_ξ are the sources of CP violation. For simplicity we neglect fermion mixings.

Evidently, baryon number asymmetries generated from S_ξ decays are determined by the partial rate difference

$$\begin{aligned}\Delta_{S_\xi}&=\Gamma(S_\xi\to e\bar{u}^{\,c})-\Gamma(\bar{S}_\xi\to\bar{e}u^{c})\\&=\Gamma(\bar{S}_\xi\to d\bar{u}^{\,c})-\Gamma(S_\xi\to\bar{d}u^{c})\ ,\end{aligned} \tag{2}$$

where $\bar{S}_\xi$, $\bar{e}$, and $\bar{u}^{\,c}$ are the CP conjugates of S_ξ, e, and u^c, respectively, and the last step of Eq. (2) follows from CPT. Unless necessary, henceforth we will not display the color indices explicitly.

It follows from Eq. (1) that S_ξ has only two decay modes with final states $e\bar{u}^{\,c}$ and $\bar{d}u^c$. Adjoining the oneshell t-channel final-state scattering $\bar{d}\,u^c\to e\bar{u}^{\,c}$ to $S_\xi\to\bar{d}u^c$ [Fig. 1(b)] corresponds to a calculation of an absorptive part of a vertex correction [6] [Fig. 1(c)]. The interference of the vertex correction [Fig. 1(c)] with the tree-level amplitude [Fig. 1(a)] yields the standard result

$$\Delta_{S_\xi}(\text{vertex})=\frac{M_\xi}{32\pi^2}\sum_{\xi'}\text{Im}(G^*_\xi G_{\xi'}F_\xi F^*_{\xi'})\times\left[1-\frac{M^2_{\xi'}}{M^2_\xi}\ln\left[1+\frac{M^2_\xi}{M^2_{\xi'}}\right]\right]\ , \tag{3a}$$

where M_ξ is the mass of S_ξ. All fermions are massless at the scale of M_ξ. Asymptotically, Δ_{S_ξ}(vertex) behaves as

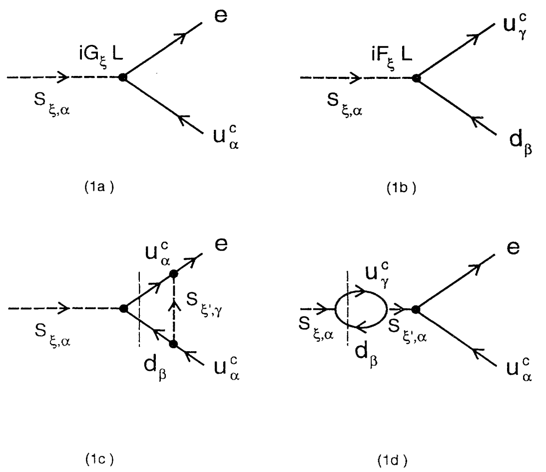

FIG. 1. Feynman diagrams for the generation of baryon number asymmetries from the decay of a colored heavy scalar $S_{\xi,\alpha}$ in an SU(5) model.

$$\Delta_{S_\xi}(\text{vertex})=\frac{M_\xi}{32\pi^2}\sum_{\xi'}\text{Im}(G_\xi^* G_{\xi'} F_\xi F_{\xi'}^*)\times\begin{cases}1-(M_{\xi'}^2/M_\xi^2)\ln(M_\xi^2/M_{\xi'}^2), & M_{\xi'}^2/M_\xi^2\to 0\ ,\\ \frac{1}{2}(M_\xi/M_{\xi'})^2, & M_{\xi'}^2/M_\xi^2\gg 1\ .\end{cases} \tag{3b}$$

It reaches its maximal value $\Delta S_\xi(\text{vertex})=O(M_\xi)$ at $(M_{\xi'}/M_\xi)\to 0$, and becomes vanishingly small in the opposite limit $(M_{\xi'}/M_\xi)\gg 1$.

In addition to the t-channel scattering, the two final states are also related by an s-channel interaction. Adjoining the on-shell s-channel amplitude $\bar{d}u^c\to e\bar{u}^c$ to $S_\xi\to\bar{d}u^c$ corresponds to a calculation of an absorptive part of a wave-function renormalization correction [Fig. 1(d)]. If the scalars are not degenerate, the calculation is very simple with the result from the interference of Figs. 1(a) and 1(d) given by

$$\Delta_{S_\xi}(\text{wave})=-\frac{M_\xi}{32\pi^2}\sum_{\xi'}\text{Im}(G_\xi^* G_{\xi'} F_{\xi'}^* F_\xi)\frac{M_\xi^2}{M_\xi^2-M_{\xi'}^2}\ . \tag{4}$$

In obtaining this result we have assumed for simplicity that

$$(M_\xi-M_{\xi'})^2\gg(\Gamma_\xi-\Gamma_{\xi'})^2\ ,$$

where Γ_ξ is the width of S_ξ. This contribution, which is of the same order as $\Delta_{S_\xi}(\text{vertex})$, has been missed by early calculations.

The significance of $\Delta_{S_\xi}(\text{vertex})$ may be illustrated as follows. From Eq. (4) one sees that $\Delta S_\xi(\text{wave})$ has a similar asymptotic behavior as $\Delta S_\xi(\text{vertex})$ but with the opposite sign:

$$\Delta S_\xi(\text{wave})=-\frac{M_\xi}{32\pi^2}\sum_{\xi'}\text{Im}(G_\xi^* G_{\xi'} F_\xi F_{\xi'}^*)\begin{cases}1+(M_{\xi'}/M_\xi)^2, & M_{\xi'}^2/M_\xi^2\to 0\ ,\\ -(M_\xi/M_{\xi'})^2, & M_{\xi'}^2/M_\xi^2\gg 1\ .\end{cases} \tag{5}$$

As a consequence,

$$\Delta S_\xi(\text{vertex})+\Delta S_\xi(\text{wave})=\frac{M_\xi}{32\pi^2}\sum_{\xi'}\text{Im}(G_\xi^* G_{\xi'} F_\xi F_{\xi'}^*)\begin{cases}-(M_{\xi'}/M_\xi)^2[1+\ln(M_\xi^2/M_{\xi'}^2)], & M_{\xi'}^2/M_\xi^2\to 0\ ,\\ \frac{3}{2}(M_\xi/M_{\xi'})^2, & M_{\xi'}^2/M_\xi^2\gg 1\ .\end{cases} \tag{6}$$

In contrast to the previously reported result (3), Eq. (6) shows that as long as there is a wide disparity between the two scalar masses, i.e., either $(M_{\xi'}/M_\xi)^2\gg 1$ or $\ll 1$, the CP-violating partial rate difference is always highly suppressed.

The relative size and sign of $\Delta_{S_\xi}(\text{vertex})$ and $\Delta_{S_\xi}(\text{wave})$ in the limit $M_{\xi'}\gg M_\xi$ can be understood easily by a Fierz transformation. In general, their final-state scattering amplitude in Figs. 1(c) and 1(d) is given by

$$\mathcal{A}(\bar{d}u^c\to e\bar{u}^c)=-iG_{\xi'}F_{\xi'}^*\epsilon^{\alpha\beta\gamma}\left[\frac{[\bar{u}_e(p_1)Lv_\gamma(k_1)][\bar{v}_\beta(k_2)Lu_\alpha(p_2)]}{(p_1-k_1)^2-M_{\xi'}^2}+\frac{[\bar{u}_e(p_1)Lu_\alpha(p_2)][\bar{v}_\beta(k_{23})Lv_\gamma(k_1)]}{(p_1+p_2)^2-M_{\xi'}^2}\right]\ , \tag{7}$$

where the u's and v's are the standard Dirac spinors. The first term arises from the t channel [Fig. 1(c)] and the second is due to the s channel [Fig. 1(d)]. Averaging over the incident momenta in the center-of-mass frame yields, for the $J=0$ partial wave (J is the total angular momentum),

$$\int_{-1}^{1}d\cos\theta\sum_{\text{spin}}\mathcal{A}=C^\alpha[\bar{u}_e(p_1)Lu_\alpha(p_2)]\left\{\left[1-\frac{M_{\xi'}^2}{s}\ln\left[1+\frac{s}{M_{\xi'}^2}\right]\right]-\frac{s}{s-M_{\xi'}^2}\right\}\ , \tag{8}$$

where $s=(k_1+k_2)^2=M_\xi^2$ and C^α is an overall factor determined by the couplings $G_{\xi'}$ and $F_{\xi'}$ and the normalization constants of the spinors. The ratio between the first and second terms is precisely

$$\Delta_{S_\xi}(\text{vertex})/\Delta_{S_\xi}(\text{wave}) .$$

Including fermion mixings the couplings G_ξ and F_ξ become matrices in flavor space. Hence, generally speaking, $\Delta_{S_\xi}(\text{vertex})$ and $\Delta_{S_\xi}(\text{wave})$ are not simply related as in Eqs. (5) and (6): for vertex corrections one will have a trace over a family matrix to the fourth power while the wave-function correction will have a product of two traces of the square of family matrices. Still, the order of magnitudes of $\Delta_{S_\xi}(\text{vertex})$ and $\Delta_{S_\xi}(\text{wave})$ are the same.

The model of baryogenesis we have considered requires more than one Higgs color triplet ($S_\xi \neq S_{\xi'}$) and does not have natural flavor conservation; i.e., both S_ξ and $S_{\xi'}$ couple to the two scalar fermion currents. The additional diagrams we have been discussing will not make any contribution without these features. On the other hand, the vertex diagrams also do not make a contribution [7] if these features are absent. Thus, in general, wave-function effects are equally important as vertex corrections.

It is interesting to notice that Fig. 1(d) is a one-particle-reducible (OPR) diagram. Even though one can introduce a renormalization scheme in which the renormalized self-energy matrix $\Sigma^{(R)}_{\xi\xi'}(p)$ vanishes on-shell, i.e.,

$$\Sigma^{(R)}_{\xi\xi'}(p)\big|_{p^2=M_\xi^2}=\Sigma^{(R)}_{\xi\xi'}(p)\big|_{p^2=M_{\xi'}^2}=0 ,$$

that $\Delta_{S_\xi}(\text{wave})\neq 0$ is because the kinetic-energy part of the renormalized Lagrangian will not have the standard normalization, and the renormalized field does not conjugate to its Hermitian conjugate (Ref. [5]), due to the non-Hermicity of the renormalized effective Lagrangian.

A situation of special interest (Ref. [3]) is that in which the heavy particle masses are nearly degenerate, i.e.,

$$(M_\xi - M_{\xi'}) - i(\Gamma_\xi - \Gamma_{\xi'})/2 \to 0 .$$

It has been suggested (Ref. [3]) that graphs similar to those we have been discussing might produce some resonance effects to enhance the CP-violating asymmetry. In that case Eq. (4) is invalid. If CP violation still can be treated perturbatively, $\Delta_{S_\xi}(\text{wave})$ can be obtained by studying the renormalization effect on unstable particle propagator

$$\langle 0|TS_\xi(x)S^\dagger_{\xi'}(y)|0\rangle .$$

An analogous example with $\xi=\xi'=1$ for the CP-violating partial rate difference of the decay $t\to bW^+, bH^+$ is discussed in Ref. [5]. Methods useful for degenerate unstable particles with large CP-violating interactions are still unfortunately unavailable.

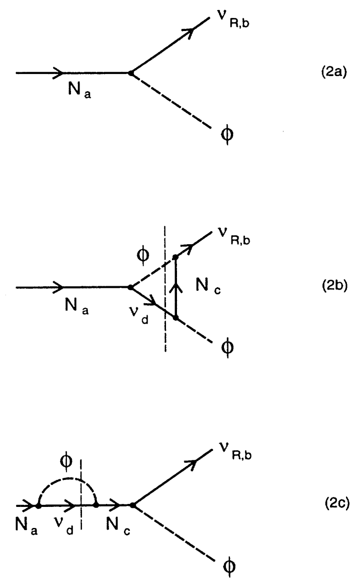

FIG. 2. Feynman diagrams for the generation of lepton number asymmetries from the decay of a heavy Majorana neutrino N_a.

Wave-function renormalization also plays an important role in leptogenesis. Consider the generation of lepton number asymmetries by heavy Majorana neutrino decay [8–12]. We will assume the neutrinos get their masses by the usual "seesaw" mechanism [13] so that we have very heavy Majorana neutrinos N_a coupled to, on the ~ 100 GeV scale, effectively massless neutrinos ν_b. The coupling between N_a and ν_b is of the form

$$\mathcal{L}_I=\bar{N}_a(V_{ab}R+V^*_{ba}L)\nu_b\phi+\text{H.c.}, \tag{9}$$

where ϕ is a neutral scalar meson. For convenience here we use $L\nu$ for ν_L and $R\nu$ for $(\nu_L)^c$. The indices $b\,(a)$ run from $1\,(n+1)$ to $n\,(2n)$, where n is the number of neutrino families, generally taken to be 3. The Majorana form of the mass matrix imposes the condition $V_{ab}=V_{ba}$. Although, strictly speaking, Majorana neutrinos do not carry lepton number, CP violation can nevertheless introduce a partial rate difference:

$$\Delta_{ab}=\Gamma(N_a\to\phi\nu_{R,b})-\Gamma(N_a\to\phi\nu_{L,b}) . \tag{10}$$

A lepton number asymmetry can therefore be generated if we assign a lepton number $L=\pm1$ to the left- and right-handed light neutrinos (the latter are of course usually referred to as antineutrinos).

The interference of the tree-level amplitude [Fig. 2(a)] and the vertex correction [Fig. 2(b)] yields

$$\Delta_{ab}(\text{vertex})=\frac{1}{64\pi^2}\sum_{c,d}\left\{m_c\,\text{Im}(V_{ab}V^*_{cb}V^*_{dc}V_{da})\left[1-\left[1+\frac{m_c^2}{m_a^2}\right]\ln\left[1+\frac{m_a^2}{m_c^2}\right]\right]\right.$$
$$\left.+m_a\,\text{Im}(V_{ab}V^*_{cb}V_{cd}V^*_{ad})\left[1-\frac{m_c^2}{m_a^2}\ln\left[1+\frac{m_a^2}{m_c^2}\right]\right]\right\} , \tag{11}$$

where m_a and m_c are, respectively, the masses of N_a and N_c, and for simplicity we have neglected scalar masses. The light neutrino masses are also neglected since they are, for all practical purposes, massless. The first term in Eq. (11) is the standard result arising from an internal mass insertion. The second, which has not been included in previous studies, is due to an external neutrino mass insertion. The final-state interaction in this vertex correction goes through a t-channel with a total angular momentum $J=\frac{1}{2}$.

Once again wave-function renormalization gives a contribution [Fig. 2(c)] of the same order as $\Delta_{ab}(\text{vertex})$. For nondegenerate heavy neutrinos we find

$$\Delta_{ab}(\text{wave})=\frac{1}{128\pi^2}\sum_{c,d}\frac{m_a^2}{m_a^2-m_c^2}[m_c\text{Im}(V_{ab}V_{cb}^*V_{dc}^*V_{da})+m_a\text{Im}(V_{ab}V_{cb}^*V_{cd}V_{ad}^*)]\ , \tag{12}$$

where partial widths have been neglected for simplicity. Here the final-state interaction goes through an s-channel with $J=\frac{1}{2}$. Asymptotically, $\Delta_{ab}(\text{vertex})$ and $\Delta_{ab}(\text{wave})$ have the simple relations

$$\begin{aligned}\Delta_{ab}(\text{wave})&=\Delta_{ab}(\text{vertex}),\quad \frac{m_c}{m_a}\gg 1\ ,\\ \Delta_{ab}(\text{wave})&=\frac{1}{2}\Delta_{ab}(\text{vertex}),\quad \frac{m_c}{m_a}\ll 1\ .\end{aligned}\tag{13}$$

Thus, neglecting $\Delta_{ab}(\text{wave})$ will underestimate the lepton number asymmetry by a factor of 2 ($\frac{3}{2}$) in the limit $m_c/m_a\gg 1$ ($m_c/m_a\ll 1$).

In conclusion, we have re-examined the generation of baryon and lepton number asymmetries by heavy particle decays. We have shown that an important piece of contribution due to wave-function renormalization has been missed by early investigations. This missing piece is generally of the same order of magnitude as the other terms calculated before, and can result, in some interesting cases, in a complete cancellation of the leading terms.

Note added. After the submission of this paper we realized that the wave-function renomalization effects in heavy scalar decays were briefly discussed previously by Ignatev, Kuzmin, and Shaposhnikov [14]. An explicit calculation in the same context was done more recently by Botella and Roldan [15]. However, their result differs from ours by a sign, with the consequence that

$$\Delta S_\xi(\text{vertex})+\Delta S_\xi(\text{wave})$$

has similar asymptotic behavior as $\Delta S_\xi(\text{vertex})$ [see Eq. (3b)] rather than the one given by Eq. (6) of this paper.

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