

Covariant perturbations of domain walls in curved spacetime

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(Received 7 April 1993)

A manifestly covariant equation is derived to describe the perturbations in a domain wall on a given background spacetime. This generalizes recent work on domain walls in Minkowski space and introduces a framework for examining the stability of relativistic bubbles in curved spacetimes.

PACS number(s): 98.80.Cq, 03.70.+k, 98.80.Hw

I. INTRODUCTION

Topologically stable field configurations would have been produced during any phase transitions that occurred in the early Universe. These could be monopoles, cosmic strings, or domain walls, depending on the model. If they were produced during or after inflation, even a small abundance of these defects could have profound cosmological consequences. In practice, however (at least until recently), calculations were based on the flimsy assumption that topological defects of high symmetry were the ones that were preponderant. Now, *perhaps* this could be justified for the initial distribution. At least in Minkowski space, the semiclassical approximation to quantum theory does appear to predict an exponential suppression in the materialization rate during a phase transition of classical configurations with low symmetry [1]. However, even then, one cannot ignore the fact that there will always be quantum fluctuations attending these classical configurations. If one is, therefore, to place any confidence in cosmological predictions that rely on the assumption that symmetrical topological defects maintain their shape in the course of their evolution, such as the collapse of cosmic strings to form black holes, one had better be sure not only that the evolution is stable but also that a bound can be placed on the perturbation to prevent it from substantially disrupting the process [2].

In this paper, we examine the evolution of small irregularities in a closed domain wall propagating on a curved background spacetime. This provides a generalization of the work of Garriga and Vilenkin, who examined the stability of domain walls in Minkowski space and equatorial domain walls in de Sitter space [3,4]. Because of the extra technical difficulty they involve, we defer the examination of lower dimensional defects to a subsequent publication [5].

We begin in Sec. II with a derivation of the equations of motion of the unperturbed domain wall in the thin wall approximation. Our approach to perturbation theory in Sec. III will be to expand the action describing domain walls in a manifestly covariant way out to second

order in the perturbation. In Sec. IV we discuss the equations of motion describing perturbations on various background spacetime and domain wall geometries and compare our results with those of Refs. [3] and [4]. We work in an arbitrary spacetime dimension N .

II. THE EQUATIONS OF MOTION

Let us consider an oriented domain wall m in the thin wall approximation. This is justified so long as the thickness of the wall is much smaller than any of its other dimensions. The wall is then described by the timelike hypersurface

$$x^\mu = X^\mu(\xi^a), \quad (2.1)$$

$\mu = 0, \dots, N-1$, $a = 0, \dots, N-2$, embedded in spacetime M , which we describe by the metric $g_{\mu\nu}$. The metric induced on the world sheet of the domain wall is then given by

$$\gamma_{ab} = X^\mu_{,a} X^\nu_{,b} g_{\mu\nu}. \quad (2.2)$$

Before proceeding any further, we need to distinguish between open domain walls possessing a physical boundary, and closed ones which do not. A closed domain wall need not be compact. However, if it is not it must be infinite in all directions. A spatial boundary at infinity can be ignored.

An oriented closed wall provides a partition of spacetime into two regions, an interior M_{int} , and an exterior M_{ext} , each supporting its own phase. Neither region need be finite in spatial extent. The action that describes the dynamics of a closed domain wall is given by the Nambu form plus a possible spacetime volume term

$$S[X^\mu, X^\mu_{,a}] = -\sigma \int_m d^{N-1} \xi \sqrt{-\gamma} + \rho \int_{M_{\text{int}}} d^N x \sqrt{-g}. \quad (2.3)$$

The first term represents the simplest generally covariant action one can associate with the wall, proportional to the area swept out by the world sheet of the wall as it evolves. The constant of proportionality σ represents the energy density of the wall in its rest frame. ρ represents the energy density deficit in the region M_{int} . In the description of the nucleation of bubbles, M_{int} will be finite. If ρ is positive (negative), the interior consists of

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true (false) vacuum. We now associate an action with M_{int} proportional to the spacetime volume enclosed by the world sheet of the wall. If this volume is infinite the associated action will be infinite. However, the change in volume corresponding to a variation in the embedding of compact support will always be finite. It will not, therefore, affect the equations of motion.

If, on the other hand, the domain wall possesses a physical spatial boundary, it clearly cannot provide a partition of spacetime. The phases on either side must coincide and it makes no sense to introduce the volume term. Now, however, in any physically realistic situation (such as a domain wall arising in the course of symmetry breaking in field theory) a cosmic string (codimension two) will bound the domain wall. We defer the treatment of the open defect to a subsequent publication.

The equations of motion of the bubble wall are given by the extrema of S subject to variations $X^\mu(\xi) \rightarrow X^\mu(\xi) + \delta X^\mu(\xi)$, which leave the initial and final configurations of the domain wall unchanged:

$$-\frac{\delta S}{\delta X^\mu} \equiv \sigma [\Delta X^\mu + \Gamma_{\alpha\beta}^\mu (X^\nu) \gamma^{ab} X_{,a}^\alpha X_{,b}^\beta] - \rho n^\mu = 0, \quad (2.4)$$

where Δ is the scalar Laplacian,

$$\Delta = \frac{1}{\sqrt{\gamma}} \partial_a (\sqrt{\gamma} \gamma^{ab} \partial_b),$$

n^μ is the unit normal to the world sheet, and $\Gamma_{\alpha\beta}^\mu$ are the spacetime Christoffel symbols evaluated on m . We comment on the derivation in the Appendix. If ρ is zero, Eq. (2.4) represents a higher dimensional generalization of the geodesic equations describing the motion of a point defect.

Despite the nice analogy, this form of the equations of motion is not very useful in practice. This is because all but one linear combination of these equations are identically satisfied. To see this, we note that, both on shell and off, Gauss's equation [6] can be cast in the form

$$\nabla_b X_{,a}^\mu + \Gamma_{\alpha\beta}^\mu (X^\nu) X_{,a}^\alpha X_{,b}^\beta = K_{ab} n^\mu, \quad (2.5)$$

where ∇_a is the world sheet covariant derivative compatible with γ_{ab} and the extrinsic curvature tensor K_{ab} is defined by

$$K_{ab} = -X_{,a}^\mu X_{,b}^\nu D_\nu n_\mu, \quad (2.6)$$

where D_ν is the spacetime covariant derivative. A consequence of Eq. (2.6) is that the tangential projections of the Euler-Lagrange derivatives of S vanish identically:

$$\frac{\delta S}{\delta X^\mu} X_{,b}^\mu = 0. \quad (2.7)$$

The geometrical reason for this redundancy is the invariance of the action with respect to world sheet diffeomorphisms. In particular, under the infinitesimal world sheet diffeomorphism,

$$\xi^a \rightarrow \xi^a + \omega^a,$$

$\delta S = 0$, which implies the (Bianchi) identities, Eq. (2.7).

It is now clear that the equations describing the world

sheet are entirely equivalent to the single equation

$$\sigma K = \rho. \quad (2.8)$$

If ρ vanishes, this is just the equation describing an extremal hypersurface. Thus, Eq. (13) in Ref. [3], which was derived for a domain wall propagating in Minkowski space, generalizes, with no surprises, to a curved spacetime. That there is only one independent equation describing the dynamics of the domain wall has nothing to do with any symmetry, spherical or otherwise, that the wall might possess.

III. THE QUADRATIC ACTION

At lowest order, the dynamics of any irregularities in the geometry of the wall will still be described by the thin wall action Eq. (2.3). One might hope that the nonlinearity of the equation of motion would serve to damp out any irregularities that might appear in the course of the bubble's evolution in much the same way as the nonlinearity of the underlying field theory is inclined to improve the thin wall approximation in certain models [1]. However, the way it turns out (for examples, see Refs. [3] and [4]), is that sometimes it does but sometimes it does not.

One way to derive the equation of motion describing the perturbation in the wall is simply to consider the linearization of Eq. (2.8),

$$\delta K = 0, \quad (3.1)$$

with respect to the displacement in the world sheet δX^μ . This is the method exploited in Ref. [3] when the background is Minkowski space. The approach we will follow will be to expand the action out to quadratic order about the classical solution satisfying Eq. (2.8). Once this is done, it becomes a simple matter to obtain the corresponding equations of motion. In addition, when physical boundaries are considered, the variational principle provides a guide to the implementation of appropriate boundary conditions.

As we have seen, variations along tangential directions correspond to world sheet diffeomorphisms. The only diffeomorphism invariant measure of the perturbation δX^μ in the wall is the scalar

$$\Phi \equiv n_\mu \delta X^\mu, \quad (3.2)$$

representing the normal projection of the spacetime displacement vector δX^μ . This single scalar will now completely characterize the perturbation in the domain wall.

The simplest way to evaluate the quadratic action is to introduce Gaussian normal coordinates for spacetime adapted to the world-sheet hypersurface. Thus, from each spacetime point P in the neighborhood of m , we drop the geodesic from P , which intersects m orthogonally at the point P' . P is then uniquely characterized by the coordinates $\xi^a(P')$ and the proper distance η along the geodesic. Permitting ourselves an abuse of notation that should not lead to confusion, we denote the nontrivial spacetime metric components $\gamma_{ab}(\eta, \xi^a)$. We note that $\gamma_{ab}(0, \xi^a) = \gamma_{ab}(\xi^a)$ and that

$$K_{ab} = -\frac{1}{2}\gamma'_{ab} , \quad (3.3)$$

where the prime denotes the proper derivative along the normal and we evaluate it at $\eta=0$. With respect to these coordinates, Φ is simply the component of the variation δX^μ along the normal, δX^η . The first and second variations in γ_{ab} are

$$\begin{aligned} \gamma_{ab}^{(1)} &= -2K_{ab}\Phi , \\ \gamma_{ab}^{(2)} &= 2\Phi_{,a}\Phi_{,b} - 2K'_{ab}\Phi^2 . \end{aligned} \quad (3.4)$$

To second order in Φ ,

$$\begin{aligned} \sqrt{-\gamma}^{(2)} &= \frac{1}{2}\sqrt{-\gamma}[\gamma^{ab}\gamma_{ab}^{(2)} \\ &\quad + \frac{1}{2}(\gamma^{(1)2} - 2\gamma^{(1)ab}\gamma_{ab}^{(1)})] . \end{aligned} \quad (3.5)$$

Dropping a divergence, and exploiting the constancy of K implied by the background equation of motion, the corresponding second order world sheet area can be written

$$A^{(2)} = -\frac{1}{2} \int_m d^D \xi \sqrt{-\gamma} [\Phi \Delta \Phi (K' + K^2) \Phi^2] . \quad (3.6)$$

This form of the second variation is not very useful because it involves K' . However, it is simple to express K' in terms of more familiar world-sheet and spacetime scalars. We note that K' appears linearly in the spacetime scalar curvature, ${}^N R$. The easiest way to eliminate K' is, therefore, to exploit the Ricci identity

$$D_\mu D_\nu n_\alpha - D_\nu D_\mu n_\alpha = {}^N R_{\mu\nu\alpha\beta} n^\beta .$$

We contract on μ and α and project onto n^ν

$$n^\nu D_\mu D_\nu n^\mu - n \cdot D(D \cdot n) = {}^N R_{\mu\nu} n^\mu n^\nu .$$

We now rewrite the first term

$$n^\nu D_\mu D_\nu n^\mu = D_\mu (n^\nu D_\nu n^\mu) - D_\mu n^\nu D_\nu n^\mu .$$

The divergence vanishes because the normal to a hypersurface is a spacetime gradient. We obtain ($n_\mu = \partial_\mu \eta$):

$$K' = K_{ab} K^{ab} + {}^N R_{\mu\nu} n^\mu n^\nu . \quad (3.7)$$

An alternative way to eliminate K' is to exploit the dynamical Einstein equation for K in the initial value formulation of general relativity with the replacement of proper time by η [7].

We must also expand the enclosed volume out to second order. We find [see Appendix, Eq. (A2)]

$$V^{(2)} = \frac{1}{2} \int_m d^{N-1} \xi \sqrt{-\gamma} K \Phi^2 . \quad (3.8)$$

We now add Eqs. (3.67) and (3.8) and again exploit the background equation of motion to cancel the K^2 term against the volume contribution:

$$\begin{aligned} S^{(2)} &= \frac{\sigma}{2} \int_m d^{N-1} \xi \sqrt{-\gamma} [\Phi \Delta \Phi + ({}^N R_{\mu\nu} n^\mu n^\nu \\ &\quad + K^{ab} K_{ab}) \Phi^2] . \end{aligned} \quad (3.9)$$

In the elimination of K' we introduced the quadratic in the extrinsic curvature, $K_{ab} K^{ab}$. We can, however, elimi-

nate this term from $S^{(2)}$ in favor of curvature scalars by exploiting the contracted Gauss-Codazzi equation:

$${}^N R_{\alpha\beta\mu\nu} h^{\alpha\mu} h^{\beta\nu} = {}^{N-1} R + K^{ab} K_{ab} - K^2 , \quad (3.10)$$

where the projection $h^{\alpha\beta} = g^{\alpha\beta} - n^\alpha n^\beta$. We then get

$$\begin{aligned} S^{(2)} &= \frac{\sigma}{2} \int_m d^{N-1} \xi \sqrt{-\gamma} [\Phi \Delta \Phi + ({}^N R_{\mu\nu} n^\mu n^\nu \\ &\quad + {}^N R_{\alpha\beta\mu\nu} h^{\alpha\mu} h^{\beta\nu} \\ &\quad - {}^{N-1} R + K^2) \Phi^2] . \end{aligned} \quad (3.11)$$

Finally, we use Eq. (2.8) to eliminate K in favor of ρ and σ and add the spacetime curvature terms

$${}^N R_{\mu\nu} n^\mu n^\nu + {}^N R_{\alpha\beta\mu\nu} h^{\alpha\mu} h^{\beta\nu} = {}^N R_{\mu\nu} h^{\mu\nu}$$

to obtain

$$\begin{aligned} S^{(2)} &= \frac{\sigma}{2} \int_m d^{N-1} \xi \sqrt{-\gamma} \left\{ \Phi \Delta \Phi + \left[{}^N R_{\mu\nu} h^{\mu\nu} - {}^{N-1} R \right. \right. \\ &\quad \left. \left. + \left[\frac{\rho}{\sigma} \right]^2 \right] \Phi^2 \right\} . \end{aligned} \quad (3.12)$$

IV. THE LINEARIZED EQUATIONS

The equation of motion for small perturbations is now given by

$$\Delta \Phi + \left[{}^N R_{\mu\nu} h^{\mu\nu} - {}^{N-1} R + \left[\frac{\rho}{\sigma} \right]^2 \right] \Phi = 0 . \quad (4.1)$$

This is a scalar wave equation for Φ on the curved background geometry of the world sheet. If this geometry is flat, ${}^N R_{\nu\alpha\beta} = 0$ and we reproduce Eq. (22) of Ref. [3]. The wave equation then involves only the intrinsic geometry of the world sheet.

Eq. (4.1) is an unconventional Klein-Gordon equation in several respects.

(1) We first note that the perturbation possesses a tachyonic mass whenever $\rho \neq 0$. When $\rho = 0$ this mass is zero.

(2) There is a nonminimal coupling of the perturbation to the scalar curvature of the background world-sheet geometry. This coupling is universal and independent of the dimension of the geometry. In particular, there is no privileged dimension in which the coupling becomes conformal.

(3) When the ambient spacetime geometry is not flat, the perturbation couples to the tangential projection of the Ricci tensor. This is the only explicit dependence of Φ on the spacetime geometry. It is this feature which distinguishes the perturbation theory we are considering from a conventional field theory on the curved spacetime described by the metric γ_{ab} . If, however, the background is a vacuum solution to the Einstein equations, this term vanishes. In particular, it will vanish on Schwarzschild spacetime. We note also that the perturbation does not couple to the Weyl part of the background curvature.

Normally, we would interpret a tachyonic mass to signal an instability. However, what is more significant is the effective mass given by

$$m^2 = {}^N R - {}^N R_{\mu\nu} h^{\mu\nu} - \left[\frac{\rho}{\sigma} \right]^2, \quad (4.2)$$

which might be positive, depending on the value of the first two terms. This could depend on the topology of the domain wall about which we are perturbing. However, even a tachyonic effective mass does not always signal an instability. This will be the case for perturbations about an equatorial bubble in de Sitter space [4]. The expansion of de Sitter space introduces a damping term into the Laplacian, which annuls the destabilizing effect of a tachyonic effective mass. Indeed, sometimes the notion of stability itself is ambiguous. An example is provided in Ref. [3] where the stability of true vacuum bubbles in Minkowski space is discussed. The Fitzgerald-Lorentz contraction in the perturbation detected by an inertial observer in Minkowski space is sufficiently large to render physically divergent perturbations of the wall apparently convergent.

In a vacuum background geometry with a cosmological constant Λ (this need not be de Sitter space), the coupling to the spacetime Ricci tensor simplifies. The background Einstein equations then read

$${}^N R_{\mu\nu} = \frac{2\Lambda}{N-2} g_{\mu\nu}, \quad (4.3)$$

so that Eq. (4.1) reduces to the form

$$\Delta\Phi + \left[2 \left[\frac{N-1}{N-2} \right] \Lambda - {}^N R + \left[\frac{\rho}{\sigma} \right]^2 \right] \Phi = 0. \quad (4.4)$$

Suppose, in particular, that the background is de Sitter space. We can express ${}^N R$ in terms of the Hubble parameter H :

$${}^N R = N(N-1)H^2. \quad (4.5)$$

Let us also suppose that $\rho=0$. We describe de Sitter space by Friedmann-Robertson-Walker (FRW) closed coordinates:

$$ds^2 = -dt^2 + H^{-2} \cosh^2(Ht) d\Omega_{N-1}^2, \quad (4.6)$$

where $d\Omega_N^2$ is the line element on a round N sphere. We now consider a spherically symmetric domain wall. In general, if the domain wall is spherically symmetric, its world sheet is isometric to an $N-1$ dimensional FRW closed cosmology described by the line element

$$ds^2 = -d\tau^2 + a(\tau)^2 d\Omega_{N-2}^2, \quad (4.7)$$

where τ represents the proper time recorded on a clock moving with the wall. The location of the wall at any time τ is specified by the polar angle χ marking the position the $N-2$ sphere of the wall is embedded on the $N-1$ sphere:

$$d\Omega_{N-1}^2 = d\chi^2 + \sin^2\chi d\Omega_{N-2}^2.$$

There are two qualitatively different kinds of trajec-

tories [2]. One of these consists of trajectories that begin with zero size at the pole $\chi=0$ and grow to a maximum value before recollapsing. The other is a bounce that consists of a bubble originating on the equator ($\chi=\pi/2$) contracting to a minimum value of χ before bouncing back to the equator. There is a solution with $\chi=\pi/2$ representing a domain wall that spans the equator. Such solutions can be interpreted as bubbles that materialize from nothing through quantum processes [2] and as such are particularly interesting. The world sheet is now an embedded $(N-1)$ -dimensional de Sitter space with the same Hubble parameter

$${}^{N-1} R = (N-1)(N-2)H^2.$$

The equation describing small perturbations about this solution is then given by

$$\Delta\Phi + (N-1)H^2\Phi = 0. \quad (4.8)$$

This reproduces the expression obtained in Ref. [3]. We stress, however, that the technique used in Ref. [3] to derive Eq. (4.2) depended sensitively on the fact that the embedded domain wall spanned the equator. Even though the effective mass in Eq. (4.2) is tachyonic, this does not appear to be significant.

In general, the world sheet of the bubble will not be an embedded $(N-1)$ -dimensional de Sitter space. If the bubble is collapsing, the scalar curvature of its world sheet will diverge. We recall that the scalar curvature corresponding to the line element (4.7) is given by

$${}^{N-1} R = 2(N-2) \left[\frac{\ddot{a}}{a} + (N-3) \left(\frac{\dot{a}^2}{a^2} + \frac{1}{a^2} \right) \right],$$

where the dot refers to a derivative with respect to τ . Both \dot{a} and \ddot{a} remain finite as $a \rightarrow 0$. At some point, therefore, the effective mass of the perturbations about the collapsing bubble will be rendered real. It is also true, however, that as we approach the final stages of collapse, the thin wall approximation will break down.

V. CONCLUSIONS

We have provided a framework for the examination of perturbations of domain walls on a given spacetime background. This analysis can be extended in at least two different directions.

The first is the treatment of perturbations on lower dimensional topological defects [5]. When the codimension of the world sheet is r , there will be r scalar fields describing the perturbation, one for the projection of δX^μ onto each of the r normal vector $n^{(i)\mu}$. What is more, the equations we obtain will generally be coupled in a non-trivial way.

The weak point in our treatment of perturbations is that the domain wall has not been treated as a source of gravity. This is a serious limitation. If we are to place any confidence in perturbation theory, we need to accommodate the back reaction [8]. Any displacement in the wall δX^μ will couple to the spacetime metric and generate gravitational waves. This is currently being examined.

ACKNOWLEDGMENTS

I have benefited enormously from conversations with Jaume Garriga and Alexander Vilenkin. The treatment of the second variation of the action is a simplified version, exploiting normal coordinates of Niall Ó Murchadha's treatment of the same variation in another context.

APPENDIX

The extreme of S with respect to variations that vanish on the boundary satisfy the Euler Lagrange equations

$$\sigma \left[\frac{\partial}{\partial \xi^a} \frac{\partial \sqrt{-\gamma}}{\partial X_{,a}^\mu} - \frac{\partial \sqrt{-\gamma}}{\partial X^\mu} \right] - \rho \frac{\delta V}{\delta X^\mu} = 0 . \quad (\text{A1})$$

Now,

$$\frac{\partial \gamma}{\partial \gamma_{ab}} = \gamma \gamma^{ab} ,$$

so that

$$\frac{\partial \sqrt{-\gamma}}{\partial X_{,a}^\mu} = \sqrt{-\gamma} \gamma^{ab} g_{\beta\mu} X_{,b}^\beta ,$$

and

$$\frac{\partial}{\partial \xi^a} \frac{\partial \sqrt{-\gamma}}{\partial X_{,a}^\mu} = g_{\mu\beta} \partial_z (\sqrt{\gamma} \gamma^{ab} X_{,b}^\beta) + \sqrt{\gamma} \gamma^{ab} g_{\mu\beta, \alpha} X_{,a}^\alpha X_{,b}^\beta .$$

We also have

$$\frac{\partial \sqrt{-\gamma}}{\partial X^\mu} = \frac{1}{2} \sqrt{\gamma} \gamma^{bc} g_{\alpha\beta, \mu} X_{,a}^\alpha X_{,b}^\beta .$$

The first derivatives of the spacetime metric appear in the combination $\Gamma_{\alpha\beta}^\mu$. The term in square brackets in Eq. (A1) reproduces the corresponding term appearing in Eq. (2.4). To complete the derivation of Eq. (2.4), we note that under the variation $X^\mu \rightarrow X^\mu + \delta X^\mu$, the volume transforms by

$$\frac{\delta V}{\delta X^\mu} = \sqrt{-\gamma} n^\mu ,$$

or, alternatively, in the notation of Sec. III,

$$\delta V = \int d^{N-1} \xi \sqrt{-\gamma} \Phi . \quad (\text{A2})$$

In this form, it is clear that the second variation is given by Eq. (3.7).

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