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Magnetic moments of the baryon decuplet in a relativistic quark model

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The magnetic moments of the baryon decuplet are calculated in a relativistic constituent quark model using the light front formalism. Of particular interest are the magnetic moments of the Ω^- and Δ^{++} for which new recent experimental measurements are available. Our calculation for the magnetic moment ratios $\mu(\Delta^{++})/\mu(p)$ and $\mu(\Omega^-)/\mu(\Lambda^0)$ is in excellent agreement with the experimental ratios.

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I. INTRODUCTION

Already two of the magnetic moments of the baryon decuplet have been measured. The most precise experiment for the Δ^{++} is a pion bremsstrahlung analysis [1]. The magnetic moment of the Ω^- has been measured by the E756 Collaboration [2] where Ω^- hyperons are produced by a polarized neutral spin transfer reaction. The final result from the succeeding experiment called E800 is expected within the year [3]. Theoretical predictions for the Δ^{++} and Ω^- magnetic moments have been given in many models. In particular, the simple, additive quark model predicts the ratio of the magnetic moment of Δ^{++} and proton to be 2, and it predicts the ratio of the magnetic moment of Ω^- and Λ^0 to be 3.

The analysis of Ref. [1] finds the ratio $\mu(\Delta^{++})/\mu(p)$ to be 1.62 ± 0.18, which is significantly smaller than the additive quark model. Most models however predict a ratio 2 or higher, a value that is only compatible with older experiments.

We recently investigated the predictive power of a relativistic constituent quark model formulated on the light front [4]. It provides a simple model wherein we have overall an excellent and consistent picture of the magnetic moments and of the semileptonic decays of the baryon octet. The parameters of this model are the constituent quark mass m and the scale parameter β, which is a measure for the size of the baryon. All parameters except β for Ω^- have been determined and fixed in Ref. [4]. We predict the magnetic moments for the baryon decuplet and find that the ratios $\mu(\Delta^{++})/\mu(p)$ and $\mu(\Omega^-)/\mu(\Lambda^0)$ are in excellent agreement with the experiment.

In Sec. II we give a brief summary of our model as described in Ref. [4]. Section III contains the explicit expressions for the magnetic moments of the baryon decuplet. The numerical results are presented in Sec. IV, and are compared with experiment and other calculations. We summarize our investigation in a concluding Sec. V.

II. QUARK-MODEL WAVE FUNCTION

The constituent quark model that we are going to use is described in a previous paper [4]. The model is formulated on the light front, which is specified by the invariant hypersurface $x^+ = x^0 + x^3 = 0$.

We can write, for instance, the Δ^{++} as

$$|\Delta^{++}\rangle = (uuu)\chi\phi, \tag{2.1}$$

with χ being the spin wave function and ϕ being the momentum distribution. For the latter we choose a function of M_0^2 defined in Eq. (2.3) of Ref. [4]. In particular we choose the same harmonic oscillator and pole-type wave functions as in Ref. [4]:

$$\begin{aligned} \phi_H &= N_H \exp\left(-M_0^2/2\beta^2\right), \\ \phi_P &= N_P (1 + M_0^2/\beta^2)^{-3.5}, \end{aligned} \tag{2.2}$$

with N_H and N_P being the normalization constants. The spin-$\frac{3}{2}$ wave functions χ with z components $\frac{3}{2}$ and $\frac{1}{2}$ are given by

$$\begin{aligned} \chi_{\frac{3}{2}} &= \uparrow\uparrow\uparrow, \\ \chi_{\frac{1}{2}} &= (\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow)/\sqrt{3}. \end{aligned} \tag{2.3}$$

In order to get the baryon to be an eigenfunction of the spin operator we still have to rotate the quark spins by the Melosh transformation [5] as given by Eqs. (3.7) and (3.10) of Ref. [4]. The wave functions for the other baryons of the decuplet are obtained by changing uuu by the corresponding flavor wave function.

III. MAGNETIC MOMENTS FOR THE BARYON DECUPLET

The electromagnetic current matrix element for spin $\frac{3}{2}$ particles can be written as

$$\langle p', s'|J^\mu(0)|p, s\rangle = \bar{u}_\alpha(p', s')\mathcal{O}^{\alpha\mu\beta}u_\beta(p, s), \tag{3.1}$$

where $u_\alpha(p, s)$ is a Rarita-Schwinger spin-vector with momentum p and spin s. The Lorentz covariant form for the tensor $\mathcal{O}^{\alpha\mu\beta}$ may be written as

$$\mathcal{O}^{\alpha\mu\beta} = g^{\alpha\beta}\left(F_1\gamma^\mu + \frac{F_2}{2M_B}i\sigma^{\mu\nu}K_\nu\right) + \frac{K^\alpha K^\beta}{2M_B^2}\left(F_3\gamma^\mu + \frac{F_4}{2M_B}i\sigma^{\mu\nu}K_\nu\right) \quad (3.2)$$

with momentum transfer $K = p' - p$ and baryon mass M_B. For $K^2 = 0$ the form factors F_1 and F_2 are, respectively, equal to the charge and the anomalous magnetic moment, and the magnetic moment is $\mu = F_1(0) + F_2(0)$. For this analysis we are therefore not interested in the form factors F_3 and F_4. We express the form factors F_1 and F_2 in terms of the plus component of the current:

$$F_1(0) = \left\langle p, \frac{3}{2}\middle| J^+ \middle| p, \frac{3}{2}\right\rangle,$$
$$K_\perp F_2(0) = 2\left[\sqrt{3}M_B\left\langle p', \frac{1}{2}\middle| J^+ \middle| p, \frac{3}{2}\right\rangle + K_\perp\left\langle p, \frac{3}{2}\middle| J^+ \middle| p, \frac{3}{2}\right\rangle\right]. \quad (3.3)$$

By inserting our wave function from Eq. (2.1) into the spin-conserving matrix element $\langle p, \frac{3}{2}|J^+|p, \frac{3}{2}\rangle$ we get the charge of the baryon. For the spin-flipping matrix element we get

$$\left\langle p', \frac{1}{2}\middle| J^+ \middle| p, \frac{3}{2}\right\rangle = K_\perp f, \quad (3.4)$$

where f is given for the different baryons as

$$\begin{aligned} f(\Delta^{++}) &= 2I_\Delta, \\ f(\Delta^+) &= I_\Delta, \\ f(\Delta^0) &= 0, \\ f(\Delta^-) &= -I_\Delta, \\ f(\Sigma^{*+}) &= (4I_{\Sigma^*}^{(2)} - I_{\Sigma^*}^{(3)})/3, \\ f(\Sigma^{*0}) &= (I_{\Sigma^*}^{(2)} - I_{\Sigma^*}^{(3)})/3, \\ f(\Sigma^{*-}) &= (-2I_{\Sigma^*}^{(2)} - I_{\Sigma^*}^{(3)})/3, \\ f(\Xi^{*0}) &= (2I_{\Xi^*}^{(3)} - 2I_{\Xi^*}^{(2)})/3, \\ f(\Xi^{*-}) &= (-I_{\Xi^*}^{(3)} - 2I_{\Xi^*}^{(2)})/3, \\ f(\Omega^-) &= -I_\Omega. \end{aligned} \quad (3.5)$$

The integral I is given by

$$I = \frac{N_c}{(2\pi)^6}\int d^3q d^3Q|\phi|^2(A_1 + A_2 + A_3)/\sqrt{3}, \quad (3.6)$$

where the quantities A_i are

$$A_1 = \frac{\eta\left(a - \frac{Q_\perp^2}{2(1-\eta)M_0}\right)}{a^2 + Q_\perp^2}\frac{c^2}{c^2 + q_\perp^2},$$

$$A_2 = \frac{\eta\left(a - \frac{Q_\perp^2}{2(1-\eta)M_0}\right)}{a^2 + Q_\perp^2}\frac{d^2}{d^2 + q_\perp^2},$$

$$A_3 = \frac{\frac{Q_\perp^2}{2M_0} - \eta b}{b^2 + Q_\perp^2}.$$

TABLE I. The parameter of the constituent quark model for the harmonic oscillator wave function (H) and for the pole-type (P) wave function. Both the quark masses m and the scale parameter β are given in units of GeV.

Parameters	H	P
$m_u = m_d$	0.26	0.263
m_s	0.38	0.38
β_Δ	0.55	0.607
β_{Σ^*}	0.60	0.75
β_{Ξ^*}	0.62	0.90
β_Ω	1.00	1.80

The quantities $a, b, c, d, \eta, q_\perp$, and $Q_\perp$ are defined in Eqs. (2.1) and (3.8) of Ref. [4]. Note that for equal u and d quark masses there is an equality $A_1 = A_2$ under the integral. The masses m_i in our equations are set as follows ($m = m_u = m_d$):

$$\begin{aligned} I_\Delta &: m_1 = m_2 = m_3 = m\ , \\ I_{\Sigma^*}^{(2)} &: m_1 = m_3 = m, \quad m_2 = m_s\ , \\ I_{\Sigma^*}^{(3)} &: m_1 = m_2 = m, \quad m_3 = m_s\ , \\ I_{\Xi^*}^{(2)} &: m_1 = m_3 = m_s, \quad m_2 = m\ , \\ I_{\Xi^*}^{(3)} &: m_1 = m_2 = m_s, \quad m_3 = m\ , \\ I_\Omega &: m_1 = m_2 = m_3 = m_s\ . \end{aligned}$$

In the nonrelativistic limit, $\beta/m \to 0$, and for equal quark masses the integral I does vanish.

IV. RESULTS AND DISCUSSIONS

We have calculated the magnetic moments of the decuplet baryons using Eqs. (3.3)–(3.6). The parameters of the model, the constituent quark mass m, and the scale parameter β, have been determined and fixed by a successful fit to the electroweak properties of the baryon octet ($\beta_\Delta = \beta_N, \beta_{\Sigma^*} = \beta_\Sigma, \beta_{\Xi^*} = \beta_\Xi$) [4]. The only free

TABLE II. Magnetic moments of the baryon decuplet. The calculations of the present work with the harmonic oscillator (H) wave function and the pole-type (P) wave function are compared with experiment (Expt), with the simple nonrelativistic quark model (NQM), with a lattice calculation (Latt), and with the Skyrme model (Skyr). The magnetic moments are given in units of the nuclear magneton. References are given in the text.

Baryon	Expt	H	P	NQM	Latt	Skyr
Δ^{++}	4.52 ± 0.50	4.76	4.93	5.56	6.09	4.53
Δ^+		2.38	2.47	2.73	3.05	2.09
Δ^0		0.00	0.00	−0.09	0.00	−0.36
Δ^-		−2.38	−2.47	−2.92	−3.05	−2.80
Σ^{*+}		1.82	1.84	3.09	3.16	2.55
Σ^{*0}		−0.27	−0.28	0.27	0.33	−0.02
Σ^{*-}		−2.36	−2.41	−2.56	−2.50	−2.60
Ξ^{*0}		−0.60	−0.56	0.63	0.58	0.40
Ξ^{*-}		−2.41	−2.41	−2.20	−2.08	−2.31
Ω^-	-1.94 ± 0.17	−2.35	−2.37	−1.84	−1.73	−1.98

TABLE III. Comparison of our calculations (H) and (P) of the magnetic moments for the Δ^{++} and Ω^- with other calculations and experiment (Expt). The calculations are the simple nonrelativistic quark model (NQM), lattice calculations (Latt), Skyrme model (Skyr), cloudy bag model (CB), Bethe-Salpeter formalism (BS), an additive quark model based on effective quark masses (EM), and a calculation including relativistic corrections (RC's). The experimental value given for Δ^{++} has some model dependence. All numbers are given in units of the nuclear magneton. References are given in the text.

Magnetic moment	Expt	H	P	NQM	Latt	Skyr	CB	BS	EM	RC's
$\mu(\Delta^{++})$	4.52 ± 0.50	4.76	4.93	5.56	6.09	4.53	6.54	4.44	–	–
$\mu(\Delta^{++})/\mu(p)$	1.62 ± 0.18	1.69	1.75	2.00	2.18	1.98	2.34	1.59	–	–
$\mu(\Omega^-)$	-1.94 ± 0.17	-2.35	-2.37	-1.84	-1.73	-1.98	-2.52	–	-1.69	-2.25
$\mu(\Omega^-)/\mu(\Lambda^0)$	3.16 ± 0.28	3.41	3.43	3.00	3.6	3.73	4.13	–	2.77	3.66

parameter is β_Ω. The parameters for both wave functions ϕ_H and ϕ_P in Eq. (2.2) are summarized in Table I. The corresponding results for the magnetic moments for both wave functions (H) and (P) are given in Tables II and III, together with other calculations.

In the simple nonrelativistic quark model (NQM) [6] the magnetic moment of a decuplet baryon is the sum of the magnetic moments of each quark composing the baryon. This is quite different to the fact that the magnetic moment is derived from the elastic electron scattering at nonzero momentum transfer. The lattice result (Latt) is taken from a recent lattice simulation of quenched QCD [7]. The other model calculations include results from a cloudy bag (CB) model [8], the Skyrme model (Skyr) [9], a Bethe-Salpeter (BS) formalism [10], an additive quark model [11] with effective quark masses (EM), and a calculation in which relativistic corrections (RC's) to the baryon magnetic moments are considered [12]. The experimental value (Expt) for the Δ^{++} is taken from a recent pion bremsstrahlung analysis [1], and the experimental value for the Ω^- is taken from a recent investigation where Ω^- hyperons are produced by a polarized neutral spin transfer reaction [2].

It is instructive to compare the different results of the ratios of the magnetic moments $\mu(\Delta^{++})/\mu(p)$ and $\mu(\Omega^-)/\mu(\Lambda^0)$. In the NQM these ratios are parameter free and given to be 2 and 3, respectively.

The experimental value for $\mu(\Delta^{++})/\mu(p)$ of Ref. [1] is lower than 2 even if we include the uncertainty of the model dependence of the measurement (± 0.16). Only the BS calculation and our result are in excellent agreement with experiment. The results from the lattice and CB models are even larger than 2, a result only compatible with older experimental values [13,14]. In this sense the magnetic moment of the Δ^{++} is a good mean to distinguish between the different models.

The experimental value for $\mu(\Omega^-)/\mu(\Lambda^0)$ of Ref. [2] is slightly higher than 3. It is noteworthy that every model beyond the NQM gives also a value larger than 3, except for EM. Only the NQM and our calculation agree with experiment. Our result for $\mu(\Omega^-)$ is quite insensitive to the choice of the parameter β. If we choose β to be 0.7 and 1.05 for the harmonic oscillator and pole type wave function, respectively, the ratio $\mu(\Omega^-)/\mu(\Lambda^0)$ would be 3.49 for both cases, only about 2% larger than the value in Table III.

V. SUMMARY

The magnetic moments of the baryon decuplet are calculated in a relativistic constituent quark model using the light front formalism. The parameters of the model are fixed by fitting the baryon octet physics, except for β_Ω. It is a challenge for every hadronic model to get consistent values for the magnetic moments for both the Δ^{++} and Ω^-. Our calculation for the magnetic moment ratios $\mu(\Delta^{++})/\mu(p)$ and $\mu(\Omega^-)/\mu(\Lambda^0)$ is in excellent agreement with the experimental ratios.

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