# Chiral effective action with heavy-quark symmetry

M. A. Nowak,* M. Rho,† and I. Zahed
*Department of Physics, State University of New York at Stony Brook, Stony Brook, New York 11794*
(Received 10 May 1993)

We derive an effective action combining chiral and heavy-quark symmetry, using approximate bosonization techniques to QCD-inspired models of spontaneous symmetry breaking. We explicitly show that the heavy-quark limit is compatible with the large $N_c$ (number of color) limit in the meson sector, and estimate the couplings between the light and heavy mesons ($D, D^*, \ldots$) and their chiral partners to order one in the heavy-quark mass. The relevance of this effective action to solitons with heavy quarks is briefly discussed.

PACS number(s): 11.30.Rd, 11.30.Hv, 14.40.Jz

The constraints of chiral symmetry on low-energy processes have led to a wealth of predictions ranging from strong to weak interactions. In general, chiral symmetry constrains the dynamics of pions and kaons by organizing the scattering amplitudes in powers of the light meson momenta. Recently, it was suggested that chiral symmetry also put constraints on the soft part of processes involving pions and heavy mesons such as $D$ and $B$ [1].

It was suggested by Shuryak [2] and more recently by Isgur and Wise [3,4] that if the mass of one quark is taken to infinity, the dynamics of the heavy quark $Q$ is independent of its mass and spin. [We will refer to this limit as the Isgur-Wise IW limit.] As a result, a new spin symmetry develops in hadronic processes involving one heavy quark, leading to a degeneracy of, say, $D$ with $D^*$ and $B$ with $B^*$. Several relations [3,4] have been recently derived showing that the excitation spectra and form factors are indeed independent of the mass and spin of the heavy quark, a result analogous to the hydrogen atom.

The purpose of this paper is to provide a short derivation of the effective action for processes involving heavy and light but nonstrange mesons, using an approximate bosonization scheme to QCD-motivated models such as the instanton liquid model [5], potential models [6], Nambu–Jona-Lasinio–type models [7] and others [8]. This approach is equivalent to the bound-state approach using zero-momentum Bethe-Salpeter kernels for the same models. Our construction shows the necessity of including the chiral partners of the heavy mesons, and is consistent with general symmetry arguments. It provides for an estimation of the axis-vector couplings, masses, and mixed axial-vector couplings of the heavy mesons and their chiral partners to order one in the heavy quark mass. In the presence of the light vector mesons our effective action differs from the one presently in use in the literature [1]. We show that the heavy-quark (IW) limit is compatible with the large $N_c$ (number of color) limit in the meson sector, and argue that heavy baryons *may* be described as solitons, as previously suggested [9] in the context of the Callan-Klebanov model [10] and recently advocated by Manohar and co-workers [11]. While our method could be readily extended to strange mesons as well, here we shall restrict our discussion to two light flavors $q=(u,d)$ and a heavy flavor ($Q$). The heavy mesons ($\bar{u}Q, \bar{d}Q$) transform as singlets under chiral symmetry.

If the mass of the heavy quark is infinitely large, then the heavy quark momentum is large and conserved $P_\mu = m_Q v_\mu$. In this limit, there is a velocity superselection rule [17]. In the effective theory with heavy quarks, this translates to a different heavy-quark (antiquark) field $Q_V^\pm(x)$ for each velocity $v$. The latter carries momenta of the order of the QCD scale $\Lambda$ and will be referred to as soft. To display this we follow Georgi [17] and define

$$Q(x)=\frac{1+\not{v}}{2}e^{-im_Q v\cdot x}Q_v^+(x)+\frac{1-\not{v}}{2}e^{im_Q v\cdot x}Q_v^-(x) \ . \quad (1)$$

As a result, the free QCD action reads

$$S=\sum_v \int d'x[\bar{q}(i\not{\partial}-m_q)q+\bar{Q}_v(i\not{v}\cdot\partial)\bar{Q}_v] \ . \quad (2)$$

The action (2) is flavor $U(2)_L \times U(2)_R$ symmetric (for $m=0$) and invariant under independent spin rotations of the quark and the antiquark (Isgur-Wise symmetry). The latter follows from the fact that the spin effects are down by powers of $1/m_Q$. Finally, the decomposition (1) is invariant under velocity shifts of the order of $\Lambda$—a point recently stressed by Luke and Manohar [18]. These conclusions are unaffected by the introduction of gluons to leading order.

Approximate bosonization schemes for QCD-inspired models have been discussed extensively in the literature [5–8]. We will apply them here to the heavy-light system. The idea consists of integrating out the short wavelength ($k \gg \Lambda$) content of the light quarks generating massive constituent quarks with multiquark interactions (as in the instanton liquid model, for instance) admixed

*Permanent address: Institute of Physics, Jagellonian University, 30-059 Cracow, Poland.

†Permanent address: Service de Physique Théorique, C.E. Saclay, 91191 Gif-sur-Yvette, France.

 

with bare but soft heavy quarks. In the long-wavelength limit, approximate bosonization schemes can be used to generate an effective action as a gradient expansion in the slowly varying fields that intermingles heavy-light dynamics.

Specifically, if we denote the heavy meson fields by

$$\hat{H}_{\pm}=\frac{1+\not{v}}{2}(\gamma^{\mu}\hat{P}^{*}_{\mu,\pm}+i\gamma_5\hat{P}_{\pm})\gamma_5^{\pm}+\text{H.c.}\ , \tag{3}$$

where $\hat{P}^{a}_{+}\sim\bar{q}^{a}_{R}Q_v$, $\hat{P}^{*a}_{\mu,+}\sim\bar{q}^{a}_{R}\gamma_{\mu}Q_r$ ($+\rightarrow-$ corresponds to $R\rightarrow L$) are the pseudoscalar and vector *bare* heavy mesons with specific light chirality, then standard arguments yield

$$S=\sum_{v}\int\bar{\psi}[\mathbb{1}_2 i\not\partial+\mathbb{1}_3 i\not{v}v\cdot\partial+\mathbb{1}_2(\hat{\not{L}}\gamma_5^{+}+\hat{\not{R}}\gamma_5^{-})$$
$$-\mathbb{1}_2(M\gamma_5^{+}+M^{\dagger}\gamma_5^{-})+\hat{H}_{+}+\hat{H}_{-}]\psi\ . \tag{4}$$

Here $\hat{L}_{\mu}\sim\bar{q}_L\gamma_{\mu}q_L$ and $\hat{R}_{\mu}\sim\bar{q}_R\gamma_{\mu}q_R$ are the bare light vector fields, valued in $U(2)_L$ and $U(2)_R$ respectively, *open* $1_2=\text{diag}(1,1,0)$, $\mathbb{1}_3=\text{diag}(0,0,1)$ are the projectors onto the light and heavy sectors, respectively, and we are using the shorthand notation $\gamma_5^{\pm}\equiv\frac{1}{2}(1\pm\gamma_5)$. The quark field $\psi$ in (4) stands for $\psi=(q;Q_v)$. The $\hat{P}$'s are off diagonal in flavor space, so that $H=H_aT^a$ with $a=1,2$ and $T^1=(\lambda_4-i\lambda_5)/2, T^2=(\lambda_6-i\lambda_7)/2$.

The action (4) enjoys both heavy-quark spin symmetry denoted $SU(2)_Q$ and rigid chiral symmetry $U(2)_L\times U(2)_R$. We have set the light-quark masses to zero and ignored the usual assumption-dependent part related to the effective potential in $M^{\dagger}M$. Here we will just assume that chiral $U(2)_L\times U(2)_R$ is spontaneously broken to $SU(2)_v$ with the appearance of three Goldstone bosons [the chiral anomaly taking care of the $U(1)_A$] around a chiral symmetric condensate. With this in mind, we will decompose the $2\times2$ complex matrix $M$ as follows (a pertinent choice of gauge to avoid doubling of the Goldstone bosons will be specified below):

$$M=\xi_L^{\dagger}\Sigma\xi_R \tag{5}$$

with the $\xi$'s as elements in the coset

$$SU(2)_L\times SU(2)_R/SU(2)_V\ .$$

In the vacuum (saddle point) $\Sigma$ is diagonal and constant along the light directions $(u,d)$. As a result, the "bosonized" action is in addition invariant under local $SU(2)_v$ symmetry: $\xi_L\rightarrow h(x)\xi_L g_L^{\dagger}$, $\xi_R\rightarrow h(x)\xi_R g_R^{\dagger}$, and $\Sigma\rightarrow h(x)\Sigma h(x)^{\dagger}$.

The constituent (dressed) quark field $\chi$ relates to the bare quark field $\psi$ through $\chi_{L,R}=(\xi_{L,R}q_{L,R};Q_v)$. In terms of the constituent field, the "bosonized" action reads

$$S=\sum_{v}\int d^4x\,\bar{\chi}(\mathbb{1}_2 i\not\partial+\mathbb{1}_3 i\not{v}v\,\partial+\mathbb{1}_2(\xi_R i\not\partial\xi_R^{\dagger}\gamma_5^{+}+\xi_L i\not\partial\xi_L^{\dagger}\gamma_5^{-})$$
$$+\mathbb{1}_2(\xi_L\hat{\not{L}}\xi_L^{\dagger}\gamma_5^{+}+\xi_R\hat{\not{R}}\xi_R^{\dagger}\gamma_5^{-})-\mathbb{1}_2(\Sigma\gamma_5^{+}+\Sigma^{+}\gamma_5^{-})+\bar{H}+H+\bar{G}+G)\chi\ . \tag{6}$$

It is invariant under local $SU(2)_V$ symmetry and global $SU(2)_Q$ symmetry. Now, let us define the dressed fields

$$L_{\mu}=\xi_L\hat{L}_{\mu}\xi_L^{\dagger}+i\xi_L\partial_{\mu}\xi_L^{\dagger}\ ,$$
$$R_{\mu}=\xi_R\hat{R}_{\mu}\xi_R^{\dagger}+i\xi_R\partial_{\mu}\xi_R^{\dagger}\ , \tag{7}$$

with

$$H=\frac{1+\not{v}}{2}(\gamma^{\mu}P^{*}_{\mu}+i\gamma_5P)$$
$$=\frac{1+\not{v}}{2}[\gamma^{\mu}(P^{*}_{\mu,+}\xi_R^{+}+P^{*}_{\mu,-}\xi_L^{+})+i\gamma_5(P_{+}\xi_R^{+}+P_{-}\xi_L^{+})], \tag{8}$$
$$G=\frac{1+\not{v}}{2}(\gamma^{\mu}\gamma_5Q^{*}_{\mu}+Q)$$
$$=\frac{1+\not{v}}{2}[\gamma^{\mu}\gamma_5(P^{*}_{\mu,+}\xi_R^{+}-P^{*}_{\mu,-}\xi_L^{+})+(P_{+}\xi_R^{+}-P_{-}\xi_L^{+})]\ ,$$

where the new $H$ and $G$ fields refer to $(D,D^{*})$ and their *chiral partners* $(\tilde{D},\tilde{D}^{*})$, respectively. The heavy vector fields are transverse $v^{\mu}P^{*}_{\mu}=v^{\mu}Q^{*}_{\mu}=0$. The "bosonized" action in the dressed fields becomes

$$S=\sum_{v}\int d^4x\,\bar{\chi}[\mathbb{1}_2(i\not\nabla_L-\Sigma)\gamma_5^{-}+\mathbb{1}_2(i\not\nabla_R-\Sigma)\gamma_5^{+}$$
$$+\mathbb{1}_3 i\not{v}v\,\partial+H+\bar{H}+G+\bar{G}]\chi\ , \tag{9}$$

where the covariant $L,R$ derivatives are $\nabla_L=\partial-iL$ and $\nabla_R=\partial-iR$. $\bar{H}$ and $\bar{G}$ are related, respectively, to $H$ and $G$ through

$$\bar{H}=\gamma^0H^{+}\gamma^0\ ,\quad \bar{G}=\gamma^0G^{+}\gamma^0\ . \tag{10}$$

The light-light and heavy-quark–light-quark dynamics follows from the dressed action (9) through a derivative expansion. The momenta are bounded from above by a cutoff $\Lambda_c$ (of the order of the chiral-symmetry-breaking scale). Our expansion of (9) will be understood in the sense of $m_Q/\Lambda_c\rightarrow\infty$.

To second order, the heavy-light induced action reads

$$S_{\mu}=N_c\text{Tr}(\mathbb{1}_2\Delta_l H\mathbb{1}_3\Delta_h\bar{H})$$
$$-N_c\text{Tr}[\mathbb{1}_2\Delta_l(\not{V}\Delta_l+\not{A}\Delta_l\gamma_5)H\mathbb{1}_3\Delta_h\bar{H}]+\cdots\ , \tag{11}$$

where the functional trace includes tracing over space, flavor, and spin indices. We have denoted $\Delta_l=(i\not\partial\Sigma)^{-1}$ and $\Delta_h=(i\not{v}v\,\partial)^{-1}$, and

$$\gamma_5^{+}\not{R}+\gamma_5^{-}\not{L}=\not{V}+\gamma_5\not{A}\ . \tag{12}$$

The ellipsis in (11) stands for higher insertions of vectors and axial vectors. A similar action is expected for the heavy chiral partners $G$'s. To the order considered, there are also cross terms generated by the axial-vector current.

After carrying to the trace over space in (4) and renormalizing the heavy-quark fields, $H \to H/\sqrt{Z_H}$ and $G \to G/\sqrt{Z_G}$, we obtain, to leading order in the gradient expansion for the $H$'s ($s_l^{P_l} = \frac{1}{2}^-$),

$$\mathcal{L}_v^H = -\frac{i}{2}\,\mathrm{Tr}(\bar{H}v^\mu\partial_\mu H - v^\mu\partial_\mu\bar{H}H) + \mathrm{Tr}V_\mu\bar{H}Hv^\mu - g_H\mathrm{Tr}A_\mu\gamma^\mu\gamma_5\bar{H}H + m_H\mathrm{Tr}\bar{H}H \tag{13}$$

and, for the $G$'s ($s_l^{P_l} = \frac{1}{2}^+$),[1]

$$\mathcal{L}_v^G = +\frac{i}{2}\,\mathrm{Tr}(\bar{G}v^\mu\partial_\mu G - v^\mu\partial_\mu\bar{G}G) - \mathrm{Tr}V_\mu\bar{G}Gv^\mu - gG\mathrm{Tr}A_\mu\gamma^\mu\gamma 5\bar{G}G + m_G\mathrm{Tr}\bar{G}G\ . \tag{14}$$

The parameters in (13) and (14) are given by ($P_l = \pm$ is the parity of the light part of $H, G$)

$$Z_{P_l} = N_c\int_0^{\Lambda_c}\frac{d^4Q}{(2\pi)^4}\left[-\frac{1}{vQ+i\epsilon}\,\frac{1}{Q^2-\Sigma^2+i\epsilon} + P_l\frac{2\Sigma}{Q^2-\Sigma^2+i\epsilon}\right],$$
$$g_{P_l} = \frac{N_c}{Z_{P_l}}\int_0^{\Lambda_c}\frac{d^4Q}{(2\pi)^4}\,\frac{1}{vQ+i\epsilon}\,\frac{\Sigma^2-Q^2/3}{(Q^2-\Sigma^2+i\epsilon)^2}\ , \tag{15}$$
$$m_{P_l} = \frac{N_c}{Z_{P_l}}\int_0^{\Lambda_c}\frac{d^4Q}{(2\pi)^4}\left[\frac{1}{Q^2-\Sigma^2+i\epsilon} + P_l\frac{1}{vQ+i\epsilon}\,\frac{\Sigma}{Q^2-\Sigma^2+i\epsilon}\right].$$

Note that $m_H$ and $m_G$ are of order $m_Q^0$ The behavior of the couplings and masses in (15) as a function of the dimensionless ratio $x = \Sigma/\Lambda_c$ is shown in Fig. 1. (The masses are plotted in units of $\Lambda_c$.) For $x \sim 0$, $g_H \sim g_G \sim \frac{1}{3}$, while for $x \sim \frac{1}{3}$, $g_H \sim \frac{1}{3}$, and $g_G \sim 1$.[2] The coupling constant $g_G$ is very sensitive to the value of the constituent quark mass following the relative cancellation in $Z_G$. These estimates are to be contrasted with those of the nonrelativistic quark model, which give $g_H = g_A$, $g_G = g_A/3$ [13], with $g_A = 0.75$ [14].

The interaction between the $H$'s and the $G$'s is given by

$$\mathcal{L}_v^{HG} = +\left[\frac{Z_H}{Z_G}\right]^{1/2}\mathrm{Tr}(\gamma_5\bar{G}Hv^\mu A_\mu) - \left[\frac{Z_G}{Z_H}\right]^{1/2}\mathrm{Tr}(\gamma_5\bar{H}G\gamma^\mu A_\mu)\ . \tag{16}$$

[1]We label $H$ and $G$ by the total angular momentum $s_l$ and parity $P_l$ of the *light* degrees of freedom.

[2]The recent CLEO Collaboration data suggest $g_H \approx 0.58$ with a large error bar. See Ref. [12] for an analysis.

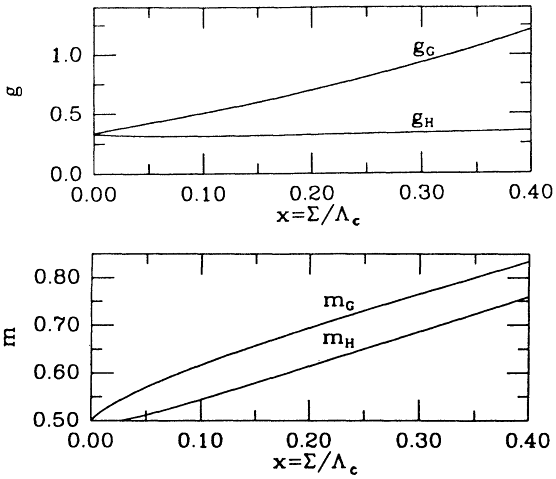


FIG. 1. Coupling constants $g_H$ and $g_G$ and mass parameters $m_H$ and $m_G$ (in units of the cutoff $\Lambda_c$) as a function of the ratio $\Sigma/\Lambda_c$, where $\Sigma$ is the constituent mass.

Notice that $Z_G/Z_H = g_H/g_G$ reduces to 1 in the heavy quark limit for $x \ll 1$. We observe that the mass splittings between the $H$'s $(D, D^*, \ldots)$ and their chiral partners the $G$'s $(\tilde{D}, \tilde{D}^*, \ldots)$ imply the following mass relations to order $m_Q^0 N_c^0$:

$$m(\tilde{D}^*) - m(D^*) = m(\tilde{D}) - m(D) = O(\Sigma)\ , \tag{17}$$

where again $\Sigma$ is the constituent mass of the light quarks in the chiral limit. This is expected since the $D$ and $D^*$ are $S$-wave mesons, while the $\tilde{D}$ and $\tilde{D}^*$ are $P$-wave mesons (not yet observed).[3] In the nonrelativistic quark model the difference is centrifugal and of order $m_Q^0$. This point has been appreciated already by Shuryak in the context of bag models and QCD sum rules [2]. In our case, this difference is of order $\Lambda_c/10$, a somehow smaller value than the naive estimate $\Sigma$ due to the presence of large $x\ln x$ corrections in (15).

Since the decomposition (5) doubles the scalar degrees of freedom, a proper gauge fixing in $\xi$'s is required. We choose the "unitary gauge" $\xi_L^\dagger = \xi_R \equiv \xi = e^{i\pi/2f_\pi}$. In a minimal model with *only* pions (model I), we have, for the vector and axial-vector currents,

$$V_\mu = +\frac{i}{2}(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi)\ ,$$
$$A_\mu = +\frac{i}{2}(\xi\partial_\mu\xi^\dagger - \xi^\dagger\partial_\mu\xi)\ . \tag{18}$$

If we were to introduce vector mesons $[\omega, \rho, a_1(1270)]$ (model II) then the vector and axial-vector currents are *entirely* vectorial:

[3]The observed $D_1(2400)$ and $D_2^*(2460)$ are components of the $s_l^{P_l} = \frac{3}{2}^+$ multiplet.

$$
\begin{aligned}
V_\mu &= \omega_\mu + \rho_\mu \;, \\
A_\mu &= \mathcal{A}_\mu - \frac{\alpha}{f\pi}\partial_\mu \pi \;,
\end{aligned}
\tag{19}
$$

where $\alpha=(m_\rho/m_A)^2$ follows after eliminating the $\pi a_1$ mixing at the tree level in the light-light sector [5–8]. The relations (19) identify the vector fields with the constituent vector currents, e.g., $\omega_\mu \sim \bar{\chi}\gamma_\mu\chi$, as can be checked explicitly in (9). They also enforce vector dominance in the light-light sector. Clearly other constructions are also theoretically possible in which the amount of pion dressing is intermediate between models I and II, using the relations (7). Of course, the issue of how the effective fields are physically defined (dressed) can only be resolved by comparing the various model predictions with experiment.

The renormalized effective action involving both $H$ and $G$ and their interactions is invariant under local SU $(2)_v$ [or more precisely U$(2)_v$ including the singlet] symmetry $(h)$, in which $V$ transforms as a gauge field, $A$ transforms covariantly and $H$, $G \rightarrow Hh^\dagger$, $Gh^\dagger$, and $\bar{H}$, $\bar{G} \rightarrow h\bar{H}$, $\bar{G}$. It is also invariant under heavy-quark symmetry SU$(2)_Q$ $(S)$, $H,G \rightarrow SH,SG$ and $\bar{H}$, $\bar{G} \rightarrow \bar{H}S^\dagger$, $\bar{G}S^\dagger$.

Our renormalized effective action (13) with only pions (model I) is entirely consistent with the one suggested by Wise and others [1] (aside from the mass term). Our derivation suggests $g_H \sim \frac{1}{3}$ with a specific *sign* assignment for the axial term. The rest of our effective action is also consistent with the effective action written down recently in Ref. [16].[4]

The introduction of vector mesons along with pions (model II) which is a more realistic description of the light-light sector yields a totally *new* effective action for the heavy-light sector. The identification of the currents is entirely vectorial in this case. The pion coupling to the heavy particles occurs *solely* through the longitudinal component of the axial-vector current. In this case the $\pi$-$HH$ coupling is quenched in comparison with the $a_1$-$HH$ coupling (1 in this case),

$$
g_{\pi HH} = (m_\rho/m_A)^2/3 \sim 0.12 \;,
$$

decreasing the decay width $D^* \rightarrow D\pi$ of model I by about $\frac{1}{2}$, in the heavy-quark limit.

Our arguments suggest that in the heavy-quark limit and in the chiral limit, a similar effective action should involve the chiral partners of the heavy pseudoscalars and vectors, here denoted by $G$. Their role in the soliton scenario might be important for the description of opposite-parity heavy-baryon states. They also allow for a qualitative estimation for the coefficients involved, and provide a rationale for the systematic expansion in both $k_\pi/(\sqrt{N_c}\Lambda)$ (derivative or $1/N_c$ expansion) and $\Lambda/m_Q$ (heavy-quark expansion).

The scaling with $N_c$ of the overall heavy-light effective action before renormalization, suggests that the heavy-light system should be entrusted with the same weight as the light-light counterpart (which is well known and hence omitted in our discussion) in the large $N_c$ limit, implying that a soliton description for heavy baryons is perhaps justified [9]. The large $N_c$ limit appears to be compatible with the heavy-quark limit in the meson sector. This point is *a priori* not obvious since terms of the form $m_Q/N_c\Lambda$ and others cannot be ruled out on general grounds. Thus our approximate bosonization scheme yields a long wavelength description that appears to be consistent with the one advocated recently in Ref. [1].

We note that in the meson sector the $HH\pi$ interaction is of order $1\sqrt{N_c}$ as expected. In the soliton sector, however, the effect of this interaction is, in *principle*, of order $N_c$, since we expect $H \sim \sqrt{N_c}$ and $\pi \sim \sqrt{N_c}$ (i.e., semiclassical field). However, in the heavy-quark limit, the heavy meson field is, *in general*, localized over a range of the order of $1/(N_c m_Q)$ affecting the energy to order $N_c^0$. Hence, one would expect heavy baryons composed of pions and $P,P^*$ (or $Q,Q^*$) to emerge to order $N_c^0$ or lower.[5] The emergency of the heavy baryon spectrum depends crucially on whether the $P$ and $P^*$ bind to the soliton to order $N_c^0$ [11,19].

*Note added.* After submitting our work for publication we became aware of the following papers where some of the issues raised in our paper are also discussed: R. Casalbuoni *et al.*, Phys. Lett. B **299**, 139 (1993); J. Schechter and A. Subbaraman, Phys. Rev. D **48**, 332 (1993); Yu. L. Kalinovsky and L. Kaschluhn, "Bilocal field theory approach and semileptonic heavy meson decays," DESY Report No. 92-071 (unpublished).

This work has been supported in part by DOE Grant No. DE-FG02-88ER40388 and by KBN Grant No. PB 2675/2. We are grateful for comments from J. Milana, D.-P. Min, N. N. Scoccola, E. Shuryak, and J. J. M. Verbaarschot.

[4]In the convention of Refs. [15,16] the $P$-wave heavy quark field $H_\alpha$ relates to our $G$ through $H_\alpha = G(v_\alpha - \gamma_\alpha)/\sqrt{3}$.

[5]The sharp localization of the heavy quark field $H$ in the heavy mass, large $N_c$ limit may be at odds with the soft character of $H$.

[1] M. B. Wise, Phys. Rev. D **45**, R2188 (1992); T.-M. Yan *et al.*, *ibid.* **46**, 1148 (1992); G. Burdman and J. F. Donoghue, Phys. Lett. B **280**, 287 (1992).
[2] E. V. Shuryak, Phys. Lett. **93B**, 134 (1980); Nucl. Phys. **B198**, 83 (1982).
[3] N. Isgur and M. B. Wise, Phys. Rev. Lett. **66**, 1130 (1991), and references given there.
[4] E.g., M. B. Wise, in *Particle Physics—The Factory Era*, Proceedings of the Lake Louise Winter Institute, Lake Louise, Canada, 1991, edited by B. A. Campbell, A. N. Kamal, P. Kitching, and F. C. Khanna (World Scientific, Singapore, 1991).

[5] D. I. Diakonov and V. Yu. Petrov, Nucl. Phys. **B272**, 457 (1986); R. Alkofer, M. A. Nowak, J. J. M. Verbaarschot, and I. Zahed, Phys. Lett. B **233**, 205 (1989).

[6] R. T. Cahill, J. Praschifka, and C. D. Roberts, Phys. Rev. D **36**, 209 (1987); Ann. Phys. (N.Y.) **188**, 20 (1988).

[7] D. Ebert and H. Reinhard, Phys. Lett. B **173**, 453 (1986); A. Dhar and S. R. Wadia, Phys. Rev. Lett. **52**, 959 (1984).

[8] See, e.g., A. Andrianov and Y. Novozhilov, Phys. Lett. **153B**, 422 (1985); A. Andrianov, *ibid.* **157B**, 425 (1985); N. I. Karchev and A. A. Slavnov, Theor. Math. Phys. **65**, 1099 (1986); P. Simic, Phys. Rev. Lett. **55**, 40 (1985); I. Zahed and G. Brown, Phys. Rep. **142**, 1 (1986); R. D. Ball, in *Skyrmions and Anomalies,* Proceedings of the Workshop, Krakow, Poland, 1987, edited by M. Jeżabek and M. Praszalowicz (World Scientific, Singapore, 1987), p. 54.

[9] M. Rho, D. O. Riska, and N. N. Scoccola, Phys. Lett. B **251**, 597 (1990); Z. Phys. A **341**, 343 (1992); Y. Oh, D.-P. Min, M. Rho, and N. N. Scoccola, Nucl. Phys. **A534**, 493 (1991).

[10] C. G. Callan and I. Klebanov, Nucl. Phys. **B262**, 365 (1985).

[11] E. Jenkins, A. V. Manohar, and M. B. Wise, Nucl. Phys. **B396**, 27 (1993); Z. Guralnik, M. Luke, and A. V. Manohar, *ibid.* **B390**, 474 (1993); E. Jenkins and A. V. Manohar, Phys. Lett. B **294**, 273 (1992).

[12] P. Cho and H. Georgi, Phys. Lett. B **296**, 408 (1992); **300**, 410(E) (1993).

[13] A. F. Falk and M. Luke, Phys. Lett. B **292**, 119 (1992).

[14] A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).

[15] A. F. Falk, Nucl. Phys. **B378**, 79 (1992).

[16] U. Kilian, J. G. Körner, and D. Pirjol, Phys. Lett. B **288**, 360 (1992).

[17] H. Georgi, Phys. Lett. B **240**, 447 (1990); in *Proceedings of the 1991 Theoretical Advanced Study Institute,* Boulder, Colorado, 1991, edited by R. K. Ellis, C. T. Hill, and J. D. Lykken (World Scientific, Singapore, 1992), p. 589.

[18] M. Luke and A. Manohar, Phys. Lett. B **286**, 348 (1992).

[19] M. A. Nowak, M. Rho, and I. Zahed, Phys. Lett. B **303**, 130 (1993).