No parity violation without R-parity violation

Ravi Kuchimanchi and R. N. Mohapatra
Department of Physics, University of Maryland, College Park, Maryland 20742
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In a class of minimal supersymmetric left-right models of weak interactions where R parity is automatically conserved, we show that the spontaneous breakdown of parity cannot occur without the spontaneous breakdown of R parity. This intriguing result connects two physically different scales in supersymmetric models.

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I. INTRODUCTION

An awkward aspect of the minimal supersymmetric standard model (MSSM) is the presence of R-parity-violating[1] terms in the superpotential [1], which are allowed by gauge invariance. These terms lead to lepton and baryon-number violation and their strengths are therefore severely limited by phenomenological [1] and cosmological constraints [2]. In fact, unless the strength of the baryon-number-violating term is less than 10^{-13}, it will lead to a contradiction with present lower limits on the lifetime of the proton. It would therefore be more appealing to have a supersymmetric theory where R-parity conservation is automatic [3,4]. It was noted some time ago [3] that if the gauge symmetry of the supersymmetric model is extended to $SU(2)_L\times U(1)_{I_{3R}}\times U(1)_{B-L}$ or $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$, the theory becomes *automatically* R-parity conserving. One can then entertain the possibility of spontaneous R-parity violation [5,6]. There are two distinct differences between spontaneous and explicit R-parity violations: The first is that the catastrophic baryon-number-violating terms are automatically absent from the Lagrangian, in the spontaneous R-parity-violation case, so that proton stability is guaranteed; and the second is that, in this case, above a certain temperature, R parity is restored so that the picture of the early Universe is very different; for instance, any preexisting lepton or baryon asymmetry of the Universe need not be erased. Furthermore, the strength of R-parity-violating terms is no longer arbitrary but is connected to the scale of R-parity breaking.

In this paper, we show that, in the minimal supersymmetric left-right (SUSYLR) model with the seesaw mechanism [7], the spontaneous breaking of parity [8], and R parity are intimately linked. By a detailed analysis of the Higgs potential of the model, we show that if R parity remains unbroken so does the parity symmetry of the model, even after one-loop radiative corrections are taken into account. Therefore, a necessary condition for parity violation to occur is the existence of the spontaneous breakdown of R parity. This is in our opinion an interesting result since it connects the breaking of two different symmetries and links their breaking scales. Since, in this class of models, the scale of parity breaking is connected to the neutrino mass [9] via the seesaw mechanism, the R-parity-breaking scale and the strength of the induced R-parity-breaking interactions get related to the neutrino masses. Some aspects of symmetry breaking for these models were studied earlier in [10] but this connection between breaking R parity and parity was not noticed in this paper.

The rest of the paper is organized as follows: In Sec. II, we write down the most general tree-level Higgs potential for the minimal SUSYLR model; in Sec. III, we derive the inequality constraints between the parameters of the tree-level Higgs potential by requiring that it be bounded from below and use these constraints to prove that there can be no parity violation without R-parity violation for the three-level potential; in Sec. IV, we show that even in certain nonminimal SUSYLR models R parity must be violated or else either electric charge is not conserved by the ground state of the theory or parity symmetry is unbroken; in Sec. V, we prove that if R parity is violated spontaneously, then in the minimal SUSYLR model itself, the ground state can violate parity and conserve electric charge; and in Sec. VI, we use the freedom of choosing a convenient renormalization scale to argue that in the minimal SUSYLR model radiative corrections do not change the tree-level result that there can be no parity violation without R-parity violation, since otherwise electric charge is not conserved.

II. THE HIGGS POTENTIAL FOR THE SUSY LEFT-RIGHT MODEL

The matter content of the minimal model is given in Table I. In this paper the doublets are represented by 2×1 column vectors, the triplets by 2×2 traceless complex matrices, and the bidoublet Φ is represented by a 2×2 complex matrix. Also note that,[2] for example, under the $SU(2)_L\times SU(2)_R$ part of the gauge transformation, $L\to U_L L$, $\Delta\to U_L\Delta U_L^\dagger$, $\tau_2 L^c\to U_R(\tau_2 L^c)$, $\tau_2\Delta^c\tau_2\to U_R(\tau_2\Delta^c\tau_2)U_R^\dagger$, and $\Phi\to U_L\Phi U_R^\dagger$, where U_L and U_R

[1]A particle of baryon number B, lepton number L, and spin S has R parity $(-1)^{3B+L+2S}$. Thus, all the familiar particles such as the quarks, leptons, and Higgs boson are R-parity even and their superpartners are R-parity odd.

[2]In this paper τ_1, τ_2, τ_3 stand for the Pauli matrices.

are the $SU(2)_L$ and $SU(2)_R$ group transformations (similarly for $\bar{\Delta}$ and $\bar{\Delta}^c$). The gauge-invariant superpotential [10,11] for the model is given by[3]

$$W=h_q Q^T\tau_2\Phi\tau_2 Q^c+hL^T\tau_2\Phi\tau_2 L^c +if(L^T\tau_2\Delta L+L^{cT}\tau_2\Delta^c L^c) +M\,\mathrm{Tr}(\Delta\bar{\Delta}+\Delta^c\bar{\Delta}^c)+\mu'\,\mathrm{Tr}(\tau_2\Phi^T\tau_2\Phi)\ . \quad (1)$$

The most general form of the Higgs potential including soft breaking terms (but omitting the Q terms) is given by [10,11] (denoting the scalar components of the superfields by $\tilde{L}$, $\tilde{L}^c$, Φ, Δ, $\bar{\Delta}$, Δ^c, and $\bar{\Delta}^c$)

$$V=V_{F\ \mathrm{terms}}+V_{\mathrm{soft}}+V_{D\ \mathrm{terms}}\ , \quad (2)$$

where

TABLE I. The matter content of minimal SUSYLR model.

Matter superfield	$SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ quantum number
Q	$(2,1,\frac{1}{3})$
Q^c	$(1,2,-\frac{1}{3})$
L	$(2,1,-1)$
L^c	$(1,2,+1)$
Higgs fields	
Δ	$(3,1,+2)$
Δ^c	$(1,3,-2)$
$\bar{\Delta}$	$(3,1,-2)$
$\bar{\Delta}^c$	$(1,3,+2)$
Φ	$(2,2,0)$

$$V_{F\ \mathrm{terms}}=|h\tilde{L}^T\tau_2\Phi\tau_2+2if\tilde{L}^{cT}\tau_2\Delta^c|^2+|h\tilde{L}^{cT}\tau_2\Phi^T\tau_2+2if\tilde{L}^T\tau_2\Delta|^2+\mathrm{Tr}|h\tilde{L}^c\tilde{L}^T+2\mu'\Phi^T|^2 +\mathrm{Tr}|if\tilde{L}\tilde{L}^T\tau_2+M\bar{\Delta}|^2+\mathrm{Tr}|if\tilde{L}^c\tilde{L}^{cT}\tau_2+M\bar{\Delta}^c|^2+|M|^2\mathrm{Tr}(\Delta^\dagger\Delta+\Delta^{c\dagger}\Delta^c)\ , \quad (3)$$

$$V_{\mathrm{soft}}=m_l^2(\tilde{L}^\dagger\tilde{L}+\tilde{L}^{c\dagger}\tilde{L}^c)+(M_1^2-|M|^2)\mathrm{Tr}(\Delta^\dagger\Delta+\Delta^{c\dagger}\Delta^c)+(M_2^2-|M|^2)\mathrm{Tr}(\bar{\Delta}^\dagger\bar{\Delta}+\bar{\Delta}^{c\dagger}\bar{\Delta}^2) +[M'^2\,\mathrm{Tr}(\Delta\bar{\Delta}+\Delta^c\bar{\Delta}^c)+\mathrm{H.c.}]+(M_\phi^2-4|\mu'|^2)\mathrm{Tr}\Phi^\dagger\Phi+\left[\frac{\mu^2}{2}\mathrm{Tr}(\tau_2\Phi^T\tau_2\Phi)+\mathrm{H.c.}\right] +[iv(\tilde{L}^T\tau_2\Delta\tilde{L}+\tilde{L}+\tilde{L}^{cT}\tau_2\Delta^c\tilde{L}^c)+e\tilde{L}^T\tau_2\Phi\tau_2\tilde{L}^c+\mathrm{H.c.}]\ , \quad (4)$$

$$V_{D\ \mathrm{terms}}=\frac{g^2}{8}\sum_m|\tilde{L}^\dagger\tau_m\tilde{L}+\mathrm{Tr}(2\Delta^\dagger\tau_m\Delta+2\bar{\Delta}^\dagger\tau_m\bar{\Delta}+\Phi^\dagger\tau_m\Phi)|^2 +\frac{g^2}{8}\sum_m|\tilde{L}^{c\dagger}\tau_m\tilde{L}^c+\mathrm{Tr}(2\Delta^{c\dagger}\tau_m\Delta^c+2\bar{\Delta}^{c\dagger}\tau_m\bar{\Delta}^c+\Phi\tau_m^T\Phi^\dagger)|^2 +\frac{g'^2}{8}|\tilde{L}^{c\dagger}\tilde{L}^c-\tilde{L}^\dagger\tilde{L}+2\,\mathrm{Tr}(\Delta^\dagger\Delta-\Delta^{c\dagger}\Delta^c-\bar{\Delta}^\dagger\bar{\Delta}+\bar{\Delta}^{c\dagger}\bar{\Delta}^c)|^2\ . \quad (5)$$

Note that in Eq. (4) the coefficient of $\mathrm{Tr}(\Delta^\dagger\Delta+\Delta^{c\dagger}\Delta^c)$ is defined such that the coefficient of this term in $V_{F\ \mathrm{terms}}+V_{\mathrm{soft}}$ is simply M_1^2. Also, in the above equations the doublets, triplets, and the bidoublets have the electric charge quantum numbers

$$\tilde{L}=\begin{bmatrix}\tilde{\nu}\\ \tilde{e}^-\end{bmatrix},\quad \tilde{L}^c=\begin{bmatrix}\tilde{\nu}^c\\ \tilde{e}^+\end{bmatrix},\quad \Delta=\begin{bmatrix}\frac{\delta^+}{\sqrt{2}} & \delta^{++}\\ \delta^0 & -\frac{\delta^+}{\sqrt{2}}\end{bmatrix},\quad \Phi=\begin{bmatrix}\phi_1^0 & \phi_2^+\\ \phi_1^- & \phi_2^0\end{bmatrix} \quad (6)$$

and similarly for other fields.

Let us note that spontaneous R-parity violation in this model arises when either $\langle\tilde{\nu}^c\rangle\neq 0$ and/or $\langle\tilde{\nu}\rangle\neq 0$. Also it is obvious that parity symmetry is spontaneously broken if

$$|\langle\Delta\rangle|,|\langle\bar{\Delta}\rangle|\ll|\langle\Delta^c\rangle|\ \text{or}\ |\langle\bar{\Delta}^2\rangle|\ . \quad (7)$$

We will show that, if $\langle\tilde{\nu}\rangle$ and $\langle\tilde{\nu}^c\rangle$ both have a zero vacuum expectation value, the inequality (7) cannot be satisfied and, in fact, the ground state of the theory corresponds to $\langle\Delta\rangle=\langle\bar{\Delta}\rangle=\langle\Delta^c\rangle=\langle\bar{\Delta}^c\rangle$. We then show that as soon as $\langle\tilde{\nu}^c\rangle\neq 0$ inequality (7) emerges for a range of parameters in the theory.

III. A PROBLEM IN THE HIGGS SECTOR: NO PARITY VIOLATION

Theorem 3.1. The lowest state of the tree-level SUSY left-right theory corresponds to

$$\langle\Delta\rangle=\langle\bar{\Delta}\rangle=\langle\Delta^c\rangle=\langle\bar{\Delta}^c\rangle=\langle\Phi\rangle=0\ \text{if}\ \langle\tilde{L}^c\rangle=\langle\tilde{L}\rangle=0\ ,$$

except on one hypersurface in the parameter space which has a volume of measure zero.

[3]Note that we are imposing an exact discrete parity symmetry in the usual way.

Proof. Let us consider Eqs. (2)–(5) with $\tilde{L}=\tilde{L}^c=0$. Since we have $SU(2)_L \times SU(2)_R$ invariance we can diagonalize Φ by using a biunitary transformation. So we will work in the Φ diagonal basis and let

$$\langle \Phi \rangle = \begin{bmatrix} \kappa & 0 \\ 0 & \kappa' \end{bmatrix} . \tag{8}$$

Further without loss of generality we can assume that M'^2 and μ^2 in Eqs. (2)–(5) are real and positive. This is because we can reabsorb their phases into a redefinition of the fields $\bar{\Delta}$, $\bar{\Delta}^c$, and Φ and other coupling constants. $M_{1,2}^2$ and M_ϕ^2 are real since the Lagrangian is real. The key point now is to recognize from Eqs. (2)–(5) that for the potential to be bounded from below for the field going to infinity the mass parameters must satisfy the constraints

$$M_{1,2}^2 \geq 0, \quad M'^2 \leq M_1 M_2 , \tag{9}$$

$$M_\phi^2 \geq 0 \quad \text{and} \quad \mu^2 \leq M_\phi^2 . \tag{10}$$

The first inequality in (9) follows by looking at the directions $\langle \Phi \rangle = 0$ and $\langle \Delta \rangle = \langle \Delta^c \rangle = v^2 \tau_1$ and $\langle \bar{\Delta} \rangle = \langle \bar{\Delta}^c \rangle = 0$ and vice versa. The D terms vanish in this direction and unless the first inequality is satisfied the potential is unbounded from below for $v \to \infty$. (Since only $M_{1,2}^2$ appear in the potential we will hereafter assume that $M_{1,2} \geq 0$ and real without loss of generality.) The second inequality in (9) follows by looking along directions $\langle \Delta^c \rangle = \langle \Delta \rangle = (v^2/M_1)\tau_1$, $\langle \bar{\Delta} \rangle = \langle \bar{\Delta}^c \rangle = -(v^2/M_2)\tau_1$, and $\langle \Phi \rangle = 0$. Once again the D terms vanish and the potential is unbounded unless the inequality is satisfied. Inequalities (10) follow similarly by looking along directions $\kappa' = -\kappa$ and real and the rest of the fields equal to zero. Inequalities (9) and (10) imply that we can define angles θ and θ' such that

$$M'^2 = M_1 M_2 \cos 2\theta \quad \text{and} \quad \mu^2 = M_\phi^2 \cos 2\theta' . \tag{11}$$

Since the vacuum expectation values (VEV's) of all other fields other than the triplets and bidoublet are zero, the Higgs potential can be rewritten in terms of θ and θ' as

$$V = \cos^2\theta \, \mathrm{Tr}(M_1 \Delta^c + M_2 \bar{\Delta}^{c\dagger})^\dagger (M_1 \Delta^c + M_2 \bar{\Delta}^{c\dagger}) + \sin^2\theta \, \mathrm{Tr}(M_1 \Delta^c - M_2 \bar{\Delta}^{c\dagger})^\dagger (M_1 \Delta^c - M_2 \bar{\Delta}^{c\dagger})$$
$$+ \cos^2\theta' M_\phi^2 (\kappa + \kappa'^*)^* (\kappa + \kappa'^*) + \sin^2\theta' M_\phi^2 (\kappa - \kappa'^*)^* (\kappa - \kappa'^*) + \Delta \to \Delta^c, \bar{\Delta} \to \bar{\Delta}^c + D \text{ terms} . \tag{12}$$

Note that every term above is a norm and is hence positive semidefinite. Thus, it follows that the absolute minimum of the Higgs potential V is $V=0$ and this is obtained when

$$\langle \Delta \rangle = \langle \bar{\Delta} \rangle = \langle \Delta^c \rangle = \langle \bar{\Delta}^c \rangle = \kappa = \kappa' = 0 .$$

A few comments are in order regarding the uniqueness of the absolute minimum. The only case in which the fields can pick up VEV's and still have $V=0$ is if the following equations are satisfied:

$$M'^2 = M_1 M_2 \quad \text{and} \quad \mu^2 = M_\phi^2 . \tag{13}$$

In this case, there is a solution of the form $\langle \Delta^c \rangle = \langle \Delta \rangle = (v^2/M_1)\tau_1$, $\langle \bar{\Delta} \rangle = \langle \bar{\Delta}^c \rangle = -(v^2/M_2)\tau_1$, and $\kappa' = -\kappa$. Such a solution is unstable under radiative corrections and in any case is electric charge (Q_{em}) violating and parity conserving. If we further restrict the coupling constant space such that $M_1 = M_2$ as well as Eqs. (13) are satisfied, then there will be a solution of the form $\Delta^c = -\bar{\Delta}^{c\dagger}$, $\Delta = -\bar{\Delta}^\dagger$, which can be parity breaking and Q_{em} conserving. However, these equality relations for the mass parameters correspond to a very constrained hypersurface in the parameter space and will occur for points of measure zero in this space. Further since they are not guaranteed by any symmetry of the theory, such equality relations are unstable under radiative corrections. Thus, the true minimum of the theory conserves parity.

In this section we have shown that the minimal SUSYLR model cannot violate parity if it does not violate R parity. In the next section we show that this conclusion holds at the tree level for several extensions of the minimal SUSYLR model as well.

IV. NONMINIMAL SUSYLR MODELS, THE PROBLEM PERSISTS

Certain extensions of minimal SUSLR models have been considered in the literature. A popular model is the SUSYLR model with two Higgs bidoublet fields instead of one [11]. Another extension [12] that has been considered is the minimal SUSYLR model with an additional parity odd singlet [13]. In this section we show that even for these models the absolute minimum of the tree-level Higgs potential cannot break parity and conserve Q_{em} if it does not break R parity.

A. Minimal SUSYLR + 1 extra bidoublet field

Let

$$\langle \Phi_1 \rangle = \begin{bmatrix} \kappa_1 & 0 \\ 0 & \kappa_1' \end{bmatrix} , \quad \langle \Phi_2 \rangle = \begin{bmatrix} \kappa_2 & 0 \\ 0 & \kappa_2' \end{bmatrix} \tag{14}$$

be the VEV's of the bidoublet fields. We choose this form since it is the most general form consistent with conserving Q_{em}. The Higgs potential of the triplet and the bidoublet fields[4] is of the form

[4]Note that the VEV's of the slepton fields are set equal to zero so that R parity remains unbroken.

$$V_{\rm Higgs}=V(\Delta,\bar{\Delta},\kappa_1,\kappa_1',\kappa_2,\kappa_2') + V(\Delta^c,\bar{\Delta}^c,\kappa_1,\kappa_1',\kappa_2,\kappa_2') + \frac{g'^2}{2}[{\rm Tr}(\Delta^\dagger\Delta-\Delta^{c\dagger}\Delta^c-\bar{\Delta}^\dagger\bar{\Delta}+\bar{\Delta}^{c\dagger}\bar{\Delta}^c)]^2 \,. \tag{15}$$

Note that the above potential is invariant under $\Delta\leftrightarrow\Delta^c$, $\bar{\Delta}\leftrightarrow\bar{\Delta}^c$. Further, if we set $g'=0$, then for *any* value of κ_1, κ_1', κ_2, κ_2', the value of the triplet fields that minimize the potential of Eq. (15) absolutely is $\Delta=\Delta^c$, $\bar{\Delta}=\bar{\Delta}^c$ due to symmetry. Now let $g'^2>0$ and be arbitrary. The last term in the Higgs potential of Eq. (15) is positive semidefinite and for $\Delta=\Delta^c$, $\bar{\Delta}=\bar{\Delta}^c$ it also has its lowest value which is zero. From this it follows that at the absolute minimum $\Delta=\Delta^c$, $\bar{\Delta}=\bar{\Delta}^c$ and parity remains unbroken. Note that the same argument can be made no matter how many triplet or bidoublet fields we have in the SUSYLR theory as long as they are the only fields that pick up VEV's. We also note that as in Sec. III there is a hypersurface in the parameter space, where parity can break but this is unstable under radiative corrections.

B. Minimal SUSYLR + parity-Odd singlet

Let σ be the parity-odd singlet.[5] Now the superpotential W has the additional terms [12]

$$W=A\sigma\,{\rm Tr}(\Delta\bar{\Delta}-\Delta^c\bar{\Delta}^c)+W(\sigma) \,. \tag{16}$$

The only additional term that this introduces in the Higgs potential that is not of the form given below in Eq. (18) is

$$V_{\rm Higgs}\ \text{the new } F \text{ term}={\rm Tr}|f(\sigma)+A(\Delta\bar{\Delta}-\Delta^c\bar{\Delta}^c)|^2 \,, \tag{17}$$

where $f(\sigma)$ is a function of σ along. The key is to note that σ does not couple directly to Φ and therefore inequality (10) still holds. This implies that the sum of the quadratic terms involving Φ along are positive semidefinite. We also note that the D terms of the Higgs potential are all positive semidefinite. Thus, the only way the Higgs potential can attain a negative value at the minimum is if some of the quadratic and the cubic terms involving the triplet and/or singlet fields are negative at the absolute minimum. The *only* way in which the triplet fields occur in these terms is in the combination

$$\begin{aligned}&{\rm Tr}(\Delta^\dagger\Delta)\,,\\ &{\rm Tr}(\bar{\Delta}^\dagger\bar{\Delta})\,,\\ &{\rm Tr}(\Delta^{c\dagger}\Delta^c)\,,\\ &{\rm Tr}(\bar{\Delta}^{c\dagger}\bar{\Delta}^c)\,,\\ &{\rm Tr}(\Delta\bar{\Delta})\,,\end{aligned} \tag{18}$$

and

$${\rm Tr}(\Delta^c\bar{\Delta}^c)$$

and their products with the singlet field. We will now show that given any configuration of the triplet fields that conserves $Q_{\rm em}$ there is a configuration that violates $Q_{\rm em}$ for which the value of these terms is *unchanged.* The most general form of the triplet fields that conserves $Q_{\rm em}$ is

$$\langle\Delta^c\rangle=\begin{bmatrix}0&0\\ \delta'&0\end{bmatrix},\quad \langle\Delta\rangle=\begin{bmatrix}0&0\\ \delta&0\end{bmatrix},\quad \langle\bar{\Delta}^c\rangle=\begin{bmatrix}0&\bar{\delta}'\\ 0&0\end{bmatrix},\quad \langle\bar{\Delta}\rangle=\begin{bmatrix}0&\bar{\delta}\\ 0&0\end{bmatrix}. \tag{19}$$

It is easy to check that the following $Q_{\rm em}$-violating form preserves the values of the terms in Eq. (18):

$$\langle\Delta^c\rangle=\frac{1}{\sqrt{2}}\begin{bmatrix}0&\delta'\\ \delta'&0\end{bmatrix},\quad \langle\Delta\rangle=\frac{1}{\sqrt{2}}\begin{bmatrix}0&\delta\\ \delta&0\end{bmatrix},\quad \langle\bar{\Delta}^c\rangle=\frac{1}{\sqrt{2}}\begin{bmatrix}0&\bar{\delta}'\\ \bar{\delta}'&0\end{bmatrix},\quad \langle\bar{\Delta}\rangle=\frac{1}{\sqrt{2}}\begin{bmatrix}0&\bar{\delta}\\ \bar{\delta}&0\end{bmatrix}. \tag{20}$$

In addition, by substituting the above $Q_{\rm em}$-violating form in the D terms of the Higgs potential it is easy to see that together with the choice $\Phi=0$, all the positive semidefinite D terms and quadratic terms in Φ that we referred to earlier take their lowest possible value which is zero. The only other term that remains is the F term in Eq. (17) and it is easy to see that this term is minimized too. Thus, the $Q_{\rm em}$-violating configuration has a lower value for the potential than the $Q_{\rm em}$-conserving form.[6] Thus, we conclude that the absolute minimum of the tree-level Higgs potential violates $Q_{\rm em}$ if R parity is not broken.

In the Appendix we investigate some more extensions of the minimal SUSYLR model and show that minimal SUSYLR + extra bidoublets + parity-odd singlet also has the same problem of violating $Q_{\rm em}$. We further show in the Appendix that minimal SUSYLR + parity even singlet cannot break parity without breaking R parity either. In the next section we show that if we violate R parity then in the minimal SUSYLR model itself, we can obtain absolute minima that violate parity and preserve $Q_{\rm em}$.

V. R PARITY VIOLATION CURES THE PROBLEM

In this section we show that by giving $\tilde{\nu}^c$ a VEV and thereby breaking R parity spontaneously, we can salvage the minimal SUSYLR theory. We will further demon-

[5]Under Parity $\sigma\to-\sigma$.

[6]Except on a hypersurface in the parameter space of the Higgs potential where the two forms have the same values for the potential. The reader is referred to the discussion at the end of Sec. III for such a case.

strate that we can have a stable nontrivial solution[7] for the absolute minimum which conserves $Q_{\rm em}$ at the tree level itself without needing quantum corrections for stability.

We will now find a region in the coupling constant space such that the absolute minimum for the tree-level Higgs potential breaks parity but conserves $Q_{\rm em}$. We will do this in perturbation by treating certain parameters as small.

Let us consider the Higgs potential in Eqs. (2)–(5) and let us choose a region in coupling constant space such that g^2 and g'^2 are much smaller than the other dimensionless coupling constants h^2 and f^2. This ensures that the troublesome D terms that were responsible for breaking $Q_{\rm em}$ and preserving parity are weaker than the quartic terms and trilinear terms between the triplet and slepton fields. Also, in order to achieve the hierarchy $\langle L^c\rangle, \langle \Delta^c\rangle \gg \langle \Phi \rangle$ we will require that the coupling constants and mass terms involving Φ are smaller than those not involving them. Under these assumptions the potential in Eqs. (2)–(5) simplifies to

$$\begin{aligned}V=&m_l^2(\tilde L^\dagger \tilde L+\tilde L^{c\dagger}\tilde L^c)+M_1^2\,\mathrm{Tr}(\Delta^\dagger\Delta+\Delta^{c\dagger}\Delta^c)+M_2^2\,\mathrm{Tr}(\bar\Delta^\dagger\bar\Delta+\bar\Delta^{c\dagger}\bar\Delta^c)+|h|^2\tilde L^{c\dagger}\tilde L^c\tilde L^\dagger\tilde L\\&+|f|^2[(\tilde L^\dagger\tilde L)^2+(\tilde L^{c\dagger}\tilde L^c)^2]+4|f|^2(|\tilde L^{cT}\tau_2\Delta^c|^2+|\tilde L^T\tau_2\Delta|^2)+M'^2\,\mathrm{Tr}(\Delta\bar\Delta+\Delta^c\bar\Delta^c+\mathrm{H.c.})\\&+[\tilde L^T\tau_2(iv\Delta+iM^*f\bar\Delta^\dagger)\tilde L+\tilde L^{cT}\tau_2(iv\Delta^c+iM^*F\bar\Delta^{c\dagger})\tilde L^c+\mathrm{H.c.}]\ .\end{aligned}\tag{21}$$

In the above we will require that inequalities (9), (10), and $|M|<M_2$ be satisfied so that there is no direction where $V\to-\infty$. We will now use the $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ invariance to choose

$$\langle \tilde L^c\rangle=\begin{bmatrix}l'\\0\end{bmatrix},\quad \langle \tilde L\rangle=\begin{bmatrix}l\\0\end{bmatrix}, \tag{22}$$

with l and l' real and positive. Now let the triplet fields have their most general form

$$\langle \bar\Delta^c\rangle=\begin{bmatrix}\frac{a}{\sqrt2} & \bar\delta'\\ b & \frac{-a}{\sqrt2}\end{bmatrix}$$

and

$$\langle \Delta^c\rangle=\begin{bmatrix}\frac{c}{\sqrt2} & d\\ \delta' & \frac{-c}{\sqrt2}\end{bmatrix}. \tag{23}$$

Substituting these VEV's into Eq. (21) it is easy to see that the only terms involving a, b, c, or d are

$$M_1^2(|c|^2+|d|^2)+M_2^2(|a|^2+|b|^2)+(M'^2(ac+bd)+\mathrm{H.c.})+2|f|^2|c|^2l'^2\ , \tag{24}$$

which are minimized when $a=b=c=d=0$ due to inequalities (9). By similar arguments we can show that for the absolute minimum, all the triplet fields *must* have the $Q_{\rm em}$-conserving form of Eq. (19). The task now is to determine the unknown VEV's in Eqs. (22) and (19) by substituting them into Eq. (21) and minimizing the potential. Since the equations for the minima will be coupled cubic equations, they are a bit complicated in general. Therefore, we will solve the equations by treating M_1^2 and M'^2 as perturbations that are smaller than the other mass parameters and will neglect them in what follows. Also we will assume that all coupling constants, including v and f, are real. These requirements, though not essential, will considerably simplify the mathematics and keep it from being messy. Also note that our main goal is to prove that there exists at least *a* region in the parameter space where the absolute minimum breaks parity and conserves $Q_{\rm em}$. Thus, the potential in Eq. (21) becomes

$$\begin{aligned}V=&m_l^2(l'^2+l^2)+M_2^2(|\bar\delta'^2|+|\bar\delta^2|)+f^2(l'^4+l^4)\\&+4f^2(l'^2|\delta'^2|+l^2|\delta^2|)+h^2l'^2l^2\\&+[v(l'^2\delta'+l^2\delta)+fM(l'^2\bar\delta'+l^2\bar\delta)\\&\quad+\text{complex conjugate}]\ .\end{aligned}\tag{25}$$

The first thing to note is that since the only terms in Eq. (25) that care about the phases of the fields are the trilinear terms, all fields will pick up real VEV's. This is because if v is positive (note that it is real by assumption), δ' will minimize the potential by being real and negative (rather than having a complex phase) and if v is negative, δ' will be positive. The same argument goes through for other fields and hence in what follows we will keep all fields real. The conditions for the extrema of Eq. (25) are

$$\begin{aligned}\frac{\partial V}{\partial l'}&=(m_l^2+4f^2\delta'^2+h^2l^2+2v\delta'+2fM\bar\delta')l'+2f^2l'^3\\&=0\ ,\\\frac{\partial V}{\partial\delta'}&=4f^2l'^2\delta'+vl'^2=0\ ,\\\frac{\partial V}{\partial\bar\delta'}&=M_2^2\bar\delta'+fMl'^2=0\ ,\end{aligned}\tag{26}$$

and three corresponding equations with $l'\leftrightarrow l$, $\delta'\to\delta$, and $\bar\delta'\to\bar\delta$. The solutions for the extrema are the following.

[7]By this we mean that at least some fields pick up nonzero VEV's and there are no directions in the tree-level potential wherein $V\to-\infty$.

(1) The trivial solution:[8]

$$l=l'=\bar{\delta}'=\bar{\delta}=0 \quad \text{for which } V=V_{\rm TS}=0 \ . \qquad (27)$$

(2) The parity-breaking solution:[9]

$$\begin{aligned} &l=\bar{\delta}=0,\quad \delta'=-\frac{v}{4f^2},\quad \bar{\delta}'=\frac{fMl'^2}{M_2^2}\ ,\\ &l'^2=l'^2_{\rm PBS}=\frac{(v^2-4f^2m_l^2)M_2^2}{8f^4(M_2^2-M^2)}\ ,\\ &V=V_{\rm PBS}=-f^2\left[1-\frac{M^2}{M_2^2}\right]l'^4_{\rm PBS}\ . \end{aligned} \qquad (28)$$

(3) The parity-preserving solution:

$$\begin{aligned} &\delta'=\delta=-\frac{v}{4f^2},\quad \bar{\delta}'=\bar{\delta}=-\frac{fMl'^2}{M_2^2}\ ,\\ &l'^2=l^2=l'^2_{\rm PPS}=\frac{(v^2-4f^2m_l^2)M_2^2}{8f^4\{M_2^2[1+(h^2/2f^2)-M^2\}}\ ,\\ &V=V_{\rm PPS}=-2\left[f^2\left[1-\frac{M^2}{M_2^2}\right]+\frac{h^2}{2}\right]l'^4_{\rm PPS}\ . \end{aligned} \qquad (29)$$

Note that, in the above, the parity-breaking solution will exist if

$$v^2-4f^2m_l^2>0\ . \qquad (30)$$

Further the parity-breaking solution will be the absolute minimum if

$$V_{\rm PBS}<V_{\rm PPS}\ . \qquad (31)$$

Substituting for $V_{\rm PBS}$ and $V_{\rm PPS}$ for Eqs. (28) and (29) we find that inequality (31) is satisfied if

$$h^2>2f^2\frac{M_2^2-M^2}{M_2^2}\ . \qquad (32)$$

Thus, we have obtained a region in parameter space such that parity is broken. In practice, this requires existence of large leptonic Yukawa couplings which can only arise if there is a fourth generation of fermions.

If we substitute the VEV's given by Eqs. (28) into the Higgs potential in Eqs. (2)–(5), then they will act as sources for the Φ field to pick up a VEV. We will now show that $\langle\Phi\rangle$ at the absolute minimum does not break $Q_{\rm em}$.

Since $g^2, g'2 \ll h^2$ the most dominant coupling term between Φ and the other right-handed fields is the positive definite term:

$$h^2\langle\tilde{L}^{c\dagger}\rangle\tau_2\Phi^\dagger\Phi\tau_2\langle\tilde{L}^c\rangle\ . \qquad (33)$$

Let $\langle\Phi\rangle$ take its most general form given by

$$\langle\Phi\rangle=\begin{bmatrix}\kappa & a\\ b & \kappa'\end{bmatrix}\ . \qquad (34)$$

Substituting for $\tilde{L}^c$ from Eqs. (22) and (28) into (33) and minimizing, we obtain $\kappa'=a=0$. The only other term in Eqs. (2)–(5) that couples Φ to the VEV of the other right-handed fields is the cross term in the D term of the Higgs potential which on substitution is

$$\frac{g^2}{4}(l'^2-2\delta'^2+2\bar{\delta}'^2)(|\kappa|^2+|b|^2)\ . \qquad (35)$$

If[10] $2\delta'^2>l'^2+2\bar{\delta}'^2$ this acts as a source for $|\kappa|^2+|b|^2$ to pick up a VEV. At this stage there are two degenerate absolute minima of the form $\kappa\neq 0$ *and* $b=0$ or $b\neq 0$ *and* $\kappa=0$. Either κ or b is zero because if both have nonzero values the D terms will contribute extra positive amounts to the potential due to cross terms between them. The reason the absolute minimum has $\kappa\neq 0$ and $b=0$ is because the trilinear terms between Φ, $\tilde{L}$, and $\tilde{L}^c$ couple only to the electrically neutral component κ and not b. The phase of κ will be determined by these terms so that they contribute the maximal negative amount to the potential and this will split the degeneracy between the two minima in favor of the $Q_{\rm em}$-conserving one. Note that a small value of $\tilde{L}$ itself is induced in this process. κ can be easily calculated by writing down all dominant terms in the Higgs potential as follows:

$$V(\text{terms involving }\kappa)=\frac{g^2}{4}\left[\left[4\frac{M_\phi^2}{g^2}+l'^2-2\delta'^2+2\bar{\delta}'^2\right]|\kappa|^2+|\kappa|^4\right]\ . \qquad (36)$$

Minimizing with respect to κ we obtain

$$|\kappa|^2=\delta'^2-\bar{\delta}'^2-\tfrac{1}{2}l'^2-2\frac{M_\phi^2}{g^2}\ . \qquad (37)$$

Note that in order that $\kappa\ll\delta'$ we have the usual problem of fine-tuning the terms in Eq. (36). Once κ picks up a VEV it acts as a source for κ' (and not a or b) to pick up a VEV due to the term $(\mu^2/2){\rm Tr}\tau_2\Phi^T\tau_2\Phi$ and we can find the values of κ' by minimizing the dominant terms: namely,

$$V(\text{dominant terms involving }\kappa')=\mu^2\kappa\kappa'+\text{H.c.}+(M_\phi^2+h^2l'^2)|\kappa'|^2\ . \qquad (38)$$

A point worth noting is the hierarchy $\kappa'\ll\kappa$ is automatically obtained in this mechanism.

Finally, we note that l, δ, $\bar{\delta}$ will pick up small VEV's due to the trilinear couplings with the fields that have already picked up VEV's. The reason these fields will pick

[8]The alert reader will notice that when the perturbation $M_1^2>0$ is turned on it will force $\delta=\delta'=0$.

[9]The perturbation $M_1^2>0$ will imply that $\delta=0$ for the minimum.

[10]This condition can be easily arranged by choosing m_l^2 positive and large enough as can be seen from Eq. (28).

up Q_{em}-conserving VEV's is that only the electrically neutral components of these fields couple to the other VEV's and so only they can be induced.

Thus, we have shown that there exists a region in parameter space of the Higgs potential for minimal SUSLR model such that parity and R parity are broken and Q_{em} is conserved. In the process we have obtained several inequality relations between the coupling constants in the Higgs potential and they can be useful for phenomenology.

VI. EFFECT OF RADIATIVE CORRECTIONS

In Sec. III we showed that at tree level there is no parity violation without R-parity violation in the minimal SUSYLR model. In this section we will argue that this result is true even after radiative corrections are taken into account. Since for phenomenological reasons $\langle \Phi \rangle \ll \langle \Delta^2 \rangle$, we will set $\langle \Phi \rangle = 0$ and will only consider VEV's for the triplets.[11] The effective potential after radiative corrections are taken into account is

$$V_{\rm eff} = V + V_{\rm rad}(Q) , \tag{39}$$

where V is the tree-level Higgs potential in Eq. (2) and $V_{\rm rad}(Q)$ is the radiative correction evaluated using the usual Coleman-Weinberg technique [14] at a renormalization scale Q. Note that all parameters of the theory depend on Q such that $V_{\rm eff}$ is independent of Q [15,16]. In other words,

$$\frac{dV_{\rm eff}}{dQ} = 0 , \tag{40}$$

where the mass parameters and the coupling constants of the theory are functions of Q. Equation (40) implies that we can choose any convenient value of Q to evaluate $V_{\rm eff}$. One of the standard ansatz [15,16] in supersymmetric theories is to choose Q such that

$$\left. \frac{\partial V_{\rm rad}(Q)}{\partial \chi} \right|_{\chi = \langle \chi \rangle, \text{rest of fields at their VEV's too}} = 0 , \tag{41}$$

where χ is a scalar field (or a linear combination of scalar fields) in the theory. Note that the mass parameters and other coupling constants of the theory are now functions of $\hat{Q}$ where $Q = \hat{Q}$ solves Eq. (41). Physically Eq. (41) implies that if we choose a configuration infinitesimally different from the vacuum configuration for the field χ, then Q is chosen such that the radiative corrections for the two configurations are the same. For our problem we will use a prescription slightly different from Eq. (41) but which is physically very similar.

Let us assume that the triplet fields pick up Q_{em}-conserving VEV's of the form given by (19) even after radiative corrections are taken into account. We will call this Q_{em}-conserving configuration of fields configuration 1. Let us consider a slightly different Q_{em}-violating configuration (configuration 2): namely,

$$\Delta^c = \delta' \begin{bmatrix} 0 & \sin\theta \\ \cos\theta & 0 \end{bmatrix} , \qquad \bar{\Delta}^2 = \bar{\delta}' \begin{bmatrix} 0 & \cos\theta \\ \sin\theta & 0 \end{bmatrix} , \tag{42}$$

where θ is any *fixed* angle. Let us choose Q such that the radiative corrections [$V_{\rm rad}$ in Eq. (39)] for configuration 1 and configuration 2 are equal at $Q = \hat{Q}$. This is our renormalization prescription. Of course, as discussed in the previous paragraph, all coupling constants in the tree-level Higgs potential in Eqs. (2)–(5) are not function of $\hat{Q}$. Let us now compare the values of the effective potential $V_{\rm eff}$ in Eq. (39) for the two configurations. Our renormalization prescription immediately implies that

$$V_{\rm eff}(\text{configuration } 2) - V_{\rm eff}(\text{configuration } 1) = V(\text{configuration } 2) - V(\text{configuration } 1) . \tag{43}$$

Substituting the forms of the triplet fields for both configurations into V given by Eqs. (2)–(5), it is easy to see that above difference is negative no matter what values the coupling constants and mass parameters in the Higgs potential V take. Note that we do not even have to assume that inequalities (9) and (10) are satisfied. However, we require $g^2(\hat{Q}) > 0$, which is actually a requirement from gauge invariance since g is a gauge coupling constant and hence is purely real. Once again it is the D term whose value is lower for the Q_{em}-violating configuration while all other terms have the same value for both configurations 1 and 2. Thus, choosing a suitable renormalization prescription we have argued that in the minimal SUSLR model Q_{em} is violated if R parity is not broken even after quantum effects are taken into account.

VII. CONCLUSION

In summary, we have shown that in a class of minimal supersymmetric left-right models where R-parity symmetry is automatic in the symmetry limit, spontaneous breakdown of parity *requires* spontaneous breakdown of R parity. We have also argued that this result established for the tree-level potential is unlikely to be effected when one-loop effects are included. This intriguing result connects two physically different scales in supersymmetric models. Its phenomenological implications will be the subject of a future publication.

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[11] We have checked that an argument similar to the one in this section can be made even if $\langle \Phi \rangle \neq 0$.

APPENDIX

In this appendix we prove that minimal SUSYLR+extra bidoublets+parity-odd singlet also has the unsatisfactory feature of having only Q_{em}-violating absolute minimum if R parity is not broken. We also show that for minimal SUSYLR+parity-even singlet model the value of the tree-level potential for parity-conserving VEV's is always lower than the value for VEV's that violate parity maximally if R parity is unbroken.

1. Minimal SUSYLR +extra bidoublets+parity-odd singlets

The terms in the Higgs potential that involve the bidoublets are of the form $\mathrm{Tr}(\Phi_i^\dagger\Phi_j)$, $\mathrm{Tr}(\tau_2\Phi_i^T\tau_2\Phi_j)$, where the indices i and j run over the bidoublets in the theory and the D terms. To keep things simple let us consider two bidoublets and let them take $Q_{\rm em}$-conserving VEV's of the form in Eq. (14). Note that the above-mentioned terms are all invariant under $\kappa_1\leftrightarrow\kappa_1'$, $\kappa_2\leftrightarrow\kappa_2'$. Because of this symmetry it is easy to see that the most general tree-level Higgs potential involving the bidoublets alone can be written in the matrix notation

$$V=(\kappa\kappa')^\dagger\begin{bmatrix}M & B\\ B & M\end{bmatrix}\begin{bmatrix}\kappa\\ \kappa'\end{bmatrix}+\frac{g^2}{4}(\kappa^\dagger\kappa-\kappa'^\dagger\kappa')^2\ , \tag{A1}$$

where the 4×4 matrices M and B are Hermitian and

$$\kappa^\dagger=(\kappa_1^*\kappa_2^*\kappa_1\kappa_2),\quad \kappa'^\dagger=(\kappa_1'^*\kappa_2'^*\kappa_1'\kappa_2')\ . \tag{A2}$$

In the direction $\kappa'=\pm\kappa$, the quartic term in Eq. (44) vanishes. This implies that if the potential is to be bounded from going to $-\infty$ the mass matrix in Eq. (44) must satisfy the following constraints for any arbitrary κ:

$$(\kappa\kappa)^\dagger\begin{bmatrix}M & B\\ B & M\end{bmatrix}\begin{bmatrix}\kappa\\ \kappa\end{bmatrix}\geq 0\ ,$$
$$(\kappa-\kappa)^\dagger\begin{bmatrix}M & B\\ B & M\end{bmatrix}\begin{bmatrix}\kappa\\ -\kappa\end{bmatrix}\geq 0\ . \tag{A3}$$

Now note that any *general* configuration $(\kappa\kappa')$ can be written as

$$\begin{bmatrix}\kappa\\ \kappa'\end{bmatrix}=\frac{1}{2}\begin{bmatrix}\kappa+\kappa'\\ \kappa+\kappa'\end{bmatrix}+\frac{1}{2}\begin{bmatrix}\kappa-\kappa'\\ -\kappa+\kappa'\end{bmatrix}\ . \tag{A4}$$

Substituting the above in Eq. (44) it is easy to see that the contribution from the quadratic terms in κ and κ' is always positive semidefinite due to inequalities (46). This was the only information from the Φ sector which was needed to prove that the absolute minimum violates $Q_{\rm em}$ in Sec. IV B. Thus, following the same steps as in Sec. IV B, we can conclude that no matter how many bidoublets or parity-odd singlets we have, $Q_{\rm em}$ is violated if R parity is not broken.

2. Minimal SUSYLR+parity-even singlet

Let ω be the parity-even singlet.[12] The superpotential W has the additional terms

$$W=B\omega\,\mathrm{Tr}(\Delta\bar{\Delta}+\Delta^c\bar{\Delta}^c)+C\omega\,\mathrm{Tr}(\tau_2\Phi^T\tau_2\Phi)+W(\omega)\ . \tag{A5}$$

The only additional term that this introduces in the Higgs potential that is not quadratic in the triplet or bidoublet fields is

$$V_{\text{new }F\text{ term}}=\mathrm{Tr}|f(\omega)+B(\Delta\bar{\Delta}+\Delta^c\bar{\Delta}^c)+C\tau_2\Phi^T\tau_2\Phi|^2\ . \tag{A6}$$

Consider a configuration that violates parity maximally, namely, $\Delta=\bar{\Delta}=0$ and $\Delta^c=X,\bar{\Delta}^c=Y$, where X and Y are $Q_{\rm em}$-conserving VEV's. We will show that regardless of the VEV of the Φ field there exists a parity-conserving configuration which lowers the value of the tree-level configuration.[13] The reader may verify that this configuration is $\Delta=\Delta^c=X/\sqrt{2}$ and $\bar{\Delta}=\bar{\Delta}^2=Y/\sqrt{2}$. Basically the parity-conserving configuration lowers the values of the positive semidefinite D terms, regardless of the value of the diagonal Φ field. The values of the quadratic terms and the new F term are invariant for the two configurations.

We will end this section here though the proof only considered initial configurations that violated parity maximally and did not consider cases where $\langle\Delta\rangle\neq0$ initially. Since phenomenologically $\langle\Delta\rangle\ll\langle\Delta^c\rangle$, our assumption that for parity-violating configuration $\Delta=0$ is not too bad.

Our conclusion then is that if we insist on a SUSYLR theory with triplet fields to ensure the seesaw mechanism for the neutrino masses and keep R parity intact, then we should go well beyond the minimal models in order to achieve breakdown of parity.

[12]Under parity $\omega\to\omega$.

[13]Except on one hypersurface in the parameter space where the two configurations have the same value for the potential.

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