Uses and abuses of effective Lagrangians

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Motivated by past and recent analyses we critically reexamine the use of effective Lagrangians in the literature to constrain new physics and to determine the "physics reach" of future experiments. We demonstrate that many calculations, such as those involving anomalous trilinear gauge-boson couplings, either considerably overestimate loop-induced effects or give ambiguous answers. The source of these problems is the use of cutoffs to evaluate the size of such operators in loop diagrams. In contrast with other critiques of these loop estimates, we prove that the inclusion of nonlinearly realized gauge invariance into the low-energy Lagrangian is irrelevant to this conclusion. We use an explicit example using known multi-Higgs-boson physics above the weak scale to underline these points. We show how to draw conclusions regarding the nature of the unknown high-energy physics without making reference to low-energy cutoffs.

I. INTRODUCTION

As experimentally accessible energies have risen above the thresholds for producing electroweak gauge bosons, it has become more and more clear that the mass scale associated with any new physics is probably at significantly higher energies. This is reflected by the great success of the standard model in predicting the results of these experiments in general and the properties of these gauge bosons in particular.

Given that the scale of physics beyond the standard model is well above the weak scale, the low-energy effects of such new physics may be parametrized in terms of an effective Lagrangian [1] in which the influence of any at-present-unknown new heavy particles is felt through the effective nonrenormalizable interactions that they generate among the lighter particles. These nonstandard interactions may be organized according to increasing operator dimension. At a practical level this method is useful only to the extent that it is possible to consider just those few interactions which have the lowest dimension. This can usually be justified by the suppression of higher-dimensional operators by extra powers of the inverse of some heavy-mass scale M.

This type of reasoning has led to considerable effort in using experimental data to constrain the coefficients of the operators in such an effective Lagrangian which parametrize deviations from the standard model. Of particular interest are those terms which correspond to anomalous couplings of the photon and the Z^0, since these are the probes that are currently the most cleanly available in collider experiments. Analyses have focused on the lowest electromagnetic and electroweak moments of the light fermions [2–4] as well as gauge-boson self-couplings [5–11] that would dominate interactions at low energies. In this way it is possible to ascertain which interactions could have hitherto escaped detection and might yet be detectable at upcoming experiments. Proponents of particular experiments can turn this argument around and estimate the scale M of new physics to which a particular proposal can be sensitive—its so-called "physics reach." The most interesting proposals are naturally those that are potentially sensitive to the highest scales and so whose physics reach is the longest.

A complication arises, however, when loop effects in the low-energy theory are important for detecting the effective interaction under study. This is because such loops are typically divergent and so can depend on *positive* powers of a large high-energy cutoff Λ. This cutoff physically describes the maximum energy to which the effective Lagrangian is expected to apply and so is frequently also taken to be of order of the new physics scale M. To the extent that this is true, the most divergent contributions to a given amplitude could be taken as indications of a strong dependence on new physics at scale M, potentially indicating a long physics reach.

Our main point in this paper is to show that the above argument can be very misleading and can even lead to conclusions which contradict general decoupling results [12]. At best, it gives [10] an ambiguous, and at worst, a false, indication of the scale of new physics to which a given experiment may be sensitive, often yielding overly stringent constraints on parameters in the effective Lagrangian. The weak link in the arguments used is the assumed connection between what can be computed (the cutoff dependence of amplitudes in the low-energy effective theory) and what is meant to be bounded (the dependence of low-energy amplitudes on physical high-energy physics scales such as heavy particle masses).

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In this paper we refine and expand on our results in Ref. [13] by exploring in detail this connection between low-energy cutoff dependence and heavy-mass dependence. We demonstrate our conclusions within the context of a multi-Higgs-boson model in which the influence of the high-energy physics is known and calculable. We show that cutoff dependence can be a very poor indicator of heavy-mass dependence, particularly where massive spin-1 particles are involved. We then indicate how to extract the dependence on high-frequency physics without resorting to arguments that rely on cutoffs.

In the literature the misidentification of heavy-mass and cutoff dependences arises most frequently in the context of anomalous three-gauge-boson vertices (TGV's). There are two reasons for this. First, since TGV's cannot yet be measured directly, the only available information concerning them arises indirectly through their contributions to loops. Second, since problems with interpreting the cutoff dependence arise most strikingly for loops involving massive spin-1 particles, TGV-induced loops are very easy to mishandle [10]. This has led to misleadingly stringent constraints on anomalous TGV's, as well as to mistaken predictions of large effects in future experiments, that is to say, long physics reach.

In addition to this confusion between cutoff behavior and new physics dependence, the waters have recently become even more muddied due to a parallel confusion that has risen within the specific context of TGV analyses. The authors of Ref. [11] agree that physics reach as regards anomalous TGV's is overstated in places in the literature. However, they go on to identify the error as being the gauge invariance (or lack thereof) of the analysis. (An alternative phrasing of this line of thought is to object to the use of the unitary gauge in performing loop calculations.)

The key question is whether the light particles in the effective theory being considered fill out a *linear* representation of the gauge group. They do not, for instance, if there is no light Higgs boson to transform with the longitudinal W and Z bosons. In this case, gauge invariance can only be realized nonlinearly. We contend here that, for gauge symmetries, such a nonlinear realization can be included, or not, simply by a change of variables, and so nothing physical can depend on this choice.

That this confusion can arise at all serves to underline a more pervasive hazard that underlies the association of a physical interpretation to divergences within an effective Lagrangian: The Lagrangians themselves, and so also the divergences they contain, are *not* invariant under field redefinitions. Conclusions that are based on them are generically marked by the same flaw, unless it is specifically demonstrated otherwise (as can be done for the S matrix, for example). Proposals which link cutoff dependence in the Lagrangian to heavy-mass dependence are therefore at best ambiguous, unless they are specifically referred to a set of variables which are to be used. They are simply wrong if the variables used are poorly chosen.

Some of these points are undoubtedly familiar to some of the effective-Lagrangian cognoscenti. They have not, however, been absorbed into the wider community which is now finding applications for these techniques. We therefore feel that an examination of the issues is timely given the present debate over the accuracy of estimates of physics reach and over the nature of the properties that should be built into low-energy Lagrangians.

We next expose all of these points in more detail, with reference to explicit underlying models for which both heavy-mass and cutoff dependences are separately calculable. We start in Sec. II by discussing the relevance of gauge symmetries for effective Lagrangians. In so doing we (re)demonstrate the equivalence between nonlinearly realized gauge symmetries and no gauge symmetries at all. This is followed in Sec. III by some general observations about how the cutoff dependence arises in low-energy effective theories. Section IV contains the guts of our criticism. We first present the arguments for thinking that cutoffs might track heavy masses and then criticize these arguments. We provide several examples which indicate how field redefinitions can alter the cutoff dependence and argue which variables are most likely to allow cutoffs to mimic the heavy-mass dependence in observables. In Sec. V we outline how to infer the heavy-mass dependence without having to rely on the cutoff dependence of low-energy graphs. This permits the retention of most applications of cutoff methods, but with the conceptual advantage of relying on a more solid foundation. Section VI then presents an explicit multi-Higgs-boson model for the underlying physics in which these ideas are explicitly worked out. Our conclusions are summarized in Sec. VII.

II. PERTINENCE OF GAUGE SYMMETRIES

Essentially, two ingredients are required to specify a low-energy effective Lagrangian: the low-energy particle content and the symmetries that their interactions preserve. Once these have been specified, all possible interactions of successively higher dimensions may generically be written down.

When considering the interactions of W and Z bosons, the most important distinction to be made concerns where the scale of the unknown new physics, M, lies in relation to the electroweak scale $v \simeq 246$ GeV. If M is much greater than roughly $4\pi v$, then the perturbative unitarity of the low-energy theory requires that it must linearly realize the electroweak gauge symmetries [14,15]. In this case the low-energy theory must contain more particles than have presently been discovered (such as the standard-model Higgs boson and top quark) in order for the known particles to fill out a linear representation of the gauge group. This is the choice that has been pursued in Refs. [4], [9], and [11].

We are mostly concerned in what follows with the other alternative in which the underlying physics we are groping for is the electroweak-breaking physics itself. In this case the particle content need not fall into linear representations of the gauge group and so could, in particular, consist only of those particles that have already been discovered. Since perturbative unitarity fails in this type of effective theory at energies of order $4\pi v \simeq 8\pi M_W/g$, we are guaranteed that the effective theory must fail at or

before this point. Below this scale, agreement exists in the literature as to the appropriate low-energy particle content that is to be chosen, but practitioners differ according to their choices for the symmetries that these particles should respect.

No gauge invariance. In the first approach [2,5–8,10], only electromagnetic gauge invariance is imposed, and all spontaneously broken gauge symmetries are simply ignored.

Nonlinearly realized gauge invariance. In the alternative framework [3,16], invariance with respect to the full electroweak gauge group is required, but with all but the unbroken $U_{em}(1)$ subgroup being nonlinearly realized. The physical motivation that underlies this second approach is the assumption that the low-energy degrees of freedom of the unknown symmetry-breaking sector contain only the three Nambu-Goldstone bosons which are absorbed by the massive W and Z particles. Given this assumption, the transformation properties of all fields are determined by general arguments [17,18] that were developed within the framework of chiral perturbation theory many years ago.

It is the point of this section to (re)demonstrate the equivalence of these last two schemes. This result is not new, appearing as it does in Refs. [14] and [18], but the reminder is worthwhile in order to put to rest more recent concerns as to the legitimacy of ignoring the broken electroweak symmetries in the effective Lagrangian. The equivalence is established by explicitly finding a change of variables that relates the two alternatives. Although our arguments can be made quite generally, we restrict ourselves here to two specific cases: a simplified toy model involving a single massive spin-1 particle, as well as the realistic case appropriate to the couplings of the electroweak gauge bosons $W^{\pm}, Z^0$ and the photon γ.

A. Toy example

In order to describe the argument within its simplest context, consider first the coupling of a single massive spin-1 particle, V_{μ}, coupled to various forms of spinless or spin-$\frac{1}{2}$ matter, ψ. We first state the two alternative forms for the effective Lagrangian and then demonstrate their equivalence.

No gauge invariance. The Lagrangian in the first formulation then takes the form

$$\mathcal{L}_1 = \mathcal{L}_1(V_{\mu}, \psi) , \tag{1}$$

in which $\mathcal{L}_1$ is *a priori* an arbitrary local Lorentz-invariant function of the fields V_{μ}, ψ and their spacetime derivatives. Since ψ and V_{μ} are independent degrees of freedom, the quantum theory could be defined in this case by a functional integral of the form

$$Z_1 = \int [d\psi][dV_{\mu}] \exp \left[i \int d^4x \, \mathcal{L}_1(V_{\mu}, \psi) \right] . \tag{2}$$

Nonlinearly realized gauge invariance. The alternative formulation is to consider a U(1) gauge theory with matter fields χ_i carrying U(1) charges q_i. The gauge symmetry transformation acting on these fields and on the gauge potential A_{μ} are the usual ones:

$$\chi_i \rightarrow e^{iq_i\omega}\chi_i , \quad gA_{\mu} \rightarrow gA_{\mu} + \partial_{\mu}\omega . \tag{3}$$

g here is the gauge coupling constant.

Symmetry breaking is incorporated by coupling these matter and gauge fields in a completely general way to a single Nambu-Goldstone boson φ for a spontaneously broken U(1). The action of the U(1) on the Nambu-Goldstone bosons may always be chosen to take a standard form [17], which becomes, in this case,

$$\varphi \rightarrow \varphi + f\omega . \tag{4}$$

f here is the Nambu-Goldstone boson's decay constant which is of the order of the scale at which the U(1) symmetry is spontaneously broken. It is related to the mass of the gauge boson by the relation $M = gf$.

The most general gauge-invariant low-energy Lagrangian may then be written in the form

$$\mathcal{L}_2 = \mathcal{L}_2(D_{\mu}\varphi, \chi') , \tag{5}$$

in which the redefined field is $\chi_i' \equiv e^{-iq_i\varphi/f}\chi_i$ is the gauge-covariant derivative for φ is given by $D_{\mu}\varphi \equiv \partial_{\mu}\varphi - gfA_{\mu}$. Note that all of the dependence on A_{μ} in $\mathcal{L}_2$ arises through this gauge-covariant derivative. For example, the gauge field strength is given by $gfF_{\mu\nu} = \partial_{\mu}D_{\nu}\varphi - \partial_{\nu}D_{\mu}\varphi$.

The corresponding functional integral defining the quantum theory then has the standard form

$$Z_2 = \int [d\chi_i'][dA_{\mu}][d\varphi]$$
$$\times \exp \left[i \int d^4x \, \mathcal{L}_2(D_{\mu}\varphi, \chi') \right] \delta[G] \text{Det} \left[\frac{\delta G}{\delta \omega} \right] , \tag{6}$$

in which the second-to-last term is the functional delta function $\delta[G]$, which enforces the gauge condition $G = 0$, and the last term is the associated Faddeev-Popov-DeWitt, or ghost, functional determinant.

It is crucial for the remainder of the argument that both χ_i' and $D_{\mu}\varphi$ be *invariant*, as opposed to being *covariant*, with respect to gauge transformations. As a result, *any* Lorentz-invariant Lagrangian, such as $\mathcal{L}_2$, that is built from these fields becomes gauge invariant automatically.

Equivalence. Now comes the main point. The two Lagrangians $\mathcal{L}_1$ and $\mathcal{L}_2$ are identical to one another. There is a one-to-one correspondence between the terms in each given by the replacement $\psi \leftrightarrow \chi_i'$ and $D_{\mu}\varphi \leftrightarrow -gfV_{\mu}$. This is only possible because *both* $\mathcal{L}_1$ and $\mathcal{L}_2$ are constrained only by Lorentz invariance and so any interaction which is allowed for one is equally allowed for the other.

More formally, the functional integral of Eq. (2) may be obtained from that of Eq. (6) by simply choosing unitary gauge, defined by the condition $G \equiv \varphi(x)$, and using the functional δ function to perform the integration over φ. The ghost "operator" is in this case $\delta G(x)/\delta \omega(x') = f\delta^4(x - x')$, and so the ghost determinant contributes just a trivial field-independent normalization factor.

The integration over the "extra" Nambu-Goldstone

degree of freedom of the gauge-invariant theory is thereby seen to be precisely compensated for by the freedom to choose a gauge.

B. Applications to the electroweak bosons

The argument as applied to a more complicated symmetry-breaking pattern, such as appears in the electroweak interactions, has essentially the same logic, although the technical details are slightly more intricate.

No gauge invariance. We take for the purposes of illustration the degrees of freedom in the low-energy effective Lagrangian for the electroweak interactions of leptons and quarks. These are the massless photon A_μ, the massive weak vector bosons W_μ and Z_μ, and the usual fermions ψ. Although other particles such as gluons may also be very simply included, we do not do so here for simplicity of notation. The general Lagrangian for these fields may be written

$$\mathcal{L}_1 = \mathcal{L}_1(A_\mu, W_\mu, Z_\mu, \psi) , \tag{7}$$

in which $\mathcal{L}_1$ is a general local and Lorentz-invariant function whose form is further constrained only by unbroken $U_{em}(1)$ invariance. All derivatives are taken to be the $U_{em}(1)$ gauge-covariant derivative D_μ, which for fermions takes the form $D_\mu \psi = \partial_\mu \psi - ieQA_\mu \psi$. Q here denotes the diagonal matrix of fermion electric charges.

The quantum theory is given in terms of a functional integral of the form

$$Z_1 = \int [dW_\mu][dW_\mu^*][dZ_\mu][dA_\mu][d\psi]$$

$$\times \exp \left[i \int d^4x \, \mathcal{L}_1 \right] \delta[G_{em}] Det \left[\frac{\delta G_{em}}{\delta \omega_{em}} \right] . \tag{8}$$

We next outline the nonlinear realization of $SU_L(2) \times U_Y(1)$.

Nonlinearly realized gauge invariance. The first step is to briefly review the formulation for the low-energy interactions of the Nambu-Goldstone bosons for the *global* symmetry-breaking pattern $SU_L(2) \times U_Y(1) \rightarrow U_{em}(1)$ [17]. We then promote the symmetry to local gauge transformations.

Consider, therefore, a collection of matter fields ψ on which $SU_L(2) \times U_Y(1)$ is represented (usually reducibly) by the matrices $G = \exp[i\omega_2^a T_a + i\omega_1 Y]$. We choose here a slightly unconventional normalization for the generators T_a and Y, viz., $\mathrm{tr}[T_a T_b] = \frac{1}{2}\delta_{ab}$, $\mathrm{tr}[T_a Y] = 0$, and $\mathrm{Tr}[Y^2] = \frac{1}{2}$. Finally, define the matrix-valued scalar field containing the Nambu-Goldstone bosons by $\xi(x) = \exp[2iX_a \varphi^a(x)/v]$, in which the three X_a's represent the spontaneously broken generators $X_1 = T_1$, $X_2 = T_2$, and $X_3 = aT_3 - bY$. Here $a^2 + b^2 = 1$, and a/b is chosen to ensure that $\mathrm{tr}[X_3 Q] = 0$, where Q is the unbroken generator: $Q = bT_3 + aY$.

The action of the gauge group $SU_L(2) \times U_Y(1)$ on ξ and ψ may be written in the standard form

$$\psi \rightarrow G\psi \quad \text{and} \quad \xi \rightarrow \xi' , \tag{9}$$

where $G\xi = \xi' H^\dagger$. Here $H = \exp[iQu(\xi, \xi', G)]$ and

$u = u(\xi, \xi', G)$ is implicitly defined by the condition that ξ' on the right-hand-side of Eq. (9) involves only the broken generators.

As was the case for the toy example, for the purposes of constructing the Lagrangian it is convenient to define new matter fields ψ' according to $\psi' \equiv \xi^\dagger \psi$ since this has the $SU_L(2) \times U_Y(1)$ transformation rule

$$\psi' \rightarrow \xi'^\dagger G \psi$$

$$= H\psi' . \tag{10}$$

Note that even for global $U_Y(1)$ rotations, for which ω_1 is constant, $u(\xi, \xi', G)$ is spacetime dependent because of its dependence on the scalar field $\xi(x)$.

The next step is the construction of the general locally $SU_L(2) \times U_Y(1)$-invariant effective Lagrangian. To this end consider the auxiliary quantity $\mathcal{D}_\mu(\xi)$, which may be defined in terms of ξ and the $SU_L(2) \times U_Y(1)$ gauge potentials $\mathbf{W}_\mu = gW_\mu^a T_a + g'B_\mu Y$ by

$$\mathcal{D}_\mu(\xi) \equiv \xi^\dagger \partial_\mu \xi - i\xi^\dagger \mathbf{W}_\mu \xi . \tag{11}$$

In terms of this quantity, it is possible to construct fields which transform in a simple way with respect to $SU_L(2) \times U_Y(1)$. Together with their $SU_L(2) \times U_Y(1)$ transformation rules, these are

$$e\mathcal{A}_\mu \equiv 2i \, \mathrm{tr}[Q\mathcal{D}_\mu(\xi)], \quad e\mathcal{A}_\mu \rightarrow e\mathcal{A}_\mu + \partial_\mu u ;$$

$$\sqrt{g'^2 + g^2} Z_\mu \equiv 2i \, \mathrm{tr}[X_3 \mathcal{D}_\mu(\xi)], \quad Z_\mu \rightarrow Z_\mu ; \tag{12}$$

$$g\mathcal{W}_\mu^\pm \equiv i\sqrt{2} \, \mathrm{tr}[T_\mp \mathcal{D}_\mu(\xi)], \quad \mathcal{W}_\mu^\pm \rightarrow e^{\pm iuQ}\mathcal{W}_\mu^\pm .$$

$T_\pm$ is defined as usual to be $T_1 \pm iT_2$. The first of these fields $\mathcal{A}_\mu(\xi)$ transforms in such a way as to permit the construction of a covariant derivative for the local transformations as realized on ψ':

$$D_\mu \psi' \equiv (\partial_\mu - ie\mathcal{A}_\mu Q)\psi' . \tag{13}$$

The main point to be appreciated here is that Eqs. (12) imply that all of the fields ψ', $D_\mu \psi'$, $\mathcal{A}_\mu(\xi)$, $Z_\mu(\xi)$, and $\mathcal{W}_\mu^\pm(\xi)$ transform purely electromagnetically under arbitrary $SU_L(2) \times U_Y(1)$ transformations. This ensures that once the Lagrangian is constructed to be invariant under the unbroken group, $U_{em}(1)$, it is *automatically* invariant with respect to the full nonlinearly realized group $SU_L(2) \times U_Y(1)$.

With these transformation rules, the most general $SU_L(2) \times U_Y(1)$-invariant Lagrangian becomes

$$\mathcal{L}_2 = \mathcal{L}_2(\mathcal{A}_\mu, \mathcal{W}_\mu, Z_\mu, \psi') , \tag{14}$$

with $\mathcal{L}_2$ restricted only by the unbroken $U_{em}(1)$ gauge invariance. The functional integral which defines the quantum theory may then be written

$$Z_2 = \int [d\mathbf{W}_\mu][d\xi][d\psi']$$

$$\times \exp \left[i \int d^4x \, \mathcal{L}_2 \right] \delta[G_a] Det \left[\frac{\delta G_a}{\delta \omega^b} \right] . \tag{15}$$

Four gauge conditions $G_a = 0$, $a = 1, \ldots, 4$, are required, one for each generator of $SU_L(2) \times U_Y(1)$.

Equivalence. The demonstration of the equivalence between Eqs. (8) and (15) proceeds along lines that are similar to those used in the Abelian toy example presented previously. As was the case in this earlier example, the equivalence works term by term in the Lagrangian. The correspondence between the field variables is

$$\mathcal{A}_\mu \leftrightarrow A_\mu, \quad \mathcal{Z}_\mu \leftrightarrow Z_\mu, \quad \mathcal{W}_\mu^\pm \leftrightarrow W_\mu^\pm, \quad \psi' \leftrightarrow \psi . \tag{16}$$

The equivalence is explicit in unitary gauge, which is defined in this case by the condition $\varphi^a(x) \equiv (0)$, or equivalently $\xi(x) \equiv 1$, throughout spacetime. As is seen from the transformation rules of Eq. (9), this condition does not completely fix the gauge. It is preserved by the unbroken electromagnetic transformations which satisfy $G = H = e^{i\omega_{\mathrm{em}}}$. In this gauge the relations for $\mathcal{Z}_\mu$, $\mathcal{W}_\mu$, and ψ indicated in Eqs. (16) above simply become equalities.

More formally, using the unitary-gauge condition to perform the functional integration over ξ in Eq. (15), gives the result

$$Z_2 = \int [d\mathbf{W}_\mu][d\psi] \exp\left[i \int d^4x \, \mathcal{L}_2\right]$$
$$\times \delta[G_{\mathrm{em}}] \mathrm{Det}\left|\frac{\delta G_{\mathrm{em}}}{\delta\omega_{\mathrm{em}}}\right| \mathrm{Det}\left|\frac{\delta\varphi^a}{\delta\omega^b}\right|\bigg|_{\varphi=0} . \tag{17}$$

Since $\mathcal{L}_2(\xi=1) = \mathcal{L}_1$, this clearly agrees with Eq. (8) apart from the final Faddeev-Popov-DeWitt ghost determinant that is associated with the choice of unitary gauge:

$$\delta\varphi^a(x)/\delta\omega^b(x') \equiv \Delta^a_b(x)\delta^4(x-x') . \tag{18}$$

The final point is that the identity $\mathrm{Det} \equiv \exp \mathrm{Tr} \ln$ may be used to rewrite this determinant as the exponential of a local, Lorentz-, and $U^{\mathrm{em}}(1)$-invariant function. As such, it may be considered as a shift in the parameters appearing in the original Lagrangian $\mathcal{L}_2$. Furthermore, since its contribution to $\mathcal{L}_2$ is proportional to $\delta^4(x=0)$, its coefficients are ultraviolet divergent and so their contribution may be absorbed into the renormalizations that are anyhow required in defining the functional integral of Eq. (17). In practice, the Faddeev-Popov determinant does not in any case arise until at least two-loop order.

The practical benefit of this equivalence is that it allows the use of the most convenient gauge for any particular application. Covariant gauges, such as the Feynman gauge, are particularly useful for making power-counting arguments, since all propagators explicitly vary like $1/p^2$ for large four-momenta, and the pathologies of the unitary-gauge propagator are put into derivative couplings. For instance, this is the simplest way to understand why QED remains renormalizable once a photon mass term is added, while the same is not true for a non-Abelian gauge theory. This distinction is most easily seen from the form of the Nambu-Goldstone boson couplings. While an invariant renormalizable Lagrangian exists for a U(1) Nambu-Goldstone boson—i.e., it is simply its kinetic term $-\frac{1}{2}D_\mu\varphi D^\mu\varphi$—the same is *not* true for a non-Abelian symmetry group. This is because the kinetic terms are in this case not by themselves invariant with respect to the nonlinearly realized symmetries. Con-

versely, unitary gauge has the simplicity of just involving physical particles, allowing a direct identification of the physical significance of the effective interactions.

C. Derivative vs Yukawa couplings

In this section we wish to make the previous arguments concrete by considering an explicit one-loop example. In addition to having applications later in the paper, the example also serves to bring out three general, but not-so-widely appreciated, features of the equivalence we have described. These general points are listed at the end of the section.

It is a basic feature of the chiral Lagrangian described above that all of the would-be Nambu-Goldstone bosons (WBNGB's) couple derivatively to all other fields (and to themselves). This expresses a completely generic feature of any Nambu-Goldstone boson interaction and is easily seen from the expansion of, e.g., $\mathcal{W}_\mu(\xi)$ [cf. Eqs. (11) and (12)] in powers of fields:

$$\mathcal{W}_\mu^\pm = W_\mu^\pm - \frac{1}{M_W}\partial_\mu\varphi^\pm + \cdots . \tag{19}$$

The second term in this expansion gives a very simple Feynman rule for WBNGB couplings: Simply contract the result for the corresponding $W^\pm$ coupling by ik^μ/M_W, where k^μ is the WBNGB four-momentum. This is equally true regardless of whether the particle to which the $W^\pm$ or $\varphi^\pm$ couples is a scalar, fermion, or a gauge boson.

Note that this type of coupling is *not* the same as what is obtained for the WBNGB's in a covariant gauge in the standard model. In the standard model, for example, the WBNGB-fermion interactions do not involve any derivatives at all, since they come from the Yukawa couplings to the Higgs multiplet. In the standard model these two formulations are physically equivalent, since it is possible to pass from one to the other by performing an appropriate field redefinition. As we shall now see, however, they can and do give rise to different types of divergences in off-shell quantities such as the effective Lagrangian. This point will become important once we begin trying to track the cutoff dependence of loops in later sections.

Consider, then, the effective interaction[1]

$$\mathcal{L}_a = -aZ_\mu(W_\nu^+ W^{-\mu\nu} + W_\nu^- W^{+\mu\nu}) . \tag{20}$$

The couplings that this interaction induces for the WBNGB's are found by substituting $W \to \mathcal{W}$ and $Z \to \mathcal{Z}$ from Eqs. (12) and expanding the result in powers of fields. We choose to compute the CP-violating Z-τ-τ vertex (or Zdm)

$$\mathcal{L}_{Zdm} = -\frac{iz}{2}\bar{\tau}\gamma_5\sigma^{\mu\nu}\tau Z_{\mu\nu} \tag{21}$$

that this coupling induces at one loop.

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[1]This interaction happens to violate CP and corresponds on shell to the interaction denoted g_4^Z in Ref. [5].

In the unitary-gauge formulation, we must evaluate the graph of Fig. 1. The W propagator that appears in each of the two internal boson lines is

$$G_U^{\mu\nu}(k) = \frac{-i}{k^2 - M_W^2} \left[g^{\mu\nu} - \frac{k^\mu k^\nu}{M_W^2} \right] . \qquad (22)$$

On the other hand, working with the chiral Lagrangian in a general covariant gauge leads not only to Fig. 1, but also to the three other graphs that are obtained from this one by replacing each W line by the corresponding WBNGB propagator. In the standard one-parameter family of covariant gauges, the two types of boson propagators that appear are

$$G_{(\eta)}(k) = \frac{i}{k^2 - \eta M_W^2} \qquad (23a)$$

and

$$G_{(\eta)}^{\mu\nu}(k) = \frac{-i}{k^2 - M_W^2} \left[g^{\mu\nu} + (\eta - 1) \frac{k^\mu k^\nu}{k^2 - \eta M_W^2} \right] . \qquad (23b)$$

The equivalence theorem of the previous sections argues that the unitary-gauge result equals the sum of the four covariant-gauge graphs. This is easy to see by using the following identity in the unitary-gauge result:

$$G_U^{\mu\nu}(k) = G_{(\eta)}^{\mu\nu}(k) + \frac{k^\mu k^\nu}{M_W^2} G_{(\eta)}(k) . \qquad (24)$$

The two factors $(ik^\mu / M_W)(-ik^\nu / M_W)$ are just what is required to reproduce the Feynman rules for the WBNGB couplings as given in Eq. (19). (The relative sign arises because momentum in at one end of the boson line corresponds to momentum out at the other.) Thus the diagrams with WBNGB's simply cancel the η dependence of the W propagators in Fig. 1. Note that it is crucial for this result to use the derivative WBNGB couplings for both the WWZ vertex *and* the W-fermion vertices.

Since the integrands in the two formulations are equal, they give the same result for the τ weak dipole moment, regardless of how the graphs are regularized. We choose here to regulate the graph by inserting the form factor $F(p, \Lambda) = -\Lambda^2 / (p^2 - \Lambda^2)$ into each internal line.[2] (This may be viewed as a higher-derivative regularization, for which a higher-derivative kinetic term has been added to the unperturbed Lagrangian for each field.) The result for the most-divergent part becomes

$$z_{\text{most div}} = -\frac{ag^2}{2304\pi^2} \frac{\Lambda^2}{M_W^4} m_\tau , \qquad (25)$$

where g is the $SU_L(2)$ coupling constant.

It is instructive to compare this result with what would have been obtained if we had used the chiral Lagrangian only for the WWZ vertex in Fig. 1 and had simply used standard-model Yukawa-type Feynman rules for the

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[2]The momentum flowing through each line must be regulated separately, or else the result will depend on how momentum is routed through the graph.
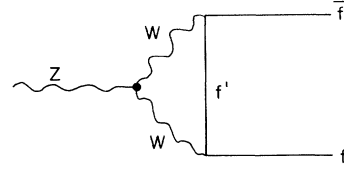

FIG. 1. Feynman graph through which the anomalous gauge-boson vertex contributes to fermion weak dipole moments.

WBNGB-fermion vertices. In this case the WBNGB graphs are less divergent, since there are fewer powers of momentum associated with each vertex. The most-divergent part of the result becomes

$$z_{\text{most div}} = -\frac{ag^2}{384\pi^2} \frac{m_\tau (m_\tau^2 - m_{\nu_\tau}^2)}{M_W^4} \ln \left[\frac{\Lambda^2}{M_W^2} \right] . \qquad (26)$$

There are three lessons to be learned from this section and from this example.

(1) First, we explicitly verify the equivalence between the chiral Lagrangian and the Lagrangian which ignores all but $U_{\text{em}}(1)$. This equivalence relies crucially on the derivative couplings of the WBNGB's in chiral perturbation theory. Criticizing the apparent gauge noninvariance of the TGV Lagrangians that are used in loop calculations (or, equivalently, of unitary gauge) in favor of chiral perturbation theory clearly misses the point. If there are problems with the large loop estimates that have been obtained, then the reason must be found elsewhere. We point this reason out in the following section.

(2) Next, we see explicitly that even the dominant cutoff dependence of off-shell quantities, such as couplings in the effective Lagrangian, depend strongly on the choice of field variables used. In particular, the two kinds of Feynman rules for the fermion-WBNGB vertex may be obtained from one another by performing a WBNGB-dependent nonlinear field redefinition on the fermion fields of the form $\psi \rightarrow f(\varphi)\psi$. (In fact, the answer would have remained unchanged if the higher-derivative terms which implement the cutoff were also transformed, since this transformation introduces new cutoff-dependent fermion-WBNGB interactions.) This is part of the intrinsic hazard of trading in off-shell divergences: They depend in detail on which field variables are regulated.

(3) But the last, and most important, point is this: Without knowing the underlying physics, which of these two answers is correct? If one interprets Λ to agree, in order of magnitude, with the new physics scale, they have very different physical implications. The difference between them could well be the difference between detecting z at the CERN e^+e^- collider LEP or not. We shall argue in the following sections that in this case it is Eq. (26) which is correct. It is clearly important to be able to decide which is right in advance.

III. CUTOFFS: GENERAL ARGUMENTS

The main point of this paper is to critically reassess the common habit of inferring heavy-mass dependence from

the cutoff dependence obtained purely within low-energy loops. In this section we make our main points. In order to do so, we start by presenting the arguments in favor of using cutoffs in this way, followed by our criticism of these arguments. We then provide a few explicit examples to illustrate the relevant points.

A. Why might one think that cutoffs track new physics?

Associating a physical interpretation with the cutoff is an almost irresistible impulse when dealing with effective Lagrangians. After all, the effective theory is from the start only meant to describe physics below some scale Λ, above which we cannot probe. Since effective theories are not renormalizable in the traditional sense, the insertion of effective vertices into loop graphs can produce very divergent results. It is natural to suppose that these divergences indicate that the amplitude in question gets its most important contributions from the highest frequencies: those just below the cutoff. Presumably this strong sensitivity is removed once all of the heavy degrees of freedom of mass $M > \Lambda$ are included, such as would happen if this underlying theory were renormalizable. As a result, so the argument goes, the strong short-distance contributions should saturate at M, leaving a result whose size is set by replacing Λ with M, at least up to an order of magnitude.

This reasoning can be made considerably more precise by rephrasing it as the following principle [19].

If there is a divergent graph in the low-energy theory, cutting it off at the scale where the theory breaks down due to new physics gives a *lower bound* to the actual value of the graph in the full theory (in the absence of fine-tuning).

What could possibly be wrong with such a physically appealing argument? The answer is that, in certain circumstances, nothing goes wrong with it. Unfortunately, it can sometimes also happen that it is completely false, and it fails because it does not take into account cancellations that are automatically built into any effective theory. We describe here what these cancellations are and return in following sections to the question of how to tell when the above reasoning will fail.

B. Curse of cancellations

Consider, then, a theory which involves two very different mass scales $M \gg m$. (We have in mind that m represents the weak scale, say, $m \sim M_W$, while M represents the scale of unknown new physics.) Suppose that within this theory we wish to compute a physical low-energy observable, such as a calculable low-energy mass shift $\delta\mu^2$ as a function of these two mass scales. An example of this type of observable in the electroweak interactions would be the deviation from unity of the ρ parameter, which is related to the comparative strength of the low-energy charged- and neutral-current weak interactions.

We are interested in the form taken by $\delta\mu^2$ in the limit where m/M is taken to be asymptotically small, with all dimensionless couplings held fixed. It is possible to make fairly general statements as to the result in this limit (in

four spacetime dimensions) if the renormalizable part of the low-energy theory is perturbative, so that all fields scale approximately as the noninteracting Lagrangian would indicate. Typically, the answer in this case takes the general form

$$\delta\mu^2(m,M) = c_0 M^2 + c_1 m^2 + c_2 \frac{M^4}{M^2} + \cdots , \qquad (27)$$

in which the ellipsis represents terms that are suppressed by more than two powers of m/M. The dimensionless coefficients are functions of the other (renormalized) dimensionless parameters of the theory, and they may also depend at most logarithmically on the large mass ratio M/m. Note that the largest power of M here is just set by dimensional analysis.

For applications to the electroweak interactions $m \sim M_W$, it is important to be aware that the above form strictly applies only asymptotically for $M_W/M \to 0$. It may therefore be expected to hold when the new physics can be at very high scales compared to the weak scale, such as if the underlying physics were a grand unified theory of some kind. Its application is less straightforward when the new physics is associated with electroweak symmetry breaking, since in this case M cannot be larger than of order $4\pi v$, and so $M/M_W \lesssim 8\pi/g$. In this case Eq. (27) must be interpreted as applying to the $g \to 0$ limit rather than for $M_W/M \to 0$ with g fixed.

Imagine now performing the same calculation, but this time dividing the contributions into a "low-energy" part and a "high-energy" part. To this end choose a cutoff Λ which satisfies $m \ll \Lambda \ll M$. First, integrating out the high-energy part of the spectrum produces a low-energy effective Lagrangian that is applicable at scales below Λ. Next, compute the physical mass shift in this low-energy effective theory. Since this simply corresponds to a particular way of organizing the calculation in the full theory, it must produce the correct answer of Eq. (27) above. The full expression may therefore be broken up as follows:

$$\delta\mu^2(m,M) = \delta\mu_{LE}^2(m,\Lambda,M) + \delta\mu_{HE}^2(m,\Lambda,M) , \qquad (28)$$

in which the first (second) term here, respectively, contains only the low-energy (high-energy) contributions. (This split between low and high frequencies may be conveniently formulated in Euclidean signature according to whether the four-momentum p for a particle of mass m_i in each internal line of a Feynman graph satisfies the condition $p^2 + m_i^2 > \Lambda^2$.)

The low- and high-energy contributions to $\delta\mu^2$ in general take the form

$$\delta\mu_{HE}^2 = c_0 M^2 + b_1 \Lambda^2 + \cdots ,$$
$$\delta\mu_{LE}^2 = b_1' \Lambda^2 + \cdots . \qquad (29)$$

In both of these equations, the ellipses represent terms that depend differently on the small mass ratios m/Λ, Λ/M, or m/M than the terms that are explicitly written. Examples in later sections include, for example, such quartically divergent terms as Λ^4/m^2. Clearly, the condition that the two contributions sum up to the full result

of Eq. (27), whose Λ independence is manifest, requires that the coefficients satisfy $b_1 + b'_1 = 0$, etc.

Now comes the main point. In order to calculate the scale of new physics that may be probed by a detailed measurement of a quantity such as $\delta\mu^2$, we require accurate knowledge of the coefficient c_0 in Eq. (27). If we only have access to the low-energy effective Lagrangian below scale Λ, then it is impossible to precisely compute c_0. In particular, knowledge of the coefficient b'_1 of the low-energy quadratic divergence gives no *a priori* information regarding c_0, since it is completely canceled by the high-energy contribution (or counterterm) b_1. There is nothing miraculous about this cancellation; it simply reflects how physics cannot depend on the intermediate steps in a calculation.

There are occasions, however, when knowledge of the coefficient of a particular divergence in the low-energy theory can be parlayed into reliable information about the heavy-mass dependence of the full result [20]. A logarithmic divergence furnishes perhaps the simplest example. Here the full and partial results for a dimensionless observable, call it A, can take the form

$$A = A_{LE} + A_{HE} = a_0 \ln\left[\frac{M^2}{m^2}\right] + \cdots , \qquad (30a)$$

while

$$A_{HE} = a'_0 \ln\left[\frac{M^2}{\Lambda^2}\right] + \cdots , \qquad (30b)$$

and

$$A_{LE} = a''_0 \ln\left[\frac{\Lambda^2}{m^2}\right] + \cdots . \qquad (30c)$$

In this case the condition that the cutoff dependence cancel requires that $a_0 = a'_0 = a''_0$ and so the coefficient of the large logarithm within the full theory may be determined simply by identifying the coefficient of the logarithmic divergence within the low-energy theory. It is important to realize that this property is *not* generically shared by other types of divergences.

C. "Good" vs "bad" variables

With the general concepts regarding cutoffs now firmly in hand, we can now demonstrate the flaw in the principle enunciated earlier (Sec. III A), which states that cutoffs furnish lower bounds for the contributions of new physics. A brief example here is instructive.

Consider the case where the standard model itself, Higgs boson and all, is the low-energy theory, as might be appropriate to a grand unified theory. In this case the bounds of Eq. (27) should apply since the low-energy theory is perturbative in the regime of interest, and we may take M_W/M to be extremely small. Suppose we choose to compute the cutoff dependence in this theory of the coefficient of the effective operator $F_{\mu\nu}\Box F^{\mu\nu}$, which contributes to the vacuum polarization of the photon. Since the standard model is renormalizable, this result in a manifestly renormalizable gauge is finite, varying like

$1/\Lambda^2$ for $\Lambda \gg M_W$. If the same result is computed in unitary gauge, however (or, equivalently, in any gauge using the derivative WBNGB couplings of chiral perturbation theory), then it diverges quadratically: $\sim \Lambda^2/M_W^4$. If taken seriously, this example would drastically overestimate the heavy-mass dependence of the underlying theory, which cannot be larger than $O(1/M^2)$.

Once again, just as in our earlier example involving the weak dipole moment of the τ, a change of variables has dramatically altered the cutoff dependence of the effective Lagrangian. These two examples illustrate the difference between what might be called "good" and "bad" variables. To see the distribution between these variables, note that for both examples the divergences of the S matrix are the same in *both* sets of variables, since the S matrix is unchanged by field redefinitions. "Bad" variables are therefore characterized by large cancellations in physical quantities, such as the S matrix, between enormous terms in the effective Lagrangian. As a result, these are variables for which the couplings in the Lagrangian do not follow the couplings that would be defined in terms of scattering amplitudes.

With this in mind, one can propose a modification of the above principle [19].

If there is a divergent graph in the low-energy theory, cutting it off at the scale where the theory breaks down due to new physics gives a *lower bound* to the actual value of the graph in the full theory (in the absence of fine-tuning), as long as "good" variables are used in the calculation.

It appears that this principle holds in all known examples. However, its utility relies on the existence of a practical algorithm for determining in advance whether the variables of interest are "good" or "bad."

This is our main criticism of the papers in Ref. [10]. In using cutoffs to regulate divergent loops involving anomalous TGV's, they obtain limits on the coefficients of these operators which depend on their choice of variables. Without knowing whether these variables are "good" or "bad," one cannot ascertain if the bounds obtained are reasonable. A second criticism of some of these papers, and indeed of some of those in Ref. [8], is that the scale of the new physics, M, is often allowed to be greater than $4\pi v$, which is not permitted if the symmetry is realized nonlinearly. This typically leads to overly stringent bounds on new operators.

IV. BANISHING CUTOFFS

Rather than searching for a practical algorithm for "good" and "bad" variables, we prefer to recast the above principle in a way which does not refer to cutoffs at all. It amounts, in essence, to the judicious use of dimensional analysis, together with any other information that may also be available purely within the low-energy theory. This information is all that is really required of any analysis of low-energy graphs and, in applications where cutoff dependence happens to track the underlying masses, produces identical answers. It has the conceptual advantage, however, of being insensitive to field redefinitions and so of never leading one badly astray

through the mistaken use of "bad" variables. In the remainder of this section, we describe this procedure, followed immediately by a detailed calculation using a known model of underlying physics with which both cutoff and our results can be compared.

Suppose then, that some physics that is associated with a heavy-mass scale M (which might, for example, denote the mass of the lightest unknown particle) is integrated out to produce a low-energy effective Lagrangian $\mathcal{L}_{\text{eff}}$:

$$\delta \mathcal{L}_{\text{eff}} = c_n \mathcal{O}_n \ . \tag{31}$$

We are interested in the M dependence of the coupling for an effective operator of scaling dimension (mass)^{d_n} which appears in this effective Lagrangian. In general, this is an ill-defined question, since the dependence of c_n on heavy physics requires a proper definition of the composite operator it multiplies, $\mathcal{O}_n$. We therefore pause here to make a brief aside concerning a particularly convenient formalism for these purposes.

A. Regularizational aside

A particularly clean and convenient scheme with which to work in an effective theory is dimensional regularization supplemented by the "decoupling subtraction" renormalization scheme [21]. This scheme consists of minimal subtraction supplemented by the explicit removal of heavy degrees of freedom as the renormalization point is lowered below the corresponding mass thresholds. This "integrating out" of the heavy particles is in practice implemented as a set of matching conditions for the appropriate effective couplings at these thresholds. The resulting couplings may then be used as initial conditions for the renormalization-group equations that define the scale dependence of such couplings in the theory below the threshold. With this scheme a logarithmic dependence on the masses of the problem (including M) is introduced into the coefficients c_n as the various effective operators are evolved between particle thresholds.

The beauty of using dimensional regularization in this way is that no confusion is possible between the cutoff and the heavy-physics scale, since within this framework no cutoff Λ arises at all. As a result, only the physical masses ever arise in effective couplings. Furthermore, more and more divergent graphs in the effective theory, which involves only light particles, simply introduce higher and higher powers of the light mass m rather than some higher scale such as Λ or M. As a result, it becomes possible (and convenient) to include within the loops of the low-energy theory all of the momenta of the light fields, right up to infinity. This leads to a real distinction in the nature of the matching between the underlying theory and the effective theory when using cutoffs and dimensional regularization. When using a cutoff, all frequencies above the scale Λ are integrated out, including all of the modes of the heavy particles as well as the high-frequency components of the light particles. In dimensional regularization, one instead integrates out only the heavy-particle contributions, leaving all of the momenta of the light fields in the low-energy theory. This allows the matching between the effective and the under-

lying theories to be made at the heavy-mass threshold itself, and so the only mass which appears due to this matching is typically this threshold mass M.

There is another practical benefit in using dimensional regularization. Dimensionally regularized graphs are much less sensitive to the field redefinitions that relate the "good" and "bad" variables of the earlier examples. For instance, if dimensional regularization is used to regularize the contribution of the WWZ interaction to the Zdm, the divergent piece is found to be

$$z_{\text{pole}} = \frac{ag^2}{384\pi} \frac{m_\tau(m_\tau^2 - m_{\nu_\tau}^2)}{M_W^4} \left[\frac{1}{\epsilon} \right] \ , \tag{32}$$

where $n = 4 - 2\epsilon$ is the dimension of spacetime. This result holds using either Yukawa-type or derivative couplings for the WBNGB's to the fermions.

Within minimal subtraction, we find therefore that the renormalized parameter z mixes with the renormalized parameter a in the following way:

$$z(\mu) = z(M) - \frac{g^2}{384\pi^2} \frac{m_\tau(m_\tau^2 - m_{\nu_\tau}^2)}{M_W^4} a(M) \ln \left[\frac{M^2}{\mu^2} \right] \ . \tag{33}$$

Note the similarity between the logarithmic dependence here and the previous results of Eq. (26).

Both terms in Eq. (33) have a clear interpretation. The logarithmic dependence corresponds to the explicit operator mixing that can be unambiguously computed purely within the low-energy effective theory. The initial conditions $z(M)$ and $a(M)$, however, are determined by matching to the underlying theory and so cannot be known until this theory is specified. At best, we can only try to estimate the size of these initial conditions, and this is the goal of the remainder of this section.

B. Generic estimate

With this definition in mind, we wish now to estimate how the couplings c_n of Eq. (31) depend on the new physics scale M. We are specifically interested here in the powers of M that arise at the threshold M rather than any logarithmic dependence. Simple dimensional analysis would indicate

$$c_n = \hat{c}_n M^{4-d_n} \ . \tag{34}$$

Without any additional information about the nature of the new physics that is responsible for this effective Lagrangian, all that can be said about the dimensionless coupling $\hat{c}_n$ is that it is of order 1 or smaller.

With more assumptions concerning the physics at M, more information can be extracted about c_n. We next illustrate how different kinds of physics can differ in their implications for c_n by contrasting two plausible alternatives for electroweak symmetry-breaking physics at M.

C. Strong coupling: Naive dimensional analysis

Suppose first that the symmetry-breaking sector is strongly coupled, with only the WBNGB's appearing at

energies much less than M. In this case chiral perturbation theory organizes their couplings according to the numbers of derivatives which appear in the Lagrangian. For applications to energies that are much less than the electroweak scale, v, simple dimensional analysis with $M \sim v$ properly describes the size of each interaction.

Of more practical interest, however, is the application of this Lagrangian to electroweak energies $E \simeq v \ll M$. In this case higher-derivative interactions should be suppressed by powers of M rather than v, and it becomes important to keep track of the powers of v/M which can appear in the coefficients c_n. A set of self-consistent statements for the sizes that can be expected for any given term in the chiral Lagrangian is called "naive dimensional analysis" (NDA) [22]. It states that a term having b WBNGB fields, f weakly interacting fermions fields, d derivatives, and w gauge fields has a coefficient whose size is

$$c_n(M) \sim v^2 M^2 \left[\frac{1}{v} \right]^b \left[\frac{1}{M^{3/2}} \right]^f \left[\frac{1}{M} \right]^d \left[\frac{g}{M} \right]^w , \quad (35)$$

with $M \lesssim 4\pi v$. (If the fermions are strongly interacting, then the appropriate factor is $1/v\sqrt{M}$ for each fermion.)

Some examples of this counting are instructive, particularly when these are compared with the alternative estimates of the next section. For instance, according to the above estimate, the mass terms for the W and Z bosons are both of order $g^2 v^2$. This indicates that the small size of the deviation from unity of the ρ parameter cannot be understood in this picture as being simply the result of a suppression by powers of v/M. Additional approximate symmetries are required in order to explain the small size of $\delta\rho$. Also, typical corrections to the charged- and neutral-current interactions for fermions are here of the order of gv^2/M^2. Finally, triple-gauge boson operators such as $\kappa W_\mu^* W_\nu Z^{\mu\nu}$ and $\lambda W^{*\mu\nu} W_{\nu\lambda} Z^\lambda{}_\mu$ are, respectively, of order $\kappa \sim g^3 v^2/M^2$ and $\lambda \sim g^3 v^2/M^4$. We next compare these estimates with the implications of an alternative scenario.

D. Weak couplings: Linearly realized Lagrangian

An alternative perspective arises if the low-energy theory fills out a linear realization of the electroweak group. In this case M need not be small compared to $4\pi v$, and the WBNGB's fall into some linear representation of this group. Again, operators can have coefficients that are suppressed by powers of v/M once the low-energy Higgs fields are given their vacuum expectation values (VEV's), and so the power that arises depends on the representation in which the symmetry-breaking order parameter transforms. Much the most plausible choice for such a linearly realized Higgs representation is one or more doublets, with the standard hypercharge assignment. In this case the dependence on v/M of any non-Higgs-boson interactions may be found by taking $c_n = \hat{c}_n M^{4-d_n}$, as before, and then replacing any Higgs multiplets in the effective operator by their VEV's. In this case the linearly realized gauge symmetry enforces relations among the coefficients of operators of a given

type, depending on how these operators fall into linearly realized multiplets.

This is best illustrated with a few examples. Consider the W and Z boson mass terms $\mathcal{O}_W = W_\mu^* W^\mu$ and $\mathcal{O}_z = \frac{1}{2} Z_\mu Z^\mu$. The lowest-dimensional operator which contains these terms is simply the dimension-4 Higgs kinetic term $(D_\mu \phi)^\dagger (D^\mu \phi)$. Just as for the standard model, replacement of ϕ by its expectation value in this operator generates the particular combination $\cos^2\theta_w \mathcal{O}_W + \mathcal{O}_z$ with a coefficient that is of order $g^2 v^2$. More general combinations arise at dimension 6, such as through the operator $(\phi^\dagger D_\mu \phi)(\phi^\dagger D^\mu \phi)/M^2$. This and similar operators ruin the mass relation $M_W = M_z \cos\theta_W$ by amounts that are of order $g^2 v^4/M^2$. In contrast with the NDA estimate, $\delta\rho$ is automatically small if $v^2/M^2 \ll 1$.

As we shall see in a later section, the smallness of the present estimate in comparison with the NDA result has a simple explanation within the context of an underlying multi-Higgs-boson model. In this case contributions such as those to $\delta\rho$ typically arise at one loop and are proportional to $g\lambda_H^2/16\pi^2$, where $\lambda_H \simeq gm_H/M_W$ is a Higgs self-coupling. If this self-coupling is weak, then the suppression by $1/(4\pi)^2$ corresponds to a factor of v^2/M^2. Once λ_H is of order 4π, however, for which $m_H \sim 4\pi v$, this suppression is lost and we obtain the NDA result.

It is not always true that NDA gives a larger estimate for effective couplings than would a linearly realized underlying theory, however. For example, both predict deviations from the standard-model charged- and neutral-current couplings that are of order gv^2/M^2. Similarly, both estimates for the coupling $\kappa W_\mu^* W_\nu Z^{\mu\nu}$ are of order $g^3 v^2/M^2$. Furthermore, for the coupling λ (which premultiplies the interaction $W^{*\mu\nu} W_{\nu\lambda} Z^\lambda{}_\mu$, the NDA estimate is actually *smaller* than that for a linearly realized model. NDA would predict $\lambda \sim g^3 v^2/M^4$, while the linearly realized estimate is $\lambda \sim g^3/M^2$, since this interaction can be embedded into the linearly realized operator $\text{Tr}[W^{\mu\nu} W_{\nu\lambda} W^\lambda{}_\mu]$ without the necessity for Higgs doublets.

E. Using loops to infer further information

These estimates that are simply based on dimensional analysis can be sharpened using additional assumptions. A dimensional estimate for $c_n(M)$ can be obtained by using loops in the low-energy theory to estimate factors of dimensionless coupling constants and $1/(16\pi^2)$ which arise from the low-energy contribution to c_n at lower scales. If these are assumed to not cancel with the high-frequency contribution, then these factors may be used to place a *lower bound* on c_n, just as in the principle that was enunciated in Sec. III C to describe the potential relevance of cutoff dependence. The main difference in the present formulation is the use of these loops purely to determine the dependence on dimensionless combinations of couplings, with only dimensional analysis being used to fix the dependence on M.

For loops which involve WBNGB's, there is one dimensionless coupling that is of particular interest. This is the dimensionless coupling which describes the interactions between WBNGB's and the other particles of the

theory. It is always possible to choose variables such that these are proportional to the ratio of the particle's mass to v, rather than using the derivative coupling of Sec. II. For instance, $\lambda_{\varphi ff} \sim g m_f / M_W$, $\lambda_{\varphi w w} \sim q M_W^2 / v$, etc. Including these couplings is important if the corresponding particle masses are large, in that they can produce what appears to be a positive power of a heavy mass. We illustrate this in more detail in the following section. Use of this coupling strength amounts to using our freedom to use field redefinitions to remove as many derivatives as possible from WBNGB couplings [23]. This is where the use of "good" variables enters our rules [19].

This procedure is clearly operationally very similar to what is usually done when using cutoffs to estimate effective interactions. In particular, it reproduces the many successful estimates that are often argued from using cutoffs. The main difference is that the power of M that contributes here is explicitly argued purely on dimensional grounds, thereby removing the uncertainty that is associated with the choice of "good" and "bad" variables.

V. KNOWN NEW PHYSICS: AN EXAMPLE

We now wish to apply this reasoning to a model for which all of the heavy-mass dependence is known and calculable. This permits a comparison of the above arguments with the known correct dependence on M, as well as with the cutoff dependence of the low-energy effective theory.

A. Explicit calculation

We consider a two-Higgs-doublet model with soft CP-breaking terms in the Higgs potential and where we imagine that the physical Higgs particles all have masses that are as large as is possible: $m_H \lesssim 4\pi v$. Larger masses are not possible here without disbelieving the perturbative analysis, since the Higgs boson masses can be made larger than v only by increasing their self-couplings. In this model the anomalous WWZ coupling of Eq. (20) arises at one loop, with a calculable coefficient. We may therefore compute the contributions which this operator makes to the Zdm of Sec. II, we well as to the ρ parameter, and contrast this with an estimate of the corresponding higher-loop graphs that are obtained within the underlying theory when the effective WWZ vertex is resolved.

Following Ref. [24], we consider a two-Higgs-doublet model in which CP is spontaneously broken. This occurs when there is a relative phase between the vacuum expectation values (VEV's) of the two Higgs doublets. In such a scenario, tree-level flavor-changing neutral currents (FCNC's) are usually generated, but these can naturally be made small if CP violation is generated via soft CP-breaking terms in the Higgs potential. The two Higgs doublets can then be written $\phi_i^T = (\phi_i^+, \phi_i^0 + v_i e^{i\theta_i})$,
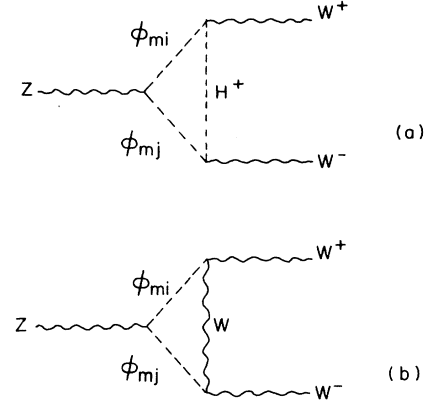
FIG. 2. Feynman graphs which generate the CP-violating anomalous gauge-boson vertex in the two-Higgs-doublet model.

$i = 1, 2$, in which $v_i e^{i\theta_i}$ are the VEV's. For calculational purposes, it is useful to change bases such that the WBNGB fields $(\varphi_Z^0, \varphi_W^+)$ are decoupled from the physical Higgs fields $(H^+, H_{1,2}^0, I_2^0)$. The new basis is

$$\phi_1'^T = (\varphi_W^+, H_1^0 + i\varphi_Z^0 + \sqrt{v_1^2 + v_2^2}\,),$$

$$\phi_2'^T = (H^+, H_2^0 + iI_2^0)\,,$$

in which only the VEV of ϕ_1' is nonzero. Although H^+ is a mass eigenstate, the neutral states $H_{1,2}^0$ and I_2^0 are not. They are related to the mass eigenstates by an orthogonal matrix d_{ij}:

$$
\begin{bmatrix} H_1^0 \\ H_2^0 \\ I_2^0 \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} \begin{bmatrix} \phi_{m1} \\ \phi_{m2} \\ \phi_{m3} \end{bmatrix}.
\tag{36}
$$

In the absence of CP violation, $d_{13} = d_{23} = d_{31} = d_{32} = 0$

The WWZ effective operator. There are two graphs which contribute at one loop to the CP-violating WWZ vertex in Eq. (20). These are shown in Fig. 2. In fact, since we are only interested in getting an idea of the dependence on the Higgs boson masses, we concentrate only on diagram (a) in Fig. 2.

If we had no knowledge of the underlying theory, we could estimate the dependence of a, the coefficient of the WWZ vertex, on the heavy-mass scale $M \sim m_H$ by using the dimensional analysis of the previous section. In order to determine the suppression by v that is appropriate, we use the estimate of NDA, since this is appropriate to the case of a strongly coupled Higgs bosons that we are considering. Since a is dimensionless, we expect (after inserting a factor of g for each vector boson)

$$
a_{\text{dim}} \sim g^3 \left[\frac{v^2}{M^2} \right] \sim g \left[\frac{g}{4\pi} \right]^2.
\tag{37}
$$

Direct calculation, on the other hand, gives

$$
a_{\text{model}} = \sum_{ij} \left\{ i \frac{g^3}{32\pi^2 \cos\theta_W} (d_{3i}d_{2j} - d_{3j}d_{2i})(d_{2i}d_{2j} + d_{3i}d_{3j})(m_i^2 - m_j^2) I_1 \right\},
\tag{38}
$$

where

$$I_i = \int_0^1 dx_1 \int_0^{x_1} dx_2 \, x_2(x_1 - x_2) \frac{1}{x_2(m_i^2 - m_j^2) + x_1(m_j^2 - m_c^2) + m_c^2 + M_W^2 x_1(x_1 - 1)} \ . \tag{39}$$

In the above equations, the sum is over the physical neutral Higgs bosons with m_i and m_c being the neutral and charged Higgs boson masses, respectively. From the above expression, it is clear that

$$a_{\text{model}} \simeq \frac{g^3}{16\pi^2} \ln \left[\frac{m_H^2}{M_W^2} \right] , \tag{40}$$

where m_H is a generic Higgs boson mass. This agrees with the estimates from dimensional analysis, for $m_H \sim M \lesssim 4\pi v$.

For our later purposes we wish to embed the anomalous WWZ interaction into loops in order to estimate their implications for other effective interactions. Since the strongest dependence on heavy masses comes from the longitudinal W particles φ_W in these loops, we pause here to present an estimate for the size of the coefficient of the anomalous $Z\varphi_W\varphi_W$ vertex. There is only one difference from the previous case: The φ_W bosons couple with a strength that is proportional to gM/M_W rather than simply to g. On dimensional grounds the largest contributions to the $Z\varphi_W\varphi_W$ coupling should be proportional to

$$a_{\text{dim}}^\varphi \sim g \left[\frac{v^2}{M^2} \right] \left[\frac{gM}{M_W} \right]^2 \sim g \ . \tag{41}$$

In this model, the lowest-dimensional anomalous $Z\varphi_W\varphi_W$ coupling arises at one loop from the graphs of Fig. 2. Keeping only terms linear in q, the four-momentum of the external Z, we find that the $Z\varphi_W\varphi_W$ vertex is

$$\sum_{ij} \left\{ -\frac{g^3}{4\cos\theta_W} (d_{3i}d_{2j} - d_{3j}d_{2i})(d_{2i}d_{2j} - d_{3i}d_{3j}) \right.$$
$$\left. \times \frac{(m_i^2 - m_j^2)(m_i^2 - m_c^2)(m_j^2 - m_c^2)}{M_W^2} I_2^\mu \right\} , \tag{42}$$

with

$$I_2^\mu = \int \frac{d^4 l (2l)^\mu (2q \cdot l)}{(2\pi)^4 [(l + K/2)^2 - m_c^2](l^2 - m_i^2)^2(l^2 - m_j^2)^2} , \tag{43}$$

where $K = k - k'$. It is not necessary to solve this integral exactly—what is important is that for the external momenta of interest (i.e., those that are $\lesssim m_H$) it is dominated by momenta of order m_H, giving an integral that is of order m_H^{-4}. This gives the result

$$a_{\text{model}}^\varphi \simeq \frac{g^3}{16\pi^2} \left[\frac{m_H}{M_W} \right]^2 \ln \left[\frac{m_H^2}{M_W^2} \right] , \tag{44}$$

which is larger by an additional factor of m_H^2 / M_W^2 in

comparison with the result for transverse W's: Eq. (40). This enhancement corresponds, in the underlying theory, to the replacement of two gauge couplings g with two WBNGB-Higgs couplings $\lambda_{HH\varphi} \sim gm_H/M_W$. It agrees with estimate (41) when $m_H \sim M \sim 4\pi v$.

The weak dipole moment. Next, consider the Zdm of Eq. (21) in this effective theory. Using only naive dimensional analysis, we can therefore only conclude

$$z_{\text{dim}} \sim \frac{gv^2}{M^3} \ . \tag{45}$$

Any further information is more model specific.

In order to sharpen our estimate, we next consider the size of the Zdm that is induced in the low-energy theory from the effective WWZ operator considered previously, via the loop of Fig. 1. The dominant short-distance behavior comes from the contributions of longitudinal W's to this graph. In this case there is now an additional factor of m_τ from the required helicity flip, as well as two factors of the longitudinal W couplings to the fermion line, $\lambda_{\varphi\tau\tau} \sim gm_\tau/M_W$. Taking our estimate for this graph as a lower bound, we therefore expect

$$z_{\text{dim}} \gtrsim \frac{a_{\text{dim}}^\varphi}{16\pi^2} \left[\frac{gm_\tau}{M_W} \right]^2 \frac{m_\tau}{M^2}$$
$$\sim \frac{g^5}{(4\pi)^2} \left[\frac{v^2}{M_W^2} \right] \left[\frac{m_\tau^2}{M_W^2} \right] \left[\frac{m_\tau}{M^2} \right]$$
$$\gtrsim \frac{g^5}{(4\pi)^4} \left[\frac{m_\tau^3}{M_W^4} \right] . \tag{46}$$

We have used $v^2/M^2 \gtrsim 1/(4\pi)^2$ in this last equation.

In the underlying theory, the Zdm appears at two loops as in Fig. 3. The strongest dependence on m_H again comes when both W's in the loop are longitudinal—in a covariant gauge they are WBNGB's. Although the full two-loop diagram is difficult to solve completely, it is sufficient for our purposes to estimate the integrals using dimensional analysis. Again, the important region in the loop integration comes from momenta $\sim m_H$, since m_H is the largest scale in the problem. Including the factor of m_τ due to the required helicity
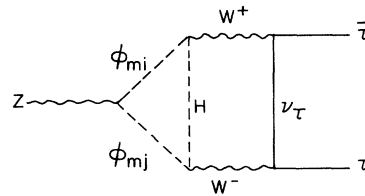

FIG. 3. Feynman graph which generates the CP-violating τ Zdm in the two-Higgs-doublet model.

flip on the fermion line and two factors of the WBNGB-τ Yukawa coupling, $\lambda_\tau \simeq g m_\tau / M_W$, we arrive at the following estimate for z:

$$z_{\text{model}} \sim g^5 \left[\frac{1}{16\pi^2} \right]^2 \frac{m_\tau^3}{M_W^4} \ln^2 \left[\frac{m_H^2}{M_W^2} \right] . \tag{47}$$

This result agrees both with our current estimate of Eq. (46), as well as with the earlier cutoff-based estimate of Eq. (26), but *not* with the "bad-variable" result of Eq. (25).

The vacuum polarization. It is instructive to also consider the contributions toward the Z vacuum polarization that are induced by the WWZ operator in this model. In addition to providing another comparison with the estimates, it furnishes an example for which there is (superficially) an *enhancement* by powers of M/M_W and for which a simple cutoff analysis in unitary gauge proves to be correct.

The required contribution to the Z vacuum polarization comes from the three-loop graph of Fig. 4. Again, in the underlying model the largest contribution comes when both W's in the inside loop are longitudinal. Then each $Z\varphi_W\varphi_W$ vertex contributes a factor of order

$$a^\varphi_{\text{model}} q , \tag{48}$$

where a^φ_{model} is given in Eq. (44) and q is the four-momentum which flows through the external Z line. The middle loop gives just the loop factor $1/16\pi^2$ times a logarithm. Therefore we find that the contribution to the Z vacuum polarization in this model has the form

$$[\delta \Pi_{ZZ}]_{\text{model}} \sim \left[\frac{g^2}{16\pi^2} \right]^3 \frac{m_H^4}{M_W^4} q^2 \ln^3 \left[\frac{m_H^2}{M_W^2} \right] . \tag{49}$$

Note the large power of m_H/M_W. This result *agrees* with the most-divergent part of the unitary-gauge cutoff dependence that is obtained by inserting two effective WWZ interactions into a one-loop vacuum polarization diagram,

$$[\delta \Pi_{ZZ}]_{\text{most div}}(q^2) = - \frac{a^2}{576\pi^2} \frac{\Lambda^4}{M_W^4} q^2 , \tag{50}$$

if we take our earlier estimate for the WWZ interaction: $a_{\text{dim}} \sim g^3 v^2 / m_H^2 \gtrsim g^3 / (4\pi)^2$.

Our dimensional estimate for this quantity, on the other hand, is

$$[\delta \Pi_{ZZ}]_{\text{dim}} \sim \frac{1}{(4\pi)^2} (a^\varphi_{\text{dim}})^2$$

$$\sim \frac{g^6}{(4\pi)^2} \left[\frac{v^4}{M^4} \right] \left[\frac{M^4}{M_W^4} \right] , \tag{51}$$
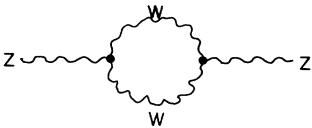

FIG. 4. Three-loop contribution to the Z-boson vacuum polarization. The blobs indicate the one-loop anomalous gauge-boson vertices whose structure is shown in Fig. 2.

which also agrees with the result of the underlying model once we use $v/M \gtrsim 1/4\pi$.

Note that, keeping in mind $g m_H \lesssim 8\pi M_W$, what appears to be an enhancement of four powers of m_H/M_W in Eq. (49) is really more than compensated for by the suppression of six powers of $g/4\pi$, as it must be in order for the result to be sensible. Thus it is misleading in this case to use the corresponding enhancement in Eq. (50) without also including the accompanying suppression that is implicit in the coefficient a.

VI. CONCLUSIONS

Effective Lagrangians are the natural way to parametrize the effects of the new physics which must lie beyond the standard model. The next generation of experiments will have the ability to probe a number of these new effective operators. Quite naturally, then, one wants to have an idea of how big these new effects might be.

Much work has gone into constraining the new operators, particularly those corresponding to trilinear gauge-boson vertices through their loop contributions to quantities which are measured at lower energies. We have argued here that these estimates [10] are typically misleading and often give bounds which are overly stringent.

Other authors [11] have made the same criticisms. However, they trace the cause of the problem to the apparent gauge noninvariance of the operators that are widely used in the literature. We argue instead that in this instance gauge invariance is a complete red herring and is not the source of the problem.

If one does not wish to explicitly include a Higgs scalar in the low-energy theory, there are two principal candidates for such an effective Lagrangian: one which requires only $U_{\text{em}}(1)$ gauge invariance, but not $SU_L(2) \times U_Y(1)$ gauge invariance, and one which imposes the full $SU_L(2) \times U_Y(1)$ gauge invariance, nonlinearly realized. We have demonstrated the equivalence of these two Lagrangians.

The same arguments as are used here may be similarly used to prove this equivalence for more general symmetry-breaking patterns $G \rightarrow H$. This shows that any effective theory containing light spin-1 particles automatically has a (spontaneously broken) gauge invariance. Alternatively, one can say that at low energies there is little to choose between a spontaneously broken gauge invariance and no gauge invariance at all.

The real source of the problems is the widespread use of cutoffs to regulate divergent graphs in the low-energy effective Lagrangian. Both the effective Lagrangian and its divergences, being off-shell quantities, are not invariant under field redefinitions. As a consequence, the result of a loop calculation will generically depend on the choice of variables if cutoffs are used to regulate the divergences.

It is in principle possible to use "good" variables in such loop calculations, in which case the cutoff behavior of the final answer accurately reflects the true dependence of the operator on the heavy-mass scale M. However, it is equally possible to choose "bad" variables, characterized by cancellations in the S matrix between large terms

in the effective Lagrangian, in which case the cutoff does *not* properly track the dependence on M. If "bad" variables are used, the bounds on effective operators inferred from such calculations are typically much too strong and completely unreliable. In the absence of an algorithm to distinguish "good" and "bad" variables, the constraints obtained from such cutoff-regulated calculations are ambiguous at best.

A separate mistake that has also been made when bounding effective interactions has been to take the scale of new physics M to be 10 TeV or higher [8,10], even when the effective theory does not *linearly* realize the electroweak gauge group. In this case the effective Lagrangian is simply being applied beyond its domain of applicability, since perturbative unitarity typically fails for such models when $M \gtrsim 4\pi v$.

If one wants to estimate the size of the new operators, we advocate dispensing with cutoffs completely. A simpler method is to just use simple dimensional arguments, supplemented by any additional information concerning dependence on coupling constants and (4π)'s that can be gleaned by inspecting underlying or low-energy graphs. These rules coincide in practice with currently used lore when this lore is sufficiently well spelled out. It has the conceptual advantage of not relying on the cutoff dependence of low-energy diagrams.

One quantity which *is* accurately calculable within the low-energy effective Lagrangian (as opposed to being an order-of-magnitude estimate) is the mixing among operators as the effective Lagrangian is evolved down from the heavy-mass scale M to low energies. This mixing, which is always logarithmic, is most easily computed using dimensional regularization, along with the decoupling-subtraction renormalization scheme. Among the beauties of dimensional regularization is that it is comparatively insensitive to the choice of "good" or "bad" variables.

By using dimensional regularization to calculate the mixing of operators and dimensional analysis to estimate the size of the initial conditions, i.e., the effective operators at scale M, one sees that it is never necessary to deal with cutoffs in a low-energy effective Lagrangian.

Note added. After this paper was submitted, we became aware of Ref. [25], whose authors present a point of view similar to our own.

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