Remarks on quark and gluon contribution to proton spin

B. Bagchi
Department of Applied Mathematics, Vidyasagar University, Midnapore 721101, West Bengal, India

S. Basu
Department of Theoretical Physics, Indian Association for the Cultivation of Science, Jadavpur, Calcutta 700 032 India
(Received 29 June 1992)

We consider a pole-model scheme to study isospin violation in the quark and gluon contribution to the proton matrix element of the flavor-singlet axial-vector current.

PACS number(s): 13.88.+e, 11.40.Ha, 14.20.Dh

A recent European Muon Collaboration (EMC) analysis [1,2] from experiments on deep-inelastic scattering shows that the portion of the proton spin carried by the quarks is significantly small. The EMC result is expressible in terms of the first moment of the polarized proton structure function which is related to the nucleonic matrix element of the flavor-singlet axial-vector current. The singlet axial-vector current $A_u^{(0)}=\bar{u}\gamma_\mu\gamma_5 u+\bar{d}\gamma_\mu\gamma_5 d+\bar{s}\gamma_\mu\gamma_5 s$ between two nucleonic states is necessarily proportional to the spin vector $\langle p's'|A_{\mu 5}^{(0)}|Ps\rangle=\Delta q' s_\mu$, where $s_\mu=\bar{\psi}i\gamma_\mu\gamma_5\psi$ is the spin vector and ψ is the wave function of the nucleon. The available data on the structure function of the polarized proton and the results on the hyperon decays lead to the estimates

$$\Delta u'=0.78\pm0.08 \ ,$$
$$\Delta d'=-0.47\pm0.08 \ , \quad (1)$$
$$\Delta s'=-0.19\pm0.08 \ ,$$

implying

$$\Delta q'\equiv\sum\Delta q'=\Delta u'+\Delta d'+\Delta s'$$
$$=0.12\pm0.24 \ . \quad (2)$$

It should be noted that SU(3) restricts $\Delta u'+\Delta d'-2\Delta s'-=3F-D$ while the Bjorken sum rule fixes $\Delta u'-\Delta d'=g_A=F+D$. Using the SU(3) relations the results [3] for the spin contribution due to the quarks differ appreciably from the unity value which is expected naively from the simple quark-model picture.

Of the two axial-vector couplings, the strength of the octet may be ascertained from the weak decays of the hyperons while the singlet may be used to determine the quark contribution of the proton spin. It is known that the EMC data, in effect, signal an approximate decoupling of the would-be U(1) Goldstone boson from the nucleon. Fritzsch [4] has shown it possible to define flavor-independent matrix elements of the anomalous divergence which are linked to the axial-vector U(1) charge and to the spin densities inside the proton. Further, the present authors [5] have shown that the axial-vector singlet charge may be related to $\Delta q'$ and the F/D ratio in a straightforward manner.

Recently, Hatsuda [6] made a careful analysis of the anomalous gluon content of the proton. Employing a large-N_C chiral dynamics approach he has evaluated the proton matrix element of the flavor-singlet axial-vector current and has noted a large isospin content in the gluon contribution to $\Delta q'$. In this paper we take a look at the same problem from a different point of view: adopting a pole dominance model, we consider the nucleonic matrix element of the axial-vector anomaly and show that it is possible to express $\Delta q'$ and the gluon helicity component Δg purely in terms of the decay constants and topological charge parameters of the (η,η',π^0) mesons. In this way we avoid any direct reference to the meson mixing angles. Furthermore, we perform a fit to the nucleonic meson coupling constants to estimate $\Delta q'$ and Δq quantitatively. In our scheme, we are guided by the solutions [7] of the anomalous Ward identities which provide reasonable estimates of the isospin and flavor breaking in QCD.

We begin by writing down the Goldberger-Treiman relation and its analogue in the singlet channel. The isovector axial-vector current $A_\mu^{(3)}$ between the protons gives the matrix element

$$\langle p|A_\mu^{(3)}|p\rangle\equiv\langle p|\bar{u}\gamma_\mu\gamma_5 u-\bar{d}\gamma_\mu\gamma_5 d|p\rangle$$
$$=\bar{\psi}(p)[G_1(q^2)\gamma_\mu\gamma_5-G_2(q^2)q_\mu\gamma_5]\psi(p') \ , \quad (3)$$

where ψ is the free Dirac spinor and (p',p) are the proton momenta.

A pion pole is present in the induced form factor $G_2(q^2)$ but this pole makes no contribution to the divergence at $q^2\to 0$. Taking the divergence yields the Goldberger-Trieman relation

$$2MG_1(0)=2Mg_A=2f_\pi g_{\pi NN} \ . \quad (4)$$

On the other hand, the isosinglet axial-vector current between the protons is

$$\langle p|\bar{u}\gamma_\mu\gamma_5 u+\bar{d}\gamma_u\gamma_5 d|p\rangle$$
$$=\bar{\psi}(p)[G_1(q^2)\gamma_\mu\gamma_5-G_2(q^2)q_\mu\gamma_5]\psi(p') \ . \quad (5)$$

Since no Goldstone boson can be attached to G_2^0, we have [4], in contrast with (4),

$$2MG_1^0=2f_\eta g_{\eta NN}+C \ , \quad (6)$$

where C represents the contribution from the continuum assuming an unsubtracted dispersion relation for the form factor.

We now turn to the divergence of the axial-vector currents [including the U(1) component], which in QCD reads as

$$\partial_\mu A_\mu = -d_{ijk} c_j \vartheta_k + \delta_{i0} a , \quad i,j,k = 0, \ldots, 8 , \tag{7}$$

where the v_j's are the standard pseudoscalar densities according to the Gell-Mann–Oakes–Renner (GMOR) scheme; $c_0 = (m_u + m_d + m_s)/\sqrt{6}$, $c_3 = (m_u - m_d)/2$, $c_8 = (m_u + m_d - 2m_s)/2\sqrt{3}$; and a stands for the gluon anomaly, $a = N_f(\propto_s/2\pi)\mathrm{Tr}G\tilde{G} = \partial_\mu k_\mu$. One may introduce a partially conserved U(1) current $\tilde{A}_u^{(0)} = A_u^{(0)} - k_u$ whose divergence is soft. However, since the current k_u depends explicitly on the gluon gauge field, $\tilde{A}_u^0$ is not a gauge-invariant quantity. From (7), one has, for the singlet component,

$$\partial_\mu A_\mu^{(0)} = 2(m_u \bar{u} i\gamma_5 u + m_d \bar{d} i\gamma_5 d + m_s \bar{s} i\gamma_5 s) + a . \tag{8}$$

Relation (8) dictates that even to lowest order q must involve a gluonic contribution:

$$\Delta q' = \Delta q - N_f \frac{\alpha_s}{2\pi} \Delta g , \tag{9}$$

where, following Cheng and Li [8], we define q and g through (setting $N_f = 3$)

$$\langle p' | \sum_i^{u,d,s} 2 m_i \bar{g}_i i\gamma_5 g_i | p \rangle = (\Delta q) 2M \bar{\psi} i\gamma_5 \psi + O(q^{n\geq 2}) , \tag{10a}$$

$$\langle p' | a | p \rangle = -\left[\frac{3}{2} \frac{\alpha_s}{\pi} \Delta g \right] 2M \bar{\psi} i\gamma_5 \psi + O(q^{n\geq 2}) . \tag{10b}$$

Hatsuda [6] has shown that under the pole dominance approximation as in Fig. 1, Δg is determinable in terms of the (η', η, π^0) mixing angles. Writing an effective Lagrangian he has studied the influence of the nucleon-nucleon coupling constants on Δq and the gluon helicity component Δg individually.

Here we consider the simplest procedure to determine Δg. We write

$$\langle N' | \phi_p | N \rangle = \frac{g_{PNN}}{m_p^2} \bar{\psi} i\gamma_5 \psi , \tag{11}$$

where N stands for the nucleon (p,n) and P stands collectively for the $I=0$ pseudoscalar mesons (η', η, π^0). Defining the gluon anomaly in terms of the meson fields as $a \equiv A_P m_P^2 \phi_P$, where the A_P's are the so-called topological charges $\langle 0 | a | P \rangle = A_P m_P^2$, we may use (10b) to translate (11) into the following sum rule:

$$-\left[\frac{3}{2} \frac{\alpha_s}{\pi} \Delta g \right] 2M = A_{\pi^0} g_{\pi^0 NN} + A_\eta g_{\eta NN} + A_{\eta'} g_{\eta' NN} , \tag{12}$$

where the positive and negative signs stand for the proton and the neutron, respectively. It is noticeable that no meson mixing angle is explicitly involved in (12). Similarly, considering the definition (10a) we can write down

$$(\Delta q) 2M = \pm f_{0\pi} g_{\pi^0 NN} + f_{0\eta} g_{\eta NN} + f_{0\eta'} g_{\eta' NN} , \tag{13}$$

where $\langle 0 | \partial_\mu A_\mu^{(0)} | P \rangle = f_{0P} m_P^2$ has been used.

The topological charge parameters $A_{\eta'}$, A_η, and A_{π^0} and the decay constants (f_{0P}, f_{8P}) may be determined by solving the Ward identities which are [7]

$$\begin{aligned}
& m_P^2 f_{3P}^2 = (m_u + m_d) v , \\
& m_P^2 f_{3P} f_{8P} = \frac{1}{\sqrt{3}} (m_u - m_d) v , \\
& m_P^2 f_{8P}^2 = \tfrac{1}{3} (m_u + m_d + 4m_s) , \\
& m_P^2 f_{3P} f_{0P} = \sqrt{2/3} (m_u - m_d) v , \\
& m_P^2 f_{8P} f_{0P} = \frac{\sqrt{2}}{3} (m_u + m_d - 2m_s) v , \\
& m_P^2 f_{0P}^2 - m_P^2 f_{0P} A_P = \tfrac{2}{3} (m_u + m_u + m_s) v , \\
& m_P^2 f_{8P} A_P = 0 , \\
& m_P^2 f_{3P} A_P = 0 , \\
& m_P (f_{0P} - A_P) A_P \geq 0 ,
\end{aligned} \tag{14}$$

where $-v = \langle \bar{u}u \rangle = \langle \bar{d}d \rangle = \langle \bar{s}s \rangle$ and the decay constants are defined by $\langle 0 | \partial_\mu A_\mu^i | P \rangle - f_{iP} m_P^2$; $i = 0$, 8, and 3. The relations (14) are expected to hold within an order of about 20–30 % accuracy that is typical of a pole dominance approximation. To determine the decay constants and gluon couplings unambiguously, one needs to seek consistency between (14) and the 2γ decay amplitudes of P, $\eta\gamma$, $\eta'\gamma$ decay widths of the J/ψ and $J/\psi\pi^0(J/\psi\eta)$ decay widths of ψ' assuming that the latter processes are mediated by two gluon matrix elements. A comprehensive treatment may be found in Ref. [7]—here we only quote the estimates of the topological charge constants and the singlet decay constants:

$$\begin{aligned}
& A_{\pi^0} = (0.64 \pm 0.06) f_\pi , \quad f_{0\pi} = (0.00 \pm 0.05) f_\pi , \\
& A_\eta = (0.86 \pm 0.08) f_\pi , \quad f_{0\eta} = (0.19 \pm 0.02) f_\pi , \\
& A_{\eta'} = (0.76 \pm 0.07) f_\pi , \quad f_{0\eta'} = (1.05 \pm 0.12) f_\pi .
\end{aligned} \tag{15}$$

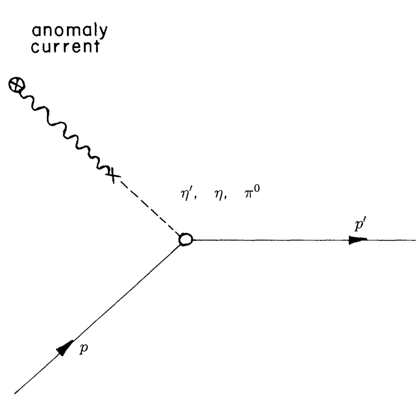

FIG. 1. Pole dominance with η', η, π^0.

For the meson-nucleon coupling constants there are various estimates in the literature. For instance, one can employ the pole dominance approximation to determine meson-nucleon coupling constants within a one-loop baryon model. The results are [9]

$$g_{\eta NN}=8.8\pm0.3\ ,\qquad g_{\eta' NN}=6.3\pm0.4\ . \tag{16}$$

As remarked by Efremov, Soffer, and Tornqvist [3], if the above estimates are compared with that of the one boson exchange potential (OBEP) model which gives $g_{\eta' NN}=7.3\pm$ a large error, it seems more reasonable to adopt the result (16). The point is that in the OBEP analysis of the *NN* scattering no ghost pole is taken into account which may generate a contact *NN* interaction: for small squared momenta transfer the latter may change $g_{\eta' NN}$ considerably.

In the following we compare the results for $[-(\alpha s/\pi)\Delta g]$ using the baryon loop model and the nucleon-nucleon potential values for $g_{\eta NN}$ and $g_{\eta' NN}$.

a. Baryon loop model. Using the values (16) we get, from (12),

$$\left[-\frac{\alpha s}{2\pi}\Delta g\right] \text{ for proton}=0.35\ ,$$

$$\left[-\frac{\alpha s}{2\pi}\Delta g\right] \text{ for neutron}=0.06\ .$$

b. Nucleon-nucleon model. Using flavor SU(3) symmetry, the results of the nucleon-nucleon potential model are $g^2_{\eta NN}/4\pi=3.68, g^2_{\eta' NN}/4\pi=4.23$. These give

$$\left[-\frac{\alpha s}{2\pi}\Delta g\right] \text{ for proton}=0.33\ ,$$

$$\left[-\frac{\alpha s}{2\pi}\Delta g\right] \text{ for neutron}=0.05\ .$$

In Fig. 2 we give a graphic illustration of the variation of Δq and Δg with the meson-nucleon coupling constants. For comparison with the results of Ref. [6] we have set $g_{\eta N}$ $(=g_{\eta' N})$. It is clear from the figures that the graphs for Δq and the gluon helicity component Δg show a marked difference in behavior for the proton and the neutron thus reflecting a large isospin breaking. However, $\Delta q'$ is comparatively much more stable. Our analysis confirms Hatsuda's observation that while individually Δq and Δg may involve sizable violations of isospin, their linear combination, namely, $\Delta q'$, is more or less insensitive to isospin breaking within a reasonable level of tolerance.

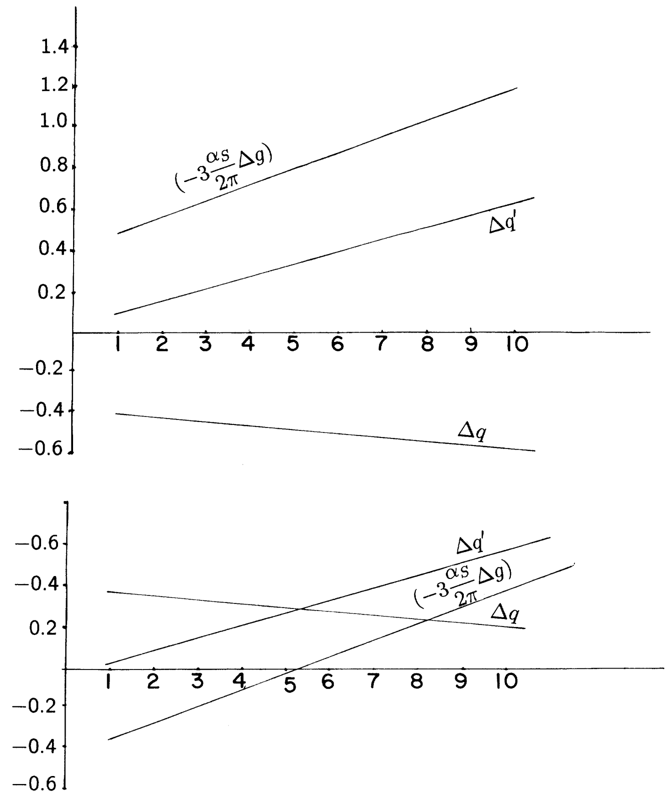

FIG. 2. (a) Variation of $-3(\alpha_s/2\pi)\Delta g$, Δq, and $\Delta q'$ with $g_{\eta NN}$ $(=g_{\eta' NN})$ for the proton. (b) Variation of $-3(\alpha_s/2\pi)\Delta g$, Δq, and $\Delta q'$ with $g_{\eta NN}$ $(=g_{\eta' NN})$ for the neutron.

[1] J. Ashman *et al.*, Phys. Lett. B **202**, 603 (1988); M. Arneodo *et al.*, *ibid.* **211**, 493 (1988).
[2] M. Bourquin *et al.*, Z. Phys. C **21**, 27 (1983).
[3] A. V. Efremov, J. Soffer, and N. A. Tornqvist, Phys. Rev. Lett. **64**, 1495 (1990).
[4] H. Fritzsch, Phys. Lett. B **242**, 451 (1990).
[5] B. Bagchi and S. Basu, Phys. Lett. B **268**, 112 (1991).
[6] T. Hatsuda, Nucl. Phys. **B329**, 376 (1990).
[7] B. Bagchi, A. Lahiri, and S. Niyogi, Phys. Rev. D **41**, 2871 (1990).
[8] T. P. Cheng and Ling-Fong Li, Phys. Rev. Lett. **62**, 1441 (1989).
[9] B. Bagchi and A. Lahiri, J. Phys. G **16**, L239 (1990).