PHYSICAL REVIEW D VOLUME 48, NUMBER 9 1 NOVEMBER 1993

Z^0 decay into charmonium via charm quark fragmentation

Eric Braaten and Kingman Cheung
Department of Physics and Astronomy, Northwestern University, Evanston, Illinois 60208

Tzu Chiang Yuan
Davis Institute for High Energy Physics, Department of Physics, University of California, Davis, California 95616
(Received 23 February 1993)

In decays of the Z^0, the dominant mechanism for the direct production of charmonium states is the decay of the Z^0 into a charm quark or antiquark followed by its fragmentation into the charmonium state. We calculate the fragmentation functions describing the splitting of charm quarks into S-wave charmonium states to leading order in the QCD coupling constant. Leading logarithms of M_Z/m_c are summed up using Altarelli-Parisi evolution equations. Our analytic result agrees with the complete leading order calculation of the rate for $Z^0 \to \psi c\bar{c}$. We also use our fragmentation functions to calculate the production rate of heavy quarkonium states in $W^\pm$, top quark, and Higgs boson decays.

PACS number(s): 13.38.+c, 12.38.Bx, 14.40.Gx

INTRODUCTION

Among the rare decay modes of the Z^0 predicted by the standard gauge theory are ones whose final states include charmonium. Of particular importance are the $^3S_1$ charmonium states J/ψ and ψ', since their decays into lepton pairs provide easily identifiable experimental signatures. The dominant production mechanism for ψ and ψ' is the decay of B hadrons; in fact, this serves as a signature for B hadron production in Z^0 decay. The direct production of ψ and ψ' is therefore important in Z^0 decays as a background to B physics. It is also of interest in its own right, since it involves both short-distance and long-distance aspects of quantum chromodynamics (QCD). The production of a charm quark and antiquark with small relative momentum in Z^0 decay is a short-distance process with a characteristic length scale that can range from $1/M_Z$ to as large as $1/m_c$. The subsequent formation of a bound state from the $c\bar{c}$ pair is a long-distance process involving all the complications of nonperturbative QCD. The methods of perturbative QCD can be used to calculate the production rates provided that it is possible to systematically separate the short-distance effects from the long-distance effects.

Most previous work on charmonium production in Z^0 decay [1–3] has focused on *short-distance* processes in which the $c\bar{c}$ pair that form the ψ is produced with a transverse separation of order $1/M_Z$. Long-distance effects involved in the formation of the bound state are factored into the nonrelativistic radial wave function at the origin $R(0)$. The best example of a short-distance process is $Z^0 \to \psi gg$, which has a branching fraction of about 10^{-7}. This small branching fraction can be partly attributed to a factor of $|R(0)|^2/(m_c M_Z^2)$, which represents the probability for a $c\bar{c}$ pair that is produced in a region of size $1/(m_c M_Z^2)$ to form a bound state. This probability factor suppresses the branching fractions for short-distance processes by m_c^2/M_Z^2, so that they can be neglected in the limit $M_Z/m_c \to \infty$.

As pointed out by Kühn and Schneider [4], the direct production of charmonium in Z^0 decay will be dominated not by short-distance processes but by *fragmentation* processes. The fragmentation mechanism is the decay of the Z^0 into a final state that includes a high energy quark or gluon, followed by the splitting of that parton into the charmonium state plus other partons. In the fragmentation mechanism, the c and $\bar{c}$ that form the charmonium state are produced with a separation of order $1/m_c$. The probability that they form a bound state is proportional to $|R(0)|^2/m_c^3$. The branching ratio for such a process is therefore not suppressed by the factor m_c^2/M_Z^2 associated with short-distance processes. The fragmentation of a parton is described by a fragmentation function $D(z,\mu)$, which gives the probability for a parton with invariant mass less than μ to split into the charmonium state with longitudinal momentum fraction z. It was recently shown that the fragmentation functions for the splitting of partons into heavy quarkonium states can be calculated using perturbative QCD [5]. The fragmentation functions $D_{g\to\psi}(z,\mu)$ and $D_{g\to\eta_c}(z,\mu)$ that describe the splitting of gluons into S-wave quarkonium states were calculated to leading order in α_s at the scale $\mu = 2m_c$. They were evolved to larger scales μ by using Altarelli-Parisi evolution equations, which sum up leading logarithms of μ/m_c. The production of ψ in Z^0 decay from the splitting of virtual gluons has been considered by Hagiwara, Martin, and Stirling [6], but they did not organize the calculation in terms of fragmentation functions and were thus unable to sum up leading logarithms of M_Z/m_c.

The production rate of ψ via the process $Z^0 \to \psi c\bar{c}$ has been calculated by Barger, Cheung, and Keung [7] with a rather surprising result: it has a branching fraction of about 10^{-5}. This is almost 2 orders of magnitude larger than $Z^0 \to \psi gg$, in spite of the fact that both rates are the same order in α_s. A similar result was found earlier in e^+e^- annihilation by Clavelli [8]. An explanation for the relatively large branching fraction of $Z^0 \to \psi c\bar{c}$ was provided in Ref. [5], where it was pointed out that this process includes a fragmentation contribution that is not suppressed by a factor of m_c^2/M_Z^2. This contribution can be factored into the rate for the Z^0 to decay into a $c\bar{c}$ pair multiplied by the probability for the c or $\bar{c}$ to fragment into ψ.

In this paper we calculate the fragmentation functions $D_{c\to\psi}(z,\mu)$ and $D_{c\to\eta_c}(z,\mu)$ for a charm quark to split into an S-wave charmonium state. The fragmentation functions at the scale $\mu = 3m_c$ are calculated to leading order in $\alpha_s(2m_c)$. Altarelli-Parisi equations are used to evolve them up to the scale $\mu = M_Z/2$ appropriate for Z^0 decay. Our simple analytic result for $Z^0 \to \psi c\bar{c}$ agrees with the complete leading order calculation of Barger, Cheung, and Keung. We also use our fragmentation functions to calculate the direct production rates for ψ in $W^\pm$ decays and for Υ in top quark and Higgs boson decays.

Z^0 DECAY VIA FRAGMENTATION

The fragmentation contribution to the inclusive decay rate of the Z^0 into charmonium is the term that survives in the limit $M_Z/m_c \to \infty$. The general form of the fragmentation contribution to the differential decay rate for the production of a ψ of four-momentum p is

$$d\Gamma(Z^0 \to \psi(p) + X) = \sum_i \int_0^1 dz\, d\widehat{\Gamma}(Z^0 \to i(p/z) + X, \mu)\, D_{i\to\psi}(z,\mu)\,, \qquad (1)$$

where the sum is over partons of type i and z is the longitudinal momentum fraction of the ψ relative to the parton. The physical interpretation of (1) is that a ψ of momentum p can be produced by first producing a parton i of larger momentum p/z which subsequently splits into a ψ carrying a fraction z of the parton momentum. The expression (1) for the differential decay rate has a factored form: all the dependence on the energy of the ψ, or equivalently on the mass M_Z, is in the parton subprocess decay rate $d\widehat{\Gamma}$, while all the dependence on the charm quark mass m_c is in the fragmentation function $D_{i\to\psi}$. To maintain this factored form in spite of the logarithms of M_z/m_c that arise in perturbation theory, a factorization scale μ must be introduced. The dependence on the arbitrary scale μ cancels between the two factors. Large logarithms of M_Z/μ in the subprocess decay rate $\widehat{\Gamma}$ can be avoided by choosing μ on the order of M_Z. Large logarithms of μ/m_c then necessarily appear in the fragmentation functions $D_{i\to\psi}(z,\mu)$, but they can be summed up by solving the evolution equations [9]

$$\mu\frac{\partial}{\partial\mu}D_{i\to\psi}(z,\mu) = \sum_j \int_z^1 \frac{dy}{y} P_{i\to j}(z/y,\mu) D_{j\to\psi}(y,\mu)\,, \qquad (2)$$

where $P_{i\to j}(x,\mu)$ is the Altarelli-Parisi function for the splitting of the parton of type i into a parton of type j with longitudinal momentum fraction x. For example, the $c \to c$ splitting function for a charm quark with energy much greater than its mass is the usual splitting function for quarks:

$$P_{c\to c}(x,\mu) = \frac{\alpha_s(\mu)}{2\pi}\left(\frac{8}{3}\,\frac{1+x^2}{(1-x)_+} + 4\,\delta(1-x)\right)\,. \qquad (3)$$

The boundary condition on the evolution equation (2) is the initial fragmentation function $D_{i\to\psi}(z,\mu_0)$ at some scale μ_0 of order m_c. As shown in Ref. [5], it can be calculated perturbatively as a series in $\alpha_s(2m_c)$.

We can easily count the order in α_s for the fragmentation contributions to ψ production in Z^0 decay. The subprocess rate $\widehat{\Gamma}$ for producing gluons is of order α_s, while that for producing quarks is of order 1. The fragmentation function for a gluon to split into ψ, which was calculated in Ref. [5], is proportional to α_s^3. A light quark can split into a ψ only by radiating a gluon which splits into a ψ, so its fragmentation function is of order α_s^4. In contrast, the fragmentation function for a charm quark to split into a ψ, which will be calculated explicitly below, is only of order α_s^2. Thus the fragmentation of charm quarks into ψ dominates by two powers of α_s over the fragmentation of light quarks or gluons.

Keeping only the charm quark and antiquark contributions to (1), the energy distribution of the ψ reduces at leading order in α_s to

$$\frac{d\Gamma}{dz}(Z^0 \to \psi(E)+X) = 2\,\widehat{\Gamma}(Z^0 \to c\bar{c})\, D_{c\to\psi}(z, M_Z/2)\,, \qquad z = \frac{2E}{M_Z}\,. \qquad (4)$$

This fragmentation formula is of course applicable only for a ψ of energy E that is a significant fraction z of the energy $M_Z/2$ of the charm quark and much greater than the mass M_ψ of the ψ. In (4), the factor of 2 accounts for the contribution from the fragmentation of the $\bar{c}$. We have set the factorization scale μ to $M_Z/2$ to avoid large logarithms from higher orders in perturbation theory. At leading order in α_s, only the diagonal term in the evolution equation (2) survives:

$$\mu\frac{\partial}{\partial\mu}D_{c\to\psi}(z,\mu) = \int_z^1 \frac{dy}{y}\, P_{c\to c}(z/y,\mu)\, D_{c\to\psi}(y,\mu)\,. \qquad (5)$$

Integrating (4) over the energy, the total rate for inclusive ψ production is

$$\Gamma(Z^0 \to \psi + X) = 2\,\widehat{\Gamma}(Z^0 \to c\bar{c}) \int_0^1 dz\, D_{c\to\psi}(z, 3m_c)\,. \qquad (6)$$

We have set the fragmentation scale equal to $3m_c$ by exploiting the fact that at leading order in α_s the Altarelli-Parisi splitting function (3) satisfies $\int_0^1 dx P_{c\to c}(x,\mu) = 0$. The evolution equation (5) then implies that the fragmentation probability $\int_0^1 dz D_{c\to\psi}(z,\mu)$ does not evolve with the scale μ.

FRAGMENTATION FUNCTION FOR $c \to \psi$

We proceed to calculate the initial fragmentation function $D_{c\to\psi}(z, 3m_c)$ for a charm quark to split into a ψ to leading order in $\alpha_s(2m_c)$. Our strategy is to isolate the contribution Γ_1 to the decay rate for $Z^0 \to \psi c\bar{c}$ that arises from the fragmentation of the charm quark. We can then obtain the fragmentation probability $\int_0^1 dz D(z)$ by dividing Γ_1 by the rate Γ_0 for $Z^0 \to c\bar{c}$:

$$\Gamma_0 = \frac{1}{2M_Z} \int [d\bar{q}][dq]\, (2\pi)^4 \delta^4(Z - \bar{q} - q)\, \frac{1}{3} \sum |A_0|^2 , \tag{7}$$

where Z, $\bar{q}$, and q are the four-momenta of the Z^0, $\bar{c}$, and c, and $[dq] = d^3q/(16\pi^3 q_0)$ is the Lorentz-invariant phase-space element. The square of the amplitude A_0 for $Z^0 \to c\bar{c}$, averaged over initial spins and summed over final spins and colors, is

$$\frac{1}{3}\sum |A_0|^2 = \left(-g^{\alpha\beta} + \frac{Z^\alpha Z^\beta}{M_Z^2}\right) \mathrm{tr}[\Gamma_\alpha\, (\not{\bar{q}} - m_c)\, \Gamma_\beta \times (\not{q} + m_c)] , \tag{8}$$

where Γ_α is the $Z^0 c\bar{c}$ vertex whose explicit form is not required. In the limit $M_Z >> m_c$, the factors of m_c in the trace can be neglected.

The rate for the decay $Z^0 \to \psi c\bar{c}$ is

$$\Gamma_1 = \frac{1}{2M_Z} \int [d\bar{q}][dp][dp']\, (2\pi)^4 \delta^4(Z - \bar{q} - p - p') \times \frac{1}{3} \sum |A_1|^2 , \tag{9}$$

where $\bar{q}$, p, and p' are the four-momenta of the $\bar{c}$, ψ, and c. The four Feynman diagrams that contribute to the amplitude A_1 at leading order in α_s are shown in Fig. 1. The contributions to the process $Z^0 \to \psi c\bar{c}$ that correspond to the fragmentation of the charm quark come from the region of phase space in which the $\psi - c$ system has large momentum $q = p + p'$ of order M_Z and small invariant mass $s = q^2$ of order m_c^2. To facilitate the extraction of the fragmentation probability, we write the three-body phase space for the outgoing particles in an iterated form by introducing integrals over q and over s:

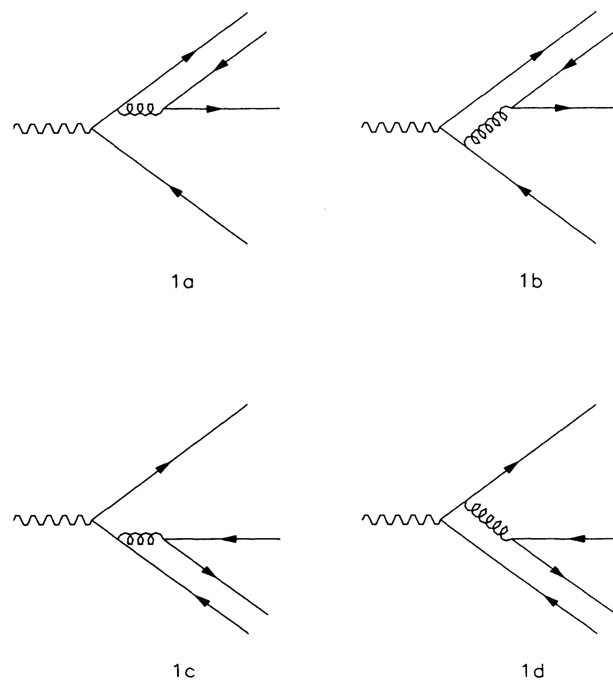

FIG. 1. The four Feynman diagrams for $Z^0 \to \psi c\bar{c}$ at leading order in α_s. The outgoing momentum assignments are $p/2$ and $p/2$ for the parallel c and $\bar{c}$ lines and p' and $\bar{q}$ for the other c and $\bar{c}$ lines.

$$\int [d\bar{q}][dp][dp']\, (2\pi)^4 \delta^4(Z - \bar{q} - p - p') = \int \frac{ds}{2\pi} \int [d\bar{q}][dq]\, (2\pi)^4 \delta^4(Z - \bar{q} - q) \int [dp][dp']\, (2\pi)^4 \delta^4(q - p - p') . \tag{10}$$

We also express the two-body phase-space integral over p and p' in terms of the longitudinal momentum fraction z of the ψ. In a frame in which the virtual charm quark has the four-momentum $q = (q_0, 0, 0, q_3)$, the longitudinal momentum fraction of the ψ is $z = (p_0 + p_3)/(q_0 + q_3)$ and its transverse momentum is $\mathbf{p}_\perp = (p_1, p_2)$. Expressed in terms of these variables, the Lorentz invariant phase space element is $[dp] = dz d^2 p_\perp/(16\pi^3 z)$. Integrating over the four-momentum p' and over $\mathbf{p}_\perp$, the two-body phase-space integral reduces to

$$\int [dp][dp']\, (2\pi)^4 \delta^4(q - p - p') = \frac{1}{8\pi} \int_0^1 dz\, \theta\left(s - \frac{4m_c^2}{z} - \frac{m_c^2}{1-z}\right) . \tag{11}$$

We have set $M_\psi = 2m_c$, which is accurate up to relativistic corrections. If $s = q^2$ is of order m_c^2, the delta function $\delta^4(q - p - p')$ constrains $\mathbf{p}_\perp$ to be of order m_c. From the mass-shell condition, the component $p_0 - p_3 = (p_\perp^2 + 4m_c^2)/(p_0 + p_3)$ is of order m_c^2/M_Z. Thus, to leading order in m_c/M_Z, we can set $p = zq$.

We proceed to isolate the contribution to the amplitude A_1 from the fragmentation of the charm quark. In covariant gauges, this contribution comes from both of the diagrams in Figs. 1(a) and 1(b), while the diagrams in Figs. 1(c) and 1(d) contain contributions from $\bar{c}$ fragmentation. In the axial gauge associated with the four-vector $n = (1, 0, 0, -1)$, the contribution from fragmentation of the charm quark comes only from the diagram shown in Fig. 1(a). The amplitude for Fig. 1(a) in this gauge can be reduced to

$$A_1 = \frac{4g_s^2 R(0)}{3\sqrt{6\pi m_c}} \epsilon_\alpha(Z)\epsilon_\mu(p)^* \frac{1}{(s-m_c^2)^2}\, \bar{u}(p') \left(2m_c\, \gamma^\mu\, (\not{q} + m_c) \; + \; \frac{s-m_c^2}{n\cdot(2q-p)}\, \not{n}\, \gamma^\mu\, (\not{p} + 2m_c) \right) \Gamma^\alpha\, v(\bar{q}) \,. \quad (12)$$

We have used standard covariant Feynman rules [1] for projecting the amplitude for production of a $c\bar{c}$ pair with equal four-momenta $p/2$ onto the amplitude for production of a ψ with four-momentum p. The parameter $R(0)$ is the value of the nonrelativistic radial wave function at the origin. Averaging over inital spins and summing over final spins and colors, the square of the amplitude reduces to

$$\frac{1}{3}\sum |A_1|^2 = \frac{128\pi\alpha_s^2 |R(0)|^2}{27 m_c} \frac{1}{(s-m_c^2)^4} \left(-g^{\alpha\beta} + \frac{Z^\alpha Z^\beta}{M_Z^2} \right) \mathrm{tr}[\Gamma_\alpha\, (\not{\bar{q}} - m_c)\, \Gamma_\beta\, D] \,, \quad (13)$$

where D is a Dirac matrix that depends on $\bar{q}$, q, and p. We need only keep the terms in D for which the Dirac trace in (13) is of order $m_c^4 M_Z^2$. While $\bar{q}$, q, and p all have components of order M_Z, $s = q^2$ is of order m_c^2 in the fragmentation region. Simplifying the Dirac matrix by dropping terms which are suppressed by powers of m_c/M_Z, it reduces to

$$D = (s^2 - 2m_c^2 s - 47 m_c^4)\, \not{q} \; - \; (s-m_c^2)(s-9m_c^2)\, \not{p}$$
$$+4\, \frac{s-m_c^2}{n\cdot(2q-p)} [(s+7m_c^2)\, n\cdot q\, \not{p} \; - \; (s-5m_c^2)\, n\cdot p\, \not{p} \; - \; 8m_c^2\, n\cdot q\, \not{q}] + 12 \left(\frac{s-m_c^2}{n\cdot(2q-p)} \right)^2 n\cdot p\, n\cdot(q-p)\, \not{p} \,. \quad (14)$$

We have exploited the fact that while $\not{p}$ and $\not{q}$ are both of order M_Z, their product $\not{p}\not{q}$ is only of order $m_c M_Z$. The coefficients of $\not{p}$ and $\not{q}$ in (14) are all manifestly of order m_c^4, so we can substitute $p = zq$ for all the remaining factors of p. The Dirac trace in (12) is then proportional to $\mathrm{tr}(\Gamma_\alpha \not{\bar{q}}\, \Gamma_\beta \not{q})$. It is now easy to divide Γ_1 by the decay rate Γ_0 given in (7) to obtain the fragmentation probability:

$$\int_0^1 dz\, D_{c\to\psi}(z) = \frac{8\alpha_s^2 |R(0)|^2}{27\pi m_c} \int_0^\infty ds\, \frac{1}{(s-m_c^2)^4} \int_0^1 dz\, \theta\left(s - \frac{4m_c^2}{z} - \frac{m_c^2}{1-z} \right)$$
$$\times \left((s^2 - 2m_c^2 s - 47m_c^4) \; - \; z(s-m_c^2)(s-9m_c^2) \; + \; 4\frac{z(1-z)}{2-z} s(s-m_c^2) \right.$$
$$\left. -4\frac{8-7z-5z^2}{2-z} m_c^2 (s-m_c^2) \; + \; 12\frac{z^2(1-z)}{(2-z)^2}(s-m_c^2)^2 \right) . \quad (15)$$

Note that the upper limit on the integral over s has been increased to ∞. Since the integrand behaves like $1/s^2$ at large s, this only changes the integral by an amount of order m_c^2/M_Z^2, which we have been systematically neglecting. Evaluating the integral over s in (15), we obtain our final expression for the initial fragmentation function:

$$D_{c\to\psi}(z, 3m_c) = \frac{8}{27\pi}\, \alpha_s(2m_c)^2\, \frac{|R(0)|^2}{m_c^3}\, \frac{z(1-z)^2(16 - 32z + 72z^2 - 32z^3 + 5z^4)}{(2-z)^6} \,. \quad (16)$$

We have set the scale μ in the fragmentation function to $\mu = 3m_c$, which is the minimum value of the invariant mass of the fragmenting charm quark. We have also set the scale in the running coupling constant to $\mu = 2m_c$, which is the minimum value of the invariant mass of the virtual gluon. Integrating over z, we obtain the total fragmentation probability

$$\int_0^1 dz\, D_{c\to\psi}(z, 3m_c) = \frac{8}{27\pi}\, \alpha_s(2m_c)^2\, \frac{|R(0)|^2}{m_c^3} \left(\frac{1189}{30} - 57\ln 2 \right) . \quad (17)$$

FRAGMENTATION FUNCTION FOR $c \to \eta_c$

The fragmentation function for a charm quark to split into the ${}^1S_0$ state of charmonium η_c can be calculated in the same way as for ψ. The starting point is the expression (9) for the decay rate for $Z^0 \to \eta_c c\bar{c}$, except that the amplitude A_1 in (12) must be replaced by

$$A_1 = \frac{4g_s^2 R(0)}{3\sqrt{6\pi m_c}} \epsilon_\alpha(Z)\epsilon_\mu(p)^* \frac{1}{(s-m_c^2)^2} \bar{u}(p') \left((\not{p} + 4m_c)\, \gamma_5\, (\not{q} + m_c) \; + \; \frac{s-m_c^2}{n\cdot(2q-p)}\, \not{n}\, \gamma_5\, (\not{p} + 2m_c) \right) \Gamma^\alpha v(\bar{q}) \,. \quad (18)$$

The square of the amplitude has the form (13), except that the Dirac matrix D reduces to

$$D = (s+3m_c^2)(s-5m_c^2)\,\not{q} \;-\; (s-m_c^2)(s-9m_c^2)\,\not{p} + 4\frac{s-m_c^2}{n\cdot(2q-p)}[(s-m_c^2)\,n\cdot q \;-\; (s-3m_c^2)\,n\cdot p\,]\not{n}$$
$$+4\left(\frac{s-m_c^2}{n\cdot(2q-p)}\right)^2 n\cdot p\; n\cdot(q-p)\;\not{n}\,. \tag{19}$$

Following the same path as in the ψ calculation, we find that the initial fragmentation function for η_c is

$$D_{c\to\eta_c}(z,3m_c) \;=\; \frac{8}{81\pi}\,\alpha_s(2m_c)^2\,\frac{|R(0)|^2}{m_c^3}\,\frac{z(1-z)^2(48+8z^2-8z^3+3z^4)}{(2-z)^6}\,. \tag{20}$$

Integrating over z, the fragmentation probability is

$$\int_0^1 dz\, D_{c\to\eta_c}(z,3m_c) \;=\; \frac{8}{27\pi}\,\alpha_s(2m_c)^2\,\frac{|R(0)|^2}{m_c^3}\left(\frac{773}{30}\;-\;37\ln 2\right)\,. \tag{21}$$

DECAY OF Z^0 INTO CHARMONIUM

From (6), the branching ratio for the decay of the Z^0 into ψ relative to the decay into $c\bar{c}$ is

$$\frac{\Gamma(Z^0\to\psi c\bar{c})}{\Gamma(Z^0\to c\bar{c})} \;=\; 0.0234\,\alpha_s(2m_c)^2\,\frac{|R(0)|^2}{m_c^3}\,. \tag{22}$$

The value of the parameter $R(0)$ can be determined from the ψ electronic width to be $|R(0)|^2 = (0.8\text{ GeV})^3$. Taking $m_c = 1.5$ GeV and $\alpha_s(2m_c) = 0.26$, we find that the branching ratio (22) is 2.4×10^{-4}. The simple result (22) agrees with the complete leading order calculation of $Z^0\to\psi c\bar{c}$ in Ref. [7] after taking into account the differences in the values of $R(0)$, α_s, and the charm quark mass. The authors of Ref. [7] used a larger value for the wave function at the origin, $|R(0)|^2 = (0.92\text{ GeV})^3$, and a smaller value for the quark mass, $m_c = 1.35$ GeV. It was also assumed implicitly in Ref. [7] that $Z^0\to\psi c\bar{c}$ is a short-distance process, so the running coupling constant was taken to be $\alpha_s(M_Z)\approx 0.15$. As we have shown, the dominant contribution comes from a fragmentation process, and the appropriate scale of the coupling constant is definitely on the order of m_c. Corrections to the fragmentation approximation are suppressed by $(4m_c/M_Z)^2$, which is about 0.4%. This is much smaller than the error due to the uncertainty in the quark mass.

The rate for production of η_c by fragmentation differs by less than 3% from that for ψ. From (21), we obtain

$$\frac{\Gamma(Z^0\to\eta_c c\bar{c})}{\Gamma(Z^0\to c\bar{c})} \;=\; 0.0227\,\alpha_s(2m_c)^2\,\frac{|R(0)|^2}{m_c^3}\,. \tag{23}$$

This agrees with the calculation of Ref. [7] after taking into account the differences in the values of α_s, $R(0)$, and m_c and an apparent algebraic error of a factor of 3.

The energy distribution of the ψ's produced by the fragmentation of charm quarks in Z^0 decay is given in (4). It is proportional to the fragmentation function evaluated at the scale $M_Z/2$. The initial fragmentation function (16) at the scale $3m_c$ is shown as a solid line in Fig. 1. It must be evolved up to the scale $M_Z/2$ using the Altarelli-Parisi equation (5) in order to sum up the leading logarithms of M_Z/m_c from higher order radiative corrections. The result is shown as the dotted line in Fig. 2. The evolution softens the energy distribution, shifting the peak in the fragmentation function from $z = 0.75$ to $z = 0.68$. The energy distribution shown in Fig. 2 should be accurate provided that the energy of the ψ is large compared to its mass, or equivalently $z \gg 0.07$. The fragmentation function for η_c production is also shown in Fig. 2. It has a slightly softer distribution, but its behavior is otherwise similar to that for the ψ.

The expression (22) also applies with minor modifications to the corresponding branching ratio for Υ production:

$$\frac{\Gamma(Z^0\to\Upsilon b\bar{b})}{\Gamma(Z^0\to b\bar{b})} \;=\; 0.0234\,\alpha_s(2m_b)^2\,\frac{|R(0)|^2}{m_b^3}\,, \tag{24}$$

where $R(0)$ is the radial wave function at the origin for the Υ, which is determined from its electronic decay rate to be $|R(0)|^2 = (1.8\text{ GeV})^3$. Taking $m_b = 4.9$ GeV and $\alpha_s(2m_b) = 0.19$, we find that the branching ratio (24) is 4.2×10^{-5}. The fragmentation approximation for Υ production in Z^0 decay is not as accurate as it is for ψ production. Corrections are approximately $(4m_b/M_Z)^2$, which is about 4%.

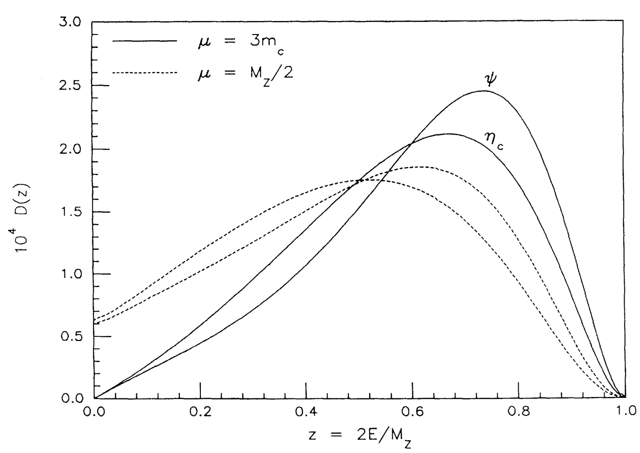

FIG. 2. The fragmentation functions $D_{c\to\psi}(z,\mu)$ and $D_{c\to\eta_c}(z,\mu)$ as a function of z for $\mu = 3m_c$ (solid lines) and $\mu = M_Z/2$ (dotted lines).

DECAY OF $W^{\pm}$ INTO ψ

About 1/3 of the decays of the W^+ will proceed through the channel $W^+ \to c\bar{s}$. The mass of the W is sufficiently large that the dominant production mechanism for charmonium will be $W^+ \to c\bar{s}$, followed by the fragmentation of the charm quark into charmonium. Fragmentation of the strange antiquark into ψ is suppressed by a factor of α_s^2. The branching ratio for decay into ψ relative to decay into $c\bar{s}$ is therefore smaller than (22) by a factor of 2:

$$\frac{\Gamma(W^+ \to \psi c\bar{s})}{\Gamma(W^+ \to c\bar{s})} = 0.0117\, \alpha_s(2m_c)^2 \, \frac{|R(0)|^2}{m_c^3} \, . \qquad (25)$$

Numerically this branching ratio is 1.2×10^{-4}. Our analytic calculation of the fragmentation contribution is consistent with the full leading order calculation of Ref. [7].

DECAY OF TOP QUARK INTO Υ

The top quark will probably decay almost exclusively into W^+b. If the top quark is heavy enough, the dominant production mechanism for bottomonium in top quark decay will be $t \to W^+b$, followed by the fragmentation of the b quark into bottomonium. The branching fraction for the direct decay into the $^3S_1$ state Υ is one half of (24):

$$\frac{\Gamma(t \to W^+\Upsilon b)}{\Gamma(t \to W^+b)} = 0.0117\, \alpha_s(2m_b)^2 \, \frac{|R(0)|^2}{m_b^3} \, , \qquad (26)$$

which has the numerical value 2.1×10^{-5}. The complete leading order calculation of the rate for $t \to W^+b\Upsilon$ gives a branching fraction of 4×10^{-7} for a top quark with a mass of 100 GeV [7]. The fragmentation formula (26) does not apply to such a small value of the top quark mass, since the maximum momentum of the $\Upsilon - b$ system is only 13 GeV, too small for the decay rate to be dominated by fragmentation. The simple result (26) is a good approximation if the mass of the top quark is closer to 150 GeV.

DECAY OF HIGGS BOSON INTO Υ

If the Higgs boson mass is below the threshold for decay into W pairs, than its dominant decay mode will be $H \to b\bar{b}$. The dominant production method for bottomonium in Higgs boson decay will be $H \to b\bar{b}$, followed by the fragmentation of the b quark or antiquark into bottomonium. The branching fraction for the direct decay into the Υ is twice (26), because both the b and $\bar{b}$ can fragment into Υ:

$$\frac{\Gamma(H \to \Upsilon b\bar{b})}{\Gamma(H \to b\bar{b})} = 0.0234\, \alpha_s(2m_b)^2 \, \frac{|R(0)|^2}{m_b^3} \, . \qquad (27)$$

This branching ratio is 4.2×10^{-5}, which is probably too small for this decay mode to be useful as a signal for an intermediate mass Higgs boson.

CONCLUSIONS

We have shown in this paper that the dominant mechanism for the direct production of charmonium in Z^0 decay is fragmentation, the production of a high energy charm quark or antiquark followed by its splitting into the charmonium state. Most previous calculations of charmonium production have considered only short-distance production mechanisms which are suppressed by a factor of m_c^2/M_Z^2. We calculated the fragmentation functions $D(z, \mu)$ for charm quarks or antiquarks to split into S-wave charmonium states to leading order in α_s. The fragmentation functions satisfy Altarelli-Parisi evolution equations which can be used to sum up large logarithms of M_Z/m_c. These fragmentation functions are universal, applying to the production of heavy quarkonium in any high energy process that can produce heavy quarks with transverse momentum large compared to their mass. We applied them to the production of charmonium and bottomonium in decays of the Z^0, $W^{\pm}$, top quark, and Higgs boson.

ACKNOWLEDGMENTS

This work was supported in part by the U.S. Department of Energy, Division of High Energy Physics, under Grant No. DE-FG02-91-ER40684.

[1] B. Guberina, J.H. Kühn, R.D. Peccei, and R. Rückl, Nucl. Phys. **B174**, 317 (1980).
[2] W.-Y. Keung, Phys. Rev. D **23**, 2072 (1981).
[3] K.J. Abraham, Z. Phys. C **44**, 467 (1989).
[4] J.H. Kühn and H. Schneider, Phys. Rev. D **24**, 2996 (1981); Z. Phys. C **11**, 263 (1981).
[5] E. Braaten and T.C. Yuan, Phys. Rev. Lett. **71**, 1673 (1993).
[6] K. Hagiwara, A.D. Martin, and W.J. Stirling, Phys. Lett. B **267**, 527 (1991); erratum (to be published).
[7] V. Barger, K. Cheung, and W.-Y. Keung, Phys. Rev. D **41**, 1541 (1990).
[8] L. Clavelli, Phys. Rev. D **26**, 1610 (1982).
[9] J.C. Collins and G. Sterman, Nucl. Phys. **B185**, 172 (1981); G. Curci, W. Furmanski, and R. Petronzio, *ibid.* **B175**, 27 (1980); J.C. Collins and D.E. Soper, *ibid.* **B194**, 445 (1982).