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Systematic biases in particle lifetime measurements

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In this paper it is shown that particle lifetime measurements based on reconstruction of charged decay products are subject to a bias arising from correlations of the tracking errors. The effects of correlated tracking errors on existing lifetime measurements of charm and bottom hadrons and the τ lepton are examined. The bias can be significant in the case of the τ.

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I. INTRODUCTION

Particle lifetimes in the range 10^{-13}–10^{-11} s are generally determined from spatial measurements of daughter tracks, rather than from time measurements. In particular, the lifetimes of charm and bottom hadrons and the τ lepton are studied in this way [1]. In this paper I show that tracking errors may introduce a bias on the measured lifetime. The bias can be significant in the case of the τ lepton.

The origin of the bias is described in Sec. II. In Sec. III, the size of the effect is estimated for various τ lifetime analysis methods. In Sec. IV, the implications of the bias for existing measurements of τ, charm, and bottom lifetimes are discussed. Some guidelines are also given for properly accounting for the bias in future measurements. The conclusions are presented in Sec. V.

Measurements of the τ lifetime have been made in high-energy e^+e^- collider experiments [2–17]. Because of the particular relevance of the present results to the τ, the usual e^+e^- coordinate system is employed, i.e., with the polar axis along the incoming beams. The signed distance of closest approach of a particle to the beam axis is denoted d; the sign of d is taken to be that of the z component of the particle angular momentum about the beam axis. Azimuthal angles ϕ are measured with respect to the horizontal plane. The angle $\psi \equiv \phi_{\text{daughter}} - \phi_{\text{parent}}$ is defined for each daughter track. (See Fig. 1.) Finally, the acoplanarity $\Delta\phi \equiv \phi_+ - \phi_- \pm \pi$ is defined for $\tau^+\tau^-$ events of 1-1 topology and for $e^+e^- \to e^+e^-$ and $\mu^+\mu^-$ events.

II. CORRELATION OF IMPACT PARAMETERS AND AZIMUTHAL ANGLES

The various methods which have been devised for measuring charm, bottom, and τ lifetimes are based on measurements of the daughter tracks. In most methods, measurements of both azimuthal angle ϕ and impact parameter d enter into the lifetime determination [18]. The nonzero lifetime introduces a positive correlation between d and ψ (i.e., between d and ϕ) of the daughter tracks:

$$d = \ell \sin\theta \sin\psi,$$

where ℓ and θ are the displacement and polar angle of the decaying particle. To some extent the lifetime is determined by measuring this correlation. However, the measurements of d and ϕ are subject to errors from many sources, including detector resolution, Coulomb scattering, nuclear interactions, bremsstrahlung, detector alignment errors, and hit-track assignment errors [19]. All of these sources influence the track measurement at some

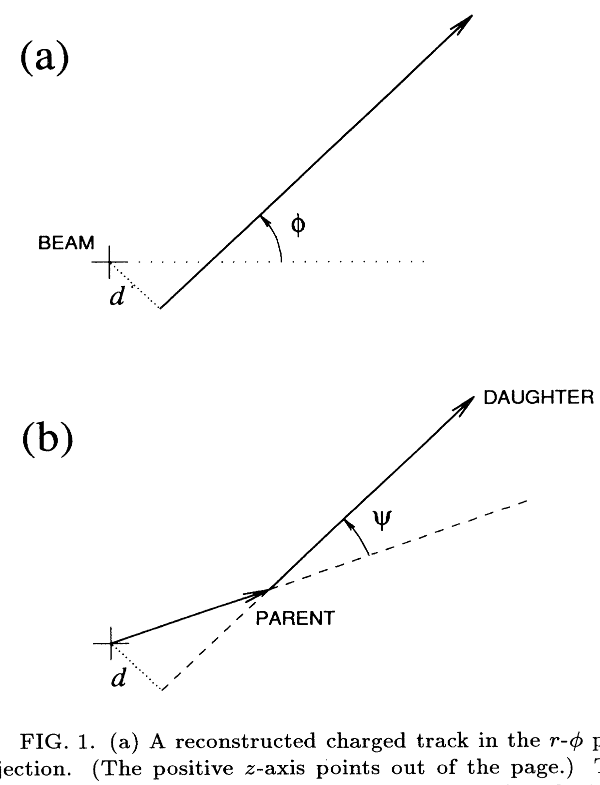

FIG. 1. (a) A reconstructed charged track in the r-ϕ projection. (The positive z-axis points out of the page.) The geometric impact parameter d and the azimuthal angle ϕ are indicated. (b) The laboratory azimuthal decay angle ψ.

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distance from the interaction point. When the reconstructed track is extrapolated back to the decay point, the d and ϕ errors have a *positive* correlation. This correlation increases the apparent lifetime.

The case of a track scattering in the beam pipe of a colliding-beam experiment is illustrated in Fig. 2. The track direction error incurred at the beam pipe leads to a positively correlated error on the impact parameter when the track is extrapolated inward. The errors on d and ϕ are also correlated if a track is extrapolated with incorrect curvature, e.g., due to an energy loss by bremsstrahlung in the detector material. In this case, the induced lifetime bias depends on the strength of the solenoidal magnetic field.

In this paper I concentrate on tracking in the r-ϕ plane. It should however be noted that the "longitudinal" track parameters θ and z also contain lifetime information [20]. For example, three-prong τ decay vertices may be fitted in three dimensions. In such situations, a similar effect involving correlations between the θ and z measurement errors is also present.

III. ESTIMATE OF BIAS IN THE MEASURED DECAY LENGTH

The effect of Coulomb scattering, nuclear interactions, etc., on the track parameters and subsequently on the measured lifetime depends on many details of detector geometry and performance and of the reconstruction and analysis programs, including tracking layer radii and resolutions, pattern recognition and track fitting algorithms, event selection, the method used to extract the lifetime from the event information, and the averaging or fitting procedure. A detailed simulation is therefore required to determine the size of the bias due to tracking errors in any particular analysis. However, it is possible to estimate the size of the effect as a function of the parameter $\langle \delta d\, \delta\phi \rangle$, which characterizes the size and correlation of the d and ϕ measurement errors. The estimation is straightforward for the case of the impact parameter difference method, which has been used to measure the τ lepton lifetime [13].

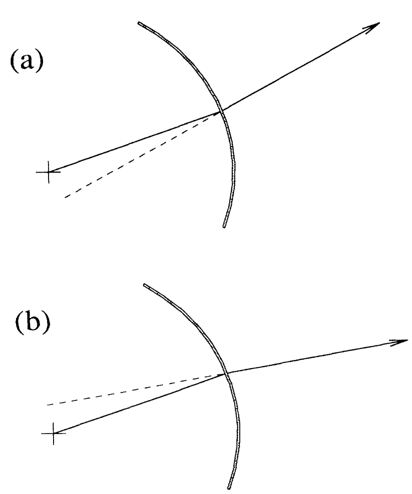

FIG. 2. Correlated tracking errors caused by scattering in the beam pipe (r-ϕ projection). The measurement errors on d and ϕ are both greater than zero in (a), and less than zero in (b).

A. Impact parameter difference method

In this method, the lifetime is determined from a sample of 1-1 topology $\tau^+\tau^-$ events. The mean decay length $\bar{\ell}$ is given by the slope of $\langle Y \rangle$ vs X, where $Y_i = d_+ - d_-$ and $X_i = \Delta\phi \sin\theta$ are measured for each event i. A least-squares linear fit gives the slope [1]

$$\bar{\ell} = \frac{S_1 S_{XY} - S_X S_Y}{S_1 S_{XX} - S_X^2},$$

where S_Q is the weighted sum of Q_i over all events. When small errors δX_i, δY_i are introduced, the change in the fitted slope is given, to leading order, by [21]

$$\Delta\bar{\ell} = \frac{\langle \delta X_i\, \delta Y_i \rangle}{\langle X_i^2 \rangle} - \bar{\ell}\,\frac{\langle (\delta X_i)^2 \rangle}{\langle X_i^2 \rangle}.$$

In all practical situations the first term on the right-hand side dominates. Considering $\theta = \pi/2$ for simplicity, one obtains $\langle \delta X_i\, \delta Y_i \rangle = 2\langle \delta d\, \delta\phi \rangle$, $\langle X_i^2 \rangle \sim 2\langle \psi^2 \rangle \sim 2/\gamma^2$, and hence

$$\Delta\bar{\ell} \sim \gamma^2 \langle \delta d\, \delta\phi \rangle \tag{1}$$

or

$$\frac{\Delta\tau}{\tau} = \frac{\Delta\bar{\ell}}{\bar{\ell}} \sim \frac{\gamma}{\beta c\tau} \langle \delta d\, \delta\phi \rangle, \tag{2}$$

where $\gamma = E_\tau / M_\tau$ and $\beta = p_\tau / E_\tau$. Roughly speaking, the relative lifetime bias is equal to the ratio of the detector-induced d-ϕ correlations and the lifetime-induced d-ϕ correlations.

Effects such as Coulomb scattering, nuclear interactions, and bremsstrahlung in the detector material can introduce large non-Gaussian errors in the track measurements. The long tails in the error distributions can significantly increase $\langle \delta d\, \delta\phi \rangle$ and hence the lifetime bias. On the other hand, the impact parameter smearing caused by the uncertainty on the τ production point (due to the nonzero beam size) does not contribute to the bias because there is no accompanying error on ϕ.

As an example, I consider a hypothetical tracking system with modest performance: $\sigma_d = 150\,\mu$m, $\sigma_\phi = 1.4$ mrad, $\langle \delta d\, \delta\phi \rangle = 0.9\, \sigma_d \sigma_\phi = 0.19\,\mu$m. For $\tau_\tau = 295$ fs, Eq. (2) then predicts $\Delta\tau/\tau \sim +0.6\%$ at $\sqrt{s} = 10$ GeV and $+5\%$ at 91.2 GeV.

Although it is clear that the fit of $\langle Y \rangle$ vs X employed in the impact parameter difference method is sensitive to detector-induced d-ϕ correlations, it is perhaps not as clear that these correlations also introduce a lifetime bias in other methods. A Monte Carlo simulation has been used to demonstrate that this is indeed the case for the classical impact parameter and decay length methods applied to τ decays; the results are described in the following sections.

B. Impact parameter method

A number of Monte Carlo $Z^0 \to \tau^+\tau^-$ events were generated [22] at $\sqrt{s} = 91.2\,\mathrm{GeV}$. A sample of 10 000 events of 1-1 topology was selected, requiring $p > 1\,\mathrm{GeV}/c$ and $|\cos\theta| < 0.85$ for both charged daughter tracks [23]. No other selection criteria were imposed. In order to reduce the statistical errors, many sets of τ decay lengths and Gaussian tracking errors were generated for each event, with the τ momenta and decay angles being reused. The tracking resolutions were assumed to be the same for all tracks, namely, $\sigma_d = 150\,\mu\mathrm{m}$ and $\sigma_\phi = 1.4\,\mathrm{mrad}$. The d and ϕ errors were generated without and with correlations ($\langle \delta d\, \delta\phi \rangle = 0$ or $0.19\,\mu\mathrm{m}$) to determine the effect of the correlations on the reconstructed lifetime. The other track parameters were not used in the analysis. The generated τ lifetime was 295 fs and the beam size was assumed to be zero.

In the impact parameter method, the lifetime-signed impact parameter D is computed for each daughter track as follows: $D = d$ if $\psi > 0$, and $D = -d$ if $\psi < 0$, where the reconstructed ψ angle is measured with respect to the event thrust axis. Thus $D > 0$ if the apparent τ decay point lies in the same hemisphere as the τ momentum direction. In the absence of measurement errors D is always greater than or equal to zero, whereas the d distribution is symmetric about zero. The mean value of D is roughly proportional to the lifetime.

In this study only 1-1 events were considered, and only the two (reconstructed) charged tracks were used in the thrust determination. In this situation, the signs of the ψ angles depend only on the sign of $\Delta\phi$, i.e., on the azimuthal angle between the two daughter tracks: if $\Delta\phi > 0$ then $\psi_+ > 0$ and $\psi_- < 0$; if $\Delta\phi < 0$ then $\psi_+ < 0$ and $\psi_- > 0$.

The sample mean of D was computed for the two Monte Carlo samples, each consisting of $100 \times 10\,000 = 10^6$ events: $\bar{D} = 50.23 \pm 0.14\,\mu\mathrm{m}$ with correlated errors and $47.76 \pm 0.14\,\mu\mathrm{m}$ without. The bias due to the tracking error correlations is therefore $(\bar{D}_{\mathrm{with}} - \bar{D}_{\mathrm{without}})/\bar{D}_{\mathrm{without}} = (+5.2 \pm 0.4)\%$, to be compared with the value $+5\%$ obtained from Eq. (2).

How do the correlations of the d and ϕ errors introduce a bias on $\bar{D}$? The bias arises from τ decays in which the daughter track momentum is nearly parallel to (i.e., within about σ_ϕ of) the reconstructed thrust direction. In these decays, the impact parameter error δd ($= d_{\mathrm{meas}} - d_{\mathrm{true}}$) tends to have the same sign as ψ_{meas}. The apparent τ decay point is therefore likely to lie in the same hemisphere as the τ momentum direction, resulting in a lifetime-signed impact parameter D which is greater than zero. (See Fig. 3.)

An attempt to eliminate the effect by removing decays with $\psi \cong 0$ would be futile: any cut on the reconstructed ψ would introduce a new bias on D. For example, the above Monte Carlo analysis was repeated with the requirement $|\Delta\phi| > 0.5^\circ$ ($\cong 6\sigma_\phi$); the bias was then $\Delta\tau/\tau = (+5.5 \pm 0.4)\%$.

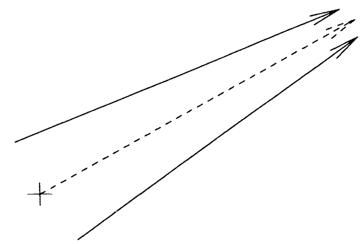

FIG. 3. The true trajectory of a particle is represented by the dashed line in this r-ϕ projection. The tracking errors on d and ϕ tend to have the same sign, as indicated by the two solid lines. In cases where the true trajectory nearly coincides with the event thrust axis, the reconstructed track is more likely to intersect that axis in the forward hemisphere, giving a positive lifetime-signed impact parameter.

C. Decay length method

A similar Monte Carlo analysis was performed with the decay length or vertex method for three-prong τ decays. A sample of 10 000 decays was selected in which all three daughter tracks satisfied $p > 0.5\,\mathrm{GeV}/c$ and $|\cos\theta| < 0.85$. Gaussian tracking errors were simulated with the same resolutions as before, again forming samples of 10^6 decays with and without correlations between the d and ϕ errors. A two-dimensional vertex fitter [24] was used. The impact parameters d and azimuthal angles ϕ were allowed to vary in the fit, while the track curvatures were assumed to be zero. The three-dimensional decay length was computed from

$$\ell = (x_V \cos\phi_\tau + y_V \sin\phi_\tau) \csc\theta_\tau,$$

where (x_V, y_V) is the fitted vertex point and ϕ_τ, θ_τ describe the true τ momentum direction. The sample mean and weighted mean [25] of each of the two ℓ distributions were then computed. Maximum likelihood fits [26] were also used to determine the means. The results are given in Table I. The observed bias due to tracking error correlations is $(\bar{\ell}_{\mathrm{with}} - \bar{\ell}_{\mathrm{without}})/\bar{\ell}_{\mathrm{true}} = (+3.2 \pm 0.6)\%$ for the sample mean, indicating that the d-ϕ error correlations introduce an offset in the fitted vertex points. The observed bias is about a factor of 2 smaller than the value obtained from Eq. (2).

Decays in which the three daughter tracks are nearly parallel in the r-ϕ projection are responsible for the positive bias. I stress that this bias is not simply caused by pattern recognition or hit assignment errors as such details were not simulated in this study. Tracking errors of all types can contribute to the bias.

In a real experiment, the acceptance for decays with small opening angles would undoubtedly be smaller than in the present Monte Carlo study. However, this reduced acceptance does not necessarily imply a reduced bias on the mean decay length. Any cut which rejects the decays in question and which operates on the reconstructed rather than the true ψ angles is likely to introduce a new bias. In order to demonstrate this, I selected

TABLE I. Sample mean, weighted mean, and fitted mean decay lengths from Monte Carlo simulation of three-prong τ decays. The generated mean decay length is 0.2240 cm. The bias is defined as $(\bar{\ell}_{\text{with}} - \bar{\ell}_{\text{without}})/\bar{\ell}_{\text{true}}$.

	Sample mean	Weighted mean	Fitted mean
$\bar{\ell}$ (cm), with correlations	0.2317 ± 0.0009	0.2338 ± 0.0004	0.2290 ± 0.0004
$\bar{\ell}$ (cm), without correlations	0.2246 ± 0.0009	0.2233 ± 0.0004	0.2238 ± 0.0004
Bias	$(+3.2 \pm 0.6)\%$	$(+4.7 \pm 0.3)\%$	$(+2.3 \pm 0.2)\%$

Monte Carlo events with $\max(\psi_i - \psi_j) > 0.01$ radian (reconstructed) and obtained $\Delta\tau/\tau = (+4.6 \pm 0.4)\%$ (using sample means).

The weighted mean of the decay length distribution is commonly used instead of the sample mean because a smaller statistical uncertainty can be obtained. The following biases may affect the weighted mean, in addition to the vertex offset noted above.

(1) The effect of Coulomb scattering on the weighted mean decay length is discussed in Ref. [9]. It was pointed out that fluctuations to wider opening angles give rise to larger event weights and tend to be associated with longer decay lengths. In the present study, the bias due to tracking error correlations (from Coulomb scattering or any other source) is indeed larger, $(+4.7 \pm 0.3)\%$, when the weighted mean is used.

(2) A positive decay length bias may be introduced if the track covariance matrices are swum to the fitted vertex position and a second fit iteration is performed. Tracks from long-lived τ's require a shorter extrapolation from the measured points in the detector to the decay vertex. The impact parameter errors therefore tend to be smaller and the weights larger for long-lived τ's [27]. In the Monte Carlo sample, the bias on the weighted mean becomes $(+11.5 \pm 0.3)\%$ when two iterations are performed in each vertex fit.

(3) The energy of a τ is sometimes less than E_{beam} due to initial- or final-state radiation. It is standard procedure to convert the measured mean decay length into a lifetime by dividing by $\langle \beta\gamma \rangle c$ as determined from a Monte Carlo simulation which includes radiative events. However, low-energy τ's tend to have wider opening angles (hence larger weights) and shorter laboratory decay lengths, so a negative lifetime bias is induced when the unweighted mean $\langle \beta\gamma \rangle$ is used with the weighted mean decay length. In the present study (at $\sqrt{s} = 91.2$ GeV) this bias is $(-0.5 \pm 0.1)\%$. A larger bias is expected in experiments operating below the Z^0 resonance, due to the larger relative energy loss via initial-state radiation.

A maximum likelihood fit is used in some experiments to extract $\bar{\ell}$ from the decay length distribution. These fits are also affected by the biases described above [28].

IV. IMPLICATIONS FOR PAST AND FUTURE LIFETIME MEASUREMENTS

A. Implications for the τ

I have shown that correlated tracking errors induce a positive bias on τ lifetime measurements made with the impact parameter difference, impact parameter, and decay length methods, and that the bias can be large for experiments operating at high $\sqrt{s}$ without a high-resolution tracking system. The Monte Carlo calculations described in Sec. III were repeated with the τ's boosted to various energies and with various detector resolutions; the parametrization of the bias given in Eqs. (1) and (2) was shown to be correct within a factor of 2 for all three methods.

It is of course possible to account for the bias in any measurement of the τ lifetime. However, the effect is explicitly mentioned and treated in few of the experimental papers. It is therefore appropriate to consider whether the bias has affected the existing measurements.

1. *Impact parameter method*

A number of different procedures have been used to extract the τ lifetime from an observed impact parameter distribution. Three of these procedures are outlined here.

A Monte Carlo sample, without detector simulation, was used in Ref. [2] to determine the lifetime from the measured $\bar{D}$. A large positive shift in $\bar{D}$, corresponding to $(9 \pm 5)\%$ of the lifetime-induced $\bar{D}$, was noted when the tracking errors were included in the simulation; this shift was accounted for in the systematic uncertainty but no correction was applied to the measured lifetime.

A maximum likelihood fit was used in Ref. [11] to extract τ_τ from the D distribution. The likelihood function was the convolution of a symmetric resolution function and the unsmeared D distribution obtained from a Monte Carlo simulation. The systematic uncertainty included a contribution based on a Monte Carlo check for bias in the procedure.

In Ref. [4], the trimmed mean of the D distribution was determined. Monte Carlo samples with full detector simulation and with various generated τ lifetimes were used to determine τ_τ, assuming $\tau_\tau = a\bar{D} + b$.

In the first two cases, the bias due to tracking errors was not accounted for in the measured lifetime but it was accounted for in the systematic uncertainty; in the third, a correction for this bias was implicitly applied using the Monte Carlo samples. Thus in all three experiments the effect was treated in a reasonable way. However, the assumption which is usually made when computing the world average τ lifetime is invalid: since the sign of the bias is always positive, one should not assume that all of the experimental errors are independent. In order to improve this situation, the following points should be considered for future measurements with the impact parameter method.

(1) The mean lifetime-signed impact parameter depends on the detector resolution.

(2) A maximum likelihood fit to the D distribution will yield a biased lifetime measurement if the resolution function for D is assumed to be symmetric about zero. A full detector simulation should be used either to account for the offset in the resolution function or to apply a correction on the fitted lifetime [29].

(3) A correction based on a detailed simulation is also required if the lifetime is to be determined from the measured $\bar{D}$ (rather than from a maximum likelihood fit). To be rigorous, one should use $\tau_\tau = a\bar{D} + b$, i.e., one should not assume $\bar{D} = 0$ for $\tau_\tau = 0$.

(4) The systematic uncertainty on the lifetime should reflect possible differences between the real and simulated tracking errors.

Finally, one additional point should be made concerning the impact parameter method: the mean lifetime-signed impact parameter is *not* expected to be zero for events from $e^+e^- \to e^+e^-$ or $\mu^+\mu^-$. It is necessary to correct the measured τ lifetime for the effect of these background processes. Bhabha and dimuon events may also be used to check for systematic errors associated with detector calibration and alignment.

It is evident from the discussion in Sec. III B that a positive offset in $\bar{D}$ is present when the lifetime-signed impact parameter is computed for Bhabha and dimuon events. The size of the offset is readily calculated under the assumptions that the two final state particles are produced exactly back to back and that the tracking errors on d and ϕ are Gaussian, with the same covariance matrix for all tracks:

$$\mathbf{C} = \begin{bmatrix} \sigma_d^2 & \sigma_{d\phi}^2 \\ \sigma_{d\phi}^2 & \sigma_\phi^2 \end{bmatrix}.$$

The result is

$$\bar{D} = \frac{\sigma_{d\phi}^2}{\sigma_\phi \sqrt{\pi}}.$$

With the parameters used in the numerical examples of Sec. III (*with* d-ϕ correlations) one finds $\bar{D} = 75\,\mu$m, i.e., the offset for Bhabha scattering events and dimuons is larger than the mean impact parameter observed in τ decays. The bias is amplified because *all* of the final state tracks are nearly parallel to the reconstructed thrust direction (Sec. III B).

The e^+e^- and $\mu^+\mu^-$ events which satisfy τ selection criteria are likely to contain large tracking errors or initial- or final-state radiation. Radiative events can be acoplanar and therefore subject to a smaller offset in $\bar{D}$. Nevertheless, it is incorrect to assume $\bar{D} = 0$ for Bhabha and dimuon backgrounds [2,11,16].

2. Decay length method

Reconstructed three-prong decay vertices provide a direct measurement of the τ decay length. For this reason, a Monte Carlo simulation is often used only to check the analysis procedure. The simulated τ decays are analyzed in the same way as the real data, and the reconstructed lifetime is compared with the input value. In most experiments, a correction is applied to the measured τ lifetime only if a statistically significant bias is found in the Monte Carlo sample; i.e., it is assumed that the bias is zero unless proven otherwise. In view of the various sources of systematic bias which are possible (Sec. III C), even for a perfectly understood detector, this assumption should not be made.

Table II contains the results of the Monte Carlo checks for all decay length method measurements published since 1985. The tabulated biases correspond to the difference between the reconstructed and input τ lifetimes ($\tau_{\text{rec}} - \tau_{\text{input}}$) when a sample of $\tau^+\tau^-$ Monte Carlo events is analyzed in the same way as the real data. These values reflect the bias due to correlated tracking errors, as well as any other possible biases which are produced by simulated phenomena. The bias tends to be greater than

TABLE II. Bias observed in Monte Carlo simulations for various experiments using the decay length method. The impact parameter resolution σ_d for tracks in Bhabha and dimuon events is given as an indication of the tracking system performance. The last column shows whether the measured τ lifetime was corrected by subtracting the bias observed in Monte Carlo events.

Experiment	Refs.	σ_d (μm)	Bias (fs)	Subtracted?
CLEO (1987)	[3]	100	$+4 \pm 3$	no
MAC (1987)	[4]	90	$+20 \pm 3$	yes
ARGUS (1987)	[5,30]	95	$+4 \pm 7$	no
HRS (1987)	[6,31]	140	0 ± 6	–
TASSO (1988)	[7]	$\sim$100	$+2 \pm 5$	no
MARK II (1988)	[8]	88	$+4 \pm 7$	no
JADE (1989)	[9]	160	$+50$	yes
L3 (1991)	[10,32]	144	$+3 \pm 4$	no
DELPHI (1991)	[11,33]	62	$+2 \pm 4$	no
ALEPH (1992)	[13,34]	131	$+6 \pm 6$	no
CLEO (1992)	[14,35]	92	$+7 \pm 2$	yes
ALEPH (1992)	[15]	28	-2 ± 4	yes
DELPHI (1993)	[16,36]	26	$+1 \pm 2$	no
OPAL (1993)	[17]	40, 18	-4 ± 3	yes

zero, particularly for experiments with larger σ_d. It is evident that the systematic errors of the various published measurements are not independent.

The following points should be considered for future measurements with the decay length method.

(1) The reconstructed mean decay length depends on the detector resolution. A full detector simulation is therefore required to correct for the bias due to tracking errors.

(2) Weighted means and maximum likelihood fits are subject to a bias related to the τ energy distribution.

(3) One should not assume that the bias is zero. The various lifetime measurements are more likely to be independent if a bias correction based on a Monte Carlo simulation is always applied.

(4) The systematic uncertainty on the lifetime should reflect possible differences between the real and simulated tracking errors.

3. Other methods

A lifetime bias due to correlated tracking errors was found in the impact parameter difference method (Sec. III A). Other methods have been developed which are also sensitive to d-ϕ correlations [15,17]. However, in each of these cases a Monte Carlo simulation was used to "calibrate" the final result, so a correction for the bias due to tracking errors was made.

B. Implications for charm

The lifetimes of weakly decaying charm hadrons have been measured with the decay length method in e^+e^- annihilation and fixed-target experiments [1]. The most precise results have been achieved in fixed-target experiments equipped with silicon microstrip detectors. A track position resolution in the xy plane (perpendicular to the beam) on the order of $15\,\mu$m is obtained, with direction errors of roughly 0.1 mrad. Assuming 100% correlation of these errors, $\langle \delta x\, \delta s_x \rangle$ (the fixed-target analogue of $\langle \delta d\, \delta\phi \rangle$) is equal to 1.5×10^{-7} cm. Equation (2) may be used to estimate the systematic bias in the vertex reconstruction expected due to the correlated tracking errors. For D^0 mesons with $p = 100\,\text{GeV}/c$, one obtains $\Delta L/L \sim +0.06\%$; i.e., the reconstructed vertices are offset by a negligible amount. (Measurements of the short-lived charm baryons would incur a somewhat larger relative bias.) More importantly, in most fixed-target experiments the mean lifetime is extracted from the data in a manner which is insensitive to the offset: a maximum likelihood fit is performed on a limited interval of the proper decay time distribution. Since the average vertex offset due to tracking errors is constant for all proper decay times, the fitted decay rate is unaffected.

Several charm lifetime measurements were made in e^+e^- experiments without high-resolution tracking systems. For these results, the expected bias induced by tracking errors is not more than about 0.5% [37]. Moreover, these measurements carry little weight in the world averages.

C. Implications for bottom

The existing measurements of bottom hadron lifetimes were made in e^+e^- experiments at center-of-mass energies up to 91 GeV [1]. The impact parameter and decay length methods (and variations thereof) were employed. In many experiments, a maximum likelihood fit with a symmetric resolution function is used to extract the mean decay length or impact parameter. This is a valid procedure because the lifetime bias induced by the tracking errors is not more than about 0.6% [38]. The bias is even smaller for experiments with high resolution vertex detectors.

In a given experiment, the relative bias is much smaller for B's than for τ's because (1) the mean B decay length is larger and (2) the B decay opening angles are wider. In other words, the lifetime-induced d-ϕ correlations are larger for bottom than for the τ.

V. CONCLUSIONS

I have studied the effects of correlated tracking errors on measurements of particle lifetimes. The correlations were found to induce a positive lifetime bias. The size of the bias in the impact parameter difference method was computed [Eq. (2)]; the effect was found to be larger for higher parent energies and proportional to $\langle \delta d\, \delta\phi \rangle$, which characterizes the size and correlation of the track position and angle measurement errors. This dependence was also observed for the traditional impact parameter and decay length methods.

A Monte Carlo simulation which accurately reproduces the tracking errors and their correlations may be used to account for this bias in any lifetime measurement. The bias was found to be negligible for the existing measurements of charm and bottom hadron lifetimes. On the other hand, a significant effect is expected in many τ lifetime measurements, but in some experiments no correction for the bias is made. Some guidelines were given to properly account for this effect in future measurements.

I have attempted to estimate the effect of uncorrected biases on the world average τ lifetime of Ref. [1], which is based on Refs. [2–12]. Assuming that the Monte Carlo simulations performed in each experiment accurately predict the tracking resolution in the real data, I find that the average would be reduced by about 2 fs if all bias corrections were applied [39].

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[17] OPAL Collaboration, P.D. Acton *et al.*, Report No. CERN-PPE/93-009, 1993 (unpublished); replaces Ref. [12].
[18] An exception is the "miss distance" method [16], in which the ϕ measurements do not enter into the lifetime determination.
[19] The fact that hit sharing introduces a positive decay length bias was pointed out by C.K. Jung, Phys. Rev. D **47**, 3994 (1993).
[20] The track parameter θ is the polar angle of the momentum; z refers to the longitudinal coordinate at the point of closest approach to the beam axis.
[21] It is assumed that the errors δX_i, δY_i are not correlated with X_i or Y_i, and that $\langle X_i \rangle = \langle \delta X_i \rangle = \langle \delta Y_i \rangle = 0$. The decay length bias $\Delta \bar{\ell}$ is then found to be a quadratic function of the errors. However, a first-order bias may also be present under certain circumstances. Detector alignment and calibration errors can cause ϕ-dependent distortions in the d and ϕ measurements. If the detector acceptance and/or the event weights are nonuniform in ϕ, the quantities $\langle X_i \, \delta Y_i \rangle$ and $\langle \delta X_i \, Y_i \rangle$ may be nonzero, yielding a first-order decay length bias. Similarly, a scale error on the dimensions of the tracking detectors is propagated directly onto the scale of measured decay lengths.
[22] Computer program KORALZ, version 3.8, courtesy of S. Jadach, B.F.L. Ward, and Z. Wąs, S. Jadach and Z. Wąs, Comput. Phys. Commun. **36**, 191 (1985); Monte Carlo Group, in *Z Physics at LEP1*, Proceedings of the Workshop, Geneva, Switzerland, 1989, edited by G. Altarelli, R. Kleiss, and C. Verzegnassi (CERN Report No. 89-08, Geneva, 1989), Vol. III; S. Jadach, B.F.L. Ward, and Z. Wąs, Comput. Phys. Commun. **66**, 276 (1991).
[23] The event selection was based on the generated (unsmeared) track parameters.
[24] The "full" vertex fit formalism given by P. Billoir and S. Qian, Nucl. Instrum. Methods **A311**, 139 (1992), was adapted to the case of two dimensions and two track parameters. The same covariance matrix used to generate the tracking errors (referred to the point of closest approach to the beam axis) was used in the vertex fitter. The fit was performed in one iteration; the e^+e^- collision point was used as the starting vertex position.
[25] The weighted mean was computed with an iterative procedure. Event i was weighted by $[(\Delta \ell_i)^2 + \bar{\ell}_0^2]^{-1}$, where $\Delta \ell_i$ is the uncertainty on the reconstructed decay length and $\bar{\ell}_0$ is an estimate of $\bar{\ell}$. The resulting mean was then used to recompute the weights for a second iteration. The value of $\bar{\ell}$ was found to converge after two or three iterations.
[26] The likelihood function for event i was the convolution of a decreasing exponential with mean $\bar{\ell}$ and a symmetric Gaussian with $\sigma = \Delta \ell_i$.
[27] MAC Collaboration, W.T. Ford *et al.*, Phys. Rev. Lett. **49**, 106 (1982).
[28] It is well known that the mean decay length obtained with a maximum likelihood fit depends on the assumed tracking resolution. On the other hand, correlated tracking errors introduce a bias on the ℓ distribution itself; this bias is present even if the tracking resolution is correctly parametrized in the vertex fit.
[29] Maximum likelihood fits may be subject to bias from yet another source. The impact parameter resolution σ_d tends to be smaller for higher-momentum daughter tracks. If the variation of σ_d with momentum is accounted for in the likelihood function on a track-by-track basis, the decays with high-momentum tracks will effectively be weighted more heavily in the fit. However, because of kinematics these decays also tend to have smaller impact parameters, so the mean lifetime is underestimated in the fit. The size of the bias depends on the size of the momentum-dependent (i.e., multiple scattering) component of σ_d relative to the total σ_d, the smearing due to the beam size, and the intrinsic width of the D distribution.
[30] B. Paul Padley, doctoral thesis, University of Toronto, 1987.
[31] P. Baringer *et al.*, Nucl. Instrum. Methods **A254**, 542 (1987).
[32] A. Biland (private communication).
[33] R. McNulty (private communication). Hadronic scattering and hit association errors were not simulated.
[34] Ll.M. Mir and S. Orteu, ALEPH internal note.
[35] J. Whitmore, doctoral thesis, Ohio State University, 1992.
[36] M. McCubbin, R. McNulty, and D. Reid, DELPHI internal note 92-174 PHYS 257.
[37] The quantity $\langle \delta d \, \delta \phi \rangle$ may be parametrized as σ_d^2/R. Roughly speaking, the distance R characterizes the radii of the coordinate measurements and the material concentrations in the inner part of the tracking system. The smearing related to the beam size is not included in σ_d. A plausible example at the SLAC e^+e^- storage ring PEP or DESY e^+e^- collider PETRA would be $\sigma_d = 120\,\mu$m and $R = 15$ cm; hence, $\langle \delta d \, \delta \phi \rangle = 0.10\,\mu$m. From Eq. (2), D^0 mesons with $p = 12\,\mathrm{GeV}/c$ would be measured with a lifetime bias of roughly $+0.5\%$ in this system.

[38] Some PEP and PETRA experiments which measured the B lifetime had impact parameter resolutions of several hundred microns. The bias could be as large as $+0.6\%$, corresponding to $\sigma_d = 400\,\mu$m, $R = 20\,$cm, and $p_B = 12\,$GeV/c.

[39] The Monte Carlo biases in Table II were used to adjust the measurements made with the decay length method, which account for 72% of the weight in the world average. Measurements made with the impact parameter method were adjusted using published Monte Carlo bias calculations or estimates based on Eq. (2). The pre-LEP world average [Particle Data Group, J.J. Hernández *et al.,* Phys. Lett. B **239**, 1 (1990)] incurred a +3 fs offset due to uncorrected biases.