# Cabibbo-allowed nonleptonic weak decays of charmed baryons

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Cabibbo-allowed nonleptonic weak decays of charmed baryons  $\Lambda_c^+$ ,  $\Xi_c^{0,4}$ ,  $\Xi_c^{+,4}$ , and  $\Omega_c^0$  into an octet baryon and a pseudoscalar meson are analyzed. The nonfactorizable contributions are evaluated under the pole approximation, and it turns out that the *s*-wave amplitudes are dominated by the low-lying  $\frac{1}{2}^$ resonances, while *p*-wave ones are governed by the ground state  $\frac{1}{2}^+$  poles. The MIT bag model is employed to calculate the coupling constants, form factors, and baryon matrix elements. Our conclusions are the following: (i) *s* waves are no longer dominated by commutator terms; the current-algebra method is not applicable to parity-violating amplitudes. (ii) Nonfactorizable *W*-exchange effects are generally important; they can be comparable to and sometimes even dominate over factorizable contributions, depending on the decay modes under consideration. (iii) The large- $N_c$  approximation for factorizable amplitudes also works in the heavy baryon sector and it accounts for the color nonsuppression of  $\Lambda_c^+ \to p\overline{K}^0$ relative to  $\Lambda_c^+ \to \Lambda \pi^+$ . (iv) A measurement of the decay rate and the sign of the  $\alpha$  asymmetry parameter of certain proposed decay modes will help discern various models. (v) *p* waves are the dominant contributions to the decays  $\Lambda_c^+ \to \Xi^0 K^+$  and  $\Xi_c^{0A} \to \Sigma^+ K^-$ , but they are subject to a large cancellation; this renders present theoretical predictions on these two channels unreliable.

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### I. INTRODUCTION

With more and more data of charmed baryon decays becoming available at ARGUS, CLEO, CERN, and Fermilab, it reaches the point that a systematical serious theoretical study of the underlying mechanism for nonleptonic decays of charmed baryons is called for [1]. The experimental progress in this area is best summarized in the recent concluding remark by Butler [2]. "Our knowledge of the charmed baryons has taken another leap forward. This is a field whose time has finally arrived." Indeed, in the past few years, new and highstatistics measurements of the nonleptonic  $\Lambda_c^+$  decays have been carried out, and new decay modes of  $\Xi_c^{0.4}$  and  $\Omega_c$  also have been seen recently.

Theoretically, all nonleptonic weak decays of mesons and baryons can be classified in terms of the following quark diagrams<sup>1</sup> [3]: the external *W*-emission diagram, the internal *W*-emission diagram, the *W*-exchange diagram, the *W*-annihilation diagram, and the *W*-loop diagram. The external and internal *W*-emission diagrams

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are usually referred to as factorizable contributions.<sup>2</sup> It is known for meson nonleptonic decays that the factorizable contribution dominates over the nonfactorizable ones such as W exchange and W annihilation. For baryon decays, *a priori* the nonfactorizable contribution can be as important as the factorizable one since W exchange, contrary to the meson case, is no longer subject to helicity and color suppression.

How do we handle the *W*-exchange contribution in the baryon decay? In principle the *W*-exchange amplitude can be expressed as a sum of all possible intermediate hadronic states. In practice, one assumes pole approximation, namely, that only one-particle intermediate states are kept; that is, the *W*-exchange contribution is assumed to be approximately saturated by pole intermediate states. Among all possible pole contributions, including resonances and continuum states, one usually concentrates on the most important poles such as the low-lying  $J^P = \frac{1}{2}^+, \frac{1}{2}^-$  states. In general, nonfactorizable *s*- and *p*-wave amplitudes are dominated by  $\frac{1}{2}^-$  low-lying baryon resonances and  $\frac{1}{2}^+$  ground-state baryon poles, respective-

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<sup>&</sup>lt;sup>1</sup>The W-annihilation diagram is absent in the baryon decay. The W-loop diagram does not contribute to the Cabibboallowed weak decays of hadrons.

<sup>&</sup>lt;sup>2</sup>In general, there are two distinct internal *W*-emission diagrams and three *W*-exchange diagrams for the nonleptonic baryon decay [4]. However, only the internal *W*-emission diagram with the meson formed along the parent quark which decays weakly is factorizable. At the hadron level, the factorizable internal *W*-emission graph corresponds to a meson-pole contribution.

ly. Evidently, the estimate of the s-wave terms is a difficult and nontrivial task since it involves weak baryon matrix elements and strong-coupling constants of  $\frac{1}{2}^{-}$  baryon states, which we know very little. Nevertheless, there is one exceptional case: For hyperon nonleptonic decays, the evaluation of s waves is no more difficult than the *p*-wave amplitudes. This comes from the fact that the emitted pion in this case is soft. As a result, the parity-violating pole amplitude of the hyperon decay is reduced, in the soft-pion limit, to the familiar equal-time commutator terms. The magic feature with this current algebra approach is that the s-wave amplitude can now be manipulated without appealing to any information of the cumbersome  $\frac{1}{2}^{-}$  poles.

Traditionally, the two-body nonleptonic weak decays of charmed baryons is studied by utilizing the same technique of current algebra as in the case of hyperon decays [5-13]. However, the use of the soft-meson theorem makes sense only if the emitted meson is of the pseudoscalar type and its momentum is soft enough. Obviously, the pseudoscalar-meson final state in charmed baryon decay is far from being "soft." Therefore, it is not appropriate to make the soft-meson limit. Moreover, since the charmed baryon is much heavier than the hyperon, it will have decay modes involving a vector meson; this is certainly beyond the realm of current algebra. Because of these two reasons, it is no longer justified to apply current algebra to heavy-baryon weak decays, especially for swave amplitudes. Thus, one has to go back to the original pole model,<sup>3</sup> which is nevertheless reduced to current algebra in the soft pseudoscalar-meson limit, to deal with nonfactorizable contributions. The merit of the pole model is obvious: Its use is very general and is not limited to the soft-meson limit and to the pseudoscalar-meson final state. Of course, the price we have to pay is that it requires the knowledge of the negative-parity baryon poles for the parity-violating transition. This also explains why the theoretical study of nonleptonic decays of heavy baryons is much more difficult than the hyperon and heavy meson decays.

Recently, a calculation of the nonfactorizable s- and pwave amplitudes of charmed baryon decays through the pole contributions from the low-lying  $\frac{1}{2}^{-}$  resonances and ground-state  $\frac{1}{2}^{+}$  baryons has been presented by us [14] and by Xu and Kamal [15]. We use the MIT bag model to tackle both  $\frac{1}{2}^{-}$  and  $\frac{1}{2}^{+}$  baryon poles. By comparing the pole-model and current-algebra results for the s waves of  $B_c \rightarrow B + P$ , we reach an important conclusion: the parity-violating amplitude of charmed baryon decays is no longer dominated by the commutator terms. That is to say, away from the soft-meson limit the correction to the commutator terms is very important. This correction will affect the magnitude and sometimes even the sign of the asymmetry parameter  $\alpha$ . Needless to say, the pole model also allows us to treat the weak decays,  $B_c \rightarrow B + V(1^-)$  on the same footing as  $B_c \rightarrow B + P(0^-)$  decays.

In a previous publication [14] we have applied the pole model to some selected decay modes, namely,  $\Lambda_c^+ \rightarrow p \overline{K}^0(\overline{K}^{*0}), \Lambda \pi^+(\rho^+), \Sigma^0 \pi^+(\rho^+), \Sigma^+ \pi^0(\rho^0)$ . The main purpose of the present paper is to complete the pole-model analysis for all two-body Cabibbo-allowed weak decays of the antitriplet charmed baryons  $\Lambda_c^+, \Xi_c^{+A}, \Xi_c^{0A}$  and the sextet charmed baryon  $\Omega_c^0$ . Owing to large theoretical uncertainties associated with the vector-meson case, as elaborated on in detail in Ref. [14], we will confine ourselves to the decay modes  $B_c \rightarrow B + P$ .

The present paper is organized as follows. The general framework of the pole model is recapitulated in Sec. II. Numerical results of the decay rate and the asymmetry parameter  $\alpha$  for Cabibbo-allowed two-body nonleptonic decays of charmed baryons are presented in Sec. III with some model details given in Appendices A-D. In Sec. IV, we then compare our results with current algebra as well as recent theoretical calculation [15,16] and then draw conclusions.

### **II. GENERAL CONSIDERATIONS**

Since the general framework for treating the nonleptonic weak decays of charmed baryons is already discussed in Ref. [14], here we will emphasize some main points which are not thoroughly discussed in a previous publication. The QCD-corrected effective weak Hamiltonian responsible for the Cabibbo-allowed charmedbaryon decays has the form

$$\mathcal{H}_{W} = \frac{G_{F}}{2\sqrt{2}} V_{cs} V_{ud} (c_{+}O_{+} + c_{-}O_{-}) , \qquad (2.1)$$

with  $O_{\pm} = (\bar{s}c)(\bar{u}d) \pm (\bar{s}d)(\bar{u}c)$ , where  $(\bar{q}_1q_2) = \bar{q}_1\gamma_{\mu}(1-\gamma_5)q_2$ , and  $V_{ij}$  are the quark mixing matrix elements. The Wilson coefficients are evaluated at the charm mass scale to be  $c_+ \cong 0.73$  and  $c_- \cong 1.90$ . In general, the decay amplitude of the baryon decay  $B_i \rightarrow B_f + P$  can be written in terms a sum over intermediate hadronic states. As far as the vacuum intermediate state is concerned, the amplitude will be factorized if the pseudoscalar meson P can be created from the quark currents of  $O_{\pm}$ . (For a review of factorization and the large  $N_c$  approach, see, e.g., Ref. [17].) Schematically,

$$M(B_i \rightarrow B_f + P) = M(B_i \rightarrow B_f + P)^{\text{fac}} + M(B_i \rightarrow B_f + P)^{\text{nf}}, \qquad (2.2)$$

where the superscript of stands for nonfactorization. It is clear from the expression of  $O_{\pm}$  that factorization occurs if the final-state meson is  $\pi^+$  or  $\overline{K}^0$ . Explicitly,

$$\begin{split} M(B_i \to B_f + \pi^+)^{\text{fac}} &= \frac{G_F}{2\sqrt{2}} V_{cs} V_{ud} \left[ c_1 + \frac{c_2}{N_c} \right] \langle \pi^+ |(\bar{u}d)|0\rangle \langle B_f |(\bar{s}c)|B_i\rangle , \\ M(B_i \to B_f + \bar{K}^0)^{\text{fac}} & (2.3) \\ &= \frac{G_F}{2\sqrt{2}} V_{cs} V_{ud} \left[ c_2 + \frac{c_1}{N_c} \right] \langle \bar{K}^0 |(\bar{s}d)|0\rangle \langle B_f |(\bar{u}c)|B_i\rangle , \end{split}$$

<sup>&</sup>lt;sup>3</sup>It is a "model" because of the assumption of pole approximation: The nonfactorizable contribution is approximately saturated by one-particle intermediate states.

where  $N_c$  is the number of quark color degrees of freedom, and

$$c_1 = \frac{1}{2}(c_+ + c_-)$$
,  $c_2 = \frac{1}{2}(c_+ - c_-)$ . (2.4)

In quark-diagram language, the  $c_1$  ( $c_2$ ) term of the factorizable  $B_i \rightarrow B_f + \pi^+$  ( $B_i \rightarrow B_f + \overline{K}^0$ ) amplitude comes from the external (internal) W-emission diagram. The W-exchange diagram is, of course, nonfactorizable.

In the content of meson nonleptonic decay, it is customary to make a further assumption, namely, the factorization (or vacuum-saturation) approximation, in which one only keeps the factorizable contributions and drops the nonfactorizable ones. This approximation can be justified in the large- $N_c$  limit [18] since the nonfactorizable amplitudes are suppressed, as far as the color factor is concerned, by powers of  $1/N_c$ . However, in the  $N_c \rightarrow \infty$  limit, one should also drop the  $1/N_c$ -suppressed factorizable contribution [see, e.g., Eq. (2.3)]. The large- $N_c$  version of factorization is thus different from the naive factorization approximation in that the Fierztransformed terms are taken into account in the latter approach. Nowadays we have learned from the nonleptonic decays of charmed and bottom mesons that the naive factorization method fails to account for the bulk of data, especially for those decay modes which are naively expected to be color suppressed. The discrepancy between theory and experiment gets much improved in the  $1/N_c$ expansion method. Does this scenario also work for the baryon sector? This issue is settled down by the experimental measurement of the Cabibbo-suppressed mode  $\Lambda_c^+ \rightarrow p\phi$ , which receives contributions only from the factorizable diagrams. We have shown in Ref. [14] that the large- $N_c$  predicted rate is in good agreement with the measured value. By contrast, its decay rate predicted by the naive factorization approximation is too small by a factor of 15. Therefore, we should take the  $1/N_c$  approach for the factorizable amplitude of  $B_i \rightarrow B_f + P$ .

We next turn to the nonfactorizable contribution. It is here we see a significant disparity between meson and baryon decays. Contrary to the meson case, the nonfactorizable amplitudes of baryon nonleptonic decays are not necessarily color suppressed in the  $N_c \rightarrow \infty$  limit [16]. Although the W-exchange diagram, for example, is down by a factor of  $1/N_c$  relative to the external W-emission diagram, this seeming suppression is compensated by the fact that the baryon contains  $N_c$  quarks in the limit of large  $N_c$ , thus allowing  $N_c$  different possibilities for W exchange between heavy and light quarks. This leads to the well-known statement that W exchange in baryon decay is subject to neither color nor helicity suppression. Using the reduction formula, the nonfactorizable amplitude can be recast to

$$M[B_i \to B_f + P^a(q)]^{\mathrm{nf}}$$

$$= \lim_{q^2 \to m_P^2} i(m_P^2 - q^2) \int d^4x \ e^{iq \cdot x} \langle B_f | T \phi^a(x) \mathcal{H}_W(0) | B_i \rangle$$
(2.5)

or

$$M[B_{i} \rightarrow B_{f} + P^{a}(q)]^{\text{nf}} = \lim_{q^{2} \rightarrow m_{P}^{2}} i(m_{P}^{2} - q^{2}) \int d^{4}x \ e^{iq \cdot x} \left[ \sum_{n} \theta(x^{0}) \langle B_{f} | \phi^{a}(x) | n \rangle \langle n | \mathcal{H}_{W}(0) | B_{i} \rangle + \sum_{n} \theta(-x^{0}) \langle B_{f} | \mathcal{H}_{W}(0) | n \rangle \langle n | \phi^{a}(x) | B_{i} \rangle \right],$$

$$(2.6)$$

where  $\phi^a$  is the interpolating field for  $P^a$ . Conventionally one considers pole approximation so that only onebaryon intermediate states are kept. Under the pole approximation, the nonfactorizable amplitude is nothing but the contribution arising from two distinct pole diagrams. This can be seen from the identity

$$\langle B_{f} | \phi^{a}(0) | B_{n} \rangle = \frac{1}{m_{P}^{2} - q^{2}} \langle B_{f} | J^{a}(0) | B_{n} \rangle ,$$
  
$$= \frac{g_{B_{f}B_{n}P^{a}}}{m_{P}^{2} - q^{2}} \overline{u}_{f} i \gamma_{5} u_{n} . \qquad (2.7)$$

Hence, for example, the first term on the right-hand side (RHS) of (2.6) represents the pole diagram in which a weak transition  $B_i - B_n$  is followed by a strong emission of the  $P^a$ . Note that since the baryon-color wave function is totally antisymmetric, only the operator  $O_-$  contributes to the baryon-baryon transition matrix element as it is antisymmetric in color indices. As shown in Ref. [14], at least for hyperon and charmed-baryon decays, the *s*-wave amplitude is dominated by the low-lying  $\frac{1}{2}^-$  resonances and the *p*-wave one governed by the ground-state  $\frac{1}{2}^+$  poles. As a result, it follows from Eq. (2.6) that [14]

$$A^{\text{nf}} = -\sum_{B_{n}^{*}(\frac{1}{2}^{-})} \left[ \frac{g_{B_{f}B_{n}^{*}b_{n}^{*}i}}{m_{i}-m_{n}^{*}} + \frac{b_{fn}^{*}g_{B_{n}^{*}B_{i}^{P}}}{m_{f}-m_{n}^{*}} \right] + \cdots,$$

$$B^{\text{nf}} = -\sum_{B_{n}} \left[ \frac{g_{B_{f}B_{n}P}a_{ni}}{m_{i}-m_{n}} + \frac{a_{fn}g_{B_{n}B_{i}P}}{m_{f}-m_{n}} \right] + \cdots,$$
(2.8)

where A and B are s- and p-wave amplitudes, respectively, ellipses in Eq. (2.8) denote other pole contributions which are negligible for our purposes, and  $a_{ij}$  as well as  $b_{i*i}$  are the baryon-baryon matrix elements defined by

$$\langle B_i | \mathcal{H}_W | B_j \rangle = \overline{u}_i (a_{ij} - b_{ij} \gamma_5) u_j , \langle B_i^*(\frac{1}{2}) | \mathcal{H}_W^{\text{PV}} | B_j \rangle = i b_{i^* j} \overline{u}_i u_j ,$$

$$(2.9)$$

with  $b_{ji} = -b_{i*j}$ . It should be stressed that Eq. (2.8) is derived only under the assumption of pole approximation, and it is valid also for vector-meson emission.

Evidently, the calculation of s-wave amplitudes is generally more difficult than the p wave owing to the troublesome negative-parity baryon resonances. Nevertheless, a simplification happens for hyperon nonleptonic decays where the final-state pion is approximately soft. Using the Goldberger-Treiman (GT) relation (C2) for the coupling constants  $g_{BBP}$  and the generalized GT relation (C8) for  $g_{B*BP}$  couplings (both relations being valid in the soft-pion limit), Eq. (2.6) leads to [14]

$$A^{CA} = \frac{\sqrt{2}}{f_{P^a}} \langle B_f | [Q_5^a, \mathcal{H}_W^{PV}] | B_i \rangle$$
(2.10)

and

$$B^{CA} = -\frac{\sqrt{2}}{f_{P^{a}}} \sum_{B_{n}} \left[ g^{A}_{B_{f}B_{n}} \frac{m_{f} + m_{n}}{m_{i} - m_{n}} a_{ni} + a_{fn} \frac{m_{i} + m_{n}}{m_{f} - m_{n}} g^{A}_{B_{i}B_{n}} \right]. \quad (2.11)$$

Traditionally, the current-algebra results<sup>4</sup> (2.10) and (2.11) are derived from Eq. (2.6) together with the PCAC (partial conservation of axial-vector current) relation

$$\pi^a = \frac{\sqrt{2}}{f_\pi m_P^2} \partial^\mu A^a_\mu \tag{2.12}$$

and the Ward identity

$$i \int d^{4}x \ e^{iq \cdot x} T \partial^{\mu} A^{a}_{\mu}(x) \mathcal{H}_{W}(0) = q^{\mu} \int d^{4}x \ T A^{a}_{\mu}(x) \mathcal{H}_{W}(0)$$
$$- \int d^{4}x \ e^{iq \cdot x} \delta(x^{0})$$
$$\times [A^{a}_{0}(x), \mathcal{H}_{W}(0)] .$$
(2.13)

Note that  $B^{CA}$  can actually be read off directly from Eq. (2.8) by substituting the GT relation for strong-coupling constants. Therefore, the parity-violating amplitude is reduced in the soft-pion limit to a simple commutator relation and is related to parity-conserving baryon-baryon matrix elements. In other words, *no* information of  $\frac{1}{2}^{-}$  poles is required for evaluating the *s*-wave amplitudes. However, as explained in the Introduction, such simplification is no longer applicable to heavy-baryon weak decays for the meson there is far from being soft; for example, the pion's momentum in the decay  $\Lambda_c^+ \rightarrow \Lambda \pi$  is 863 MeV, which is much larger than its own mass. Writing

$$A = A^{CA} + (A - A^{CA}), \qquad (2.14)$$

it has been demonstrated in Refs. [14,15] that the on-shell correction  $(A - A^{CA})$  is very important for charmedbaryon decays, and this clearly indicates that the *s*-wave amplitude is not dominated by the commutator term.

To summarize, the dynamics of heavy-baryon decays is more complicated than the meson decay because of the

$$B^{\rm CA} = \lim_{q \to 0} \left[ B^{\rm nf} - i \frac{\sqrt{2}}{f_{P^a}} q^{\mu} T_{\mu} \right] + i \frac{\sqrt{2}}{f_{P^a}} q^{\mu} T_{\mu} ,$$

which can be shown to be equivalent to Eq. (2.11).

importance of nonfactorizable contributions, and is more difficult to treat than the hyperon decay owing to the presence of  $\frac{1}{2}^{-}$  poles for s waves. In short, Eq. (2.8) is the starting point for handling the nonfactorization amplitudes of heavy-baryon decays.

## **III. NUMERICAL RESULTS**

We employ the MIT bag model [20] to evaluate the form factors appearing in factorizable amplitudes and the strong-coupling constants and baryon transition matrix elements relevant to nonfactorizable contributions. Some model details are given in Appendixes A–D. In this section we will first discuss the evaluation of the aforementioned ingredients and then present the results of decay rates and the  $\alpha$  asymmetry parameter for Cabibbo-allowed nonleptonic weak decays of charmed baryons.

### A. Baryon-Baryon transition matrix elements

Among the two four-quark operators  $O_{\pm}$  given in Eq. (2.1),  $O_{+}$  is symmetric in color indices where  $O_{-}$  is antisymmetric. Therefore, the operator  $O_{+}$  does not contribute to baryon transition matrix elements since the baryon-color wave function is totally antisymmetric. The parity-conserving (PC) matrix elements  $a_{ij}$  and the parity-violating (PV) ones  $b_{i*j}$  have the expression

$$a_{ij} = \frac{h}{2\sqrt{2}} \langle B_i | O_-^{\text{PC}} | B_j \rangle c_- ,$$
  

$$b_{i*j} = -i \frac{h}{2\sqrt{2}} \langle B_i(1/2^-) | O_-^{\text{PV}} | B_j \rangle c_- ,$$
(3.1)

with  $h \equiv G_F V_{cs} V_{ud}$ . Note that  $b_{ji*} = -b_{i*j}$ . With the bag integrals  $X_1 = -3.58 \times 10^{-6}$  GeV<sup>3</sup> and  $X_2 = 1.74 \times 10^{-4}$  GeV<sup>3</sup> [14], the PC transitions are (in units of  $c_h$  GeV<sup>3</sup>)

$$a_{\Sigma^{+}\Lambda_{c}^{+}} = a_{\Sigma_{c}^{0}\Lambda^{0}} = -3.76 \times 10^{-3} ,$$
  

$$a_{\Xi_{c}^{0}\Lambda_{\Xi^{0}}} = -3.81 \times 10^{-3} ,$$
  

$$a_{\Sigma_{c}^{+}\Sigma^{+}} = -a_{\Sigma_{c}^{0}\Sigma^{0}} = a_{\Xi_{c}^{0}\Sigma_{\Xi^{0}}} = -6.58 \times 10^{-3} ,$$
  
(3.2)

where the superscripts A and S denote antitriplet and sextet charmed baryons, respectively.

In the bag model the low-lying negative-parity baryon states are made of two quarks in the ground  $1S_{1/2}$  eigenstate and one quark excited to  $1P_{1/2}$  or  $1P_{3/2}$ . Consequently, the evaluation of the  $\frac{1}{2} - \frac{1}{2}^+$  baryon matrix elements  $b_{i*_j}$  becomes much more involved owing to the presence of  $1P_{1/2}$  and  $1P_{3/2}$  bag states. Assuming that the  $\frac{1}{2}^-$  resonances are dominated by the low-lying negative-parity states, we have four (70, L=1) states  ${}^{2}8_{1/2}$ ,  ${}^{4}8_{1/2}$ ,  ${}^{2}10_{1/2}$ ,  ${}^{1}1_{1/2}$  (see Appendix A for notation) for uncharmed baryons and two states  ${}^{2}6_{1/2}$ ,  ${}^{2}\overline{3}_{1/2}$  for charmed baryons. With the bag integrals given by Eq. (3.7) of Ref. [14], it follows from Eqs. (3.1), (A3), (B2)-(B5), and Eq. (A7) of Ref. [14] that [note that the PV matrix elements  $b_{\sum_{c}^*\Lambda}$ ,  $b_{\sum_{c}^*\Sigma}$  presented in Ref. [14] are for wrong SU(3) presentation (see also the footnote in Appendix A); they are corrected here in Eq. (3.3)]

<sup>&</sup>lt;sup>4</sup>As stressed in Ref. [14], the current-algebra expression for the parity-conserving wave should read

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$$\begin{split} b_{\Sigma^{+}(^{2}8)\Lambda_{c}^{+}} &= -1.76 \times 10^{-3} , \ b_{\Sigma^{+}(^{4}8)\Lambda_{c}^{+}} = -4.21 \times 10^{-3} , \ b_{\Sigma^{+}(^{2}10)\Lambda_{c}^{+}} = 1.47 \times 10^{-4} , \\ b_{\Xi^{0}(^{2}8)\Xi_{c}^{0,4}} &= 4.77 \times 10^{-5} , \ b_{\Xi^{0}(^{4}8)\Xi_{c}^{0,4}} = -1.72 \times 10^{-3} , \ b_{\Xi^{0}(^{2}10)\Xi_{c}^{0,4}} = -1.41 \times 10^{-3} , \\ b_{\Xi^{0}(^{2}8)\Xi_{c}^{0,5}} &= 3.55 \times 10^{-3} , \ b_{\Xi^{0}(^{4}8)\Xi_{c}^{0,5}} = 7.02 \times 10^{-3} , \ b_{\Xi^{0}(^{2}10)\Xi_{c}^{0,5}} = 2.48 \times 10^{-4} , \\ b_{\Sigma_{c}^{0}}(^{2}6)\Lambda^{0} &= -7.97 \times 10^{-4} , \ b_{\Sigma_{c}^{0}(^{2}6)\Sigma^{0}} = 1.46 \times 10^{-3} , \ b_{\Sigma_{c}^{+}(^{2}6)\Sigma^{+}} = -1.46 \times 10^{-3} , \\ b_{\Xi_{c}^{0}(^{2}6)\Xi^{0}} = 1.46 \times 10^{-3} , \ b_{\Xi_{c}^{0}(^{2}\overline{3})\Xi^{0}} = -2.55 \times 10^{-3} , \\ b_{\Sigma_{c}^{0}(^{2}6)\Sigma^{0}} = 1.46 \times 10^{-3} , \ b_{\Sigma_{c}^{+}(^{2}\overline{3})\Sigma^{+}} = -1.46 \times 10^{-3} , \end{split}$$

expressed in units of  $c_h$  GeV<sup>3</sup>.

### B. Form factors and strong-coupling constants

Using the bag parameters given in Ref. [14], we obtain the following values for the overlap bag integrals appearing in Eq. (D4):

$$\int d^{3}r(u_{s}u_{c}+v_{s}v_{c})=0.95, \int d^{3}r(u_{u}u_{c}+v_{u}v_{c})=0.88,$$

$$\int d^{3}r(u_{s}u_{c}-\frac{1}{3}v_{s}v_{c})=0.86, \int d^{3}r(u_{u}u_{c}-\frac{1}{3}v_{u}v_{c})=0.77.$$
(3.4)

The form factors  $f_1$  and  $g_1$  [see Eq. (D1)] at  $q^2 = q_{\text{max}}^2 = (m_i - m_f)^2$  can then be determined directly from Eq. (D4) and extrapolated to the desired  $q^2$  using Eq. (D3).

In current algebra, strong-coupling constants are related to the axial-vector form factors at  $q^2=0$  via the Goldberger-Treiman (GT) relations given by (C2) and (C8). With the bag integrals

$$Z_1 = 0.052$$
,  $Z_2 = 0.056$ , (3.5)

the numerical values for the form factors  $g_{B'B}^A$  and  $g_{B'B_c}^A$  can be read off immediately from Eqs. (C4) and (C6). Note that unlike the form factor  $g_{B_cB}^A$  (i.e.,  $g_1$ ), the  $q^2$  dependence of  $g_{B'B}^A$  and  $g_{B'_cB_c}^A$  is quite weak because of smallness of  $q^2$ . In what follows we list the coupling constants  $g_{B'B,P}$  calculated in this way:

$$g_{\Xi_{c}^{+} A \Xi_{c}^{0S} \pi^{+}} = -14.3 , \quad g_{\Xi_{c}^{0} A \Xi_{c}^{0S} \pi^{0}} = 10.3 , \quad g_{\Sigma_{c}^{+} \Lambda_{c}^{+} \overline{K}^{0}} = 12.5 ,$$

$$g_{\Lambda_{c}^{+} \Sigma_{c}^{0} \pi^{+}} = 19.0 , \quad g_{\Xi_{c}^{+} A \Sigma_{c}^{+} \overline{K}^{0}} = 12.6 ,$$

$$g_{\Xi_{c}^{0} A \Sigma_{c}^{+} K^{-}} = 12.6 , \quad g_{\Lambda_{c}^{+} \Xi_{c}^{0S} K^{+}} = 12.4 , \quad g_{\Xi_{c}^{0} A \Sigma_{c}^{0} \overline{K}^{0}} = 17.8 , \quad g_{\Xi_{c}^{0} A \Omega_{c}^{0} \overline{K}^{0}} = -18.7 ,$$

$$g_{\Xi_{c}^{0S} \Omega_{c}^{0} \overline{K}^{0}} = 22.5 , \quad g_{\Lambda_{c}^{+} \Xi_{c}^{0A} K^{+}} = 0 .$$
(3.6)

The  $g_{B'BP}$  coupling computed by the method of Ref. [21] are summarized in (C1). The reader may check that the current-algebra predictions for  $g_{B'BP}$  are smaller than those in (C1) by roughly a factor of  $\sqrt{2}$ .

The coupling constants  $g_{B^*BP}$  are obtained from Eq. (D9) of Ref. [14] together with the generalized GT relation (C8). Taking the masses<sup>5</sup>

$$m_{\Sigma^{(2_8)}} = 1620 \text{ MeV} , \quad m_{\Sigma^{(4_8)}} = 1750 \text{ MeV} , \quad m_{\Sigma^{(2_{10})}} \simeq 1700 \text{ MeV} , \quad m_{\Xi^{(2_8)}} \simeq 1720 \text{ MeV} ,$$
  
$$m_{\Xi^{(4_8)}} \simeq 1900 \text{ MeV} , \quad m_{\Xi^{(2_{10})}} \simeq 1800 \text{ MeV} , \quad m_{\Sigma_c^{(2_6)}} \simeq 2750 \text{ MeV} , \quad m_{\Xi_c^*} \simeq 2770 \text{ MeV} ,$$
  
(3.7)

for low-lying  $\frac{1}{2}^{-}$  resonances with  $\Xi_{c}^{*}$  denoting  $\Xi_{c}(^{2}6)$  or  $\Xi_{c}(^{2}\overline{3})$ , we obtain

<sup>&</sup>lt;sup>5</sup>The mass of  $\Sigma(^{2}8)$  and  $\Sigma(^{4}8)$  is taken from the Particle Data Group [22]. In Ref. [14] we took  $m_{\Sigma(^{2}10)} \simeq 2$  GeV, which is unlikely to be the mass of the lowest-lying (70, <sup>2</sup>10) state.

$$\begin{split} g_{\Sigma^{+}(^{2}8)p\overline{K}^{0}} &= 0.52 , \ g_{\Sigma^{+}(^{4}8)p\overline{K}^{0}} = 2.49 , \ g_{\Sigma^{+}(^{2}10)p\overline{K}^{0}} = 0.81 , \ g_{\Sigma^{+}(^{2}8)\Xi^{0}K^{+}} = -0.47 , \\ g_{\Sigma^{+}(^{4}8)\Xi^{0}K^{+}} &= 0.33 , \ g_{\Sigma^{+}(^{2}10)\Xi^{0}K^{+}} = -0.15 , \ g_{\Sigma^{+}(^{2}8)\Lambda^{0}\pi^{+}} = -0.63 , \ g_{\Sigma^{+}(^{4}8)\Lambda^{0}\pi^{+}} = 1.59 , \\ g_{\Sigma^{+}(^{2}10)\Lambda^{0}\pi^{+}} &= -1.11 , \ g_{\Sigma^{+}(^{2}8)\Sigma^{0}\pi^{+}} = 1.55 , \ g_{\Sigma^{+}(^{4}8)\Sigma^{0}\pi^{+}} = 0.81 , \ g_{\Sigma^{+}(^{2}10)p\overline{K}^{0}} = -0.59 , \\ g_{\Xi^{0}(^{2}8)\Sigma^{0}\overline{K}^{0}} &= -0.57 , \ g_{\Xi^{0}(^{4}8)\Sigma^{0}\overline{K}^{0}} = 0.39 , \ g_{\Xi^{0}(^{2}10)\Sigma^{0}\overline{K}^{0}} = -0.17 , \ g_{\Xi^{0}(^{2}8)\Xi^{-}\pi^{+}} = 0.41 , \\ g_{\Xi^{0}(^{4}8)\Xi^{-}\pi^{+}} &= 2.38 , \ g_{\Xi^{0}(^{2}10)\Xi^{-}\pi^{+}} = 0.49 , \ g_{\Xi^{0}(^{2}8)\Xi^{0}\pi^{0}} = 0.29 , \ g_{\Xi^{0}(^{4}8)\Xi^{0}\pi^{0}} = 1.68 , \\ g_{\Xi^{0}(^{2}10)\Xi^{0}\pi^{0}} = 0.35 , \ g_{\Xi^{0}(^{2}8)\Lambda\overline{K}^{0}} = 0.57 , \ g_{\Xi^{0}(^{4}8)\Lambda\overline{K}^{0}} = 0.74 , \ g_{\Xi^{0}(^{2}10)\Lambda\overline{K}^{0}} = -0.32 , \end{split}$$

for couplings  $g_{B'*BP}$ , and

$$g_{\Sigma_{c}^{+}(^{2}6)\Xi_{c}^{0}A_{K}^{+}} = 0.13 , \quad g_{\Sigma_{c}^{+}(^{2}6)\Xi_{c}^{+}A_{\overline{K}^{0}}} = 0.13 , \quad g_{\Sigma_{c}^{0}(^{2}6)\Xi_{c}^{0}A_{\overline{K}^{0}}} = 0.18 , \quad g_{\Xi_{c}^{0}(^{2}6)\Xi_{c}^{+}A_{\pi}^{-}} = -0.15 ,$$

$$g_{\Xi_{c}^{0}(^{2}\overline{3})\Xi_{c}^{+}A_{\pi}^{-}} = -0.26 , \quad g_{\Xi_{c}^{0}(^{2}6)\Xi_{c}^{0}A_{\pi}^{0}} = 0.11 , \quad g_{\Xi_{c}^{0}(^{2}\overline{3})\Xi_{c}^{0}A_{\pi}^{0}} = 0.18 , \quad g_{\Xi_{c}^{0}(^{2}6)\Lambda_{c}^{+}K^{-}} = 0.23 ,$$

$$g_{\Xi_{c}^{0}(^{2}\overline{3})\Lambda_{c}^{+}K^{-}} = -0.39 , \quad g_{\Sigma_{c}^{0}(^{2}6)\Lambda_{c}^{+}\pi^{-}} = -0.72 , \quad g_{\Xi_{c}^{0}(^{2}6)\Omega_{c}^{0}\overline{K}^{0}} = 0.01 , \quad g_{\Xi_{c}^{0}(^{2}\overline{3})\Omega_{c}^{0}\overline{K}^{0}} = -0.17 ,$$

$$(3.9)$$

for  $g_{B'_c B_c P}$  coupling constants, where use has been made of the bag integrals

$$\tilde{Y}_1 = 0.056$$
,  $\tilde{Y}'_1 = 0.058$ ,  $\tilde{Y}_{1s} = 0.051$ , (3.10)

and Eq. (A7) of Ref. [14], and  $\widetilde{Y}'_1 = \int r^2 dr (\widetilde{u}_u u_s - \widetilde{v}_u v_s)$ .

## C. Decay rate and asymmetry parameter

Armed with all the necessary ingredients we are in a position to compute the PC and PV amplitudes from Eq. (2.8) and Eq. (2.8) of Ref. [14] for all Cabibbo-allowed nonleptonic decays  $B_c \rightarrow B + P$  with  $B_c = \Lambda_c^+$ ,  $\Xi_c^{0A}, \Xi_c^{+A}, \Omega_c^0$ . The calculated results are summarized in Table I. (The formulas for the decay rate  $\Gamma$  and the de-

cay asymmetry parameter  $\alpha$  are given in Ref. [14].) In order to have a feeling for the size of the branching ratio, we also calculate this quantity using the lifetimes (except for  $\Omega_c$ )

$$\tau(\Lambda_c^+) = 1.9 \times 10^{-13} \text{ s} ,$$
  

$$\tau(\Xi_c^{0.4}) = 1.0 \times 10^{-13} \text{ s} ,$$
  

$$\tau(\Xi_c^{+.4}) = 4.1 \times 10^{-13} \text{ s} ,$$
  
(3.11)

where  $\tau(\Lambda_c^+)$  is taken from the Particle Data Group [22],  $\tau(\Xi_c^{+A})$  and  $\tau(\Xi_c^{0A})$  from the central values of recent E687 measurements<sup>6</sup> [23]. Experimental results for the decay rates of  $\Lambda_c^+ \rightarrow p\overline{K}^0, \Lambda\pi^+, \Sigma^0\pi^+, \Xi^0K^+$  (see Table III) are from Refs. [22,24].

TABLE I. Numerical values of the predicted s- and p-wave amplitudes of  $B_c \rightarrow B + P$  decays in the pole model in units of  $G_F V_{cs} V_{ud} \times 10^{-2} \text{ GeV}^2$ . The predicted  $\alpha$  asymmetry parameter, decay rates (in units of  $10^{11} \text{ s}^{-1}$ ), and branching ratios (in percent) are given in the last three columns. Lifetimes of charmed baryons are taken from Eq. (3.11). As discussed in Sec. IV C, no reliable predictions can be made for the decays  $\Lambda_c^+ \rightarrow \Xi^0 K^+$  and  $\Xi_c^0 A \rightarrow \Sigma^+ K^-$ .

made for the d	couys m <sub>c</sub>	/ w 11 u	<u>nu = c</u> /	<u>~ n .</u>					
Reaction	$A^{ m fac}$	$A^{\mathrm{pole}}$	$A^{\mathrm{tot}}$	$B^{ m fac}$	<b>B</b> <sup>pole</sup>	$B^{tot}$	α	Г	(Br) <sub>theory</sub>
$\Lambda_c^+ \rightarrow p \overline{K}^0$	-5.73	3.91	-1.82	14.33	3.23	17.56	-0.49	0.63	1.20
$\Lambda_c^+ \rightarrow \Lambda \pi^+$	-5.40	2.10	-3.30	18.09	-4.87	13.22	-0.95	0.44	0.84
$\Lambda_c^+ \rightarrow \Sigma^0 \pi^+$	0	2.20	2.20	0	14.63	14.63	0.78	0.36	0.68
$\Lambda_c^+ \rightarrow \Sigma^+ \pi^0$	0	-2.20	-2.20	0	-14.63	-14.63	0.78	0.36	0.68
$\Lambda_c^+ \rightarrow \Xi^0 K^+$	0	0.10	0.10	0	4.26	4.26			
$\Xi_c^{+A} \rightarrow \Sigma^+ \overline{K}^0$	-3.75	0.01	-3.74	12.99	-12.50	0.49	-0.09	0.19	0.78
$\Xi_c^+ A \rightarrow \Xi^0 \pi^+$	6.99	-0.06	6.93	-24.90	14.24	-10.66	-0.77	0.89	3.65
$\Xi_c^{0A} \rightarrow \Lambda \overline{K}^0$	1.34	0.31	1.64	-4.72	-6.04	-10.76	-0.73	0.24	0.24
$\Xi_c^{0A} \rightarrow \Sigma^0 \overline{K}^0$	2.65	0.12	2.77	-9.19	6.39	-2.80	-0.59	0.12	0.12
$\Xi_c^{0A} \rightarrow \Sigma^+ K^-$	0	0.16	0.16	0	3.53	3.53			
$\Xi_c^{0A} \rightarrow \Xi^0 \pi^0$	0	1.13	1.13	0	-12.76	-12.76	-0.54	0.25	0.25
$\Xi_c^{0A} \rightarrow \Xi^- \pi^+$	-7.18	1.55	-5.63	25.10	-3.80	21.30	-0.99	1.12	1.12
$\Omega_c^0 \rightarrow \Xi^0 \overline{K}^0$	1.72	-0.06	1.66	2.75	-10.20	-7.45	-0.93	0.13	

<sup>6</sup>The average value of  $\tau(\Xi_c^{+A}) = (3.0^{+1.0}_{-0.6}) \times 10^{-13}$  s cited by the Particle Data Group [22] is pulled low by the old NA-32 result  $\tau(\Xi_c^{+A}) = (0.20^{+0.11}_{-0.06})$  ps.

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It is clear from Table III that the pole-model predictions are in good agreement with experiment except for the decay  $\Lambda_c^+ \rightarrow \Xi^0 K^+$ . In Sec. IV C we will argue that presently we cannot make reliable predictions for the decay modes  $\Lambda_c^+ \rightarrow \Xi^0 K^+$  and  $\Xi_c^{0,A} \rightarrow \Sigma^+ K^-$ . A detailed discussion of our results and a comparison with other works will be presented in Sec. IV.

#### IV. DISCUSSION AND CONCLUSION

Before drawing conclusions and implications from our predictions for charmed-baryon nonleptonic decays, it is pertinent to compare our results with the traditional approach (pre-1992), namely, current algebra, and the most recent theoretical calculation (post-1992) presented in Refs. [15,16].

#### A. Comparison with current algebra

Except for Refs. [5,10] most previous studies on the dynamics of charmed-baryon two-body weak decays are based on the current-algebra technique. The predictions are shown in Table II. The factorizable amplitudes are the same as Table I. As for nonfactorizable contributions, the s-wave amplitudes are calculated by using the commutator terms [Eqs. (E3) and (E4)], while the p-wave one by Eq. (2.11).

Although the current-algebra and PCAC methods were widely employed before for the study of  $B_c \rightarrow B + P$ , several important improvements are made in the present current-algebra calculation.

(1) As discussed in Sec. II, the large- $N_c$  approximation

rather than the naive factorization approximation, the former being supported by the experimental measurement of  $\Lambda_c^+ \rightarrow p\phi$  decay, is utilized for describing the factorizable amplitudes. This has an important consequence that the factorizable amplitude of  $B_c \rightarrow B + \overline{K}^0$ , which is naively expected to be color suppressed, is no longer subject to color suppression and has an equal weight as the factorizable amplitude of  $B_c \rightarrow B + \pi^+$ . This helps explain why the observed ratio of  $\Gamma(\Lambda_c^+ \rightarrow \Lambda \pi^+)/\Gamma(\Lambda_c^+ \rightarrow p\overline{K}^0)$  is smaller than unity.

(2) Form factors  $f_1$  and  $g_1$  evaluated by the static bag or quark model, for example, Eqs. (C3) and (D2), are interpreted as the predictions obtained at maximum  $q^2$ since static bag- or quark-model wave functions best resemble the hadron state at  $q^2 = (m_i - m_f)^2$  where both baryons are static. As a result, form factors at  $q^2 = 0$  become smaller than previously estimated. The decay rate of  $\Lambda_c^+ \rightarrow \Lambda \pi^+$ , which was overestimated before by an order of magnitude or so (see Table III of Ref. [14]), is now significantly reduced.

(3) Strong-coupling constants and baryon matrix elements are calculated using the bag model so that their relative signs are fixed. The relative signs are important when different pole contributions are combined. In many earlier publications, couplings and hadron matrix elements are often related to each other through SU(3) symmetry. Sometimes this will result in a wrong relative sign if care is not taken. A prominent example is the decay  $\Lambda_c^+ \rightarrow \Xi^0 K^+$ , which receives very little contribution for its s waves (see Table II) and has the p-wave amplitude given by

$$B^{CA}(\Lambda_{c}^{+} \to \Xi^{0}K^{+}) = -\frac{1}{f_{K}} \left[ g^{A}_{\Sigma^{+}\Xi^{0}} \frac{m_{\Sigma^{+}} + m_{\Xi^{0}}}{m_{\Lambda_{c}^{+}} - m_{\Sigma^{+}}} a_{\Sigma^{+}\Lambda_{c}^{+}} + a_{\Xi^{0}\Xi_{c}^{0,A}} \frac{m_{\Xi_{c}^{0,A}} + m_{\Lambda_{c}^{+}}}{m_{\Xi^{0}} - m_{\Xi_{c}^{0,A}}} g^{A}_{\Xi_{c}^{0,A}\Lambda_{c}^{+}} + a_{\Xi^{0}\Xi_{c}^{0,S}} \frac{m_{\Xi_{c}^{0,S}} + m_{\Lambda_{c}^{+}}}{m_{\Xi^{0}} - m_{\Xi_{c}^{0,S}}} g^{A}_{\Xi_{c}^{0,A}\Lambda_{c}^{+}} + a_{\Xi^{0}\Xi_{c}^{0,S}} g^{A}_{\Xi_{c}^{0,A}\Lambda_{c}^{+}} + a_{\Xi^{0}\Xi_{c}^{0,A}} g^{A}_{\Xi_{c}^{0,A}} + a_{\Xi^{0}\Xi_{c}^{0,A}} + a_{\Xi^{$$

Since  $g_{\Xi_c^0 A_A_c^+}^A = 0$  [see Eq. (C7)], only the first and third terms in (4.1) contribute to the current-algebra PC amplitude. From Eqs. (3.2), (C4), and (C6) we find a large cancellation between these two pole terms. By contrast, a large constructive interference was found in Ref. [6] owing to wrong relative signs.

(4) The PC amplitude derived from the pole contribution of  $i\sqrt{2}q^{\mu}/f_{p}$  has the familiar expression [19]

$$B = -\frac{\sqrt{2}}{f_P}(m_i + m_f) \sum_{B_n} \left[ \frac{g_{B_f B_n}^A a_{ni}}{m_i - m_n} + \frac{a_{fn} g_{B_n B_i}^A}{m_f - m_n} \right].$$
(4.2)

Reaction	$A^{ m fac}$	$A^{\mathrm{com}}$	$A^{\rm tot}$	$B^{ m fac}$	$B^{\mathrm{pole}}$	$B^{\mathrm{tot}}$	α	Γ	(Br) <sub>theor</sub>
$\Lambda_c^+ \rightarrow p \overline{K}^0$	-5.73	-4.44	-10.17	14.33	2.10	16.43	-0.90	1.82	3.46
$\Lambda_c^+ \rightarrow \Lambda \pi^+$	-5.40	0	-5.40	18.09	-4.14	13.95	-0.99	0.73	1.39
$\Lambda_c^+ \rightarrow \Sigma^0 \pi^+$	0	-7.66	-7.66	0	6.42	6.42	-0.49	0.88	1.67
$\Lambda_c^+ \rightarrow \Sigma^+ \pi^0$	0	7.66	7.66	0	-6.42	-6.42	-0.49	0.88	1.67
$\Lambda_c^+ \rightarrow \Xi^0 K^+$	0	-0.06	-0.06	0	-2.98	-2.98			
$\Xi_c^{+A} \rightarrow \Sigma^+ \overline{K}^0$	-3.75	4.47	0.72	12.99	-12.50	0.49	0.43	0.01	0.03
$\Xi_c^{+A} \rightarrow \Xi^0 \pi^+$	6.99	- 5.48	1.51	-24.90	14.24	-10.66	-0.77	0.19	0.78
$\Xi_c^{0A} \rightarrow \Lambda \overline{K}^0$	1.34	5.55	6.89	-4.72	-6.78	-11.50	-0.88	0.89	0.89
$\Xi_c^{0A} \rightarrow \Sigma^0 \overline{K}^0$	2.65	-3.20	-0.55	-9.19	6.13	-3.06	0.85	0.02	0.02
$\Xi_c^{0A} \rightarrow \Sigma^+ K^-$	0	0.06	0.06	0	-0.92	-0.92			
$\Xi_c^{0A} \rightarrow \Xi^0 \pi^0$	0	7.76	7.76	0	-12.20	-12.20	-0.78	1.12	1.12
$\Xi_c^{0A} \rightarrow \Xi^- \pi^+$	-7.18	5.49	-1.69	25.10	-2.74	22.36	-0.47	0.74	0.74
$\Omega_c^0 \rightarrow \Xi^0 \overline{K}^0$	1.72	-11.05	-9.33	2.75	-9.54	-6.79	0.44	0.98	

TABLE II. Same as Table I except that predictions are made by current algebra.

As discussed in footnote 4, the contribution due to  $\lim_{q\to 0} [B^{nf} - i(\sqrt{2}/f_P)q^{\mu}T_{\mu}]$  should be taken into account and it leads to Eq. (2.11) when combined with (4.2). This correction is important for the decay modes  $\Lambda_c^+ \rightarrow \Sigma^0 \pi^+, \Xi^0 K^+$  and  $\Xi_c^{0A} \rightarrow \Sigma^0 \overline{K}^0, \Sigma^+ K^-, \Lambda \overline{K}^0$ .

Recall that the predicted ratio of  $\Gamma(\Lambda \pi^+)/\Gamma(p\overline{K}^0)$  in earlier attempts is considerably larger than unity, ranging from 2.3 to 13 (see Table III of Ref. [14]), while experimentally it is only  $0.36\pm0.20$  [21]. The improved current-algebra computation yields a value of 0.40 for this ratio and a smaller absolute decay rate for  $\Lambda_c^+ \rightarrow \Lambda \pi^+$ , both being in the right ballpark.

We now compare our work with current algebra. To compute the PC amplitudes from Eq. (2.8) we actually apply the GT relation for the  $g_{B'_cB_cP}$  couplings and Eq. (C1) for the coupling constants  $g_{B'BP}$ . The difference between Tables I and II for the nonfactorizable p waves thus comes from the difference between  $g_{B'BP}$  and  $\sqrt{2}(m_{B'}+m_B)g_{B'B}^{A'}/f_P$ . It is clear that PC amplitudes in Tables I and II are generally the same except for the channels  $\Lambda_c^+ \rightarrow \Sigma^0 \pi^+, \Sigma^+ \pi^0, \Xi^0 K^+$  and  $\Xi_c^{0A} \rightarrow \Sigma^+ K^-$ . In both approaches, the nonfactorizable W-exchange effects are not negligible; they are as important as the factorizable ones in the decays  $\Xi_c^{+A} \rightarrow \Sigma^+ \overline{K}^0, \Xi^0 \pi^+, \Xi_c^{0A} \rightarrow \Sigma^0 \overline{K}^0$  and even dominate in the reactions  $\Xi_c^{0A} \rightarrow \Lambda \overline{K}^0$  and  $\Omega_c \rightarrow \Xi^0 \overline{K}^0$ .

The crucial difference between current algebra and the pole model lies in the PV sector. By comparing Table I with Table II, it is evident that (i) the s-wave amplitudes are no longer dominated by the commutator terms, that is, the on-shell correction  $(A - A^{CA})$  is quite important and has a sign opposite to that of  $A^{CA}$  [14,15]; (ii) the sign of the nonfactorizable PV amplitudes is opposite to that predicted by current algebra for the decays  $\Lambda_c^+ \rightarrow p \overline{K}^0, \Sigma^0 \pi^+, \Sigma^+ \pi^0$ , indicating that  $|A - A^{CA}| > |A^{CA}|$  in these cases; and (iii) for  $\Xi_c^{+A}$  and  $\Xi_c^{0A}$  decays, the commutator terms are of the same equal weight as factorizable contributions, whereas nonfactorizable s waves are always suppressed in the pole model.

The current-algebra method for s waves is drastically simple as it does not require the knowledge of excited  $\frac{1}{2}^{-}$ resonances. However, we see that such a simplification is certainly not applicable for describing the PV amplitudes of charmed-baryon weak decays as the pseudoscalar meson is no longer soft. We also see that the predicted signs of the total s-wave amplitudes of  $\Lambda_c^+ \rightarrow \Sigma^0 \pi^+, \Sigma^+ \pi^0$ ,  $\Xi_c^{0A} \rightarrow \Sigma^0 \overline{K}^0$ , and  $\Omega_c^0 \rightarrow \Xi^0 \overline{K}^0$  relative to the corresponding p waves are different in the pole model and current algebra. Hence, even a measurement of the sign of the  $\alpha$ symmetry parameter in above-mentioned decays would provide a very useful test on various models. Experimentalists are thus urged to perform such measurements.

### B. Comparison with most recent theoretical calculation

There are two recent works [15,16] in which a complete analysis of  $B_c \rightarrow B + P$  is performed and factorizable amplitudes are evaluated under the large- $N_c$  approximation. Among these two works, the framework adopted by Xu and Kamal (XK) [15] is most close to ours, while Körner and Krämer (KK) [16] chose to use the covariant quark model to tackle the three-body transition amplitudes (instead of two-body transitions) directly. In this section, a comparison of our work with Refs. [15,16] will be made in order.

Though XK employ the current-algebra expression Eq. (4.2) to evaluate the nonfactorizable *p*-wave amplitudes, they do consider the  $\frac{1}{2}$  pole contributions to the s waves. Their s-wave pole formula Eq. (14) is identical to our Eq. (2.8) after applying the generalized GT relation (C8) for the couplings  $g_{B^*B_P}$  and  $g_{B^*B_P}$ . XK used SU(3) and SU(4) symmetries to relate the form factors  $g_{B'BP}^{A}$  and  $g_{B'_{c}B_{c}P}^{A}$  to the SU(3) parameters F and D, which are in turn determined from a fit to hyperon semileptonic decays, and the diquark model to calculate the PC baryon matrix elements. It is the s-wave sector where the XK's work deviates mostly from ours. XK argued that the product of form factors and PV matrix elements for  $\frac{1}{2}^{-}-\frac{1}{2}^{+}$  transitions can be related to PC baryon matrix elements. Moreover, under the assumption that  $(F^-+D^-)/(F^--D^-)\approx 0$ , with  $F^-$  and  $D^-$  being the analogous of the F and D parameters for  $\frac{1}{2} - \frac{1}{2}^+$  transition form factors, they claimed that the s-wave pole contributions are completely determined from the commutator terms of the masses of  $\frac{1}{2}^{-}$  resonances without introducing further new parameters. In our analysis, we have applied the MIT bag model to compute all the form factors and baryon-baryon matrix elements involving  $\frac{1}{2}$  intermediate states.

A comparison of Table I with Tables I and II of Ref. [15] shows that we are more or less in agreement with XK on the  $A^{\text{pole}}$  amplitude in  $\Lambda_c^+$  decays except for  $\Lambda_c^+ \rightarrow p\overline{K}^0$ , but our  $A^{\text{pole}}$  for  $\Xi_c^{0,A}, \Xi_c^{+,A}$  decays are dramatically different from those of XK not only in sign but also in magnitude: ours being smaller by roughly an order of magnitude. It is not clear to us what is the source of discrepancy.<sup>7</sup> Since XK has a larger  $A^{\text{pole}}$  for  $\Lambda_c^+ \rightarrow p\overline{K}^0$ , which dominates over  $A^{\text{fac}}$ , their  $\alpha$  is opposite to ours in sign (see Table III). Hence, a measurement of the sign of  $\alpha(\Lambda_c^+ \rightarrow p\overline{K}^0)$  will furnish a useful test on the importance of on-shell corrections to the *s*-wave amplitude. Finally, we note that in spite of the disparity on the  $\alpha$  parameter, the predicted decay rates by XK are nevertheless in accordance with ours within a factor of 2.

We next switch to the work of KK. Instead of decomposing the decay amplitude into products of strong couplings and two-body weak transitions, KK analyze the nonleptonic weak processes using the spin-flavor structure of the effective Hamiltonian and the wave functions of baryons and mesons described by the covariant quark model. The nonfactorizable amplitudes are then obtained in terms of two wave-function overlap parameters  $H_2$  and  $H_3$ , which are, in turn, determined by fitting to the exper-

<sup>&</sup>lt;sup>7</sup>A possibility is that the PV  $\frac{1}{2}^+ - \frac{1}{2}^+$  baryon matrix elements  $b_{ij}$  [cf. Eq. (2.9)] are important for  $\Xi_c^{+A}$ ,  $\Xi_c^{0A}$ ,  $\Omega_c^0$  decays. It has been shown [8,9] that  $b_{ij}$  are in general small for  $\Lambda_c^+ \rightarrow B + P$  decays, but they have not yet been examined for other antitriplet charmed baryon decays.

TABLE III. The predicted decay rates (in units of  $10^{11} \text{ s}^{-1}$ ) and the  $\alpha$  asymmetry parameter (in parentheses) for  $B_c \rightarrow B + P$  decays in various models.

Reaction	Current algebra (This work)	Pole model (This work)	Xu and Kamal [15]	Körner and Krämer [16]	Experiment [22,24]
$\overline{\Lambda_c^+ \rightarrow p \overline{K}^0}$	1.82(-0.90)	0.63(-0.49)	0.60(0.51)	input (-1.0)	0.84±0.21
$\Lambda_c^+ \rightarrow \Lambda \pi^+$	0.73(-0.99)	0.44(-0.95)	0.81(-0.67)	input $(-0.70)$	$0.30\pm0.08(-1.03\pm0.29)$
$\Lambda_c^+ \rightarrow \Sigma^0 \pi^+$	0.88(-0.49)	0.36(0.78)	0.17(0.92)	0.16(0.70)	0.29±0.14
$\Lambda_c^+ \rightarrow \Sigma^+ \pi^0$	0.88(-0.49)	0.36(0.78)	0.17(0.92)	0.16(0.70)	
$\Lambda_c^+ \rightarrow \Xi^0 K^+$			0.05(0)	0.13(0)	0.13±0.04
$\Xi_{c}^{+A} \rightarrow \Sigma^{+} \overline{K}^{0}$	0.01(0.43)	0.19(-0.09)	0.10(0.24)	1.46(-1.0)	
$\Xi_c^+ A \rightarrow \Xi^0 \pi^+$	0.19(-0.77)	0.89(-0.77)	0.76(-0.81)	0.80(-0.78)	
$\Xi_{c}^{0A} \rightarrow \Lambda \overline{K}^{0}$	0.89(-0.88)	0.24(-0.73)	0.33(1.0)	0.11(-0.76)	
$\Xi_{c}^{0A} \rightarrow \Sigma^{0} \overline{K}^{0}$	0.02(0.85)	0.12(-0.59)	0.09(-0.99)	1.05(-0.96)	
$\Xi_c^{0A} \rightarrow \Sigma^+ K^-$			0.11(0)	0.11(0)	
$\Xi_c^{0A} \rightarrow \Xi^0 \pi^0$	1.12(-0.78)	0.25(-0.54)	0.50(0.92)	0.03(0.92)	
$\Xi_c^{0A} \rightarrow \Xi^- \pi^+$	0.74(-0.47)	1.12(-0.99)	1.55(-0.38)	0.93(-0.38)	
$\underline{\Omega_c^0} \to \Xi^0 \overline{K}^0$	0.98(0.44)	0.13(-0.93)		1.75(0.51)	

imental data of  $\Lambda_c^+ \rightarrow p\overline{K}^0$  and  $\Lambda_c^+ \rightarrow \Lambda \pi^+$ , respectively. Despite the absence of first-principles calculation of the parameters  $H_2$  and  $H_3$ , this quark-model approach has fruitful predictions for not only  $B_c \rightarrow B + P$ , but also  $B_c \rightarrow B + V$ ,  $B^*(\frac{3}{2}^+) + P$ , and  $B^*(\frac{3}{2}^+) + V$  decays. Another advantage of this analysis is that each amplitude has one-to-one quark-diagram interpretation.

It is clear from Table III that the predicted decay rates of  $\Xi_c^{+A} \rightarrow \Sigma^+ \overline{K}^0$ ,  $\Xi_c^{0A} \rightarrow \Sigma^0 \overline{K}^0$ ,  $\Omega_c \rightarrow \Xi^0 \overline{K}^0$  by KK are larger than ours and that of XK by an order of magnitude, whereas the decay  $\Xi_c^{0A} \rightarrow \Xi^- \pi^+$  is strongly suppressed in the scheme of KK. Therefore, a measurement of the ratios

$$R_{1} = \frac{\Gamma(\Xi_{c}^{+A} \to \Sigma^{+} \overline{K}^{0})}{\Gamma(\Xi_{c}^{+A} \to \Xi^{0} \pi^{+})} ,$$

$$R_{2} = \frac{\Gamma(\Xi_{c}^{0A} \to \Xi^{0} \pi^{0})}{\Gamma(\Xi_{c}^{0A} \to \Xi^{-} \pi^{+})} ,$$

$$R_{3} = \frac{\Gamma(\Xi_{c}^{0A} \to \Sigma^{0} \overline{K}^{0})}{\Gamma(\Xi_{c}^{0A} \to \Xi^{-} \pi^{+})} ,$$
(4.3)

which are predicted to be, respectively, 0.21, 0.22, 0.11 in the pole model, and 1.83, 0.03, 1.13 in the covariant quark model, will be quite helpful to test those two schemes.

# C. $\Lambda_c^+ \rightarrow \Xi^0 K^+$ and $\Xi_c^{0A} \rightarrow \Sigma^+ K^-$

The decays  $\Lambda_c^+ \to \Xi^0 K^+$  and  $\Xi_c^{0,4} \to \Sigma^+ K^-$  share some common features that they do not receive factorizable contributions and that their *s*-wave amplitudes are very small and *p*-wave ones are subject to a large cancellation. More explicitly,

$$B^{\text{pole}}(\Lambda_{c}^{+} \to \Xi^{0}K^{+}) = - \left[ \frac{g_{\Sigma^{+}\Xi^{0}K^{+}}a_{\Sigma^{+}\Lambda_{c}^{+}}}{m_{\Lambda_{c}^{+}} - m_{\Sigma^{+}}} + \frac{a_{\Xi^{0}\Xi_{c}^{0,4}}g_{\Lambda_{c}^{+}\Xi_{c}^{0,4}K^{+}}}{m_{\Xi^{0}} - m_{\Xi_{c}^{0,4}}} + \frac{a_{\Xi^{0}\Xi_{c}^{0,5}}g_{\Lambda_{c}^{+}\Xi_{c}^{0,5}K^{+}}}{m_{\Xi^{0}} - m_{\Xi_{c}^{0,5}}} \right],$$

$$B^{\text{pole}}(\Xi^{0,4}, \Sigma^{+}K^{-}) = - \left[ \frac{g_{\Sigma^{+}\Xi^{0}K^{+}}a_{\Sigma^{+}}}{m_{\Xi^{0}} - m_{\Xi_{c}^{0,5}}} \right],$$

$$(4.4)$$

 $B^{\text{pole}}(\Xi_c^{0A} \rightarrow \Sigma^+ K^-)$ 

$$= - \left[ \frac{g_{\Xi^0 \Sigma^+ K^-} a_{\Xi^0 A \Xi^0}}{m_{\Xi^0 A}^- m_{\Xi^0}} + \frac{a_{\Sigma^+_c \Sigma^+} g_{\Xi^0 A \Sigma^+_c K^-}}{m_{\Sigma^+} - m_{\Sigma^+_c}} \right].$$

[The first line of Eq. (4.4) is identical to Eq. (4.1) after the use of the GT relations.] A substitution of Eqs. (C1), (3.2), and (3.6) into Eq. (4.4) clearly indicates a large destructive interference in the *p*-wave amplitudes, resulting in rather small decay rates for both modes. The fact that the naive prediction  $\Gamma(\Lambda_c^+ \rightarrow \Xi^0 K^+)=1.1 \times 10^9 \text{ s}^{-1}$  is too small compared to the recent CLEO measurement [24]  $(1.3\pm0.4)\times10^{10} \text{ s}^{-1}$  shows that our predictions for those two decay modes are unreliable. The situation becomes even worse in the framework of current algebra<sup>8</sup> (see Table II).

The CLEO data thus suggest that the destructive interference in the p wave of  $\Lambda_c^+ \rightarrow \Xi^0 K^+$  is not as severe as originally expected. There are several possibilities for allowing the alleviation of large cancellation. For example, the Goldberger-Treiman relation for the coupling

<sup>&</sup>lt;sup>8</sup>The discrepancy is improved in Ref. [15], but the prediction there is still too small by a factor of 3 (see Table III). As noted in passing, the *p*-wave formula used in Ref. [15] is that of Eq. (4.2).

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 $g_{\Lambda_c^+ \Xi_c^{0S} K^+}$  may not work well, or the  $g_{\Lambda_c^+ \Xi_c^{0S} K^+}$  coupling constant is not strictly zero, or the  $\frac{1}{2}^+$  resonances may make important contributions to the PC amplitudes or it requires the combination of above mechanisms. This issue should be seriously concerned in future study.

It is worth mentioning that the predicted  $\Gamma(\Lambda_c^+ \to \Xi^0 K^+)$  by KK is in remarkable agreement with experiment. In the scheme of KK, the decay modes  $\Lambda_c^+ \to \Xi^0 K^+$  and  $\Xi_c^{0.4} \to \Sigma^+ K^-$  receive contributions only from the quark diagrams IIa and III (see Ref. [16] for notation). KK observed that the effect of diagram III is strongly suppressed relative to IIa. In other words, these two decay modes proceed essentially through diagram IIa; strong cancellation occurs only in diagram III.

In the pole model, diagram IIa corresponds to the pole diagram in which a weak transition is followed by a strong emission of a meson, while diagram III contributes to both different pole diagrams. Unfortunately, we do not know how to separate diagram IIa from diagram III in pole language. At any rate, our goal is to understand the suppression of diagram III in the pole model in order to resolve the aforementioned problem.

### **D.** Conclusion

We now draw some conclusions from our analysis of nonleptonic weak decays of charmed baryons into an octet baryon and a pseudoscalar meson.

(i) The large- $N_c$  approximation for factorizable amplitudes, which works well in the charmed- and bottommeson sector, is also effective in the heavy baryon sector as borne out by the experimental measurement of  $\Lambda_c^+ \rightarrow p\phi$ . This accounts for the color nonsuppression of the decay  $\Lambda_c^+ \rightarrow p\overline{K}^0$  relative to  $\Lambda_c^+ \rightarrow \Lambda \pi^+$ .

(ii) Nonfactorizable contributions are evaluated under pole approximation so that they are saturated by oneparticle intermediate states. It turns out that s-wave amplitudes are dominated by the excited  $\frac{1}{2}^{-}$  resonances, and p-wave ones by the ground-state  $\frac{1}{2}^{+}$  poles. In the soft pseudoscalar-meson limit, the parity-violating amplitude is reduced to the current-algebra commutator term. We find that s waves in charmed-baryon decays are no longer dominated by commutator terms; this is not surprising since the meson is far from being soft. The important on-shell correction  $(A - A^{CA})$  will affect the  $\alpha$  asymmetry parameter and changes its sign for the decays  $\Lambda_c^+ \rightarrow \Sigma^0 \pi^+, \Sigma^+ \pi^0, \Xi_c^0 A \rightarrow \Sigma^0 \overline{K}^0$ , and  $\Omega_c \rightarrow \Xi^0 \overline{K}^0$ . Hence, even a measurement of the sign of  $\alpha$  in these decay modes will be helpful to discern current-algebra and other theoretical models.

(iii) Nonfactorizable *W*-exchange effects are not negligible; they are comparable to the factorizable ones in the decays  $\Xi_c^{+A} \rightarrow \Sigma^+ \overline{K}^0, \Xi^0 \pi^+, \Xi_c^{0A} \rightarrow \Sigma^0 \overline{K}^0$ , and even dominate in the reactions  $\Xi_c^{0A} \rightarrow \Lambda \overline{K}^0$  and  $\Omega_c \rightarrow \Xi^0 \overline{K}^0$ .

(iv) Form factors  $f_1$  and  $g_1$  evaluated by the static bag or quark model are interpreted as the predictions obtained at maximum  $q^2$  where both baryons are static. Consequently, form factors become smaller at  $q^2=0$  than previously expected. The decay rate of  $\Lambda_c^+ \rightarrow \Lambda \pi^+$ , which was largely overestimated before, is now significantly reduced. (v) The decays  $\Lambda_c^+ \to \Xi^0 K^+$  and  $\Xi_c^{0A} \to \Sigma^+ K^-$  receive dominant contributions from nonfactorizable p waves. Owing to a large cancellation in the pole amplitude, we cannot make reliable predictions on their decay rates and asymmetry parameters. An effort to resolve this problem is urgently needed.

Note added. After the paper was submitted for publication, we learned that the decay asymmetry parameter  $\alpha$  in the decay  $\Lambda_c^+ \rightarrow \Sigma^+ \pi^0$  has been measured by CLEO and a result of  $\alpha = -0.43 \pm 0.23 \pm 0.20$  was reported [28]. At first glance, this experimental result seems to agree with the prediction of current algebra, but is in disagreement in sign and magnitude with most recent theoretical calculations (see Table III). However, a more detailed analysis gives a different theoretical implication. Using the CLEO result for the decay rate [29]

$$\Gamma(\Lambda_c^+ \to \Sigma^+ \pi^0) = (0.33 \pm 0.08 \pm 0.07) \times 10^{11} \text{ s}^{-1}$$
 (N1)

together with the measured value of  $\alpha$ , we find two solutions for s and p waves (only central values are quoted):

(i) A = 1.06, B = -15.21, (ii) A = 4.73, B = -3.42, (N2)

in units of  $G_F V_{cs} V_{ud} 10^{-2} \text{ GeV}^2$ , where the relative sign is fixed by  $\alpha$ . For a comparison, theoretical predictions are given by (see Tables I and II)

pole model : A = -2.20 , B = -14.63 , current algebra : A = 7.66 , B = -6.42 . (N3)

It is evident that the current-algebra prediction is inconsistent with both of the experimental values given in (N2) since it predicts a large decay rate  $\Gamma(\Lambda_c^+ \rightarrow \Sigma^+ \pi^0) = 0.88 \times 10^{11} \text{ s}^{-1}$ . In order to pin down

 $\Gamma(\Lambda_c^+ \to \Sigma^+ \pi^0) = 0.88 \times 10^{11} \text{ s}^{-1}$ . In order to pin down which solution of (N2) is chosen by nature, experimentally one has to measure another asymmetry parameter:

$$\gamma = \frac{(E_f + M_f) |A|^2 - (E_f - M_f) |B|^2}{(E_f + M_f) |A|^2 + (E_f - M_f) |B|^2} , \qquad (N4)$$

where  $E_f(M_f)$  is the energy (mass) of the daughter baryon in the rest frame of the charmed baryon. Unfortunately, a measurement of  $\gamma$ , which characterizes the transverse polarization of the daughter baryon, is difficult at present since it requires the charmed baryon be polarized. Theoretically, we believe that the real experimental value is that of the first set in (N2). Clearly, the p-wave amplitude predicted by the pole model is in excellent agreement with "experiment." In the pole model, the swave amplitude receives important on-shell corrections as the pion is no longer soft [see Eq. (2.14)]. The on-shell effect is calculated in the MIT bag model to be  $A_{\text{pole}} - A_{\text{CA}} = -2.20 - 7.66 = 9.86$ . The experimental value of A = 1.06, when compared to the soft-pion prediction  $A_{CA} = 7.66$ , clearly indicates that on-shell effects are indeed destructive but not large enough to change the sign of the PV amplitudes. We conclude that (i) the agreement of current algebra with experiment for the decay asymmetry parameter  $\alpha$  is somewhat an accident because its predicted decay rate is too large, and (ii) if the first set of (N2) is proved in the future to be the true experimental result, this will imply that the pole model is indeed in the right ballpark for both s and p waves. A large and destructive on-shell effect is inevitable for understanding the PV amplitudes. The moral we learned from the above analysis is that the present bag-model calculation seems to overestimate the on-shell corrections. In retrospect, this is perhaps not surprising since it is known that the bag model is not successful in reproducing the quantitative features of the observed spectrum of low-lying negative-parity  $\frac{1}{2}^{-}$  baryon resonances [26]. Therefore, a better understanding of the  $\frac{1}{2}^{-}$  states, which are responsible for s waves, is called for.

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## APPENDIX A: BARYON WAVE FUNCTION

To fix the relative sign of the coupling constants, form factors, parity-conserving, and parity-violating matrix elements, it is very important to employ the baryon wave functions consistently. In the present paper, we use the isospin baryon-pseudoscalar coupling convention given in Ref. [25] (see Appendix C) to fix the sign of the groundstate  $\frac{1}{2}^+$  octet baryon wave functions. In the following, we list those wave functions relevant to our purposes:

$$\begin{split} p &= \frac{1}{\sqrt{3}} \left[ uud\chi_s + (13) + (23) \right], \\ \Sigma^+ &= -\frac{1}{\sqrt{3}} \left[ uus\chi_s + (13) + (23) \right], \\ \Sigma^0 &= \frac{1}{\sqrt{6}} \left[ (uds + dus)\chi_s + (13) + (23) \right], \\ \Lambda^0 &= -\frac{1}{\sqrt{6}} \left[ (uds - dus)\chi_A + (13) + (23) \right], \\ \Xi^0 &= \frac{1}{\sqrt{3}} \left[ ssu\chi_s + (13) + (23) \right], \\ \Xi^- &= \frac{1}{\sqrt{3}} \left[ ssd\chi_s + (13) + (23) \right], \\ \Lambda_c^+ &= -\frac{1}{\sqrt{6}} \left[ (udc - duc)\chi_A + (13) + (23) \right], \\ \Sigma_c^+ &= \frac{1}{\sqrt{6}} \left[ (udc + duc)\chi_s + (13) + (23) \right], \\ \Sigma_c^0 &= \frac{1}{\sqrt{3}} \left[ ddc\chi_s + (13) + (23) \right], \\ \Xi_c^{0A} &= \frac{1}{\sqrt{6}} \left[ (dsc - sdc)\chi_A + (13) + (23) \right], \\ \Xi_c^{0S} &= \frac{1}{\sqrt{6}} \left[ (dsc - sdc)\chi_s + (13) + (23) \right], \\ \Xi_c^{-A} &= \frac{1}{\sqrt{6}} \left[ (usc - suc)\chi_A + (13) + (23) \right], \\ \Xi_c^+ &= \frac{1}{\sqrt{6}} \left[ (usc - suc)\chi_s + (13) + (23) \right], \\ \Xi_c^+ &= \frac{1}{\sqrt{6}} \left[ (usc + suc)\chi_s + (13) + (23) \right], \\ \Omega_c^0 &= \frac{1}{\sqrt{3}} \left[ ssc\chi_s + (13) + (23) \right], \end{split}$$

where  $abc\chi_s = (2a^{\uparrow}b^{\uparrow}c^{\downarrow} - a^{\uparrow}b^{\downarrow}c^{\uparrow} - a^{\downarrow}b^{\uparrow}c^{\uparrow})/\sqrt{6}$  and  $abc\chi_A = (a^{\uparrow}b^{\downarrow}c^{\uparrow} - a^{\downarrow}b^{\uparrow}c^{\uparrow})/\sqrt{2}$ , and the superscripts A and S indicate antitriplet and sextet charmed baryons, respectively.

The low-lying negative-parity  $\frac{1}{2}^{-}$  noncharmed baryons belong to the (70, L = 1) multiplet in the flavor-spin SU(6) basis, which can be decomposed into SU(3) multiplets as

$$|70, L = 1\rangle = |70, {}^{2}8_{1/2}\rangle \oplus |70, {}^{4}8_{1/2}\rangle$$
$$\oplus |70, {}^{2}10_{1/2}\rangle \oplus |70, {}^{2}1_{1/2}\rangle , \qquad (A2)$$

where the superscript and subscript denote the quantum numbers 2S + 1 and J, respectively. In the MIT bag model these states are made of two quarks in the ground  $1S_{1/2}$  state and one quark excited to  $1P_{1/2}$  or  $1P_{3/2}$ . That is, the SU(6) (70, L = 1) states can be constructed from the  $|\mathbf{8}, P_{1/2}\rangle_a$ ,  $|\mathbf{8}, P_{1/2}\rangle_b$ ,  $|\mathbf{8}, P_{3/2}\rangle$ ,  $|\mathbf{10}, P_{1/2}\rangle$ , and configurations  $|10, P_{3/2}\rangle$ [26], where  $P_{1/2} \equiv (1S_{1/2})^2 1P_{1/2}$ ,  $P_{3/2} \equiv (1S_{1/2})^2 1P_{3/2}$ . The explicit wave functions for the  $\frac{1}{2}^-$  resonances of  $\Sigma^+$  are given by Eq. (A8) of Ref. [14]. The wave functions for the lowlying negative-parity states of the octet baryons can be easily obtained from that of  $\Sigma^+(\frac{1}{2})$  by an appropriate replacement of quarks. For example, the  $\Xi^{0}(\frac{1}{2}^{-})$  wave functions may be obtained from  $\Sigma^+(\frac{1}{2}^-)$  wave functions by the substitution  $u \leftrightarrow s$ .

As for the charmed baryons, the charmed quark in the low-lying  $\frac{1}{2}^{-}$  state does not get excited, while the two light quarks of the charmed baryons are either in the symmetric sextet or antisymmetric antitriplet state in the SU(3) flavor space.<sup>9</sup> The wave function of the  $\frac{1}{2}^{-}$  sextet charmed baryon, say  $\Sigma_c^0(\frac{1}{2}^{-})$ , is simply given by

$$\begin{split} |^{2}\mathbf{6}_{1/2}, \frac{1}{2}^{-}\rangle &= -\frac{\sqrt{8}}{3} |\mathbf{6}, P_{3/2}\rangle - \frac{1}{3} |\mathbf{6}, P_{1/2}\rangle , \qquad (A3) \\ \mathbf{\Sigma}_{c}^{0}(\mathbf{6}, P_{1/2}) &= \frac{1}{6} \{ 2(\tilde{a}^{\downarrow} d^{\uparrow} c^{\uparrow} + d^{\uparrow} \tilde{a}^{\downarrow} c^{\uparrow}) - \tilde{a}^{\uparrow} d^{\downarrow} c^{\uparrow} \\ &- \tilde{a}^{\uparrow} d^{\uparrow} c^{\downarrow} - d^{\uparrow} \tilde{a}^{\uparrow} c^{\downarrow} - d^{\downarrow} \tilde{a}^{\uparrow} c^{\uparrow} \\ &+ (13) + (23) \} , \qquad (A4) \\ \mathbf{\Sigma}_{c}^{0}(\mathbf{6}, P_{3/2}) &= \frac{1}{6} \{ \sqrt{3} (d^{\downarrow} \underline{d}^{\uparrow} c^{\downarrow} + \underline{d}^{\uparrow} d^{\downarrow} c^{\downarrow}) - d^{\uparrow} \underline{d}^{\uparrow} c^{\downarrow} \\ &- d^{\downarrow} \underline{d}^{\uparrow} c^{\uparrow} + d^{\uparrow} \underline{d}^{\downarrow} c^{\uparrow} - \underline{d}^{\uparrow} d^{\uparrow} c^{\downarrow} \\ &- d^{\downarrow} \underline{d}^{\uparrow} c^{\uparrow} + d^{\uparrow} \underline{d}^{\downarrow} c^{\uparrow} - \underline{d}^{\uparrow} d^{\uparrow} c^{\downarrow} \\ &- \underline{d}^{\uparrow} d^{\downarrow} c^{\uparrow} + \underline{d}^{\downarrow} d^{\uparrow} c^{\uparrow} + (13) + (23) \} , \end{split}$$

where the  $1P_{1/2}$   $(1P_{3/2})$  quark is denoted by a tilde (undertilde), the  $s_z = \frac{3}{2}$  quark state is remarked by  $\underline{q}^{\uparrow}$ , and (ij) means permutation for the quark in place *i* with the quark in place *j*. The low-lying  $\frac{1}{2}^{-}$  resonance of the anti-triplet charmed baryon, e.g.,  $\Xi_c^{0.4}(\frac{1}{2}^{-})$  has the form

<sup>&</sup>lt;sup>9</sup>The SU(3) representation of charmed baryons given in Ref. [14] is erroneous.

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$$\Xi_{c}^{0}({}^{2}\overline{\mathbf{3}}_{1/2}, {}^{\frac{1}{2}^{-}}) = \Xi_{c}^{0}(\overline{\mathbf{3}}, {}^{\frac{1}{2}^{-}}, \boldsymbol{P}_{1/2})$$

$$= \frac{1}{2\sqrt{6}} \left\{ d^{\dagger}\overline{s}^{\dagger}c^{\downarrow} - d^{\downarrow}\overline{s}^{\dagger}c^{\uparrow} + \widetilde{d}^{\dagger}s^{\downarrow}c^{\uparrow} - \widetilde{d}^{\dagger}s^{\dagger}c^{\downarrow} - s^{\dagger}\widetilde{d}^{\dagger}c^{\downarrow} + s^{\downarrow}\widetilde{d}^{\dagger}c^{\uparrow} - \widetilde{s}^{\dagger}d^{\downarrow}c^{\uparrow} + \widetilde{s}^{\dagger}d^{\uparrow}c^{\downarrow} + (13) + (23) \right\}. \quad (A5)$$

The explicit spatial wave functions of the quark states  $1S_{1/2}$ ,  $1P_{1/2}$ , and  $1P_{3/2}$  are given in Appendix A of Ref. [14].

### **APPENDIX B: PARITY-CONSERVING AND -VIOLATING MATRIX ELEMENTS**

Since the evaluation of the parity-conserving (PC) and parity-violating (PV) matrix elements in the MIT bag model is already elaborated on in detail in Appendix B of Ref. [14], here we just summarize the matrix elements relevant to the present paper. The PC matrix elements are found to  $be^{10}$ 

$$\langle \Sigma^{+} | O_{-}^{PC} | \Lambda_{c}^{+} \rangle = \langle \Lambda^{0} | O_{-}^{PC} | \Sigma_{c}^{0} \rangle = -\frac{4}{\sqrt{6}} (X_{1} + 3X_{2}) (4\pi) ,$$

$$\langle \Sigma^{+} | O_{-}^{PC} | \Sigma_{c}^{+} \rangle = -\langle \Sigma^{0} | O_{-}^{PC} | \Sigma_{c}^{0} \rangle$$

$$= \frac{2\sqrt{2}}{3} (-X_{1} + 9X_{2}) (4\pi) ,$$
(B1)

$$\langle \Xi^{0} | O_{-}^{PC} | \Xi_{c}^{0A} \rangle = \frac{4}{\sqrt{6}} (X_{1} - 3X_{2})(4\pi) ,$$
  
$$\langle \Xi^{0} | O_{-}^{PC} | \Xi_{c}^{0S} \rangle = -\frac{4}{3\sqrt{2}} (X_{1} + 9X_{2})(4\pi) ,$$

where  $X_1$  and  $X_2$  are the four-quark overlap bag integrals defined by Eq. (B3) of Ref. [14].

The evaluation of the parity-violating matrix elements for  $\frac{1}{2}^+ - \frac{1}{2}^-$  transitions is much more involved because the physical  $\frac{1}{2}^-$  baryon states are linear combinations of  $(S_{1/2})^2 P_{1/2}$  and  $(S_{1/2})^2 P_{3/2}$  quark eigenstates. Consequently, the number of the related bag overlap integrals is largely increased. The relevant PV matrix elements for our purposes are

$$\begin{split} \langle \Xi^{0}(\mathbf{8}, P_{1/2})_{a} | O_{-}^{\text{PV}} | \Xi_{c}^{0.4} \rangle \\ &= i 2 \sqrt{2} (4\pi) (\frac{1}{3} \tilde{X}_{1} + \tilde{X}_{2} + \frac{1}{3} \tilde{X}_{1s} - \tilde{X}_{2s}) , \\ \langle \Xi^{0}(\mathbf{8}, P_{1/2})_{b} | O_{-}^{\text{PV}} | \Xi_{c}^{0.4} \rangle = i 2 \sqrt{2} (4\pi) (\frac{1}{3} \tilde{X}_{1} + \tilde{X}_{2} - \frac{2}{3} \tilde{X}_{1s}) , \\ \langle \Xi^{0}(\mathbf{10}, P_{1/2}) | O_{-}^{\text{PV}} | \Xi_{c}^{0.4} \rangle \qquad (B2) \\ &= i 2 \sqrt{2} (4\pi) (\frac{1}{3} \tilde{X}_{1} + \tilde{X}_{2} + \frac{1}{3} \tilde{X}_{1s} + \tilde{X}_{2s}) , \end{split}$$

$$\langle \Xi^{0}(\mathbf{8}, P_{3/2}) | O_{-}^{PV} | \Xi_{c}^{0A} \rangle = i \frac{8}{9} \sqrt{2\pi} (2\underline{X}_{1} + \underline{X}_{1s}) ,$$
  
$$\langle \Xi^{0}(\mathbf{10}, P_{3/2}) | O_{-}^{PV} | \Xi_{c}^{0A} \rangle = i \frac{8}{9} \sqrt{4\pi} (\underline{X}_{1} - \underline{X}_{1s}) ,$$

for  $\Xi^{0}(\frac{1}{2}^{-}) - \Xi_{c}^{0.4}$  transitions,  $\langle \Xi^{0}(\mathbf{8}, P_{1/2})_{a} | O_{-}^{PV} | \Xi_{c}^{0.S} \rangle$   $= i \frac{4}{\sqrt{6}} (4\pi) (\frac{1}{3} \tilde{X}_{1} - 3 \tilde{X}_{2} - \frac{1}{3} \tilde{X}_{1s} - 3 \tilde{X}_{2s}) ,$   $\langle \Xi^{0}(\mathbf{8}, P_{1/2})_{b} | O_{-}^{PV} | \Xi_{c}^{0.S} \rangle = i \frac{4}{\sqrt{6}} (4\pi) (\frac{1}{3} \tilde{X}_{1} - 3 \tilde{X}_{2} + \frac{2}{3} \tilde{X}_{1s}) ,$   $\langle \Xi^{0}(\mathbf{10}, P_{1/2}) | O_{-}^{PV} | \Xi_{c}^{0.S} \rangle$  (B3)  $= i \frac{4}{-} (4\pi) (\frac{1}{2} \tilde{X}_{1} - 3 \tilde{X}_{2} - \frac{1}{2} \tilde{X}_{1s} + 3 \tilde{X}_{2s}) ,$ 

$$\langle \Xi^{0}(\mathbf{8}, P_{3/2}) | O_{-}^{PV} | \Xi_{c}^{0S} \rangle = i \frac{8}{9\sqrt{3}} \sqrt{2\pi} (-2\underline{X}_{1} - \underline{X}_{1s}) ,$$
  
$$\langle \Xi^{0}(\mathbf{10}, P_{3/2}) | O_{-}^{PV} | \Xi_{c}^{0S} \rangle = i \frac{8}{9\sqrt{3}} \sqrt{4\pi} (-\underline{X}_{1} + \underline{X}_{1s}) ,$$

for  $\Xi^{0}(\frac{1}{2}^{-}) - \Xi_{c}^{0S}$  transitions,

$$\langle \Sigma_{c}^{0}(\mathbf{6}, P_{1/2}) | O_{-}^{PV} | \Lambda^{0} \rangle = i 2 \sqrt{3} (4\pi) (\frac{1}{3} \tilde{X}_{1}' + \tilde{X}_{2}') ,$$

$$\langle \Sigma_{c}^{0}(\mathbf{6}, P_{3/2}) | O_{-}^{PV} | \Lambda^{0} \rangle = i \frac{4 \sqrt{6}}{9} \sqrt{4\pi} (-\tilde{X}_{1}') ,$$

$$\langle \Sigma_{c}^{0}(\mathbf{6}, P_{1/2}) | O_{-}^{PV} | \Sigma^{0} \rangle = i 2 (4\pi) (\frac{1}{3} \tilde{X}_{1}' - 3 \tilde{X}_{2}') ,$$

$$\langle \Sigma_{c}^{0}(\mathbf{6}, P_{3/2}) | O_{-}^{PV} | \Sigma^{0} \rangle = i \frac{8}{9} \sqrt{2\pi} (-\tilde{X}_{1}') ,$$

$$\langle \Sigma_{c}^{+}(1/2^{-}) | O_{-}^{PV} | \Sigma^{+} \rangle = - \langle \Sigma_{c}^{0}(1/2^{-}) | O_{-}^{PV} | \Sigma^{0} \rangle ,$$

$$\text{for } \Sigma_{c}^{0} (\frac{1}{2}^{-}) - \Lambda^{0} \text{ and } \Sigma_{c} (\frac{1}{2}^{-}) - \Sigma \text{ transitions, and}$$

$$\langle \Xi_{c}^{0}(\mathbf{6}, P_{1/2}) | O_{-}^{PV} | \Xi^{0} \rangle = i 2 (4\pi) (\frac{1}{3} \tilde{X}_{1}' - 3 \tilde{X}_{2}') ,$$

$$\langle \Xi_{c}^{0}(\mathbf{6}, P_{3/2}) | O_{-}^{PV} | \Xi^{0} \rangle = i \frac{8}{9} \sqrt{2\pi} (-\tilde{X}_{1}') ,$$

$$(B5)$$

$$\langle \Xi_c^0(\bar{\mathbf{3}}, P_{1/2}) | O_{-}^{\mathrm{PV}} | \Xi^0 \rangle = i 2 \sqrt{3} (4\pi) (\frac{1}{3} \chi_1' - \chi_2') ,$$

for  $\Xi_c^0(\frac{1}{2}^-) - \Xi^0$  transitions, where the bag integrals  $\tilde{X}_1, \tilde{X}_2, \tilde{X}_{1s}, \tilde{X}_{2s}, \tilde{X}_1, \tilde{X}_{1s}, \tilde{X}'_i, \tilde{X}'_i$  are defined in Appendix B of Ref. [14]. The PV matrix elements for  $\Sigma^+(\frac{1}{2}^-) - \Lambda_c^+$  transitions can be found in Appendix C of Ref. [14].

## APPENDIX C: STRONG-COUPLING CONSTANTS

The octet baryon-pseudoscalar meson BBP coupling constants can be reliably evaluated using the method of Ref. [21] which employs the null result of Coleman and Glashow for the tadpole-type symmetry breaking. In addition to (D1) of Ref. [14], the results related to the present paper are

 $<sup>^{10}</sup>$ Note that there is a sign misprint in Eq. (B4) of Ref. [14] which is corrected here in Eq. (B1).

where the sign of the coupling constants is fixed by the isospin coupling convention given in Ref. [25]. As shown in Ref. [21], the above  $g_{B'BP}$  couplings are in good agreement with experiment. The quantity of interest in the approach of current algebra is  $g_{B'B}^A$ , the axial-vector form factor at  $q^2=0$ , which is related to the *BBP* coupling constant via the Goldberger-Treiman (GT) relation

$$g_{B'BP^a} = \sqrt{2} \frac{m_{B'} + m_B}{f_{P^a}} g^A_{B'B} ,$$
 (C2)

where  $f_{P^a}$  is the decay constant of the pseudoscalar meson  $P^a$   $(a=1,\ldots,8)$  in the SU(3) representation. Note that the axial-vector current corresponding to, for example,  $P^3$ , is  $\frac{1}{2}(\bar{u}\gamma_{\mu}\gamma_5 u - \bar{d}\gamma_{\mu}\gamma_5 d)$ . In the bag model the axial form factor in static limit is given by

$$g_{B'B}^{A} = \langle B' \uparrow | b_{q1}^{\dagger} b_{q2} \sigma_{z} | B \uparrow \rangle \int d^{3} r (u_{q1} u_{q2} - \frac{1}{3} v_{q1} v_{q2}) ,$$
(C3)

where u(r) and v(r) are, respectively, the large and small components of the wave function for the quark state  $1P_{1/2}$ . We find

$$g_{\Sigma^{+}p}^{A} = \frac{1}{5}g_{\Sigma^{+}\Xi^{0}}^{A} = -\frac{\sqrt{2}}{5}g_{\Sigma^{0}\Xi^{0}}^{A} = \sqrt{2/3}g_{\Xi^{0}\Lambda^{0}}^{A} = \frac{4\pi}{3}Z_{1} ,$$

$$g_{\Xi^{0}\Xi^{-}}^{A} = 2g_{\Xi^{0}\Xi^{0}}^{A} = -\frac{1}{\sqrt{6}}g_{\Sigma^{+}\Lambda^{0}}^{A}$$

$$= \frac{1}{2\sqrt{2}}g_{\Sigma^{+}\Sigma^{0}}^{A} = -\frac{1}{2}g_{\Sigma^{+}\Sigma^{+}}^{A} = -\frac{4\pi}{3}Z_{2} ,$$
(C4)

where

$$Z_{1} = \int r^{2} dr (u_{u}^{2} - \frac{1}{3}v_{u}^{2}) ,$$
  

$$Z_{2} = \int r^{2} dr (u_{u}u_{s} - \frac{1}{3}v_{u}v_{s}) .$$
(C5)

As for charmed-baryon-pseudoscalar  $B_c B_c P$  coupling, we will rely on the GT relation (C2). The results are

$$g_{\Xi_{c}^{+}\Xi_{c}^{0s}}^{A} = -2g_{\Xi_{c}^{0}A\Xi_{c}^{0s}} = -\frac{1}{\sqrt{2}}g_{\Sigma_{c}^{+}A_{c}^{+}}^{A} = -\frac{1}{\sqrt{2}}g_{A_{c}^{+}\Sigma_{c}^{0}}^{A} = -\frac{4\pi}{\sqrt{3}}Z_{1},$$

$$g_{\Xi_{c}^{+}\Sigma_{c}^{+}}^{A} = -g_{\Xi_{c}^{0}A\Sigma_{c}^{+}}^{A} = -g_{\Xi_{c}^{0}A_{c}^{+}}^{A} = \frac{1}{\sqrt{2}}g_{\Xi_{c}^{0}A\Omega_{c}^{0}}^{A} = -\frac{1}{\sqrt{2}}g_{\Xi_{c}^{0}A\Sigma_{c}^{0}}^{A} = -\frac{\sqrt{3}}{2\sqrt{2}}g_{\Xi_{c}^{0}\Omega_{c}^{0}}^{A} = -\frac{4\pi}{\sqrt{3}}Z_{2},$$
(C6)

and

$$g_{B_{\overline{3}}B_{\overline{3}}}^{A} = 0$$
 for  $B_{\overline{3}} = \Lambda_{c}^{+}, \Xi_{c}^{0A}, \Xi_{c}^{+A}$ . (C7)

Interestingly, Eq. (C7) is a rigorous and modelindependent statement in the infinite charmed-quark mass limit. This comes from the fact that the light diquark in the  $\overline{3}$  multiplet has spin parity  $0^+$  and that the pseudoscalar meson is emitted solely from the light quarks in the heavy-quark limit. Since the transition  $0^+ \rightarrow 0^+ + P$  does not conserve parity, it leads to vanishing  $B_{\overline{3}}B_{\overline{3}}P$  coupling.

To evaluate the s-wave amplitudes we also need to know the  $B^*BP$  coupling constants ( $B^*$ :  $\frac{1}{2}^-$  resonance). We shall use the generalized GT relation

$$g_{B^*BP^a} = \sqrt{2} \frac{m_{B^*} - m_B}{f_{Pa}} g_B^A *_B , \qquad (C8)$$

to estimate the couplings  $g_{B^*BP^a}$ . Note that  $g_{BB^*P} = g_{B^*BP}$ , while  $g_{BB^*}^A = -g_{B^*B}^A$ . For an evaluation of  $g_{B^*BP}$  in the bag model, see Appendix D of Ref. [14].

### **APPENDIX D: FORM FACTORS**

To evaluate the factorizable amplitudes of baryon weak decays requires the information on the form factors  $f_1$  and  $g_1$  defined by

$$\langle B_f | V_{\mu} - A_{\mu} | B_i \rangle = \overline{u}_f(p_f) [f_1 \gamma_{\mu} + i f_2 \sigma_{\mu\nu} q^{\nu} + f_3 q_{\mu} - g_1 \gamma_{\mu} \gamma_5 - i g_2 \sigma_{\mu\nu} q^{\nu} \gamma_5 - g_3 q_{\mu} \gamma_5] u_i(p_i)$$
(D1)

with  $q_{\mu} = (p_i - p_f)_{\mu}$ . In the static limit  $f_1$  and  $g_1$  are derived in the bag model to be

$$f_{1}^{B_{f}B_{i}} = \langle B_{f} \uparrow | b_{q1}^{\dagger} b_{q2} | B_{i} \uparrow \rangle \int d^{3}r(u_{q1}u_{q2} + v_{q1}v_{q2})$$
(D2)

and Eq. (C3). However, contrary to the conventional interpretation, (D2) and (C3) should be regarded as the bag-model predictions obtained at maximum fourmomentum transfer squared, i.e.,  $q^2 = (m_i - m_f)^2$ . This is because the static-bag wave functions best resemble hadronic states in the frame where both baryons are *static*. This can be achieved by choosing the Breit frame where  $\mathbf{p}_i = \mathbf{p}_f = \mathbf{q}/2 = 0$ .

For definiteness, we will assume a dipole  $q^2$  dependence for the form factors

$$f_1(q^2) = \frac{f_1(0)}{(1-q^2/m_V^2)^2}, \ g_1(q^2) = \frac{g_1(0)}{(1-q^2/m_A^2)^2}, \ (D3)$$

where the pole masses are  $m_V(1^-)=2.01$  GeV,  $m_A(1^+)=2.42$  GeV for the pole with the quark content (cd) [22], and  $m_V(1^-)=2.11$  GeV and  $m_A(1^+)=2.54$  GeV for the pole with the  $(c\bar{s})$  quark content. We find at  $q^2=q_{\max}^2=(m_i-m_f)^2$  that

 $g_{\pi^0\pi^0\pi^0} = -4.3$ ,

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$$f_{1}^{\Lambda_{c}^{+}\Lambda} = (\frac{2}{3})^{1/2} f_{1}^{\Xi_{c}^{0}A\Xi^{-}} = -(\frac{2}{3})^{1/2} f_{1}^{\Xi_{c}^{+}A\Xi^{0}} = \int d^{3}r(u_{s}u_{c} + v_{s}v_{c}) ,$$

$$f_{1}^{\Omega_{c}^{0}\Xi^{0}} = -(\frac{2}{3})^{1/2} f_{1}^{\Lambda_{c}^{+}p} = -(\frac{2}{3})^{1/2} f_{1}^{\Xi_{c}^{+}A\Sigma^{+}} = \frac{2}{\sqrt{3}} f_{1}^{\Xi_{c}^{0}A\Sigma^{0}} = 2f_{1}^{\Xi_{c}^{0}A} = \int d^{3}r(u_{u}u_{c} + v_{u}v_{c}) ,$$
(D4)

$$g_{1}^{\Lambda_{c}^{+}\Lambda} = (\frac{2}{3})^{1/2} g_{1}^{\Xi_{c}^{0}\Lambda\Xi^{-}} = -(\frac{2}{3})^{1/2} g_{1}^{\Xi_{c}^{+}\Lambda\Xi^{0}} = \int d^{3}r(u_{s}u_{c} - \frac{1}{3}v_{s}v_{c}) ,$$
  
$$-3g_{1}^{\Omega_{c}^{0}\Xi^{0}} = -(\frac{2}{3})^{1/2} g_{1}^{\Lambda_{c}^{+}P} = -(\frac{2}{3})^{1/2} g_{1}^{\Xi_{c}^{+}\Lambda\Sigma^{+}} = \frac{2}{\sqrt{3}} g_{1}^{\Xi_{c}^{0}\Lambda\Sigma^{0}} = 2g_{1}^{\Xi_{c}^{0}\Lambda} = \int d^{3}r(u_{u}u_{c} - \frac{1}{3}v_{u}v_{c}) .$$

With the overlap bag integrals given by Eq. (3.4) it is straightforward to check that our numerical results for form factors extrapolated to  $q^2=0$  are in agreement with Table VI of Ref. [27] for  $\Lambda_c^+ \rightarrow \Lambda$ ,  $\Xi_c^{+A} \rightarrow \Xi^0$ , and  $\Xi_c^{0A} \rightarrow \Xi^-$  transitions. Form factors induced by the  $c \rightarrow u$ current are not given in Ref. [27]. If (D2) and (C3) were interpreted as bag predictions at  $q^2=0$ , the calculated branching ratio of the exclusive  $\Lambda_c^+ \rightarrow \Lambda$  decay would have been enhanced by a factor of 3.5, which is in violent disagreement with experiment [14]. This is another indication that the static-bag calculation of form factors is indeed carried out at maximum  $q^2$  rather than at  $q^2=0$ .

### APPENDIX E: CURRENT-ALGEBRA COMMUTATOR TERMS

In current algebra the nonfactorizable s-wave amplitude of the decay  $B_c \rightarrow B + P^a$  in the soft-meson limit is governed by the commutator term

$$A^{\rm CA} = -\frac{\sqrt{2}}{f_{P^a}} \langle B | [Q_5^a, \mathcal{H}^{\rm PV}] | B_c \rangle , \qquad (E1)$$

where  $f_{\pi} = 132$  MeV and  $f_{\underline{K}} = 1.22 f_{\pi}$ . As an example, consider the decay  $\Xi_c^{0A} \rightarrow \Lambda \overline{K}^0$ ,

$$A^{CA}(\Xi_c^{0A} \to \Lambda \overline{K}^0) = -\frac{1}{f_K} \langle \Lambda | Q^{K^0} \mathcal{H}^{PC} | \Xi_c^{0A} \rangle . \quad (E2)$$

From Eq. (A1) we obtain  $\langle \Lambda | Q^{K^0} = (\frac{3}{2})^{1/2} \langle \Xi^0 |$  and hence

$$A^{\operatorname{CA}}(\Xi_c^{0A} \to \Lambda \overline{K}^0) = -(\frac{3}{2})^{1/2} \frac{1}{f_K} \langle \Xi^0 | \mathcal{H}^{\operatorname{PC}} | \Xi_c^{0A} \rangle .$$
(E3)

The remaining *s*-wave commutator terms are summarized below:

$$\begin{split} A^{\rm CA}(\Lambda_c^+ \to p\overline{K}^0) &= \frac{1}{f_K} \left\langle \Sigma^+ |\mathcal{H}^{\rm PC}|\Lambda_c^+ \right\rangle ,\\ A^{\rm CA}(\Lambda_c^+ \to \Lambda \pi^+) &= 0 ,\\ A^{\rm CA}(\Lambda_c^+ \to \Sigma^0 \pi^+) &= -A^{\rm CA}(\Lambda_c^+ \to \Sigma^+ \pi^0) \\ &= \frac{\sqrt{2}}{f_\pi} \left\langle \Sigma^+ |\mathcal{H}^{\rm PC}|\Lambda_c^+ \right\rangle ,\\ A^{\rm CA}(\Lambda_c^+ \to \Xi^0 K^+) &= -\frac{1}{f_K} \left\langle \Sigma^+ |\mathcal{H}^{\rm PC}|\Lambda_c^+ \right\rangle \\ &\quad + \frac{1}{f_K} \left\langle \Xi^0 |\mathcal{H}^{\rm PC}| \Xi_c^{0A} \right\rangle ,\\ A^{\rm CA}(\Xi_c^{+A} \to \Xi^0 \pi^+) &= \frac{1}{f_\pi} \left\langle \Xi^0 |\mathcal{H}^{\rm PC}| \Xi_c^{0A} \right\rangle ,\\ A^{\rm CA}(\Xi_c^{+A} \to \Sigma^+ \overline{K}^0) &= -\frac{1}{f_K} \left\langle \Sigma^+ |\mathcal{H}^{\rm PC}| \Lambda_c^+ \right\rangle , \end{split}$$

$$\begin{split} A^{\operatorname{CA}}(\Xi_{c}^{0\,A} \rightarrow \Sigma^{0}\overline{K}^{0}) &= \frac{1}{\sqrt{2}f_{K}} \left\langle \Xi^{0}|\mathcal{H}^{\operatorname{PC}}|\Xi_{c}^{0\,A} \right\rangle ,\\ A^{\operatorname{CA}}(\Xi_{c}^{0\,A} \rightarrow \Sigma^{+}K^{-}) &= -\frac{1}{f_{K}} \left\langle \Xi^{0}|\mathcal{H}^{\operatorname{PC}}|\Xi_{c}^{0\,A} \right\rangle \\ &\quad + \frac{1}{f_{K}} \left\langle \Sigma^{+}|\mathcal{H}^{\operatorname{PC}}|\Lambda_{c}^{+} \right\rangle ,\\ A^{\operatorname{CA}}(\Xi_{c}^{0\,A} \rightarrow \Xi^{0}\pi^{0}) &= -\frac{\sqrt{2}}{f_{\pi}} \left\langle \Xi^{0}|\mathcal{H}^{\operatorname{PC}}|\Xi_{c}^{0\,A} \right\rangle ,\\ A^{\operatorname{CA}}(\Xi_{c}^{0\,A} \rightarrow \Xi^{-}\pi^{+}) &= -\frac{1}{f_{\pi}} \left\langle \Xi^{0}|\mathcal{H}^{\operatorname{PC}}|\Xi_{c}^{0\,A} \right\rangle ,\\ A^{\operatorname{CA}}(\Omega_{c}^{0} \rightarrow \Xi^{0}\overline{K}^{0}) &= \frac{\sqrt{2}}{f_{K}} \left\langle \Xi^{0}|\mathcal{H}^{\operatorname{PC}}|\Xi_{c}^{0\,S} \right\rangle . \end{split}$$

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