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Inclusive nonleptonic decays of B and D mesons

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Inclusive nonleptonic decays of B and D mesons are investigated to leading order and next-to-leading order in $1/m_Q$. QCD corrections to the leading order (m_Q^5) term give the most essential contribution to the observed enhancement. Various inclusive bound-state production processes are dealt with in the factorization approximation. A good semiquantitative understanding of the data has been achieved and the role of the spectator quark for the lifetime difference between D^0 and D^+ is clarified. A search for missing decays in the $b \to c\ \bar{c}\ s$ channel is suggested.

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I. INTRODUCTION

In recent years much progress has been made in the understanding of exclusive weak decay processes, in particular, of the pattern of dominant two-body nonleptonic decays. The strong nonperturbative correlations between quarks which manifest themselves in decay constants and form factors are the decisive elements for the successful semiquantitative description which reaches from strangeness-changing transitions with its $\Delta I = 1/2$ rule to the dominant decays of heavy hadrons [1–3]. However, a rigorous treatment based solely on first principles is still out of reach. Model assumptions are necessary and lead to theoretical errors which are hard to quantify.

Inclusive decays, on the other hand, have always seemed amenable to a more quantitative treatment [4, 5]. For a sufficiently large mass m_Q of the decaying quark one expects only the short distance properties of QCD to be essential. In recent publications [5–7] particular emphasis has been given to a systematic expansion of inclusive decay rates in inverse powers of m_Q. The leading order term goes as m_Q^5 and describes the decay rate of an isolated quark. By going down to next-to-leading order (m_Q^3) the matrix element of the chromomagnetic operator has to be considered. It can be expressed in terms of the hyperfine splitting of the decaying hadron [7, 8]. In B decays, because of the small mass splitting between B^* and B, this amounts only to a 3% correction of the leading contribution. This result indicates that for our present aim in accuracy, even higher order corrections which start at m_Q^{-3} relative to the leading part are not of importance. On the other hand, the contribution from the chromomagnetic operator alone is too small to provide for a sizable cancellation of the (negative) $1/N_c$ term (N_c is the number of quark colors) of the leading contribution. A full cancellation was hoped for in order to get an increased nonleptonic width and thereby a better agreement with the data [5, 8].

It is the purpose of this paper to analyze nonleptonic decay rates in some detail. In order to avoid dependence on the precise value of the Kobayashi-Maskawa (KM) matrix elements and on the Fermi motion of the heavy quark inside the mesons and to be less sensitive to quark masses, we consider the ratio of nonleptonic to semileptonic rates and take the semileptonic branching ratio from experiment. We find that $\simeq 80\%$ of the enhancement of the nonleptonic B-decay rate is caused by QCD corrections to the leading order quark decay formula. At present, these corrections are only known in the limit of vanishing quark masses. This approximation is, presumably, the reason for the larger part of the remaining discrepancy. Together with a more phenomenological approach which contains the effect of finite quark masses and describes the direct generation of bound states in quark decay processes, we gain an understanding of the total and various partial inclusive rates. The same approach is also employed for the evaluation of inclusive D decays. Even though the $1/m_Q$ expansion is presumably poorly converging in this case we find that the QCD correction of the quark decay formula together with the matrix element of the chromomagnetic operator accounts for most of the huge enhancement factor observed in D^0 decays. In D^+ decays the QCD enhancement is suppressed again by the intriguing influence of the $\bar{d}$ spectator quark—essentially by its pure presence and kinematic role.

II. INCLUSIVE B DECAYS AND THE OPERATOR PRODUCT EXPANSION

The inclusive decay width of a hadron H of four momentum p is given by

$$\Gamma = \frac{1}{m_H} \text{Im}\ T(H \to X \to H) = \frac{1}{2m_H} \langle H|\mathcal{F}|\mathcal{H}\rangle, \qquad (1)$$
$$\mathcal{F} = \int d^4x e^{ipx} \mathcal{H}_{\text{eff}}(x/2)\mathcal{H}_{\text{eff}}(-x/2).$$

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In this formula $\mathcal{H}_{\rm eff}$ denotes the effective Hamiltonian density relevant for the process considered. We are interested in cases where the hadron H contains a heavy quark with mass $m_Q \approx m_H$. In this circumstance an operator product expansion can be applied giving an expansion in terms of decreasing powers of the heavy quark mass m_Q [6, 7]. Because the square of the Fermi constant has dimension -4, the leading term in this expansion goes as m_Q^5. It is proportional to the operator $\bar{Q}Q$ of dimension 3 where Q stands for the field operator of the heavy quark. To zeroth order in α_s one simply recovers the result of free quark decay:

$$\Gamma_0 = \frac{G_F^2}{192\pi^3}|V_{\rm KM}|^2 m_Q^5 z_0 \frac{\langle H|\bar{Q}Q|H\rangle}{2m_H}. \tag{2}$$

Here, $V_{\rm KM}$ stands for the relevant Kobayashi-Maskawa matrix element and z_0 is a phase space factor which goes to 1 for $m_Q \to \infty$ but must be kept in applications. For decays to a quark with mass m_q and two light particles one has

$$z_0 = 1 - 8y + 8y^3 - y^4 - 12y^2 \ln y,$$
$$y = (m_q/m_Q)^2. \tag{3}$$

So far there is no difference between semileptonic and nonleptonic decay rates (apart from a color factor). A sizable difference arises after the effect of perturbative QCD is taken into account.[1] The semileptonic transition rate becomes [9]

$$\Gamma_{\rm sl} = \Gamma_0\left(1 - \frac{2\alpha_s}{3}\pi + \frac{25}{6}\frac{\alpha_s}{\pi}\right). \tag{4}$$

The effective Hamiltonian for nonleptonic transitions changes into a sum of operators. For our applications we can safely neglect Penguin operators and keep only the well-known combinations $\mathcal{O}_\pm$ corresponding to the color antitriplet and color sextet weak scattering channels, respectively. The corresponding scale-dependent Wilson coefficients $c_\pm$ are normalized to one at the W-boson mass. According to [10] one finds,[2] apart from KM factors,

$$\Gamma_{\rm nl}/\Gamma_0 = 2c_+^2\left(1 - \frac{2}{3}\alpha_s\pi + \frac{43}{12}\frac{\alpha_s}{\pi}\right) + c_-^2\left(1 - \frac{2}{3}\alpha_s\pi + \frac{25}{3}\frac{\alpha_s}{\pi}\right) + O\left(\frac{1}{m_Q^2}\right),$$
$$c_\pm(\mu) = c_\pm^{\rm LL}(\mu)\left(1 + \frac{\alpha_s(\mu) - \alpha_s(m_W)}{\pi}\rho_\pm\right), \tag{5}$$

where $c_\pm^{\rm LL}(\mu)$ denotes the leading logarithmic approximation.

At next-to-leading order of the operator product expansion an operator with dimension 5 appears giving rise to order m_Q^3 contributions [8, 7]. It is the chromomagnetic operator

$$\bar{Q}i\sigma GQ \equiv \bar{Q}i\gamma^\mu\gamma^\nu G^a_{\mu\nu}\frac{\lambda^a}{2}Q. \tag{6}$$

Here, $G^a_{\mu\nu}$ denotes the gluon field strength tensor and λ^a describe the Gell-Mann color SU(3) matrices. (The kinetic energy operator $\bar{b}D^2b/2m$ appears in the nonrelativistic expansion of $\bar{b}b$. We need not make this expansion because the same matrix element appears in the nonleptonic and semileptonic decays, and thus cancels in the ratio.) The coefficient multiplying the chromomagnetic operator is different for semileptonic and nonleptonic decays. According to [7] the semileptonic rate is modified by the factor

$$\zeta_{\rm sl} = 1 - \frac{z_1}{z_0}r \quad \text{where } z_1 = (1-y)^4$$

and

$$r = \frac{1}{m_Q^2}\langle H|\bar{Q}i\sigma GQ|H\rangle/\langle H|\bar{Q}Q|H\rangle. \tag{7}$$

The corresponding factor for nonleptonic decays is

$$\zeta_{\rm nl} = 1 - \frac{z_1}{z_0}r + 2\,\frac{z_2}{z_0}\,\frac{c_-^2 - c_+^2}{2c_+^2 + c_-^2}\,r, \tag{8}$$
$$z_2 = (1-y)^3.$$

From Eqs. (4)–(8) we obtain, for the ratio of nonleptonic to semileptonic inclusive rates,

$$\Gamma_{\rm nl}/\Gamma_{\rm sl} = 2c_+^2\left(1 - \frac{7}{12}\frac{\alpha_s}{\pi} - \frac{z_2}{z_0}r\right) + c_-^2\left(1 + \frac{25}{6}\frac{\alpha_s}{\pi} + 2\frac{z_2}{z_0}r\right) + O(1/m_Q^3). \tag{9}$$

Obviously, at this order the semileptonic and the nonleptonic decay rates are still independent of the isospin of the decaying hadron. We also note the important fact that in Eq. (9) the strong m_Q dependence has dropped out.

The matrix element of the chromomagnetic operator is related to the hyperfine splitting in the hadron states. In the case of meson decay this is the splitting between the vector and the pseudoscalar $Q\bar{q}$ bound states. In this case the factor r is given by

$$r = \frac{3}{2}\frac{m_H^2(1^-) - m_H^2(0^-)}{m_Q^2}. \tag{10}$$

Equation (9) can be rewritten by setting $c_+ = c_1 + c_2$ and $c_- = c_1 - c_2$. The coefficient c_1 becomes 1 at $\mu = m_W$ (where it is the factor of the unperturbed nonleptonic Hamiltonian) while $c_2(m_W) = 0$:

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[1] We quote here and in the following the α_s corrections for zero mass particles, because the full ($m_q \neq 0$) corrections are not yet available for nonleptonic decays.

[2] In (4) and (5) α_s and the coefficients $c_\pm$ are taken at the scale m_Q. For a discussion of the scale dependence and the definition of the quantities $\rho_\pm$ see [10].

$$\Gamma_{\rm nl}/\Gamma_{\rm sl} = 3\left\{ (c_1^2+c_2^2)\left(1+\frac{\alpha_s}{\pi}\right) + \frac{2}{3}c_1c_2\left(1-\frac{16}{3}\frac{\alpha_s}{\pi}-4\frac{z_2}{z_0}r\right)\right\} + O(1/m_Q^3). \quad (11)$$

Compared to the often used naive formula

$$\Gamma_{\rm nl}/\Gamma_{\rm sl} = N_c\left(c_1^2+c_2^2+\frac{2}{N_c}c_1c_2\right), \quad (12)$$

there are two important changes: (i) the α_s correction in the first line of (11) arising from the QCD attraction within color singlet currents and (ii) the suppression of the nonleading term in $1/N_c$ in the second line caused by sizably different QCD corrections in the color sectors and by the $1/m_Q^2$ correction described by the factor r. Since c_2/c_1 is negative both (i) and (ii) *enhance* the nonleptonic rate.

In the literature, there is much discussion about the nonleading $1/N_c$ contribution in exclusive and inclusive decays. First, it was observed that a replacement of color singlet quark currents by hadron currents and subsequent factorization leads to a successful semiquantitative description of many exclusive D-decay processes [1]. This was then understood as a cancellation of factorizable and nonfactorizable $1/N_c$ terms [11] which could indeed be shown to occur in QCD sum rule calculations [12]. Since in D decays most of the inclusive decay width is dominated by exclusive two-body decays, it was suggested that the $1/N_c$ terms also be dropped for inclusive decays and thus improving the theoretical D^0 decay rate [5]. More recently in the framework of the $1/m_Q$ expansion, $1/N_c$ terms in the m_Q^3 part of the decay width have been considered [8] for the modification of the $1/N_c$ term in (12).

Equation (11) gives now the answer to these questions for inclusive decays in the framework of perturbative QCD and provides a solid basis for the calculation of the enhancement of nonleptonic decay rates.

For the phase space correction factors in (11) and for later applications we have to fix quark mass values. We use current quark masses which provide reasonable values for the semileptonic B-decay rate using (2) and (4):

$$m_b = 4.8\ \text{GeV}, \quad m_c = 1.35\ \text{GeV}, \quad m_s = 0.15\ \text{GeV}, \quad m_u = m_d = 0. \quad (13)$$

We take $\alpha_s(m_W) = 0.12$ and correspondingly, from two-loop expressions [13],

$$\alpha_s(m_b) = 0.215, \quad c_1(m_b) = 1.13, \quad c_2(m_b) = -0.28. \quad (14)$$

Our main application is to B-meson decays where the $1/m_Q$ expansion should work best. In fact, using the experimental mass splitting $m_B^* - m_B \simeq 46$ MeV for calculating r from (10), the influence of this correction (of order m_Q^2) on Eq. (11) is only 3%. The main difference from Eq. (12) is due to QCD corrections of the leading term. Together with the r term they reduce the $1/N_c$ term by 54%. From the full expression (11) one obtains an enhancement relative to the naive formula (12) by 18%. For the decay channel $b \to c\,\bar{u}\,d$, Eq. (11) predicts

$$\frac{\Gamma_{\rm nl}(\bar{B}\to X_{cud})}{\Gamma_{\rm sl}(\bar{B}\to X_c e^-\bar{\nu})} = 4.05\cos^2\theta_C + O(1/m_Q^3), \quad (15)$$

where θ_C is the Cabibbo angle.

This result is independent of the Kobayashi-Maskawa matrix element V_{cb} and is very little dependent on quark mass ratios. Using the measured semileptonic branching ratio of $10.7 \pm 0.5\%$ the nonleptonic branching ratio to final states with a single c quark is predicted to be[3]

$$B_{\rm nl}(\bar{B}\to X_c) = 43 \pm 2\%. \quad (16)$$

The corresponding experimental number can be extracted from the measured inclusive rates [14] (we neglect systematic errors)

$$\begin{aligned} &B(B\to XD^{\pm}) = 20.2 \pm 1.3\%,\\ &B(B\to XD^0/\bar{D}^0) = 59.1 \pm 2.3\%,\\ &B(B\to XD_s) = 8.3 \pm 0.9\%,\\ &B(B\to c\ \text{baryons}) = 6.7 \pm 1.9\%,\\ &B(B\to e^-,\mu^-,\tau^-) = 24 \pm 2\%,\\ &B(B\to XJ/\psi) = 1.12 \pm 0.16\%. \end{aligned} \quad (17)$$

Summing the inclusive D rates and charmed baryon rates and correcting for double counting by removing the D_s and semileptonic rates, we conclude, for transitions to one c quark,

$$B_{\rm nl}(\bar{B}\to X_c)\,|_{\rm expt} = 54 \pm 4\%. \quad (18)$$

Because of the large experimental errors, there is no strong disagreement between (16) and (18). Nevertheless, it appears that the theoretical number is about 20% too low, as we will find more convincingly below.

For the decay channel $b \to c\,\bar{c}\,s$ one obtains, from (11) and from the appropriate phase space reduction,

$$B_{\rm nl}(\bar{B}\to X_{c\bar{c}}) \simeq 19 \pm 1\%. \quad (19)$$

This prediction depends more severly on quark mass ratios [for instance, a strange quark mass (pole mass) of 0.5 GeV instead of 0.15 GeV gives 16% in place of the above 19%].

The corresponding experimental number can be estimated in several ways. By subtracting the branching ratio (18) and the semileptonic branching ratio given in

[3] We put $\theta_C = 0$ to account for Cabibbo-suppressed decays. b to u contributions in the semileptonic decays can decrease (16) by at most 2%.

(17) from the total width one obtains

$$B_{\rm nl}(\bar{B} \to X_{c\bar{c}})\,|_{\rm expt} = 22 \pm 5\%. \tag{20}$$

One may also multiply the result quoted in (18) by the relative phase space reduction factor 0.434 and obtain the branching ratio $23 \pm 2\%$. But if we add the branching ratios of the observed inclusive $c\bar{c}$ channels $X\bar{D}_s$ and XJ/ψ and take account of unseen decays to η_c and χ states[4] by multiplying $B(J/\psi)$ by a factor 4 we find only

$$B_{\rm nl}(\bar{B} \to X_{c\bar{c}})\,|_{\rm expt} \simeq 13 \pm 2\%. \tag{21}$$

This last number, however, cannot be trusted. Its smallness is related to the fact that, neglecting errors, a branching ratio of about 9% is missing in (17). (Adding the inclusive D rates, the rates to baryons and 4 times the rates to J/ψ gives only 91%.) It is likely that it is the $c\bar{c}$ channel which is not fully understood. For instance, a $c\bar{c}$ quark pair at low energy and in a color octet state may annihilate on a hadronic scale (by a "long range Penguin graph") and not form $D\bar{D}_s$ or quasistable η_c, χ, or J/ψ states.

In any case, the theoretically calculated total nonleptonic branching ratio from (16) and (19) is $62 \pm 2\%$. The experimental number obtained by simply subtracting the semileptonic decays from the full rate is $76 \pm 2\%$ which is a factor 1.23 higher.

Thus, about 20% of the nonleptonic width is not accounted for by perturbative QCD. Part of the discrepancy could be due to higher order contributions in the $1/m_Q$ expansion, i.e., by strong spectator effects. But it in view of the fact that $1/m_b^2$ corrections are a few percent and additional spectator effects arise at order $1/m_Q^3$ only, it is unlikely that these account for so much. More serious is the neglect of the c-quark mass in the calculation of the QCD correction which led to Eq. (11) and nonperturbative effects not contained in low order QCD calculations. It is known from examples that large quark masses give rise to QCD corrections of the form $\alpha_s\pi$ besides α_s/π terms [17]. In the next section we will study, therefore, typical nonperturbative contributions, taking mass effects into account.

III. BOUND-STATE PRODUCTION IN INCLUSIVE B DECAYS

In nonleptonic decays of quarks, two of the three outgoing quarks can form bound or strongly correlated states. For instance, in the decay $b \to c\ \bar{c}\ s$ there is a phase space region where $\bar{c}$ and s form dominantly $\bar{D}_s$ and $\bar{D}_s^*$ mesons and the three-body quark decay process reduces to the two-body decay $b \to c\ \bar{D}_s^{(*)}$. Such reactions are caused by the direct generation of hadrons by currents forming the weak Hamiltonian and was found to be important for inclusive as well as for exclusive processes [1, 3, 15]. The theoretical description uses an appropriate factorization of the weak interaction with one factor generating the bound state from the vacuum. The corresponding matrix element defines a "decay constant," a dimensionful and truly nonperturbative constant which brings a new scale to the problem. In a strict QCD treatment of inclusive quark decays, the emission of bound states should still be contained in the leading (m_Q^5) term of the operator product expansion. Only on the hadron level, where field operators of bound states can be introduced, does the new scale provide for a formally different power behavior, namely, $f^2 m_Q^3$ or lower, where f stands for the relevant decay constant.

We will use this approach to estimate the bound-state emission processes and to shed light on its significance. This method will also allow us to study various semi-inclusive reactions separately and independently, which cannot be done in perturbative calculations.

In order to remain independent of the KM matrix element V_{cb} and less sensitive to the quark masses, we present the decay rates to bound states relative to $\Gamma_{\rm sl}$. This has the additional advantage that the corresponding semi-inclusive B-meson decay rates at leading order are obtainable from the semileptonic rate with the b-quark motion inside the B meson accounted for.

For the decay $b \to c\bar{D}_s^*$ we find

$$\Gamma(b \to c\bar{D}_s^*)/\Gamma_{\rm sl} = \frac{12\pi^2}{z_0({\rm sl})}\cos^2\theta_C \frac{f_{D_s^*}^2 a_1^2}{m_Q^2}[(1-x-y)^2 - 4xy]^{1/2} \times [1 + x - 2x^2 - y(2-x-y)], \tag{22}$$

$$x = \left(\frac{m_{D_s^*}}{m_b}\right)^2, \quad y = \left(\frac{m_c}{m_b}\right)^2.$$

In writing this formula we left out α_s corrections in the numerator and denominator and introduced the phenomenological coefficient a_1. This constant and the constant a_2 needed below are formally related to c_1 and c_2 by

$$a_1 = c_1 + \frac{1}{N_c}c_2, \qquad a_2 = c_2 + \frac{1}{N_c}c_1. \tag{23}$$

The validity of these relations depend, however, on the validity of the factorization approximation and on the appropriate factorization scale. They cannot fully be trusted [3].

The D_s^* decay constant $f_{D_s^*}$ is defined in analogy to the well-known $\rho^\pm$ -meson decay constant ($f_\rho \approx 0.206$ GeV).

For the decay $b \to c\bar{D}_s$ the last factor in (22) has to be replaced by

$$1 - x - y(2 + x - y) \tag{24}$$

and, of course, $f_{D_s^*}$ by f_{D_s} and $m_{D_s^*}$ by m_{D_s}.

The decay constants f_{D_s} and $f_{D_s^*}$ are not known from leptonic decays of D_s and D_s^*. However, an analysis of exclusive B-meson decays to $D\bar{D}_s$, $D^*\bar{D}_s$, $D\bar{D}_s^*$, $D^*\bar{D}_s^*$ has been performed in Ref. [3] setting $f_{D_s} \approx f_{D_s^*}$, with the result

[4]In a preliminary report by M. Whadhwa (L3 Collaboration) in Moriond 93 the ratio of J/ψ to χ production is 0.8 ± 0.3. A theoretical estimate [15] gives $\simeq 3.7$.

$$f_{D_s^*} a_1 = 0.316 \pm 0.050 \text{ GeV}. \tag{25}$$

From (22), (24) together with the measured semileptonic branching ratio of $10.7 \pm 0.5\%$ we find, for the direct production of D_s and D_s^* in semi-inclusive B decays,

$$B(\bar{B} \to X_c \bar{D}_s^* \text{ direct}) = 6.6 \pm 1.5\%,$$
$$B(\bar{B} \to X_c \bar{D}_s \text{ direct}) = 4.7 \pm 1.5\%. \tag{26}$$

The sum of both decays represents a branching ratio of $11 \pm 2\%$. Since D_s^* decays to D_s by γ emission, we can now compare this number with the measured inclusive D_s branching ratio of $10 \pm 2\%$ noted in (17). The comparison shows that the direct production of low-lying states is a dominant mechanism in B decays to charm-quark–charm-antiquark states.

This conclusion is supported by the calculation of the semi-inclusive processes $b \to s\ J/\psi$ and $b \to s\ \psi'$ [3, 15, 18]. Here one can use again Eq. (22). We simply have to change the masses and to replace $f_{D_s^*} a_1$ by $f_{J/\psi} a_2$ and $f_{\psi'} a_2$, respectively. For these processes the decay constants are known [3]: $f_{J/\psi} = 0.384 \pm 0.014$ GeV, $f_{\psi'} = 0.282 \pm 0.014$ GeV. The value of the quantity a_2, on the other hand, is less clear because of the partial cancellation of the $1/N_c$ term by c_2. The measured exclusive decay $\bar{B} \to \bar{K}^* J/\psi$ suggests, however, $|a_2| = 0.21 \pm 0.03$ [3]. With this number and by noting again that b-quark decay can be related to B-meson decay if we use (22) together with the semileptonic branching ratio, we obtain

$$B(B \to X_s J/\psi \text{ direct}) = 0.40 \pm 0.12\%,$$
$$B(B \to X_s \psi' \text{ direct}) = 0.12 \pm 0.04\%. \tag{27}$$

Since ψ' decays to J/ψ with a branching ratio of 57%, we can compare the corresponding sum of both channels, namely, $0.47 \pm 0.12\%$ with the measured inclusive J/ψ production of $1.12 \pm 0.16\%$. Again, a large fraction of the inclusive J/ψ production occurs through direct bound-state production. An additional test is provided by the measurement of the inclusive J/ψ polarization. In the Appendix we will present the relevant formulas for the polarization of a vector particle emitted in a quark decay process. Also discussed there is the effect of the Fermi motion for the momentum distribution in such a two-body decay. The prediction for the longitudinal polarization in the decay $b \to s\ J/\psi$ is 54% which compares well with the one reported by a recent CLEO experiment, namely, $56 \pm 5 \pm 5\%$ [16]. Figure 1 shows theoretical J/ψ momentum distributions assuming Gaussian wave functions for the Fermi motion.

In the $b \to c\ \bar{u}\ d$ channel the direct production of low-lying hadron states is no longer dominant because of the larger phase space available. Using the decay constant for the a_1^- particle to be the same as for the ρ^- meson and setting $a_1 = 1.1$ we find, from the analogue of Eq. (22),

$$B(B \to X_c, (\pi^-, \rho^-, a_1^-) \text{ direct}) \simeq 9.4\%. \tag{28}$$

The inclusive nonleptonic branching ratio to single charm states is around 53% (or around 46% if B decays to charmed baryons are subtracted) and thus a factor of 5–6 larger.

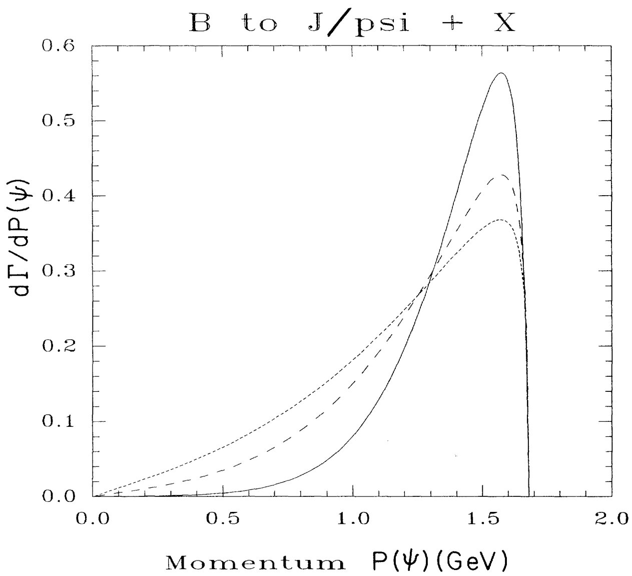

FIG. 1. J/ψ momentum spectrum in the decay $B \to X + J/\psi$ in a spectator model with Gaussian wave function. The root-mean-square Fermi momenta 0.4, 0.6, and 0.8 GeV correspond respectively to the solid curve, dashed curve (long dashes), and dashed curve (short dashes).

The direct production of D^0 and D^{*0} mesons in the $b \to c\ \bar{u}\ d$ channel is found to be negligible (below 1%) since these rates are governed by the small number a_2^2.

In addition to the formation of color singlet hadrons, QCD forces also generate bound-state-like correlations between two quarks in color antitriplet states, diquarks. The weak Hamiltonian creates such quark pairs in spin $0^\pm$ states: $b \to \bar{u}\ (cd)_{0^\pm}$ [2, 3, 19]. Because of the kinematics relevant for the emission of scalar particles, quark-quark correlations are most important in hyperon and K decays but less so in heavy quark decays. The inclusive rate goes as m_Q^{-4} with respect to free quark decay. However, because of the diquark masses and decay constants [19], we find this rate is still not negligible and of particular relevance for B-meson decays to baryons [20]. It seems reasonable to assume that the direct production of low-lying bound diquark states will lead to a final baryon whenever this is energetically possible. We obtain

$$\Gamma(b \to \bar{u}(cd) \text{ direct})/\Gamma_{\text{sl}}$$
$$= 12 \frac{\pi^2}{z_0(\text{sl})} \cos^2\theta_C \ \frac{4}{3} \frac{g_{(cd)}^2 c_-^2}{m_b^4} \ (1-x)^2,$$

with

$$x = \left(\frac{m_{(cd)}}{m_b}\right)^2. \tag{29}$$

The diquarks decay constant $g_{(cd)}$ has dimension 2 and is approximately given by $g_{(cd)} \approx f_D\ m_{(cd)0^+}^2/(m_c + m_d)$. By taking $f_D = 0.22$ GeV, $m_{(cd)0^+} = 2.0$ GeV, $m_{(cd)0^-} = 2.6$ GeV, $\theta_C = 0$, $g_{0^+} = g_{0^-}$, and by using the semileptonic branching ratio as before, we find

$$B(B \to c \text{ baryons}) \approx 6\% \ . \tag{30}$$

Although we cannot give a solid theoretical error to this number, it certainly shows the right magnitude for the inclusive baryon production in B decays, which is $7 \pm 1\%$.

In this section we have seen that the nonperturbative process of bound-state production provides for a successful description of several specific inclusive decay channels. But this technique can be used in limited regions of the phase space only. Furthermore, it is difficult to combine the corresponding results with the perturbative treatment given in the previous section. Because of the danger of double counting, it is not possible simply to add bound-state production rates to the perturbative results. For instance, we cannot add the rate for the direct production of π, ρ, a_1, given by (28), to the perturbative result (16) for the $c\ \bar{u}\ d$ channel, because the perturbative contribution in the relevant phase space region pretty much equals that of the resonances; it is dual to it. Indeed, we can show using τ-decay results that perturbative QCD for color singlet $\bar{u}\ d$ configurations matches closely the resonance contributions in this channel. (In τ decay the fraction of three-body phase space above 1.5 GeV invariant mass in the $\bar{u}d$ channel is only 10%.)

A different situation prevails, however, for the diquark (baryon) contribution to inclusive decays to the $c\ \bar{u}\ d$ channel. Because of the nonzero c-quark mass involved, a perturbative QCD calculation for massless quarks does not contain this contribution, as seen from the $f_D^2\ ({m_D}^2/m_c)^2/m_b^4$ dependence relative to the leading term. Thus, it is reasonable to add the 6% found above to our perturbative result given by Eq. (16). The inclusive nonleptonic branching ratio for B decays to final states with a single c quark is then $\approx 50\%$ in good agreement with the experimental number $54 \pm 4\%$ given in Eq. (18).

From our treatment of the $c\ \bar{c}\ s$ decay channel we learned that bound-state production accounts for the entire inclusive rates to D_s and for a large fraction of the inclusive rate to J/ψ. Since bound-state production needs only a limited phase space region (in about 50% of the three-body phase space the $\bar{c}s$ system has an energy over 2.8 GeV) roughly twice as large, a total decay rate is to be expected beyond the observed rate in this channel. Therefore, we can take our results as a strong indication that the missing branching ratio of roughly 9% mentioned below Eq. (21) should indeed be found here. A possible experiment would be to look for resonances associated with energetic K and K^* production in B decays. If found, such a decay mode would also be of interest in connection with experiments to detect CP-violating transitions [21].

IV. INCLUSIVE D DECAYS

Because of the relatively small mass of the c quark, the operator product expansion is of only limited value for the calculation of inclusive D-decay rates. From the observed lifetime difference of D^0 and D^+ it is evident that strong spectator effects are present. This requires the calculation of higher order terms in the operator product expansion which is, however, out of our means at present. Nevertheless, it is of interest to establish how close low order calculations bring us to the experimental data, how important the production of bound states is, and to what extent we can understand the lifetime difference of D^0 and D^+.

Let us first look again at Eq. (11) which is valid to order α_s and $1/m_Q^2$. Both the α_s value and in particular the value of the quantity r obtained from the mass difference between D^* and D are now larger than in the case of B decays. For the quark masses, at the lower scale we use new values which give according to (4) a good description of the measured semileptonic rate: $m_c = 1.52$ GeV, $m_s = 0.17$ GeV, $m_u = m_d = 0$, and correspondingly

$$\begin{aligned} &\alpha_s(m_c) = 0.36, \quad r = 0.36, \\ &c_1(m_c) = 1.32, \ c_2(m_c) = -0.58. \end{aligned} \tag{31}$$

From these numbers it is immediately clear that the corrections are drastic. The $1/N_c$ term [second line in (11)] now becomes positive and larger in magnitude. We find, from (11),

$$\Gamma_{\rm nl}/\Gamma_{\rm sl} \simeq 8.7 + O(1/m_c^3) \tag{32}$$

in place of the pure color factor 3.

A. D^0 decays

If the result (32) is used for D^0 decay together with $B(D^0 \to Xe^-\bar{\nu}) = 7.7 \pm 1.2\%$, one gets

$$B(D^0 \to X_{\rm nl}) \simeq 67 \pm 10\%. \tag{33}$$

The experimental result is

$$\Gamma(D^0 \to X_{\rm nl})/\Gamma(D^0 \to Xe^+\nu)|_{\rm expt} = 11 \pm 1.7, \tag{34}$$

$$B(D^0 \to X_{\rm nl})|_{\rm expt} = 85 \pm 1.7\%. \tag{35}$$

Thus, a sizable fraction of the enormous enhancement factor is already accounted for by the low order calculation.

For c-quark decays we can, as we did for b-quark decay, estimate special inclusive decays involving the direct production of a meson or resonance: $c \to s\ (\pi^+, \rho^+, a_1^+(\text{direct}))$ $c \to \bar{d}\ (\bar{K}^0, \bar{K}^{0*}(\text{direct}))$. The calculation is straightforward by using the analogue of Eqs. (22) and (24). Only the a_1^+ production is difficult to estimate because of the large width of this particle and the limited phase space available to it. Instead, we use the measured $D^0 \to K^- a_1^+$ rate.

Summing up and converting to the corresponding D^0 branching ratio we obtain

$$B(D^0 \to X(\pi^+, \rho^+, a_1^+, \bar{K}^0, \bar{K}^{0*}\text{direct})) \approx 59\%. \tag{36}$$

This result can only be used for an orientation because it is very sensitive to the phenomenological coefficient a_1 and, to a smaller degree, to the coefficient a_2 (we took

$a_1 = 1.2$, $|a_2| = 0.5$ [1, 3]). It shows that direct bound-state formation caused by strong nonperturbative forces between quarks and antiquarks is a dominant process in D^0 decays. The bigger part of all hadronic decays are quasi-two-body decays to meson resonances.

Let us also consider the quark decay process $c \to \bar{d}\ (us)_{0^+}$ where the u and s quarks are correlated and form a color antitriplet diquark state. In D decays this process can occur as a virtual process before long range forces rearrange the quarks and cause hadronization, presumably for the most part to many-body states. Taking $g^+_{(us)} = 0.20$ GeV^2, $m_{(us)}(0^+) = 0.8$ GeV, one expects a corresponding D^0-meson branching ratio of $\approx 19\%$. In spite of the fact that this contribution has a m_Q^{-4} dependence we cannot add it, at least not fully, to our perturbative result given in (33) since the perturbative calculation performed for massless quarks should be dual to the resonance contributions for states containing the very light s, u, and d quarks. As a transition to correlated states we may add it, however, to the bound-state production rate (36). This leads to a a hadronic branching ratio of $\approx 78\%$ leaving only a $\approx 7\%$ branching ratio for other decay mechanisms such as "weak annihilation." In this very simple model the fraction of meson resonance production to total hadron production is $\approx 59 : 78$, i.e., $\approx 76\%$.

B. D^+ decays

The ratio of nonleptonic to semileptonic inclusive decays of D^+ is

$$\Gamma(D^+ \to X_{\rm nl})/\Gamma(D^+ \to Xe^+\bar{\nu})|_{\rm expt} \simeq 3.2. \qquad (37)$$

Since it is close to the number of colors, this ratio was often considered to be quite normal and as expected. But it is certainly *not* normal in view of the large QCD effects at the scale m_c which causes the factor 8.7 of Eq. (32) already at $O(\alpha_s)$. If one would multiply the D^+ semileptonic branching ratio of $17.2 \pm 1.9\%$ with this factor, one would obtain nonsense, namely, a branching ratio of 150%.

Quite early, as a possible solution of this problem, a strong interference between quark clusters has been proposed [22]. Indeed, it was soon found out that in exclusive D^+ decays there is a destructive interference between two amplitudes in the most prominent two-body decays [1, 3]. For instance, the decay $D^+ \to \bar{K}^0\rho^+$ can occur through the generation of the ρ^+ by the $(\bar{u}d)$ current together with the $D^+ \to \bar{K}^0$ transition, but also by the generation of the $\bar{K}^0$ by the $(\bar{s}d)$ current in conjunction with $D^+ \to \rho^+$. The first decay is governed by the coefficient c_1, the second by the negative number c_2 leading to the reduced amplitude proportional to $c_1 + c_2 = c_+$. To these decays the operator $\mathcal{O}_-$ with its strong quark-quark correlation in color antitriplet states does not contribute. Thus, in D^+ decays the term multiplying c_-^2 in Eq. (9) will not contribute to current generated two-body decays which form a major part of D^0 decays as we have seen.

This suppression can also be understood on the quark level. In two-body decays in the rest frame of the D meson the s and u quarks generated after c-quark decay appear in different mesons and thus have momenta similar in magnitude but opposite in sign. Thus, the s-u diquark system is nearly at rest and, because of the structure of the weak Hamiltonian, in a Lorentz scalar state. If this is the case, the remaining two $\bar{d}$ quarks must also be in a state of total angular momentum zero with the Pauli exclusion principle requiring it to be symmetric with respect to color. Consequently, the s-u system itself has to be in a color sextet state giving a vanishing contribution for the matrix element of the operator $\mathcal{O}_-$.

In general, one would not have expected a large effect of the Pauli principle. It was thought to play only a minor role [23]. However, the two $\bar{d}$ quarks are strongly correlated, not because of any dynamical peculiarity, but because of the kinematical situation in D^+ decays and the structure of the effective weak Hamiltonian.

Returning to our perturbation calculation it is clear that, in the case of D^+ decays, the matrix element of the operator $\mathcal{O}_-$, i.e., the second line of Eq. (9), has to be strongly suppressed. In order to reproduce the experimental ratio Eq. (32) we have to suppress this piece by the factor 3.1. This number shows that the isospin partners of a large fraction of hadronic final states appearing in D^0 decays, namely, about 70%, do not contribute to matrix elements of the operator $\mathcal{O}_-$ in D^+ decays. They are the quasi-two-body final states just as obtained in our model estimate.

V. SUMMARY

We studied inclusive nonleptonic decays by the operator product expansion method. We considered their relation to semileptonic decays in order to minimize the effect of quark masses, the Fermi motion of the decaying quark, and the less well-known Kobayashi Maskawa matrix elements.

For B decays we could show the following.

(i) QCD corrections of the *leading* term in the $1/m_Q$ expansion are the strikingly dominate cause of the nonleptonic enhancement.

(ii) These corrections, together with the smaller effect of the color magnetic operator appearing at next-to-leading order in $1/m_Q$, reduce the color-suppressed combination of Wilson coefficients by $\simeq 54\%$. The QCD corrections applied are the corrections for massless quarks since the very involved and tricky calculations for massive quarks are not yet available. The remaining discrepancy with experiment, or most of it, is presumably due to this approximation.

(iii) In fact, the addition of a diquark term explicitly due to the c-quark mass removes the discrepancy for the $c\ \bar{u}\ d$ channel in a satisfactory way.

(iv) A very simple model for bound-state production is in accord with observed branching ratios for the inclusive decays to $B \to \bar{D}_s$, J/ψ, and baryons.

(v) We expect additional decay modes originating from $b \to c\ \bar{c}\ s$ decays with a branching ratio of $\approx 9\%$. Supporting evidence for a missing decay channel is provided by a proper summation of experimentally determined inclusive rates which do not fully add up to the total rate. In treating D decays we also applied the operator product expansion method even though no good convergence could be expected:

(vi) The dramatic enhancement of the nonleptonic mode in D^0 decay can be understood. It is again due to the leading and next-to-leading term in the $1/m_Q$ expansion.

(vii) The color-suppressed combination of Wilson coefficients changes sign and gives a positive contribution to the decay width.

(viii) Resonance production is dominant in D^0 decays. This allows for a simplified way to get an estimate of the nonleptonic decay rate.

(ix) In D^+ decays the matrix elements of the operator $\mathcal{O}_-$ (which gives the largest contribution to D^0 decays) are severly suppressed. This is due to the special kinematical situation prevailing in D decays giving the presence of the spectator quark and the Pauli exclusion principle an unexpected large efficiency.

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APPENDIX: POLARIZATION AND MOMENTUM DISTRIBUTIONS IN INCLUSIVE DECAYS

The decay rate $Q \to q + V_{(L \text{ or } T)}$ to a polarized final state vector meson of mass m_V in the longitudinal (L) or transverse states (T) is easily calculated in terms of $x = (m_V/m_Q)^2$ and $y = (m_q/m_Q)^2$ and results in the following decomposition of the decay energy dependence into transverse and longitudinal parts [15]:

$$[1 + x - 2x^2 - y(2 - x - y)]_L = 1 - x - y(2 + x - y),$$
$$[1 + x - 2x^2 - y(2 - x - y)]_T = 2x(1 - x + y). \quad \text{(A1)}$$

The transverse polarization fractions is therefore given by

$$\frac{\Gamma_T}{\Gamma} = \frac{2x(1 - x + y)}{1 + x - 2x^2 - y(2 - x - y)}. \quad \text{(A2)}$$

Thus we have $\Gamma_T/\Gamma = 31\%$ for $b \to D_s^* + c$ and $\Gamma_T/\Gamma = 46\%$ for $b \to c + J/\psi$. The J/ψ polarization compares well with that reported by a recent CLEO experiment, $\Gamma_L/\Gamma = 56 \pm 7\%$ for the inclusive polarization. In this experiment the J/ψ momentum spectrum is also measured. Our approach to inclusive decays such as $B \to X + J/\psi$ is to model them by the corresponding *two*-body decay $b \to s + J/\psi$. In this model the J/ψ momentum is fixed kinematically. This spike will be smeared by the Fermi motion of the b quark and the spectator. CLEO has recently reported the inclusive momentum spectrum, which is broadly distributed from soft momenta to the kinematic limit. Part of this smearing is due to the experimental momentum resolution and to the fact that the B meson is not at rest. We have calculated a simple model for the spectator effect, following the formalism of Peccei and Rückl [24] and Rückl [4] using the interaction

$$\phi(p_{\bar{u}})\epsilon_\mu(p_\psi)\bar{u}(p_s)\gamma^\mu(1-\gamma^5)[\gamma\cdot(P_B-p_{\bar{u}})+M_b]\gamma^5 v(p_{\bar{u}}), \quad \text{(A3)}$$

where for the wave function ϕ we use a Gaussian distribution of momenta. The result of this calculation is shown in Fig. 1 for various Fermi momenta. The spectrum at Fermi momentum 0.6 GeV is reasonably similar to the one reported by CLEO but the data show significantly more events at low momentum. (An improved model of this spectrum in a parton model framework will shortly be reported by Paschos and one of us [25].)

[1] M. Bauer and B. Stech, Phys. Lett. **152B**, 380 (1985); M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C **34**, 103 (1987). For a review of the BSW model see M. Wirbel, Prog. Part. Nucl. Phys. **21**, 33 (1988).

[2] M. Neubert and B. Stech, Phys. Rev. D **44**, 775 (1991); B. Stech and Q.P. Xu, Z. Phys. C **49**, 491 (1991); B. Stech, Mod. Phys. Lett. A **6**, 3113 (1991).

[3] B. Stech, in *CP-Violation*, edited by C. Jarlskog, Advanced Series on Directions in High Energy Physics Vol. 3 (World Scientific, Singapore, 1989), p. 680, and literature quoted there; M. Neubert, V. Rieckert, B. Stech, and Q.P. Xu, in *Heavy Flavours*, edited by A.J. Buras and M. Lindner, Advanced Series in High Energy Physics Vol. 10 (World Scientific, Singapore, 1992), p. 286.

[4] For an early report already describing most ideas used at present see R. Rückl, *Weak Decays of Heavy Flavours* (Habilitationsschrift, University of Munich, Munich, 1983).

[5] R. Rückl, Report No. MPI-Ph/89-36 (unpublished).

[6] M. Voloshin and M. Shifman, Yad. Fiz. **41**, 187 (1985) [Sov. J. Nucl. Phys. **41**, 120 (1985)]; Zh. Eksp. Teor. Fiz. **91**, 1180 (1986) [Sov. Phys. JETP **64**, 698 (1986)]; B. Guberina, R. Rückl, and J. Trampetic, Z. Phys. C **33**, 297 (1986).

[7] I.I. Bigi, N.G. Uraltsev, and A.I. Vainshtein, Phys. Lett. B **293**, 430 (1992); I.I. Bigi, B. Blok, M. Shifman, N.G. Uraltsev, and A.I. Vainshtein, Report No. NSF-ITP-92-156 (unpublished).

[8] B. Blok and M. Shifman, Nucl. Phys. **B399**, 441 (1993); **B399**, 459 (1993); Report No. ITP-92-146 (unpublished).

[9] N. Cabibbo and L. Maiani, Phys. Lett. **79B**, 109 (1978).

[10] G. Altarelli *et al.*, Nucl. Phys. **B187**, 461 (1981); G. Altarelli and S. Petrarca, Phys. Lett. B **261**, 303 (1991).

[11] A.J. Buras, J.-M. Gérard, and R. Rückl, Nucl. Phys. **B268**, 16 (1986).
[12] B.Y. Blok and M.A. Shifman, Yad. Fiz. **45**, 221 (1987); **45**, 478 (1987); **45**, 841 (1987).
[13] G. Altarelli *et al.*, Nucl. Phys. **B187**, 461 (1981); A.J. Buras and P.H. Weisz, *ibid.* **B333**, 66 (1990).
[14] CLEO Collaboration, D. Bortoletto *et al.*, Phys. Rev. D **45**, 21 (1992); S. Stone, in Proceedings of the 5th International Symposium on Heavy Flavour Physics, Montreal, Canada, 1993 (unpublished).
[15] T. DeGrand and D. Toussaint, Phy. Lett **89B**, 256 (1980); J.H. Kühn, S. Nussinov, and R. Rückl, Z. Phys. C **5**, 117 (1980); J.H. Kühn and R. Rückl, Phys. Lett. **135B**, 477 (1984); Phys. Lett. B **258**, 499(E) (1991).
[16] CLEO Collaboration, in *Proceedings of the XXVIth International Conference on High Energy Physics*, Dallas, Texas, 1992, edited by J. Sanford, AIP Conf. Proc. No. 272 (AIP, New York, 1993).
[17] E. Bagan, P. Ball, V.M. Braun, and H.G. Dosch, Phys. Lett. B **278**, 457 (1992).
[18] P.H. Cox, S. Hovater, S.T. Jones, and L. Clavelli, Phys. Rev. D **32**, 1157 (1985); S.T. Jones and P.H. Cox, *ibid.* **35**, 1064 (1987).
[19] B. Stech, Phys. Rev. D **36**, 975 (1987); G. Dosch, M. Jamin, and B. Stech, Z. Phys. C **42**, 167 (1989); M. Jamin and N. Neubert, Phys. Lett. B **238**, 387 (1990); B. Stech, in *Diquarks*, edited by M. Anselmino and E. Predazzi (World Scientific, Singapore, 1989), p. 277.
[20] B. Stech, in *Heavy Flavours*, Proceedings of the Workshop, San Miniato, Italy, 1987, edited by F.L. Navarria [Nucl. Phys. B (Proc. Suppl.) **1B**, 17 (1988)]; P. Ball and H.G. Dosch, Z. Phys. C **51**, 445 (1991).
[21] J. Bernabeu and C. Jarlskog, Phys. Lett. B **301**, 275 (1993).
[22] B. Guberina, S. Nussinov, R.D. Peccei, and R. Rückl, Phys. Lett. **89B**, 111 (1979); Y. Koide, Phys. Rev. D **20**, 1739 (1979).
[23] G. Altarelli and L. Maiani, Phys. Lett. **118B**, 414 (1982).
[24] R.D. Peccei and R. Rückl, in *Proceedings of the Symposium on Special Topics in Gauge Field Theories*, Ahrenshoop, Germany, 1981 (Akademie der Wissenschaften, Berlin, 1981).
[25] W. Palmer and E. Paschos (unpublished).