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Direct and indirect *CP* violation in the decay $K_L \to \pi^+\pi^- e^+ e^-$

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The decay $K_L \to \pi^+\pi^- e^+ e^-$ is analyzed in a model containing (i) a *CP*-conserving amplitude associated with the $M1$ transition in $K_L \to \pi^+\pi^-\gamma$, (ii) an indirect *CP*-violating amplitude related to the bremsstrahlung part of $K_L \to \pi^+\pi^-\gamma$, and (iii) a direct *CP*-violating term associated with the short-distance interaction $s\bar{d} \to e^+e^-$. Interference of the first two components produces a large *CP*-violating asymmetry ($\sim 14\%$) in the distribution of the angle Φ between the e^+e^- and $\pi^+\pi^-$ planes. The full angular distribution contains two further *CP*-violating observables. Effects of direct *CP* violation are found to be numerically small.

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I. INTRODUCTION

The decay $K_L \to \pi^+\pi^- e^+ e^-$ can be envisaged, in the first instance, as a conversion process related to the decay $K_L \to \pi^+\pi^-\gamma$. The latter is empirically known to contain two components: a bremsstrahlung piece related to the *CP*-violating decay $K_L \to \pi^+\pi^-$ and a *CP*-conserving magnetic dipole component. Interference of these terms produces a *CP*-violating circular polarization of the photon in $K_L \to \pi^+\pi^-\gamma$. The conversion process $K_L \to \pi^+\pi^- e^+ e^-$ may be viewed as a means of probing this polarization by studying the correlation of the e^+e^- plane relative to the $\pi^+\pi^-$ plane.

In a recent paper [1], a calculation of the decay $K_L \to \pi^+\pi^- e^+ e^-$ was carried out in which the amplitude was determined by the two empirically known components of the radiative decay [2]. In addition, a virtual photon component $K_L \to \pi^+\pi^-\gamma^*$ (absent for a real photon) was introduced, in the form of a K^0 charge-radius contribution. The branching ratio was determined to be $\sim 3\times 10^{-7}$. A significant *CP*-violating asymmetry was found in the Φ distribution of the process, Φ being the angle between the e^+e^- and $\pi^+\pi^-$ planes:[1]

$$\mathcal{A} = \frac{\int_0^{\pi/2} \frac{d\Gamma}{d\Phi} d\Phi - \int_{\pi/2}^{\pi} \frac{d\Gamma}{d\Phi} d\Phi}{\int_0^{\pi/2} \frac{d\Gamma}{d\Phi} d\Phi + \int_{\pi/2}^{\pi} \frac{d\Gamma}{d\Phi} d\Phi} = 15\% \sin[\Phi_{+-} + \delta_0(m_K^2) - \bar{\delta}_1] \approx 14\% \ . \tag{1}$$

Here Φ_{+-} is the phase of the *CP*-violating parameter η_{+-}, $\delta_0(M_K^2)$ is the $I=0$ $\pi\pi$ s-wave phase shift at $s_\pi = M_K^2$, and $\bar{\delta}_1$ is an average $\pi\pi$ p-wave phase shift in the domain $0 < s_\pi < M_K^2$. The result (1) represents one of the largest calculable *CP*-violating effects in the decays of the K^0-$\bar{K}^0$ system.

The effect found in Ref. [1] arose entirely from the bremsstrahlung decay of the K_1 admixture in the K_L wave function. In this sense, it is an example of "indirect" *CP* violation. One of the purposes of the present paper is to examine the consequences of a "direct" *CP*-violating amplitude[2] in $K_L \to \pi^+\pi^- e^+ e^-$ associated with the short-distance interaction $s\bar{d} \to e^+e^-$. In addition, we extend the analysis of Ref. [1], by looking at the complete angular distribution of the final state. This enables us to identify two further *CP*-violating observables. The method of calculation adopted here is quite different from that followed in Ref. [1], and permits an independent check of the results presented there.

II. MATRIX ELEMENT

The decay amplitude of

$$K_L(\mathcal{P}) \to \pi^+(p_+)\pi^-(p_-)e^+(k_+)e^-(k_-)$$

in our model has the form

$$\mathcal{M}(K_L \to \pi^+\pi^- e^+ e^-) = \mathcal{M}_{\rm br} + \mathcal{M}_{\rm mag} + \mathcal{M}_{\rm CR} + \mathcal{M}_{\rm SD}^{V,A} \ , \tag{2}$$

where

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[1]An error in Ref. [1], which led to a cosine instead of a sine factor in Eq. (1), and a correspondingly lower asymmetry ($\sim 4\%$), was corrected in the Erratum [1].

[2]For a recent review of direct *CP* violation, see [3].

$$\mathcal{M}_{\rm br}=e|f_S|g_{\rm br}\left[\frac{p_{+\mu}}{p_+\cdot k}-\frac{p_{-\mu}}{p_-\cdot k}\right]\frac{e}{k^2}\bar{u}(k_-)\gamma^\mu v(k_+)\ ,$$

$$\mathcal{M}_{\rm mag}=e|f_S|\frac{g_{M1}}{M_K^4}\epsilon_{\mu\nu\rho\sigma}k^\nu p_+^\rho p_-^\sigma\frac{e}{k^2}\bar{u}(k_-)\gamma^\mu v(k_+)\ , \tag{3}$$

$$\mathcal{M}_{\rm CR}=e|f_S|\frac{g_P}{M_K^2}[k^2\mathcal{P}_\mu-(\mathcal{P}\cdot k)k_\mu]\times\frac{1}{s_\pi-M_K^2}\frac{e}{k^2}\bar{u}(k_-)\gamma^\mu v(k_+)\ ,$$

$$M_{\rm SD}^{V,A}=-\frac{G_F}{\sqrt{2}}\sin\Theta_C\alpha\frac{1}{M_K}g_{\rm SD}(p_+-p_-)_\mu\times\bar{u}(k_-)\gamma^\mu(c_V-c_A\gamma_5)v(k_+)\ ,$$

The terms $\mathcal{M}_{\rm br}$, $\mathcal{M}_{\rm mag}$, and $\mathcal{M}_{\rm CR}$ denote the bremsstrahlung, magnetic dipole, and K^0 charge radius contributions discussed in Ref. [1], and the coefficients appearing therein are[3]

$$g_{\rm br}=\eta_{+-}e^{i\delta_0(M_K^2)}\ ,$$
$$g_{M1}=i(0.76)e^{i\delta_1}\ , \tag{4}$$
$$g_P=-\tfrac{1}{3}\langle R^2\rangle_{K_0}M_K^2e^{i\delta_0(s_\pi)}\ ,$$

with $|f_S|$ defined by

$$\Gamma(K_S\to\pi^+\pi^-)=\frac{|f_S|^2}{16\pi m_K}\left[1-\frac{4m_\pi^2}{m_K^2}\right]^{1/2}\ . \tag{5}$$

The new term in the matrix element is the direct *CP*-violating term $\mathcal{M}_{\rm SD}^{V,A}$, originating in the short-distance Hamiltonian describing the transition $s\bar{d}\to e^+e^-$:

$$H_{\rm SD}^{V,A}=\frac{G_F}{\sqrt{2}}\sin\Theta_C\alpha[\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{e}\gamma^\mu(F_V-F_A\gamma_5)e]\ . \tag{6}$$

Here F_V and F_A are complex functions depending on the quark-mixing angles and the mass of the top quark. The derivation of the amplitude $\mathcal{M}_{\rm SD}^{V,A}$ from the short-distance Hamiltonian is explained in the Appendix.[4] The coefficient $g_{\rm SD}^{V,A}$ is given by

$$g_{\rm SD}^{V,A}=i(s_2s_3s_\delta)\sqrt{2}\left[\frac{M_K}{f_\pi}\right]e^{i\delta_1(s_\pi)} \tag{7}$$

and the couplings c_V and c_A are approximately $c_V\approx c_A\approx 0.5$ for $m_t=150$ GeV [4]. (For numerical purposes, we take $s_2s_3s_\delta=0.5\times10^{-3}$.)

The phase factors $e^{i\delta}$ appearing in the coefficients $g_{\rm br}$, g_{M1}, g_P and $g_{\rm SD}^{V,A}$ are characteristic of final-state interactions in the $\pi\pi$ system. The phase of $g_{\rm br}$ is that of $K_L\to\pi^+\pi^-$, which is an exact result for low-energy photons, and an approximation in general. The phases of g_{M1} and $g_{\rm SD}$ are those of *p*-wave $I=1$ $\pi\pi$ scattering, which is the leading partial wave in these amplitudes. The charge radius term g_P has the phase of $K_S\to\pi^+\pi^-$ at the relevant $\pi\pi$ invariant mass. The factor of *i* in the *CP*-conserving magnetic dipole amplitude g_{M1} is a consequence of *CPT* invariance [5]. The factor "*i*" in the short-distance term $g_{\rm SD}^{V,A}$ is a signal of *CP* violation. The relative phases of the various terms in the matrix element $\mathcal{M}$ can be checked by confirming that in the absence of final-state interactions the terms $\mathcal{M}_{\rm br}$, $\mathcal{M}_{\rm mag}$, $\mathcal{M}_{\rm CR}$, $\mathcal{M}_{\rm SD}^{V,A}$ transform homogeneously under the *CPT* transformation ($\mathbf{p}_\pm\to\mathbf{p}_\mp$, $\mathbf{k}_\pm\to\mathbf{k}_\mp$ plus complex conjugation).

In the subsequent discussion, we have considered also a modification of the K^0 charge radius term g_P that takes account of the off-shell behavior of the $K_S\to\pi^+\pi^-$ amplitude predicted by chiral symmetry [6]: namely,

$$\mathcal{A}(K_S(p_K)\to\pi^+(p_+)\pi^-(p_-))\sim 2p_K^2-p_+^2-p_-^2\ . \tag{8}$$

For a virtual K_S and real pions, this amounts to replacing g_P with

$$g_P'=g_P\frac{s_\pi-m_\pi^2}{M_K^2-m_\pi^2}\ . \tag{9}$$

For further analysis, it is expedient to rewrite the matrix element (2) in a form that is reminiscent of the matrix element for K_{l4} decay [7]:

$$\mathcal{M}(K_L\to\pi^+\pi^-e^+e^-)=-\frac{G_F}{\sqrt{2}}\sin\Theta_C\left\{\left[\frac{1}{M_K}\left[f(p_++p_-)_\lambda+g(p_+-p_-)_\lambda+i\frac{h}{M_K^2}\epsilon_{\lambda\mu\nu\sigma}p_{K\mu}(p_++p_-)_\nu(p_+-p_-)_\sigma\right]\right]\times\bar{u}\gamma^\lambda(1-\gamma_5)v+\left[\frac{1}{M_K}\left[\tilde{f}(p_++p_-)_\lambda+\tilde{g}(p_+-p_-)_\lambda+i\frac{\tilde{h}}{M_K^2}\epsilon_{\lambda\mu\nu\sigma}p_{K\mu}(p_++p_-)_\nu(p_+-p_-)_\sigma\right]\right]\bar{u}\gamma^\lambda(1+\gamma_5)v\right\}\ . \tag{10}$$

[3]The factor *i* in g_{M1} was initially missed in Ref. [1], and inserted in the Erratum.

[4]A further term of the form

$$H_{\rm SD}^M=\frac{G_F}{\sqrt{2}}\sin\Theta_C\alpha[\bar{s}(im_s\sigma_{\mu\nu}q^\nu(1-\gamma_5)+im_d\sigma_{\mu\nu}q^\nu(1+\gamma_5))d]\frac{1}{k^2}[\bar{e}\gamma^\mu F_Me]$$

in the short-distance Hamiltonian is also possible, in principle, and is discussed in the Appendix.

The coefficients f, $\tilde{f}$, g, $\tilde{g}$, h, $\tilde{h}$ are given by

$$f=\tilde{f}=\left[-\frac{G_F}{\sqrt{2}}\sin\Theta_C\frac{1}{M_K}\right]^{-1}\pi\alpha|f_S|\left\{g_{\rm Br}\left[\frac{1}{p_+\cdot k}-\frac{1}{p_-\cdot k}\right]\frac{1}{s_l}+2\frac{g_P}{M_K^2}\frac{1}{s_\pi-M_K^2}\right\},$$

$$g=\left[-\frac{G_F}{\sqrt{2}}\sin\Theta_C\frac{1}{M_K}\right]^{-1}\pi\alpha|f_S|g_{\rm br}\left[\frac{1}{p_+\cdot k}+\frac{1}{p_-\cdot k}\right]\frac{1}{s_l}+M_K\alpha(s_2s_3s_\delta)\frac{i}{\sqrt{2}}e^{i\delta_1}\frac{1}{f_\pi}(c_V+c_A),$$

$$\tilde{g}=\left[-\frac{G_F}{\sqrt{2}}\sin\Theta_C\frac{1}{M_K}\right]^{-1}\pi\alpha|f_S|g_{\rm br}\left[\frac{1}{p_+\cdot k}+\frac{1}{p_-\cdot k}\right]\frac{1}{s_l}+M_K\alpha(s_2s_3s_\delta)\frac{i}{\sqrt{2}}e^{i\delta_1}\frac{1}{f_\pi}(c_V-c_A),$$

$$h=\tilde{h}=\left[+\frac{G_F}{\sqrt{2}}\sin\Theta_C\frac{1}{M_K}\right]^{-1}\pi\alpha|f_S|\frac{(-i)g_{M1}}{M_K^2}\frac{1}{s_l}. \quad (11)$$

Since $c_V\approx c_A\approx\frac{1}{2}$, we will replace c_V+c_A by unity in g, and omit the term proportional to c_V-c_A in $\tilde{g}$. We proceed to discuss the differential decay rate in terms of the form factors f, $\tilde{f}$, g, $\tilde{g}$, h, and $\tilde{h}$.

III. DIFFERENTIAL DECAY RATE

Using the formalism developed for K_{l4} decay [7], one can obtain from the matrix element (10) the decay rate of $K_L\to\pi^+\pi^-e^+e^-$ as a function of the following five variables: $s_\pi=(p_++p_-)^2=$ invariant mass of $\pi^+\pi^-$ pair; $s_l=(k_++k_-)^2=$ invariant mass of l^+l^- pair; $\Theta_\pi=$ angle between $\mathbf{p}_+$ and $(\mathbf{k}_++\mathbf{k}_-)$ as measured in the $\pi^+\pi^-$ c.m. frame; $\Theta_l=$ angle between $\mathbf{k}_+$ and $(\mathbf{p}_++\mathbf{p}_-)$ as measured in the e^+e^- c.m. frame; Φ is the angle between the normals to the $\pi^+\pi^-$ and e^+e^- planes. The precise definition of the angles Θ_π, Θ_l, and Φ is the following [8]: Let $\mathbf{p}_1$ be the three-momentum of the π^+ in the $\pi^+\pi^-$ center-of-mass system and $\mathbf{p}_l$ the three-momentum of the e^+ in the e^+e^- center-of-mass system. Furthermore, let $\mathbf{v}$ be a unit vector along the direction of flight of the dipion in the K_L rest system, and $\mathbf{c}(\mathbf{d})$ a unit vector along the projection of $\mathbf{p}_1(\mathbf{p}_l)$ perpendicular to $\mathbf{v}(-\mathbf{v})$:

$$\mathbf{c}=(\mathbf{p}_1-\mathbf{v}\mathbf{v}\cdot\mathbf{p}_1)/[\mathbf{p}_1^2-(\mathbf{p}_1\cdot\mathbf{v})^2]^{1/2},$$

$$\mathbf{d}=(\mathbf{p}_l-\mathbf{v}\mathbf{v}\cdot\mathbf{p}_l)/[\mathbf{p}_l^2-(\mathbf{p}_l\cdot\mathbf{v})^2]^{1/2}.$$

The angles are now given by

$$\cos\Theta_\pi=\mathbf{v}\cdot\mathbf{p}_1/|\mathbf{p}_1|,\quad \cos\Theta_l=-\mathbf{v}\cdot\mathbf{p}_l/|\mathbf{p}_l|,$$

$$\cos\Phi=\mathbf{c}\cdot\mathbf{d},\quad \sin\Phi=(\mathbf{c}\times\mathbf{v})\cdot\mathbf{d}. \quad (12)$$

The differential decay rate is

$$d\Gamma=\frac{G_F^2}{2^{12}\pi^6M_K^5}\sin^2\Theta_C X\sigma_\pi\left[1-\frac{4m_l^2}{s_l}\right]^2 I(s_\pi,s_l,\Theta_\pi,\Theta_l,\Phi)ds_\pi ds_l d\cos\Theta_\pi d\cos\Theta_l d\Phi,$$

where

$$\sigma_\pi=\left[1-\frac{4m_\pi^2}{s_\pi}\right]^{1/2},\quad X=(s^2-s_\pi s_l)^{1/2},\quad s=\frac{1}{2}(M_K^2-s_\pi-s_l). \quad (13)$$

In Eq. (13) I is a quadratic function of the form factors f, g, $\tilde{g}$, and h which are functions of s_π, s_l, and $\cos\Theta_\pi$ only, and may be rewritten as

$$f=\tilde{f}=CM_K^4\left\{|\eta_{+-}|e^{i(\delta_0+\Phi_{+-})}\frac{1}{s_l}\frac{-4\beta\cos\Theta_\pi}{s(1-\beta^2\cos^2\Theta_\pi)}+2\frac{g_P}{M_K^2}e^{i\delta_0(s_\pi)}\frac{1}{s_\pi-M_K^2}\right\},$$

$$g=CM_K^4|\eta_{+-}|e^{i(\delta_0+\Phi_{+-})}\frac{1}{s_l}\frac{4}{s(1-\beta^2\cos^2\Theta_\pi)}+i\eta_d e^{i\delta_1},$$

$$\tilde{g}=CM_K^4|\eta_{+-}|e^{i(\delta_0+\Phi_{+-})}\frac{1}{s_l}\frac{4}{s(1-\beta^2\cos^2\Theta_\pi)},$$

$$h=\tilde{h}=-CM_K^2\frac{1}{s_l}(0.76)e^{i\delta_1},$$

where

$$CM_K^4=\left[-\frac{G_F}{\sqrt{2}}\sin\Theta_C\frac{1}{M_K}\right]^{-1}\pi\alpha|f_S|\approx-0.04M_K^4,\quad \beta=X\sigma_\pi/s\ ,$$
$$\eta_d=\frac{M_K}{f_\pi}\frac{\alpha}{\sqrt{2}}s_2s_3s_\delta\approx0.02s_2s_3s_\delta,\quad \delta_0=\delta_0(s_\pi=M_K^2)\ . \tag{14}$$

Following Ref. [7] we define the following linear combinations of these form factors:

$$F_1=Xf+\sigma_\pi s\cos\Theta_\pi g\ ,$$
$$F_2=\sigma_\pi(s_\pi s_l)^{1/2}g\ , \tag{15}$$
$$F_3=\sigma_\pi X(s_\pi s_l)^{1/2}\frac{h}{M_K^2}\ ,$$

and an analogous set $\tilde F_1,\tilde F_2,\tilde F_3$ obtained by replacing f,g,h by $\tilde f,\tilde g,\tilde h$. The distribution I then takes the form [7]

$$I=I_1+I_2\cos2\Theta_l+I_3\sin^2\Theta_l\cos2\Phi$$
$$+I_4\sin2\Theta_l\cos\Phi+I_5\sin\Theta_l\cos\Phi$$
$$+I_6\cos\Theta_l+I_7\sin\Theta_l\sin\Phi$$
$$+I_8\sin2\Theta_l\sin\Phi+I_9\sin^2\Theta_l\sin2\Phi\ , \tag{16}$$

where the functions $I_1\cdots I_9$ are given by (dropping terms proportional to m_l^2)

$$I_1=\tfrac14[\{|F_1|^2+\tfrac32(|F_2|^2+|F_3|^2)\sin^2\Theta_\pi\}+(F_{1,2,3}\to\tilde F_{1,2,3})]\ ,$$
$$I_2=-\tfrac14[\{|F_1|^2-\tfrac12(|F_2|^2+|F_3|^2)\sin^2\Theta_\pi\}+(F_{1,2,3}\to\tilde F_{1,2,3})]\ ,$$
$$I_3=-\tfrac14[\{|F_2|^2-|F_3|^2\}+(F_{1,2,3}\to\tilde F_{1,2,3})]\ ,$$
$$I_4=\tfrac12\mathrm{Re}(F_1^*F_2)\sin\Theta_\pi+(F_{1,2,3}\to\tilde F_{1,2,3})\ ,$$
$$I_5=-\{\mathrm{Re}(F_1^*F_3)\sin\Theta_\pi-(F_{1,2,3}\to\tilde F_{1,2,3})\}\ , \tag{17}$$
$$I_6=-\{\mathrm{Re}(F_2^*F_3)\sin^2\Theta_\pi-(F_{1,2,3}\to\tilde F_{1,2,3})\}\ ,$$
$$I_7=-\{\mathrm{Im}(F_1^*F_2)\sin\Theta_\pi-(F_{1,2,3}\to\tilde F_{1,2,3})\}\ ,$$
$$I_8=\tfrac12\mathrm{Im}(F_1^*F_3)\sin\Theta_\pi+(F_{1,2,3}\to\tilde F_{1,2,3})\ ,$$
$$I_9=-\tfrac12[\mathrm{Im}(F_2^*F_3)\sin^2\Theta_\pi+(F_{1,2,3}\to\tilde F_{1,2,3})]\ .$$

The coefficients $I_{5,6,7}$ vanish if the lepton current is pure V: this is the reason for the minus sign between the $V-A$ contributions (involving $F_{1,2,3}$) and the $V+A$ contributions (involving $\tilde F_{1,2,3}$). Integrating over the angular variables $\cos\Theta_l$, $\cos\Theta_\pi$, and Φ gives the distribution in the invariant mass variables s_π and s_l:

$$\frac{d\Gamma}{ds_\pi ds_l}=\frac{G_F^2}{2^9\pi^5M_K^5}\sin^2\Theta_C X\sigma_\pi\left[1-\frac{4m_l^2}{s_l}\right]^2$$
$$\times\frac13(H_1+H_2+H_3)+(H_{1,2,3}\to\tilde H_{1,2,3})\ , \tag{18}$$

with

$$H_1=X^2C^2M_K^8\left\{8|\eta_{+-}|^2\frac{1}{s_l^2}\frac{\beta^2}{s^2}A_3+4\frac{|g_P|^2}{M_K^4}\frac{1}{(s_\pi-M_K^2)^2}\right\}$$
$$+s^2\sigma_\pi^2\left\{8C^2M_K^8|\eta_{+-}|^2\frac{1}{s_l^2s^2}A_3+\frac13\eta_d^2+4CM_K^4|\eta_{+-}|\eta_d\frac{1}{s_ls}A_2\sin\alpha\right\}$$
$$-Xs\sigma_\pi\left\{16C^2M_K^8|\eta_{+-}|^2\frac{\beta}{s_l^2s^2}A_3+4CM_K^4|\eta_{+-}|\eta_d\frac{\beta}{s_ls}A_2\sin\alpha\right\}\ ,$$
$$H_2=s_\pi s_l\sigma_\pi^2\left\{8C^2M_K^8|\eta_{+-}|^2\frac{1}{s_l^2s^2}(A_4-A_3)+\frac23\eta_d^2+4CM_K^4|\eta_{+-}|\eta_d\frac{1}{s_ls}(A_1-A_2)\sin\alpha\right\}\ ,$$
$$H_3=\frac23 s_\pi\sigma_\pi^2X^2C^2\frac{1}{s_l}(0.76)^2\ ,$$
$$A_1=\frac1\beta\ln\frac{1+\beta}{1-\beta}\ ,\quad A_2=\frac{1}{\beta^2}\left[-2+\frac1\beta\ln\frac{1+\beta}{1-\beta}\right]\ ,$$
$$A_3=\frac{1}{\beta^3}\left[\frac{\beta}{1-\beta^2}-\frac12\ln\frac{1+\beta}{1-\beta}\right],\quad A_4=\frac1\beta\left[\frac{\beta}{1-\beta^2}+\frac12\ln\frac{1+\beta}{1-\beta}\right]\ ,$$

where $\alpha=\delta_0+\Phi_{+-}-\delta_1$ and $\tilde{H}_{1,2,3}$ are obtained from $H_{1,2,3}$ by setting $\eta_d=0$. The resulting spectra in s_π and s_l are shown in Figs. 1(a) and 1(b). These are in good agreement with those in Ref. [1].[5] Note that $d\Gamma/ds_l$ is dominated by small values of s_l, while $d\Gamma/ds_\pi$ has a broad distribution. These spectra are essentially insensitive to the charge radius and direct *CP*-violating contributions being dominated by the bremsstrahlung and *M*1 terms in the amplitude. The integrated decay rate is

$$B(K_L\to\pi^+\pi^-e^+e^-)=(1.1\times10^{-7})_{\rm br}+(1.7\times10^{-7})_{\rm mag}+(4.6\times10^{-9})_{\rm CR}\approx2.8\times10^{-7}\,. \quad (19)$$

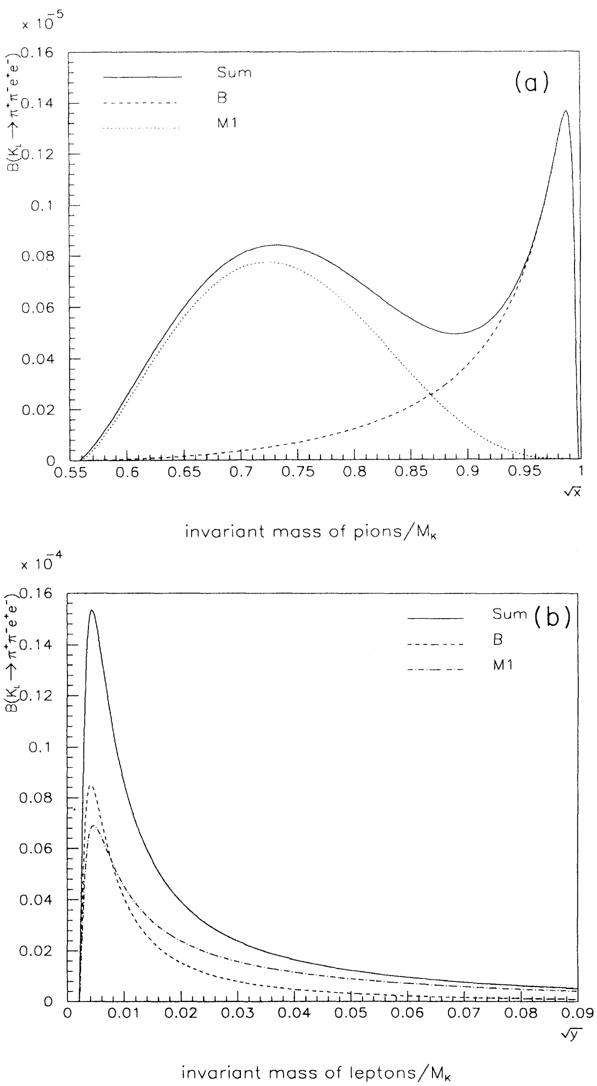

FIG. 1. Differential spectrum (a) $d\Gamma/d\sqrt{x}$ and (b) $d\Gamma/d\sqrt{y}$ for $K_L\to\pi^+\pi^-e^+e^-$, where $\sqrt{x}$ and $\sqrt{y}$ are the invariant masses of $\pi^+\pi^-$ and e^+e^-, normalized to M_K.

[5]In Fig. 2(b) of Ref. [1], the threshold should be at $\sqrt{s_l}=0.002M_K$.

Replacing g_P by g_P', as indicated in Eq. (9), changes the third term to 9.2×10^{-10}. The contribution of direct *CP* violation to the branching ratio is 1.8×10^{-16} (case *V*, *A*) and 6.4×10^{-13} (case *M*).

IV. *CP*-VIOLATING OBSERVABLES

As shown in the previous section [Eq. (12)], the differential decay spectrum of $K_L\to\pi^+\pi^-e^+e^-$ has the form

$$d\Gamma\sim I(s_\pi,s_l,\cos\Theta_\pi,\cos\Theta_l,\Phi)ds_\pi ds_l d\cos\Theta_\pi d\cos\Theta_l d\Phi\,,$$

where *I* has the expansion given in Eq. (16). To identify the *CP*-violating terms in this expansion, we note that under the *CP* transformation $\mathbf{p}_\pm\to-\mathbf{p}_\mp$, $\mathbf{k}_\pm\to-\mathbf{k}_\mp$ so that

$$\begin{aligned}&\cos\Theta_\pi\to-\cos\Theta_\pi\,,\\&\sin\Theta_\pi\to+\sin\Theta_\pi\,,\\&\cos\Theta_l\to-\cos\Theta_l\,,\\&\sin\Theta_l\to+\sin\Theta_l\,,\\&\cos\Phi\to+\cos\Phi\,,\\&\sin\Phi\to-\sin\Phi\,.\end{aligned} \quad (20)$$

It follows that the terms I_4, I_6 I_7, and I_9 are CP violating. Referring to the expression for I_6 in terms of the form factors f, $\tilde{f}$, g, $\tilde{g}$, h, and $\tilde{h}$ we find that $I_6\sim[\mathrm{Re}(F_2^*F_3)-\mathrm{Re}(\tilde{F}_2^*\tilde{F}_3)]$ is zero. We are thus left with three observable *CP*-violating coefficients: I_4, I_7, and I_9. Similarly, the *CP*-conserving term $I_5\sim[\mathrm{Re}(F_1^*F_3)-\mathrm{Re}(\tilde{F}_1^*\tilde{F}_3)]$ vanishes, so that there are only four *CP*-conserving coefficients: $I_{1,2,3,8}$.

Integrating over s_π, s_l, and $\cos\Theta_\pi$, we have

$$\frac{d\Gamma}{d\cos\Theta_l d\Phi}=K_1+K_2\cos2\Theta_l+K_3\sin^2\Theta_l\cos2\Phi+K_4\sin2\Theta_l\cos2\Phi+K_5\sin\Theta_l\cos\Phi+K_6\cos\Theta_l+K_7\sin\Theta_l\cos\Phi+K_8\sin2\Theta_l\times\sin\Phi+K_9\sin^2\Theta_l\sin2\Phi\,. \quad (21)$$

CP violation manifests itself in the constants K_4, K_7, and K_9. These constants have been evaluated numerically and are listed in Table I).[6] The only significant *CP*-violating coefficient is K_9. This is the term that is responsible for the *CP*-violating asymmetry in the Φ distribution which was calculated in Ref. [1]. The value of K_9/K_1 corresponds to an asymmetry $\mathcal{A}$ [defined in Eq. (1)] equal to

[6]The results given in Table I are compatible with the preliminary results for K_i/K_1 reported in [9], except that K_5/K_1 should be zero.

TABLE I. (a) *CP*-conserving coefficients in the differential decay spectrum of Eq. (21), normalized to K_1, for different values of the K^0 charge radius coefficient. (b) *CP*-violating coefficients, normalized to K_1. (c) Ratio of direct to indirect *CP* violation in the coefficients K_4 and K_9 for the cases $H_{\rm SD}^{V,A}$ and $H_{\rm SD}^{M}$ for $\sqrt{s_l} > 2m_e$, $\sqrt{s_l} > 100$ MeV, and $\sqrt{s_l} > 180$ MeV, respectively. The total branching ratio for the three different cuts in the invariant e^+e^- mass is also indicated.

(a) *CP*-conserving coefficients

	$g_P=0$	g_P	g_P'
K_2/K_1	0.297	0.282	0.294
K_3/K_1	0.180	0.178	0.180
K_8/K_1	0	-3.1×10^{-3}	-2.8×10^{-3}

(b) *CP*-violating coefficients

	$g_P=0$	g_P	g_P'	Comment
K_4/K_1	0	-1.33×10^{-2}	-8.68×10^{-3}	Dominant indirect $\not{C}P$
$\lvert K_7/K_1\rvert_{V,A}$	0	2.1×10^{-6}	0.9×10^{-6}	Direct $\not{C}P$ only
$\lvert K_7/K_1\rvert_{M}$	0	0	0	
K_9/K_1	-0.309	-0.305	-0.308	Dominant indirect $\not{C}P$

(c) Direct versus indirect *CP* violation

		$\sqrt{s_l} > 2m_l$	$\sqrt{s_l} > 100$ MeV	$\sqrt{s_l} > 180$ MeV
$\frac{(K_4)\ \text{direct}}{(K_4)\ \text{indirect}}$	V,A:	0.15×10^{-5}	3.64×10^{-5}	4.5×10^{-4}
	M:	5.43×10^{-4}	1.39×10^{-4}	9.07×10^{-4}
$\frac{(K_9)\ \text{direct}}{(K_9)\ \text{indirect}}$	V,A:	0.95×10^{-5}	2.06×10^{-4}	5.29×10^{-4}
	M:	4.18×10^{-4}	8.43×10^{-4}	1.07×10^{-3}
$B(K_L\to\pi^+\pi^-e^+e^-)$		2.8×10^{-7}	4.0×10^{-9}	2.3×10^{-11}

$$\mathcal{A} = -\frac{2}{\pi}\left\{\frac{\frac{2}{3}(K_9/K_1)}{1-\frac{1}{3}(K_2/K_1)}\right\} \approx 14\% \tag{22}$$

in complete agreement with the result in [1]. The dependence of this asymmetry on $\sqrt{s_\pi}$ and $\sqrt{s_l}$ is shown in Figs. 2(a) and 2(b), where we have differentiated between the cases g_P and g_P'. Note that the asymmetry for large e^+e^- masses is particularly sensitive to the magnitude of the charge radius term. Since the rate is dominated by small values of $\sqrt{s_l}$, however, the integrated asymmetry is almost insensitive to the choice g_P or g_P'.

It may be noted that the coefficient K_7 depends on the existence of an axial-vector electron current $\bar{e}\gamma_\mu\gamma_5 e$ in the matrix element of $K_L\to\pi^+\pi^-e^+e^-$, which is induced by the short-distance Hamiltonian. In this sense, K_7 is a measure of direct *CP* violation. As seen from Table I, $(K_7/K_1)_{V,A}\approx10^{-6}$, which is negligibly small. The *CP*-violating ratios K_4/K_1, K_7/K_1, and K_9/K_1 are also plotted as functions of $\sqrt{s_\pi}$ and $\sqrt{s_l}$ in Figs. 3(a) and 3(b).[7]

A perusal of Table I shows that the ratio of direct to indirect *CP* violation in the coefficients K_4 and K_9 is at most of order 10^{-4}–10^{-3}. This is the case for the V,A type short-distance interaction given in Eq. (6), as well as the "magnetic"-type interaction discussed in the Appendix.

It is encouraging to note that 20 events of the decay $K_L\to\pi^+\pi^-e^+e^-$ have recently been recorded [10], and that considerable increase of statistics is expected. It is likely that some of the characteristics of this decay calculated in this paper (and in Ref. [1]) can soon be compared with data. It would be gratifying if the large asymmetry in the Φ distribution, given in Eq. (22), could be verified, since it is one of the few cases of a *CP* -violating observable where a quantitative prediction is possible.

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[7]The ratios refer to the coefficients which appear in an expansion of $d\Gamma/d\cos\Theta_l d\Phi ds_\pi$ or $d\Gamma/d\cos\Theta_l d\Phi ds_l$ analogous to Eq. (21).

APPENDIX

1. Short-distance matrix element: Case *V*, *A*

The short-distance Hamiltonian given in Eq. (6),

$$H_{\rm SD}^{V,A}=\frac{G_F}{\sqrt{2}}\sin\Theta_C\alpha[\bar{s}\gamma_\mu(1-\gamma_5)d][\bar{e}\gamma^\mu(F_V-F_A\gamma_5)e]\ , \tag{A1}$$

is a local interaction of the current $\bar{s}\gamma_\mu(1-\gamma_5)d$ with a linear combination of the V and A currents $\bar{e}\gamma_\mu e$ and $\bar{e}\gamma_\mu\gamma_5 e$. The coefficients F_V and F_A are complex functions depending on the quark mixing parameters and the mass of the top quark. The amplitude of the decay $K^0\to\pi^+\pi^-e^+e^-$ is

$$A(K^0\to\pi^+\pi^-e^+e^-)=-\frac{G_F}{\sqrt{2}}\sin\Theta_C\alpha\langle\pi^+\pi^-|\bar{s}\gamma_\mu(1-\gamma_5)d|K^0\rangle\bar{u}\gamma^\mu(F_V-F_A\gamma_5)v\ . \tag{A2}$$

We parametrize the matrix element $\langle\pi^+\pi^-|\bar{s}\gamma_\mu(1-\gamma_5)d|K^0\rangle$ in the standard way:

$$\langle\pi^+\pi^-|\bar{s}\gamma_\mu(1-\gamma_5)d|K^0\rangle=\frac{i}{M_K}\left[F(p_++p_-)_\mu+G(p_+-p_-)_\mu+i\frac{H}{M_K^2}\epsilon_{\mu\nu\rho\sigma}p_K^\nu(p_++p_-)^\rho(p_+-p_-)^\sigma\right]\ , \tag{A3}$$

where F, G, H are real, in the absence of final state phases. The amplitude (A2) then becomes

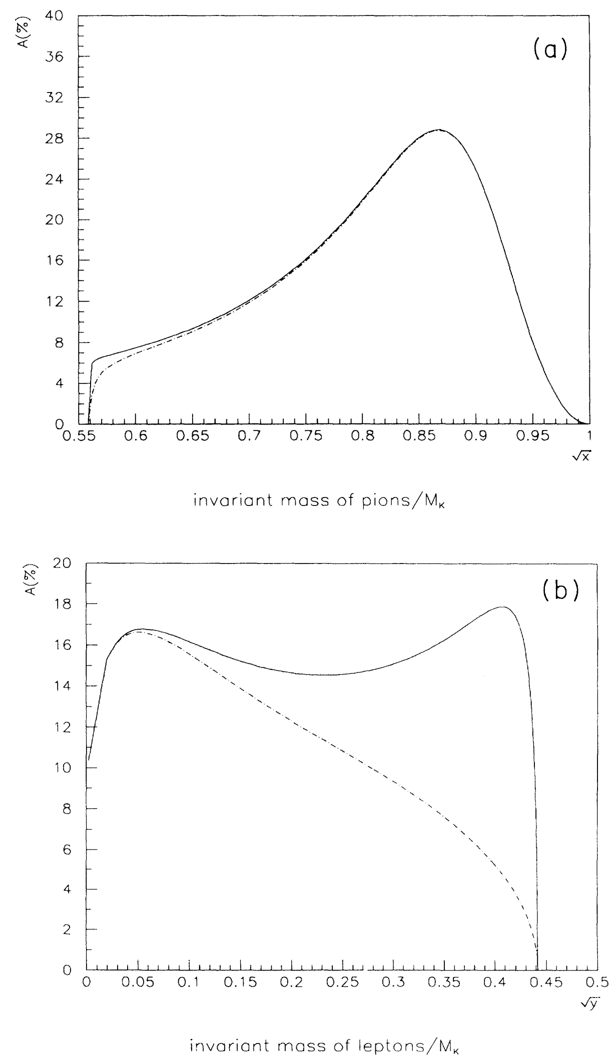

FIG. 2. *CP*-violating asymmetry in the Φ distribution as function of (a) $\sqrt{x}$ and (b) $\sqrt{y}$ for g_P (solid line) and g_P' (dotted line).

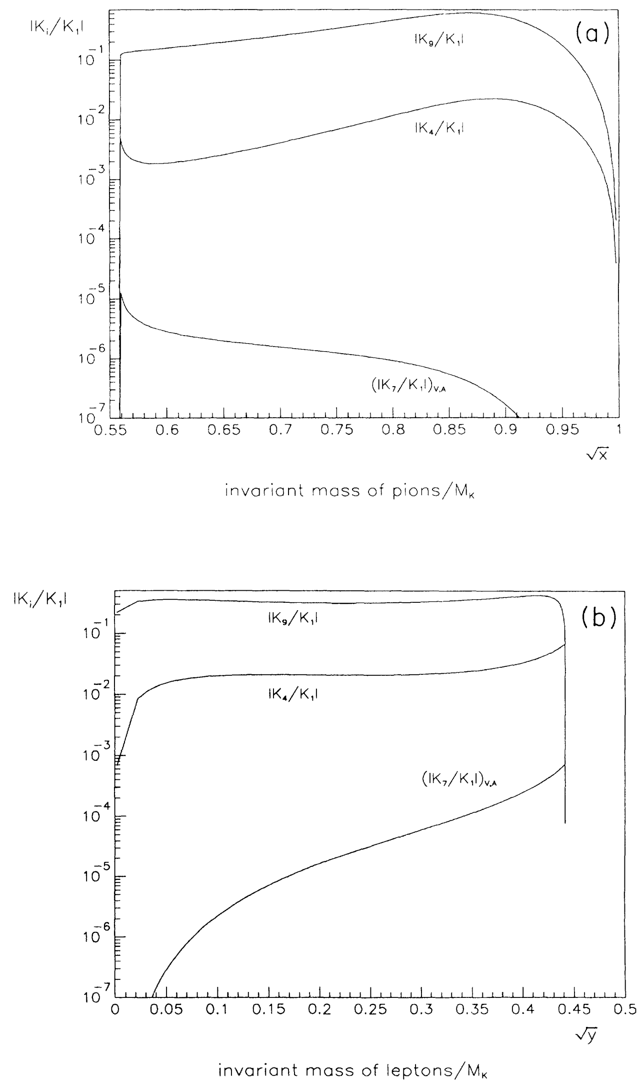

FIG. 3. *CP*-violating ratios K_4/K_1, K_7/K_1, and K_9/K_1 as functions of (a) $\sqrt{x}$ and (b) $\sqrt{y}$.

$$A(K^0\to\pi^+\pi^-e^+e^-)=-\frac{G_F}{\sqrt{2}}\sin\Theta_C\alpha\frac{i}{M_K}\left[F(p_++p_-)_\mu+G(p_+-p_-)_\mu + i\frac{H}{M_K^2}\epsilon_{\mu\nu\rho\sigma}p_K^\nu(p_++p_-)^\rho(p_+-p_-)^\sigma\right]\cdot\bar{u}\gamma^\mu(F_V-F_A\gamma_5)v \ . \quad \text{(A4)}$$

We now use *CPT* invariance to obtain the amplitude for $\bar{K}^0\to\pi^+\pi^-e^+e^-$:

$$A(\bar{K}^0\to\pi^+\pi^-e^+e^-)=+\frac{G_F}{\sqrt{2}}\sin\Theta_C\alpha\frac{i}{M_K}\left[F(p_++p_-)_\mu-G(p_+-p_-)_\mu + i\frac{H}{M_K^2}\epsilon_{\mu\nu\rho\sigma}p_K^\nu(p_++p_-)^\rho(p_+-p_-)^\sigma\right]\bar{u}\gamma^\mu(F_V^*-F_A^*\gamma_5)v \ . \quad \text{(A5)}$$

Taking the difference of (A4) and (A5), we obtain the decay amplitude of $K_2=(K^0-\bar{K}^0)/\sqrt{2}i$ as

$$A(K_2\to\pi^+\pi^-e^+e^-)=-\frac{G_F}{\sqrt{2}}\sin\Theta_C\alpha\frac{1}{M_K}\frac{1}{\sqrt{2}}\left[F(p_++p_-)_\mu 2\,\mathrm{Re}\bar{u}\gamma^\mu(F_V-F_A\gamma_5)v + iG(p_+-p_-)_\mu 2\,\mathrm{Im}\bar{u}\gamma^\mu(F_V-F_A\gamma_5)v + \frac{i}{M_K^2}H\epsilon_{\mu\nu\rho\sigma}p_K^\nu(p_++p_-)^\rho(p_+-p_-)^\sigma 2\,\mathrm{Re}\bar{u}\gamma^\mu(F_V-F_A\gamma_5)v\right] . \quad \text{(A6)}$$

The terms proportional to F and H in Eq. (A6) are *CP*-conserving, representing $I=0$ s-wave and $I=1$ p-wave configurations of the $\pi^+\pi^-$ pair (analogous to the charge-radius and magnetic dipole contributions). Our interest resides in the third term proportional to G, which involves the imaginary parts of the functions F_V and F_A, and accordingly represents a direct *CP*-violating effect. In the notation of Dib, Dunietz, and Gilman [4], the imaginary parts of F_V and F_A are

$$\mathrm{Im}F^{V,A}=\mathrm{Im}\left\{\frac{V_{ts}^*V_{td}V_{us}V_{ud}^*}{|V_{us}^*V_{ud}|^2}\right\}(c_{7,t}^{V,A}-c_{7,c}^{V,A}) = s_2s_3s_\delta c_{V,A} \ . \quad \text{(A7)}$$

For a top quark of mass 150 GeV, the parameters $c_{V,A}$ are approximately $c_V\approx c_A\approx\frac{1}{2}$ [4]. The form factor G which appears in the matrix element $\langle\pi^+\pi^-|\bar{s}\gamma_\mu(1-\gamma_5)d|K^0\rangle$ can be related by isospin to the corresponding form factor in the matrix element $\langle\pi^+\pi^-|\bar{s}\gamma_\mu(1-\gamma_5)u|K^+\rangle$ which describes K_{l4} decay. This yields $G=M_K/f_\pi$ (with $f_\pi=130$ MeV). Altogether, therefore, the direct *CP*-violating contribution to the decay $K_2\to\pi^+\pi^-e^+e^-$ is

$$\mathcal{M}_{\mathrm{SD}}^{V,A}=-\frac{G_F}{\sqrt{2}}\sin\Theta_C\alpha\frac{1}{M_K}g_{\mathrm{SD}}^{V,A}(p_+-p_-)_\mu \times\bar{u}(k_-)(c_V-c_A\gamma_5)v(k_+) \quad \text{(A8)}$$

with

$$g_{\mathrm{SD}}^{V,A}=i(s_2s_3s_\delta)\sqrt{2}\left[\frac{M_K}{f_\pi}\right]e^{i\delta_1(s_\pi)} \quad \text{(A9)}$$

which is the result given in Eq. (3) and (7).

2. Short-distance matrix element: Case M

In addition to the local V,A coupling given by Eq. (A1), the short-distance interaction gives rise to a magnetic coupling of the form $s\to d+\gamma$, which produces an effective Hamiltonian for $s\to de^+e^-$:

$$H_{\mathrm{SD}}^M=\frac{G_F}{\sqrt{2}}\alpha\sum_{q=u,c,t}V_{qs}^*V_{qd}F_M(x_q)Q_\mu^M\frac{1}{k^2}\bar{e}\gamma^\mu e \ , \quad \text{(A10)}$$

where

$$Q_\mu^M=\bar{s}[im_s\sigma_{\mu\nu}q^\nu(1-\gamma_5)+im_d\sigma_{\mu\nu}q^\nu(1+\gamma_5)]d \ , \quad \text{(A11)}$$

and the function F_M is given by [4]

$$F_M(x_q)=\frac{(8x_q^2+5x_q-7)x_q}{24\pi(x_q-1)^3}-\frac{(3x_q-2)x_q^2}{4\pi(x_q-1)^4}\ln x_q \ , \quad \text{(A12)}$$

with $x_q=m_q^2/m_W^2$. For a top-quark mass of 130 GeV, we have $F_M(x_t)\approx0.051$, which is the dominant contribution to Eq. (A10).

To evaluate the contribution of H_{SD}^M of the decay $K_L\to\pi^+\pi^-e^+e^-$, we require the matrix element $\langle\pi^+\pi^-|Q_\mu^M|K^0\rangle$. For a rough estimate we use

$$\langle\pi^+\pi^-|Q_\mu^M|K^0\rangle\approx\frac{2(m_s-m_d)}{f_\pi^2}[p_-\cdot kp_{+\mu}-p_+\cdot kp_{-\mu}] \ , \quad \text{(A13)}$$

which is suggested by the work of Dib and Peccei [11]. This leads to the following short-distance matrix element for $K_L\to\pi^+\pi^-e^+e^-$:

$$\mathcal{M}_{\rm SD}^{M} = -\frac{G_F}{\sqrt{2}}\sin\Theta_C\alpha(0.051)(s_2s_3s_\delta)\sqrt{2}i\frac{2(m_s-m_d)}{f_\pi^2}e^{i\delta_1(s_\pi)}$$
$$\times\frac{1}{2}\{-(p_+-p_-)\cdot k(p_++p_-)_\mu+(p_++p_-)\cdot k(p_+-p_-)_\mu\}\frac{1}{s_l}\bar{u}\gamma_\mu v\ , \qquad \text{(A14)}$$

where $f_\pi = 93$ MeV.

Essentially, the operator (A11) induces a *CP*-violating $E1$ amplitude in $K_L\to\pi^+\pi^-\gamma$, which in turn gives the Dalitz pair amplitude in Eq. (A14). As compared to the V, A matrix element (A8), the case M has a factor $1/s_l$, which tends to enhance its effects at small e^+e^- masses. In the notation of Eq. (14), the form factors f, $\tilde{f}$, g, $\tilde{g}$ in the present model are

$$f=\tilde{f}=CM_K^4\left\{|\eta_{+-}|e^{i(\delta_0+\Phi_{+-})}\frac{1}{s_l}\frac{-4\beta\cos\Theta_\pi}{s(1-\beta^2\cos^2\Theta_\pi)}+2\frac{g_P}{M_K^2}e^{i\delta_0(s_\pi)}\frac{1}{s_\pi-M_K^2}\right\}-i\eta_M\frac{1}{s_l}s\beta\cos\Theta_\pi e^{i\delta_1}\ ,$$
$$g=\tilde{g}=CM_K^4|\eta_{+-}|e^{i(\delta_0+\Phi_{+-})}\frac{1}{s_l}\frac{4}{s(1-\beta^2\cos^2\Theta_\pi)}+i\eta_M\frac{M_K^2-s_\pi}{s_l}e^{i\delta_1}\ , \qquad \text{(A15)}$$

where

$$\eta_M=M_K\frac{2(m_s-m_d)}{f_\pi^2}\frac{\alpha}{\sqrt{2}}s_2s_3s_\delta(0.051)\approx 2.27\times 10^{-6} \qquad \text{(A16)}$$

(taking $m_s-m_d=150$ MeV, $s_2s_3s_\delta=0.5\times 10^{-3}$).

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