Vector boson distributions inside the quark and gluon

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(Received 21 June 1993)

The strong interaction effect on the vector boson distributions is discussed by using modified Altarelli-Parisi equations. The moments of the distributions in the leading approximation and in the next-to-leading approximation are calculated. The x dependence of the distributions is obtained by using a method based on the Jacobi polynomial expansion. The result shows that the strong interaction effect modifies not only the x dependence but also the Q dependence for the vector boson distributions even at the leading approximation, which is different from the result in the discussion of photon distributions.

PACS number(s): 13.85.−t, 12.38.−t, 13.88.+e, 14.80.Er

I. INTRODUCTION

In very high energies, a new class of quasireal vector boson collisions becomes relevant for discussion [1–5]. These processes will constitute a new tool for exploring the deep properties of electroweak interactions, the true nature of $W^{\pm}$ and Z^0 bosons, the origin of gauge symmetry breaking and the still unknown scalar sector [6].

The effective vector boson approximation [1,2] is an extension to massive weak gauge bosons of the Fermi-Weizacker-Williams approximation in QED. Some authors have discussed the strong interaction effect for this kind of approximation in QED. By using various methods including the operator product expansion [7], ladder techniques [8], and modified Altarelli-Parisi (AP) equations [9–11], they have found that the strong interaction effect does not modify the Q^2 dependence but changes the x dependence.

Similarly, we can consider the strong interaction effect on the vector boson distribution (VBD). By using AP-type equations and the Jacobi polynomial expansion procedure, we consider the VBD inside a quark and gluon and obtain their apparent simple expansions in this paper. The VBD inside a proton can be calculated by convolution. In interactions with energies in the TeV range, the parton sea of a proton also contains the $W^{\pm}$ and Z^0 gauge bosons. The apparent simple expression of VBD inside a proton is useful for establishing the general idea for this kind of parton distribution, and it makes the calculation involving gauge bosons in the intermediate states considerably simpler.

II. VBD INSIDE QUARK DUE TO WEAK INTERACTION

We will use the full distribution derived in Ref. [2], but we will generalize it to the case in which the parent quark is polarized. The parity violation and the polarization effect may be the essential features of the interaction, and our result can also be used in the discussion of such a problem. The vertex among quarks and weak bosons can be expressed

$$ig\gamma_\mu(C_V - C_A\gamma_5) , \tag{1}$$

where $g=e/\sin\theta_W$, $C_V=C_A=1/2\sqrt{2}$ for charged weak bosons $W^{\pm}$ and $C_V=(T_3-2e_f\sin^2\theta_W)/2\cos\theta_W$, $C_A=T_3/2\cos\theta_W$ for Z^0 bosons. Following the procedure in Ref. [2], we find that the distributions of vector boson V with transverse polarization ($a=1,2$) or longitudinal polarization ($a=0$) inside a quark with helicity h ($=\pm 1$) can be expressed by

$$F^a_{V/q(h)}(x,r)=\frac{\alpha_B}{2\pi}f^a_{V/q(h)}(x,r) , \tag{2}$$

where

$$\begin{aligned} xf^1_{V/q(h)} &= 2[(A-Bh)(I_1+I_2)+(B-Ah)(I_1-I_2)] , \\ xf^2_{V/q(h)} &= 2[(A-Bh)(I_1+I_2)-(B-Ah)(I_1-I_2)] , \\ xf^0_{V/q(h)} &= 4(A-Bh)I_3 . \end{aligned} \tag{3}$$

In these formulas, $\alpha_B=\alpha/4\sin^2\theta_W$, $A=|C_V|^2+|C_A|^2$, $B=2\,\mathrm{Re}(C_V C_A^*)$,

$$\begin{aligned} I_1(x,r) &= (1+2r)\{[(1-x+rx)^2+2rx(1-r)(1-x+rx)]\ln[(1-x+rx)/rx]-(1-2r)\ln x+(1-r)(1-x)\} , \\ I_2(x,r) &= (1+2r)\{\ln[(1-x+rx)/r]-(1-r)(1-x)/(1-x+rx)\} , \\ I_3(x,r) &= 2(1+2r)\{(1-r)(1-x)-r\ln[(1-x+rx)/r]\} , \end{aligned} \tag{4}$$

and $r=M_V^2/Q^2$, where the variable Q will characterize the energy scale at which the effective vector boson approximation is used. To deduce Eqs. (2)–(4) the following consideration has also been noticed: they are applica-

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ble only if $r \ll 1$, but we need to have the initial condition $f^a_{V/q}(x, r=1)=0$.

The helicity average distributions are

$$f^a_{V/q}=\frac{1}{2}(f^a_{V/q(+)}+f^a_{V/q(-)}) , \tag{5}$$

and we have

$$\begin{aligned} xf^1_{V/q}&=\alpha_1 I_1+\alpha_2 I_2 , \\ xf^2_{V/q}&=\alpha_2 I_1+\alpha_1 I_2 , \\ xf^0_{V/q}&=\alpha_0 I_3 , \end{aligned} \tag{6}$$

where

$$\alpha_1=2(A+B) , \quad \alpha_2=2(A-B) , \quad \alpha_0=4A . \tag{7}$$

If we set $\boldsymbol{\alpha}=(\alpha_1,\alpha_2,\alpha_0)$, we find that $\boldsymbol{\alpha}=(1,0,1)$ for charged boson $W^{\pm}$ distributions, $\alpha=(0.3122,0.0611,0.3732)$ for Z^0 distributions inside p-type flavor quarks (u,c,t), and $\alpha=(0.4655,0.0158,0.4808)$ for Z^0 distributions inside n-type flavor quarks (d,s,b).

In leading approximation ($r \to 0$) we have

$$I_1=\ln\frac{Q^2}{M_V^2}(1-x)^2 , \quad I_2=\ln\frac{Q^2}{M_V^2} , \quad I_3=(1-x) . \tag{8}$$

The two independent transverse distributions of W bosons in this approximation are

$$F^1_{W/q}=\frac{\alpha_B}{2\pi}\ln\frac{Q^2}{M_V^2}\frac{1}{x} , \quad F^2_{W/q}=\frac{\alpha_B}{2\pi}\ln\frac{Q^2}{M_V^2}\frac{(1-x)^2}{x} . \tag{9}$$

Another two independent distributions can be introduced by the definition

$$F^T_{W/q}=F^1_{W/q}+F^2_{W/q} , \quad F^{\bar{T}}_{W/q}=F^1_{W/q}-F^2_{W/q} , \tag{10}$$

and then we have ($r \to 0$)

$$\begin{aligned} F^T_{W/q}&=\frac{\alpha_B}{2\pi}\ln\frac{Q^2}{M_V^2}\frac{1+(1-x)^2}{x} , \\ F^{\bar{T}}_{W/q}&=\frac{\alpha_B}{2\pi}\ln\frac{Q^2}{M_V^2}(2-x) . \end{aligned} \tag{11}$$

F^T and $F^{\bar{T}}$ are the average transverse and parity-violating transverse distributions, respectively, which are the names given by some authors [4,6].

III. MASTER EQUATIONS

Consider the effect of strong interaction on the VBD means that we should consider also the VBD in the gluon. Similar to the case for the quark and gluon distributions inside a photon or the photon distributions inside a quark and gluon [9–11], we assume that the master equations for VBD in a quark V^a_q and in a gluon V^a_g are

$$\begin{aligned} \frac{dV^a_q(x,t)}{dt}&=\frac{1}{2\pi b}(V^a_q\otimes P_{qq}+V^a_g\otimes P_{gq}) \\ &\quad+\frac{\alpha_B}{2\pi}\frac{d}{dt}f^a_{V/q}(x,t) , \\ \frac{dV^a_g(x,t)}{dt}&=\frac{1}{2\pi b}\left[\sum_q^{2f}V^a_q\otimes P_{qg}+V^a_g\otimes P_{gg}\right] , \end{aligned} \tag{12}$$

where $t=\ln[\ln(Q^2/\Lambda^2)/\ln(Q_0^2/\Lambda^2)]$, $Q_0=M_V$, and $b=(33-2f)/12\pi$. The P_{AB} are the AP evolution functions [9]. For the charged boson W^+ (or W^-) distributions, $f=3$, because the p-type flavor quark and n-type antiquark (or n-type quark and p-type anti-quark) are involved. For Z^0 distributions, $f=6$. The Born term in the modified AP-type equations (12),

$$\frac{\alpha_B}{2\pi}\frac{d}{dt}f^a_{V/q}(x,t) ,$$

comes from the pure electroweak interaction. As expected, in the absence of strong interaction $[\alpha_S=1/b\ln(Q^2/\Lambda^2)=0]$ we have no chance of finding the weak boson in a gluon and

$$V^a_q=\frac{\alpha_B}{2\pi}f^a_{V/q}(x,t) , \quad V^a_g=0 .$$

The functions V^a_q and $f^a_{V/q}$ can be written as

$$\begin{aligned} V^a_q&=(V^a_q)^{\mathrm{NS}}+\frac{1}{2f}\Sigma^a_q , \\ f^a_{V/q}&=(f^a_{V/q})^{\mathrm{NS}}+\langle f^a_V\rangle , \end{aligned} \tag{13}$$

where

$$\langle f^a_V\rangle=\frac{1}{f}\sum_q^f f^a_{V/q} . \tag{14}$$

Equation (12) can then be expressed by

$$\frac{d(V^a_q)^{\mathrm{NS}}}{dt}=\frac{1}{2\pi b}(V^a_q)^{\mathrm{NS}}\otimes P_{qq}+\frac{\alpha_B}{2\pi}\frac{d}{dt}(f^a_{V/q})^{\mathrm{NS}} , \tag{15}$$

and

$$\begin{aligned} \frac{d\Sigma^a_q(x,t)}{dt}&=\frac{1}{2\pi b}(\Sigma^a_q\otimes P_{qq}+2fV^a_g\otimes P_{gq}) \\ &\quad+\frac{\alpha_B}{2\pi}2f\frac{d}{dt}\langle f^a_V\rangle , \\ \frac{dV^a_g(x,t)}{dt}&=\frac{1}{2\pi b}(\Sigma^a_q\otimes P_{qg}+V^a_g\otimes P_{gg}) . \end{aligned} \tag{16}$$

To solve the master equations, we can use the moment method. After taking the moment, Eqs. (15) and (16) can be written as

$$\frac{d[V^a_q(n,t)]^{\mathrm{NS}}}{dt}=d^{qq}_n[V^a_q(n,t)]^{\mathrm{NS}}+\frac{\alpha_B}{2\pi}\frac{d}{dt}[f^a_{V/q}(n,t)]^{\mathrm{NS}} , \tag{17}$$

$$\frac{d\Sigma_q^a(n,t)}{dt}=d_n^{qq}\Sigma_q^a(n,t)+2fd_n^{gq}V_g^a(n,t)+\frac{\alpha_B}{2\pi}2f\frac{d}{dt}\langle f_V^a(n,t)\rangle ,$$
$$\frac{dV_g^a(n,t)}{dt}=\frac{d_n^{qg}}{2f}\Sigma_q^a(n,t)+d_n^{gg}V_g^a(n,t) , \tag{18}$$

where

$$\begin{bmatrix} d_n^{qq} \\ d_n^{gq} \\ d_n^{qg} \\ d_n^{gg} \end{bmatrix}=\frac{1}{2\pi b}\int_0^1 dx\, x^{n-1}\begin{bmatrix} P_{qq} \\ P_{gq} \\ 2fP_{qg} \\ P_{gg} \end{bmatrix} , \tag{19}$$

and

$$f_{V/q}^a(n,t)=\int_0^1 dx\, x^{n-1} f_{V/q}^a(x,t) . \tag{20}$$

The moments of Born term $f_{V/q}^a(n,t)$ can be expressed by

$$f_{V/q}^1(n,t)=\ln\frac{Q^2}{M_V^2}(\alpha_1 C_n^1+\alpha_2 C_n^2)\equiv\ln\frac{Q^2}{M_V^2}C_{n,V/q}^1 ,$$
$$f_{V/q}^2(n,t)=\ln\frac{Q^2}{M_V^2}(\alpha_2 C_n^1+\alpha_1 C_n^2)\equiv\ln\frac{Q^2}{M_V^2}C_{n,V/q}^2 , \tag{21}$$
$$f_{V/q}^0(n,t)=\alpha_0 C_n^0\equiv C_{n,V/q}^0$$

and

$$\ln\frac{Q^2}{M_V^2}C_n^i=\int_0^1 dx x^{n-2}I_i(x,t)\quad (i=1,2,3) . \tag{22}$$

In the leading approximation, from Eqs. (6) and (8) we have

$$C_n^1=2/n(n-1)(n+1) ,\quad C_n^2=1/(n-1) ,$$
$$C_n^0=2/n(n-1) . \tag{23}$$

IV. SOLUTION OF MOMENT EQUATION

Equation (17) for the nonsinglet sector of the VBD has the general solution

$$[V_q^a(n,t)]^{\rm NS}=\frac{\alpha_B}{2\pi}e^{d_n^{qq}t}\left[\int dt\, e^{-d_n^{qq}t}\frac{d}{dt}[f_{V/q}^a(n,t)]^{\rm NS}+\beta_0^a\right] , \tag{24}$$

where β_0^a is given by the initial condition $[V_q^a(n,0)]^{\rm NS}=0$. In the leading-logarithm approximation, we have

$$[V_q^{1,2}(n,t)]^{\rm NS}=\frac{\alpha_B}{2\pi}\ln\frac{Q^2}{M_V^2}\frac{(C_{n,V/q}^{1,2})^{\rm NS}}{(1-d_n^{qq})}[1-e^{-(1-d_n^{qq})t}] . \tag{25}$$

Equation (18) is a set of linear inhomogeneous equations and the characteristic roots of its associate homogeneous equations are

$$\lambda_\pm=\frac{1}{2}\{d_n^{gg}+d_n^{qq}\pm[(d_n^{gg}-d_n^{qq})^2+4d_n^{qg}d_n^{gq}]^{1/2}\} . \tag{26}$$

By using the variation coefficient procedure, we can find a solution of Eq. (18):

$$\Sigma_q^a(n,t)=C_1^a(t)e^{\lambda_+t}+C_2^a(t)e^{\lambda_-t} ,$$
$$V_g^a(n,t)=\frac{1}{2fd_n^{gq}}[C_1^a(t)(\lambda_+-d_n^{qq})e^{\lambda_+t}+C_2^a(t)(\lambda_--d_n^{qq})e^{\lambda_-t}] , \tag{27}$$

where

$$C_1^a(t)=\frac{\alpha_B}{2\pi}2f\frac{\lambda_--d_n^{qq}}{\lambda_--\lambda_+}\left[\int dt\, e^{-\lambda_+t}\frac{d}{dt}\langle f_V^a(n,t)\rangle+\beta_1^a\right] ,$$
$$C_2^a(t)=\frac{\alpha_B}{2\pi}2f\frac{\lambda_+-d_n^{qq}}{\lambda_+-\lambda_-}\left[\int dt\, e^{-\lambda_-t}\frac{d}{dt}\langle f_V^a(n,t)\rangle+\beta_2^a\right] , \tag{28}$$

β_1^a and β_2^a can be determined from the initial conditions $\Sigma_q^a(n,0)=0$, $V_g^a(n,0)=0$.

In the case of the leading-logarithm order the apparent solution of Eq. (27) for the transverse sector can be obtained and expressed as

$$V_q^{1,2}(n,t)=\frac{\alpha_B}{2\pi}\ln\frac{Q^2}{\Lambda^2}\left\{\frac{(C_{n,V/q}^{1,2})^{\rm NS}}{1-d_n^{qq}}(1-e^{-(1-d_n^{qq})t})+\frac{\langle C_{n,V}^{1,2}\rangle}{\lambda_--\lambda_+}\left[\frac{\lambda_--d_n^{qq}}{1-\lambda_+}(1-e^{-(1-\lambda_+)t})-(\lambda_+\leftrightarrow\lambda_-)\right]\right\} ,$$
$$V_g^{1,2}(n,t)=\frac{\alpha_B}{2\pi}\ln\frac{Q^2}{\Lambda^2}\frac{d_n^{qg}\langle C_{n,V}^{1,2}\rangle}{\lambda_+-\lambda_-}\left[\frac{1-e^{-(1-\lambda_+)t}}{1-\lambda_+}-(\lambda_+\to\lambda_-)\right] . \tag{29}$$

If we consider the case of $Q_0=\Lambda$ and then $t\to\infty$, noting $\lambda_+, \lambda_- \le 0$, we have $[B\to\gamma, C_{n,V/q}\to e_q^2 V(n)]$

$$\gamma_q(n,t)=\frac{\alpha}{2\pi}\ln\frac{Q^2}{\Lambda^2}V(n)\left[\frac{(e_q^2)^{\rm NS}}{1-d_n^{qq}}+\langle e_q^2\rangle\frac{1-d_n^{gg}}{(1-\lambda_+)(1-\lambda_-)}\right],$$
$$\gamma_g(n,t)=\frac{\alpha}{2\pi}\ln\frac{Q^2}{\Lambda^2}V(n)\frac{\langle e_q^2\rangle d_n^{qg}}{(1-\lambda_+)(1-\lambda_-)}. \tag{30}$$

The above results have been obtained in the discussion of the photon distributions in quark and gluon [9–11]. The strong interaction effect does not modify the Q^2 dependence of the effective photon approximation in QED ($\sim\ln Q^2/\Lambda^2$), but it changes the x dependence. However, the picture is quite different for the VBD. Equation (29) shows that even for the leading-logarithm approximation the strong interaction in color space affects not only its x dependence, but also its Q^2 dependence.

In the leading approximation, the moments for the longitudinal component of the VBD in quark and gluon (V_q^0, V_g^0) are zero because $f_{V/q}^0$ is independent of Q^2.

V. DISTRIBUTION IN LEADING APPROXIMATION

In order to recover the x-dependent distributions we could perform the inverse Mellin transformation. Instead we use the method proposed early in Ref. [10]. After factoring out the singular behavior at $x\simeq 0$, the distribution can be expressed in a series of Jacobian polynomials.

We first find the distributions in leading-logarithm approximation which can reveal the feature of the problem concerned. We know from Eq. (23) that the rightmost singularity in the complex n plane of the moments in Eq. (29) occurs at $n=1$, and then $V_j^a(x,t) (j=q,g)$ can be written as

$$xV_j^a(x,t)=\frac{\alpha_B}{2\pi}\ln\frac{Q^2}{M_V^2}\phi(x,t), \tag{31}$$

$$\phi(x,t)=\sum_n^N b_n(t)P_n(x). \tag{32}$$

Now the Jacobian polynomials P_n have a very simple form:

$$P_n(x)=\sum_{k=0}^n p(n,k)x^k,$$
$$p(n,k)=(-1)^{k+n}(2n+1)^{1/2}\frac{(n+k)!}{(k!)^2(n-k)!}. \tag{33}$$

Using the orthogonal condition

$$\int_0^1 dx\, P_n(x)P_{n'}(x)=\delta_{nn'}, \tag{34}$$

we have

$$b_n(t)=\int_0^1 dx\,\phi(x,t)P_n(x)=\sum_{k=0}^n p(n,k)\phi(k+2,t). \tag{35}$$

We have also

$$\phi(x,t)=\sum_{n=0}^N C_n(t)x^n,$$
$$C_n(t)=\sum_{l\ge n}^N b_n(t)p(l,n). \tag{36}$$

The three-term approximation for the distributions of vector boson V ($W^\pm$ or Z^0) with helicity $a=1,2$ inside parton j (q or g) can be expressed by

$$xV_j^a(x,t)=\frac{\alpha_B}{2\pi}\ln\frac{Q^2}{M_V^2}(C_0+C_1x+C_2x^2). \tag{37}$$

The coefficients of three-term expansion for the VBD inside quark and gluon in leading-logarithm approximation are listed in Table I. Similarly we can also obtain, for example, five-term or ten-term expansions. In Fig. 1, we plot the three-term and five-term expressions for W (W^+ or W^-) distribution with transverse polarizations 1 and 2 in a quark. It shows that by keeping only three terms in the expansion we already have an adequate description of distributions. We also wish to point out that Eq. (37) can reproduce the values of the moments given by Eq. (29) quite precisely. For the first ten moments of W_q^1 or W_q^2,

TABLE I. The coefficients of three-term expansion for the VBD inside quark and gluon in leading-logarithm approximation.

	$Q=2$ TeV			$Q=20$ TeV		
V_j^a	C_0	C_1	C_2	C_0	C_1	C_2
W_q^1	0.9815	−2.0989	1.1312	0.9751	−2.1454	1.1911
W_q^2	0.9850	0.1363	−0.3777	0.9931	0.1042	−0.4411
W_g^1	0.0667	−0.2046	0.1475	0.0984	−0.3105	0.2279
W_g^2	0.1043	−0.1847	0.0865	0.1641	−0.3366	0.1844
Z_p^1	0.3694	−0.6708	0.3500	0.3723	−0.7097	0.3840
Z_p^2	0.3771	−0.1197	−0.0402	0.3914	−0.1827	−0.0195
Z_n^1	0.4740	−1.0009	0.5469	0.4747	−1.0436	0.5927
Z_n^2	0.4818	0.0042	−0.1650	0.4962	−0.0591	−0.1599
Z_g^1	0.0751	−0.2220	0.1575	0.1098	−0.3378	0.2456
Z_g^2	0.1115	−0.2237	0.1202	0.1730	−0.3977	0.2404

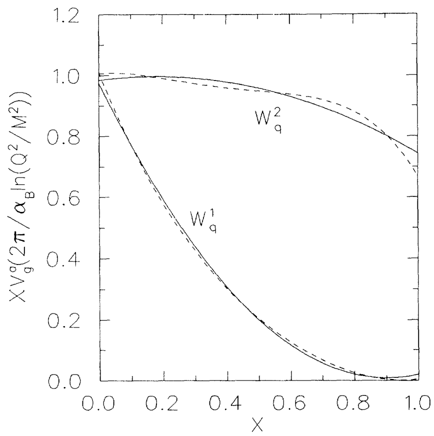

FIG. 1. Comparison between three-term (solid line) and five-term (dashed line) expressions for W_q^1 and W_q^2.

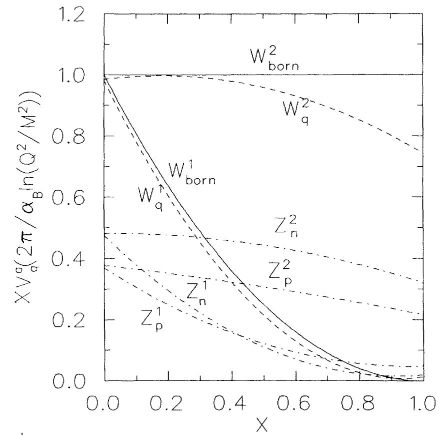

FIG. 2. Distributions for W (W^+ or W^-) and Z with the transverse polarization a (1 or 2) inside quark ($Q=20$ TeV) in leading approximation.

for example, the difference between the value given by Eqs. (29) and (37) is less than 2%.

The functions V_q^a in leading approximation at $Q=20$ TeV are also presented in Fig. 2. For comparison we plot the Born approximation

$$xW_q^{1,2}|_{\text{Born}} \equiv xF_{W/q}^{1,2} = \frac{\alpha_B}{2\pi}\ln\frac{Q^2}{M_V^2} \times \begin{cases} 1 \\ (1-x)^2 \end{cases} . \tag{38}$$

Q^2 dependence of the VBD comes from two aspects: the factor $\ln(Q^2/M_V^2)$ and the expansion coefficients C_i. In the leading approximation, the Q^2 dependence of the VBD is mainly due to the logarithm factor. The transverse VBD inside the gluon will show in the next section because we will prove that the transverse VBD inside the gluon in the leading and the next-to-leading approximation are the same.

VI. DISTRIBUTIONS IN NEXT-TO-LEADING APPROXIMATION

It is known in the case of weak interaction that the difference for the VBD between the leading and the full expressions should be noticed. In principle, we could obtain the moments for the full distribution inside quark and gluon by using Eqs. (4), (6), (24), (27), and (28). However, it is a difficult job because we do not know the correct formula of the VBD near the convolution start point ($t \sim 0$).

Let us now try to find the VBD in the next-to-leading approximation. After taking an integration by parts on Eqs. (24) and (28), the moments of the VBD in a quark and gluon can be expressed by

$$V_q^a(n,t) = \frac{\alpha_B}{2\pi}\left[f_{V/q}^a(n,t) + A_0^a e^{d_n^{qq}t} + \frac{\lambda_- - d_n^{qq}}{\lambda_- - \lambda_+} A_1^a e^{\lambda_+ t} + \frac{\lambda_+ - d_n^{qq}}{\lambda_+ - \lambda_-} A_2^a e^{\lambda_- t} \right] , \tag{39}$$

$$V_g^a(n,t) = \frac{\alpha_B}{2\pi}\frac{d_n^{qg}}{(\lambda_+ - \lambda_-)}\left(A_1^a e^{\lambda_+ t} - A_2^a e^{\lambda_- t} \right) , \tag{40}$$

where

$$\begin{aligned} A_0^a &= d_n^{qq} \int dt\, e^{-d_n^{qq}t}[f_{V/q}^a(n,t)]^{\text{NS}} + \beta_0^a , \\ A_1^a &= \lambda_+ \int dt\, e^{-\lambda_+ t}\langle f_V^a(n,t)\rangle + \beta_1^a , \\ A_2^a &= \lambda_- \int dt\, e^{-\lambda_- t}\langle f_V^a(n,t)\rangle + \beta_2^a . \end{aligned} \tag{41}$$

It is obvious that the terms, other than $f_{V/q}^a(n,t)$, in Eqs. (39) and (40) illustrate the strong interaction effect on the VBD.

Now let us consider the following approximation: the functions $f_{V/q}^a(n,t)$ in the integrand of A_i^a in Eq. (41) are taken to be their leading approximation [Eqs. (21)–(23)]. We then have

$$\begin{aligned} V_q^{1,2}(n,t) = \frac{\alpha_B}{2\pi}\Bigg\{ & f_{V/q}^{1,2}(n,t) + (C_{n,V/q}^{1,2})^{\text{NS}} \ln\frac{Q^2}{\Lambda^2}\left[e^{-t} + \frac{d_n^{qq} - e^{-(1-d_n^{qq})t}}{1-d_n^{qq}} \right] \\ & + \langle C_{n,V}^{1,2}\rangle \ln\frac{Q^2}{\Lambda^2}\left[e^{-t} + \frac{(\lambda_- - d_n^{qq})(\lambda_+ - e^{-(1-\lambda_+)t})}{(\lambda_- - \lambda_+)(1-\lambda_+)} + \frac{(\lambda_+ - d_n^{qq})(\lambda_- - e^{-(1-\lambda_-)t})}{(\lambda_+ - \lambda_-)(1-\lambda_-)} \right] \Bigg\} \end{aligned} \tag{42}$$

TABLE II. The coefficients of three-term expansion for the VBD inside quark and gluon in next-to-leading approximation.

	$Q=2$ TeV			$Q=20$ TeV		
V_j^a	C_0	C_1	C_2	C_0	C_1	C_2
W_q^1	2.1236	-5.0468	3.0657	1.6376	-3.8596	2.3177
W_q^2	0.7856	0.4060	-1.0995	0.8700	0.2959	-0.9033
W_q^0	0.3109	-0.4591	0.1449	0.1854	-0.3158	0.1312
W_g^0	0.0564	-0.1608	0.1111	0.0518	-0.1575	0.1134
Z_p^1	0.7138	-1.5749	0.9100	0.5716	-1.2332	0.7075
Z_p^2	0.3846	-0.2158	-0.1473	0.3934	-0.2275	-0.0951
Z_p^0	0.1221	-0.2070	0.0855	0.0767	-0.1566	0.0828
Z_n^1	1.0026	-2.3692	1.4363	0.7811	-1.8384	1.1096
Z_n^2	0.4070	0.0832	-0.4704	0.4493	0.0030	-0.3572
Z_n^0	0.1545	-0.2567	0.1023	0.0954	-0.1895	0.0969
Z_g^0	0.0602	-0.1785	0.1266	0.0532	-0.1688	0.1245

for the transverse VBD inside quark. For the transverse VBD inside gluon, it can be proved from Eq. (40) that the result in the leading-logarithm and the next-to-leading approximation are the same. For the longitudinal component we have

$$V_q^0=\frac{\alpha_B}{2\pi}\left\{f_{V/q}^0(n,t)-(C_{n,V/q}^0)^{\rm NS}(1-e^{d_n^{qg}t})-\langle C_{n,V}^0\rangle\left[1-\left[\frac{\lambda_--d_n^{qq}}{\lambda_--\lambda_+}\right]e^{\lambda_+t}-\left[\frac{\lambda_+-d_n^{qq}}{\lambda_+-\lambda_-}\right]e^{\lambda_-t}\right]\right\}, \tag{43}$$

$$V_g^0=\frac{\alpha_B}{2\pi}\frac{d_n^{qg}}{(\lambda_+-\lambda_-)}\langle C_{n,V}^0\rangle\left[e^{\lambda_+t}-e^{\lambda_-t}\right]. \tag{44}$$

Using the procedure in Sec. V, we obtain the apparent expressions of the VBD in the next-to-leading approximation. The coefficients of the three-term expression at $Q=2$ and 20 TeV are listed in Table II. The curves for the VBD in quarks and gluons at $Q=20$ TeV are shown in Fig. 3 and Fig. 4 respectively. In numerical calculation, we find that the contributions of all the terms which illustrate the effect of strong interaction on the moments of the VBD inside a quark in Eqs. (42) and (43) are negative. Their relative values are several percentages for $n=2$ and increase with increasing n.

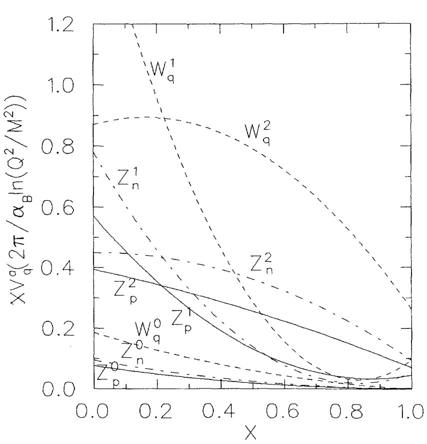

FIG. 3. Distributions for W (W^+ or W^-) and Z with the polarization a (1,2,0) inside quark ($Q=20$ TeV) in the next-to-leading approximation.

VII. VBD IN PROTON AND HIGGS PRODUCTION

At a certain energy scale Q, there are definitely distributions of quarks and gluons in a proton. If we consider now the VBD in a proton at that energy scale, they can be found by convolution:

$$F_{V/p}^a(x,Q^2)=\int_x^1\frac{d\eta}{\eta}\sum_j F_{j/p}(\eta,Q^2)V_j^a\left[\frac{x}{\eta},Q^2\right]. \tag{45}$$

The numerical results are shown in Fig. 5 for the distributions of charged boson $W^{\pm}$, and in Fig. 6 for Z^0 at $\sqrt{s}=40$ TeV. The quark and gluon distributions in pro-

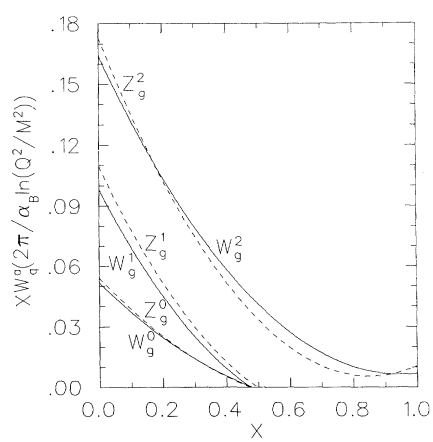

FIG. 4. Distributions for W (W^+ or W^-) and Z with the polarization a (1,2,0) inside gluon ($Q=20$ TeV).

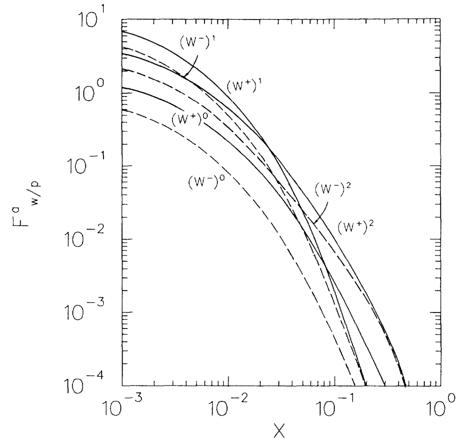

FIG. 5. Distributions for W^+ and W^- with the polarization a (1,2,0) in a proton ($Q^2=xs$, $\sqrt{s}=40$ TeV).

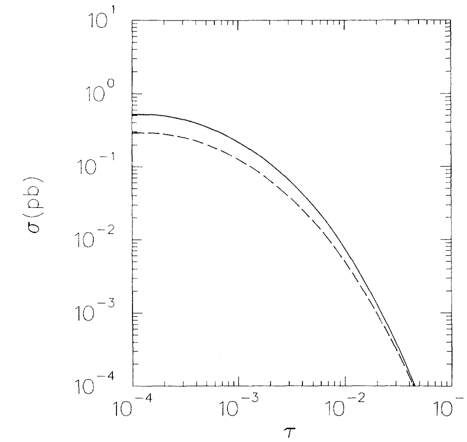

FIG. 7. Cross sections for Higgs production in pp collision ($M_H=\sqrt{\tau s}$, $\sqrt{s}=40$ TeV), from $(W^+)^0(W^-)^0$ (solid line) and $(Z^0)^0(Z^0)^0$ (dashed line).

ton $F_{j/p}(\eta,Q^2)$ we use are those of Ref. [12], which have been applied to the discussion of the problems at supercollider energies. Since W^+ originates from the u quark while W^- from the d quark, W^+ bosons are more than W^- in a proton. Z^0 is emitted by the two valence quarks which compensate for the smaller coupling and approximately $V^q_{W^\pm/p}\sim V^a_{Z^0/p}$.

As an example of application of the VBD, we consider Higgs boson production from W^+W^- or Z^0Z^0 fusion. The cross section σ_H for the process $pp\to VV\to H+X$ can be given by the evolution of the luminosity function $(dL/d\tau)(pp\to W^+W^-$ or $Z^0Z^0)$ and the fusion cross section

$$\sigma_H(s)=\int_{\tau_{\min}}^1\frac{d\tau}{\tau}\sum_{ab}\left[\frac{dL}{d\tau}\right]^{ab}\sigma^{ab}(\tau,\hat{s})\ , \tag{46}$$

and

$$\left[\frac{dL}{d\tau}\right]^{ab}=\int_\tau^1\frac{dx}{x}F^a_{V/p}(x,\hat{s})F^b_{V/p}\left[\frac{\tau}{x},\hat{s}\right]\ , \tag{47}$$

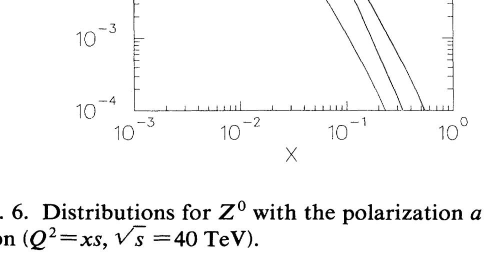

FIG. 6. Distributions for Z^0 with the polarization a (1,2,0) in a proton ($Q^2=xs$, $\sqrt{s}=40$ TeV).

with $\hat{s}=\tau s$. The nonzero subprocess cross section to be used in Eq. (46) is [4]

$$\hat{\sigma}^{11}=\hat{\sigma}^{22}=\frac{1}{2}N\ ,\quad \sigma^{00}=N\frac{(\hat{s}-2M_V^2)^2}{4M_V^2}\ , \tag{48}$$

where

$$N=\frac{16\pi^2\alpha_B M_V^2}{\sqrt{\hat{s}(\hat{s}-4M_V^2)}}\delta(\hat{s}-M_H^2)\ . \tag{49}$$

It is obvious from Eq. (48) that at high energies, $\hat{s}\gg M_V^2$, the Higgs boson production cross section is dominated by the contribution from longitudinal boson, $(W^\pm)^0$ and $(Z^0)^0$. So we have

$$\sigma^H=\frac{4\pi^2\alpha_B}{M_V^2}\frac{\tau(1-2x)^2}{\sqrt{1-4x}}\left[\frac{dL}{dx}\right]^{00}\ ,\quad x=M_V^2/\tau s\ . \tag{50}$$

The numerical results of σ_H due to $(W^+)^0(W^-)^0$ fusion and $(Z^0)^0(Z^0)^0$ fusion in pp collisions are shown in Fig. 7. The Higgs boson mass produced in this process is $M_H=\sqrt{\tau s}$. Our results can be compared with the results of the papers [4] in which only electroweak interaction is considered. In general, our results are slightly smaller.

VIII. CONCLUSION

We have discussed the strong interaction effect on the VBD in a scheme of modified AP equations. The apparent expressions of the VBD inside a quark and gluon at some energy scales are given. The result shows that the strong interaction effect modifies not only the x dependence but also the Q dependence for VBD even at the leading approximation and it is very different from the result in the discussion of photon distributions.

Finally, as an example we have calculated the VBD in a proton and considered the Higgs production cross section from the subprocess of vector boson fusion in Sec. VII. Because of the softer VBD, the cross section is slightly smaller than in other calculations in which only the electroweak interaction is considered. The softening for $W_q^{1,2}$ due to the strong interaction corrections in the leading approximation, for example, can be clearly seen in Fig. 2.

ACKNOWLEDGMENT

I would like to thank the University of Victoria and Professor H. Fearing and other members of the TRIUMF theory group for their hospitality.

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