

## Hadronic production of the $B_c$ meson at TeV energies

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Based on a reliable mechanism, the cross sections of the  $B_c$  hadronic production at the TeV energies of the CERN  $Spp\bar{S}$ , Fermilab Tevatron, CERN LHC, and SSC, as well as their possible fixed target experiments, are calculated. The results are presented numerically and indicate that the cross sections at the energies reached at the Tevatron, LHC, and SSC or higher are sufficiently large for further considering experimental studies of the  $B_c$  meson at these colliders.

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The physics of the  $B_c$  meson is very interesting, due to the nature of its structure. Comparing it with  $\eta_c, J/\psi$ , and  $\eta_b, \Upsilon$ , etc., the  $B_c$  meson, on one hand the same as them, is a double heavy-quark-antiquark bound state, so the QCD-inspired potential model should work well, and, on the other hand different from them, it carries flavors explicitly; thus it may decay by the weak interaction only, and has a much longer lifetime than the others [1-3,15]. The  $B_c$  meson (or its antiparticle) probably is the only object having the above twofold nature in the hadron world if there are only three generations of fundamental fermions as indicated by the standard model, and a very heavy top  $m_t$  indicated by 140 GeV. The reason is as follows: In addition to the  $c\bar{b}$  or  $\bar{c}b$  bound states, of the three families a double heavy-quark-antiquark system with this feature must contain a top or top antiquark inside. However, the top quark probably would not have time to form any hadron before decaying itself: The lowest bound of the top quark mass is proved up to  $m_t \geq 91$  GeV [7] now, and the top's lifetime will become shorter as the bound is increasing. It is easy to estimate that  $m_t \geq 140$  GeV happens to be the case that the top quark would not have time to form any hadron before decaying itself. Therefore the  $B_c$ , as well as its antiparticle, probably is the unique object having this nature. The properties of the meson  $B_c$  may potentially make it become a fruitful "laboratory" for testing potential models and understanding the weak decay mechanism for heavy flavors.

The reason the experimental study of  $B_c$  physics has not really started yet, interesting though the physics may be, is due to the difficulty of producing the  $B_c$  mesons. The difficulty may be understood by inspecting the following most possible mechanisms of the production. The first is to produce the meson  $B_c$  or its antiparticle  $\bar{B}_c$  by the so-called fragmentation mechanism. In fact it is practically impossible by this mechanism to produce a number of the mesons, although it is very important for

the production of the single heavy mesons (with one heavy quark and one light quark) such as  $D^0, \bar{D}^0, D^+, D^-, D_s, \bar{D}_s, B^+, B^-, B^0$ , and  $\bar{B}^0$ , etc. The reason is that the possibility to create a pair of heavy quarks from the vacuum (in the  $B_c$  production case) is much smaller than that to create a pair of light ones (in the latter case). According to the hadron string model [4], the relative possibilities for producing quark pairs of various flavors from the vacuum (the breaking patterns) are

$$u:d:s:c = 1.0:1.0:0.3-0.4:10^{-10}-10^{-11}. \quad (1)$$

Thus one can understand how rare it is to produce the  $B_c$  mesons by this mechanism. The second is to produce a pair of mesons  $B_c$  and  $\bar{B}_c$  via a highly excited state  $\Upsilon(nS)$ , where  $n$  is given, located above the threshold of the pair production of the meson, such as the production of the mesons  $B^+, B^-, B^0$ , and  $\bar{B}^0$  via the excited state  $\Upsilon(4S)$  at  $e^+e^-$  colliders. This is impossible too, because it is a great problem if such a highly excited state  $\Upsilon(nS)$  exists. Even if such a highly excited state had existed, a similar suppression factor as the fragmentation as pointed out above must play a certain role, although the resonance effects here may distort the suppression strongly. The third is to produce the meson  $B_c$  or  $\bar{B}_c$  by the mechanism of a direct coupling to a virtual  $W^+$  or  $W^-$  boson, respectively. This also is practically impossible because there is a very strong suppression from the Cabibbo-Kobayashi-Maskawa (CKM) matrix element  $V_{cb} \sim 0.04$ , in addition to that of a typical weak production, i.e., those suppression factors from the virtual  $W^+$  or  $W^-$  boson's production and propagator. Therefore the  $B_c$  meson and its antiparticle cannot be produced by the most common mechanisms.

However, a mechanism, i.e., two heavy quark pairs ( $b\bar{b}$  and  $c\bar{c}$ ) are produced in terms of a hard subprocess first, and then a single  $B_c$  (or  $\bar{B}_c$ ) meson (but not the  $B_c$  and  $\bar{B}_c$  pair) is formed by two of the four quarks with a sizable possibility of matching a color singlet object, has received attention recently [1,2]. One may imagine that as the energy is increasing it becomes favorable to produce the meson  $B_c$  or  $\bar{B}_c$ .

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In Refs. [1,2], the authors considered the mechanism for the production of the  $B_c$  and  $\bar{B}_c$  mesons in an  $e^-e^+$  collider, especially at  $Z^0$  resonance. The results show that several hundreds, even thousands, of  $B_c$  can be produced at the CERN  $e^+e^-$  collider LEP I (depending on the integrated luminosity collected at LEP I). From an optimistic point of view, this number of events may be enough to search for a  $B_c$  meson when a reasonable reconstruction efficiency is taken into account. However, it is not possible for investigating the properties of the  $B_c$  meson experimentally, with such a limited number of the events only.

In order to find where a thorough experimental study of the very interesting physics may be accessible, one should examine the other cases, especially the hadronic production with the most favorable mechanism. If the rate of the  $B_c$  production is large enough, although there exist various kinds of backgrounds, a thorough experimental study on the properties of the  $B_c$  meson may become accessible, provided the experiment(s) has a strong ability to reject the backgrounds. To see how great of the cross sections becomes a primary task if one is planning such a kind of experimental study seriously.

In this paper, we will take the task theoretically. We are to apply the mechanism (maybe the most favorable one) to calculate the production cross sections and the distributions of the produced  $B_c$  mesons at various energies of the hadronic colliders CERN Super Proton Synchrotron ( $Spp\bar{S}$ ), Fermilab Tevatron, CERN Large Hadron Collider (LHC), and Superconducting Super Collider (SSC), and their possible fixed experiments. We should emphasize here that the method adopted to calculate this mechanism is very reliable, because it is based on (i) the perturbative QCD for the heavy flavor quark production, (ii) the Mandelstam method for the bound state effects of the  $B_c$  meson production, (iii) the Bethe-Salpeter (BS) wave function, required in the calculations with the Mandelstam method, is connecting to the Schrödinger one of the heavy quark system under the framework of the QCD-inspired potential model by applying the instantaneous approximation to the BS equation. As a result, we do find that the cross sections are so large, especially at a comparatively high energy of the hadron-hadron colliders Tevatron, LHC, and SSC, and their fixed target experiments. Hence the experimental study of the properties of the  $B_c$  meson becomes accessible as the ability for rejecting the backgrounds is increasing and the new colliders have finished constructing in the near future.

The  $B_c$  meson production at hadronic colliders has been estimated by some authors [3,5]. The authors of Ref. [3] simply estimated the cross sections in analogy with the  $J/\psi$  production; however, it is well known that the hadronic production of the  $J/\psi$  is very complicated and it still is an open problem so far, and furthermore, there must be a great difference between the  $B_c$  meson (flavored explicitly and comparatively heavy) and the  $J/\psi$  (flavored hidden and comparatively light) productions. In Ref. [5], the cross sections of the  $B_c$  meson production are estimated by running HERWIG code and a Monte Carlo simulation of QCD parton showers. The

program of Ref. [5] is proved to be phenomenologically successful for the production of those mesons which contain at least one light quark (light and single heavy mesons), but we are not sure if this program is still effective for the production of the mesons which contain two heavy quarks (double heavy mesons). The authors of Ref. [5], additionally, introduced two parameters  $p_1$  and  $p_2$  in the program, to which the result is sensitive.

Now let us return to our approach. Under the leading order, the factorization theorem of the perturbative QCD should work; hence the  $B_c$  meson production may be estimated quite reliably via calculating the cross sections of the processes of gluon-gluon fusion  $g+g \rightarrow B_c + b + \bar{c}$  and light quark-antiquark fusion  $q + \bar{q} \rightarrow B_c + b + \bar{c}$ , then making convolutions of the cross sections for each with the corresponding structure functions of the colliding hadrons respectively.

There are 36 Feynman diagrams for the first subprocess and 7 for the second one under the lowest order approximation; thus, we are dealing with a very complicated computation problem. To carry out the computation, it is necessary to employ a powerful technique.

Now we proceed to calculate the gluon-gluon fusion process first. The typical Feynman diagrams of the subprocess are shown in Fig. 1(a). Under the taken approximation, the hadronization vertex may be described by means of the BS wave function at origin, and for present usage we just quote it from Refs. [2,6]. In order to compare with others, we should note (i) the wave function(s) in Refs. [2,6] which we are using has not received the QCD corrections yet that matches the lowest order approximation of QCD and (ii) the wave function at the origin is related to the decay constants. Under the convention  $f_\pi = 132$  MeV, the wave function at the origin obtained in Refs. [2,6] corresponds to  $f_{B_c} \simeq 480$  MeV. The color factors can be conveniently accounted for in the spirit of the dual amplitude method [8]. According to the color structures, the total amplitude of the subprocess for gluon-gluon fusion can be explicitly written as

$$A(a, b, i, j) = \sum_{\alpha=1}^6 C_{\alpha ij}^{ab} M_{\alpha}(\epsilon_1, \epsilon_2, s_1, s_2), \quad (2)$$

where each of the  $C_{\alpha ij}^{ab}$  ( $\alpha=1, 2, \dots, 6$ ) is a product of the Gell-Mann matrices [9]. However, not all these color factors are independent, because there exists a relation among them, that is

$$C_{3ij}^{ab} - C_{5ij}^{ab} = C_{4ij}^{ab} - C_{6ij}^{ab}. \quad (3)$$

It follows that only five color factors are independent, and we may choose them as

$$\begin{aligned} C_{mij}^{ab} &= C_{mij}^{ab} \quad (\text{when } m = 1, \dots, 4); \\ C_{5ij}^{ab} &= C_{3ij}^{ab} - C_{5ij}^{ab}. \end{aligned} \quad (4)$$

Thus the amplitude may be rewritten as

$$A(a, b, i, j) = \sum_{k=1}^5 C_{kij}^{ab} M_k'(\epsilon_1, \epsilon_2, s_1, s_2). \quad (5)$$

Being independent of the coefficients of the color factor

$C'_{kij}$ , the subamplitudes  $M'_k$  ( $k=1,2,\dots,5$ ) are individually gauge invariant [8]. In fact, owing to the fact that each of the amplitudes  $M'_k$  is related to certain Feynman diagrams of the 36 precisely, thus the explicit formulas of  $M'_k$  ( $k=1,2,\dots,5$ ) may be written down directly, based on the rules of the dual amplitude method [8]. After summing up the color indices, the absolute squared matrix element of the whole amplitude becomes

$$|A|^2 = \frac{64}{27}|8M'_1 - M'_3|^2 + \frac{64}{27}|8M'_2 - M'_4|^2 - \frac{16}{27}|(8M'_1 - M'_3)(8M'_2 - M'_4)| + \frac{88}{3}|M'_5|^2 + \frac{16}{3}|(8M'_1 + 8M'_2 - M'_3 - M'_4)M'_5| \quad (6)$$

As here the goal is to calculate the numerical values of the cross sections; thus, we adopt the direct amplitude method [10,11] to evaluate the amplitudes  $M'_k$  and the absolute squared matrix element  $|A|^2$  by Eq. (6) further.

As for the subprocess of the light-quark-antiquark fusion [Fig. 1(b)] with the same way as the above, the corresponding result may be obtained. However, here the amplitude involves three independent color factors  $D_{aijkl}$  [12] only; thus, we choose the first three of the naive four ( $\alpha=1,2,3,4$ ) and the fourth one is always expressed in terms of the linear combination

$$D_{4ijkl} = D_{3ijkl} + D_{1ijkl} - D_{2ijkl} \quad (7)$$

The amplitude can be written as

$$B(i,j,k,l) = \sum_{\alpha=1}^3 D_{aijkl} B_{\alpha}(s_1, s_2, s_3, s_4) \quad (8)$$

Summing up the color indices, the absolute squared amplitude  $|B(i,j,k,l)|^2$  may be written as follows by means of the functions  $f_m$  ( $m=1,2$ ):

$$|B|^2 = 9|f_1|^2 + 9|f_2|^2 + 6|f_1 \cdot f_2| \quad (9)$$

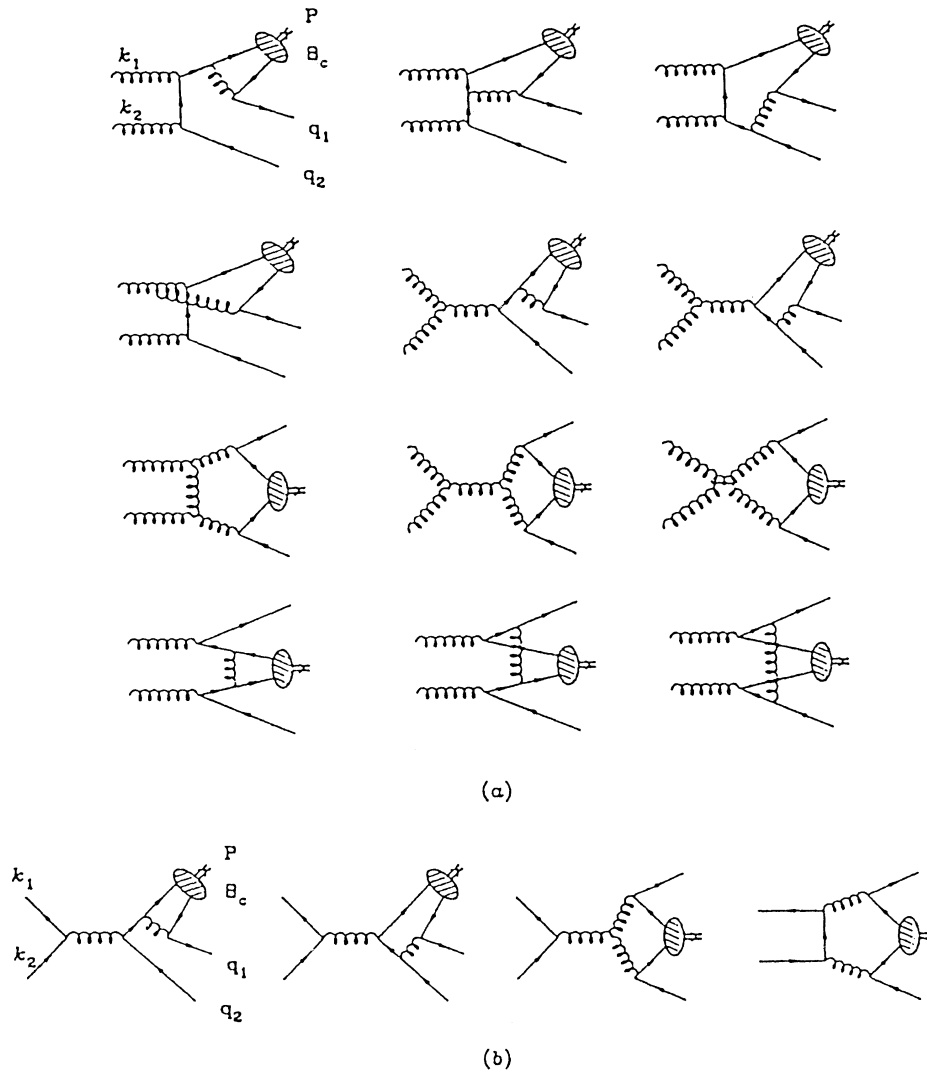


FIG. 1. The Feynman diagrams for the subprocesses. (a) The typical Feynman diagrams for the gluon-gluon fusion. (b) The Feynman diagrams for the quark-antiquark fusion.

TABLE I. The total cross sections for the productions of the  $B_c$  meson and its excited states (in nb for the colliders and in pb for their fixed target experiments).

Expt. Energy	Collider				Fixed target		
	$S\bar{p}\bar{p}S$	Tevatron	LHC	SSC	Tevatron	LHC	SSC
	0.63	1.8	16	40	0.041	0.12	0.19
$B_c(1^1S_0)$	0.18	0.86	9.8	25	0.013	4.1	16
$B_c^*(1^3S_1)$	0.55	2.54	29	69	0.034	13	48
$B_c^*(2^1S_0)$	0.10	0.48	5.5	14	0.007	2.3	9.0
$B_c^*(2^3S_1)$	0.31	1.42	16	39	0.019	7.3	27
Total	1.14	5.30	60	147	0.073	27	100

where

$$f_1 = \frac{8}{9}B_1(s_1, s_2, s_3, s_4) - \frac{1}{9}B_2(s_1, s_2, s_3, s_4) + \frac{10}{9}B_3(s_1, s_2, s_3, s_4),$$

$$f_2 = -\frac{8}{3}B_1(s_1, s_2, s_3, s_4) + \frac{1}{3}B_2(s_1, s_2, s_3, s_4) - \frac{2}{3}B_3(s_1, s_2, s_3, s_4).$$

The amplitudes  $B_\alpha$  ( $\alpha=1,2,3$ ), similar to those of  $M'_k$  ( $k=1, \dots, 5$ ), may be easily calculated numerically with the direct amplitude method [10,11].

In order to guarantee the calculations, we have our programs checked in many ways. One of the checks is the gauge invariance for each of the independent subamplitudes.

In the numerical calculations, the PAPANENO code [13] is adopted for the phase space integrations. Connecting the matrix elements squared to the PAPANENO code, the total cross sections at various hadron-hadron colliders have been calculated. The results are presented in Table I. As the energy of the  $S\bar{p}\bar{p}S$  collider is low ( $\sqrt{S} \sim 0.63$  TeV), the cross section for its fixed target experiments is so small ( $\approx 10^{-8}$  nb) that we do not put it into Table I, but only the collider's ( $S\bar{p}\bar{p}S$ ) result. In the calculations, the set II of the Eichten-Hinchliffe-Lane-Quigg (EHLQ) structure functions [14] with the scale  $Q^2 = \hat{s}/4$  ( $\hat{s}$  is the c.m. energy squared of the subprocesses respectively) are used, and the masses  $m_c = 1.5$  GeV,  $m_b = 4.9$  GeV, and  $M_{B_c} = 6.4$  GeV [2,6] have been taken.

We note two points: first, the contribution to the cross sections from the gluon-gluon fusion subprocess is dominant over that from the quark-antiquark fusion one even for  $p\bar{p}$  collision and at not very high energies, such as those of the fixed target experiments of Tevatron; hence we do not present it explicitly; second, the total cross sections are sensitive to the quark masses a bit, especially to the  $c$  quark mass  $m_c$ ; i.e., if the mass is taken to be larger, then the cross section becomes smaller. Here for consistency with that determined by the potential model, we have chosen the value being slightly larger than the usual one taken by the heavy quark productions. If concentrating on the second point only, the obtained cross sections here are underestimated.

In order to explore the features of the production signals, we present the results not only for the total cross sections but also for the transverse momentum  $p_T$  and rapidity  $Y$  distributions as well. The  $p_T$  distributions of the

produced  $B_c$  mesons for various energies are shown in Fig. 2. It is easy to see that the  $p_T$  distributions have a maximum around 2–4 GeV and the most important contribution comes from the region for the  $p_T$  of the  $B_c$  mesons less than 25 GeV. The rapidity  $Y$  distributions are shown in Fig. 3. At the considered colliders, the rapidity distributions are restrained within the range of  $-5 \rightarrow 5$  in Fig. 3 and it is easy to see that the  $B_c$  mesons are produced essentially in the region of small rapidity. Note that the total cross sections will drop down to  $\frac{1}{3}$  if a rapidity cut  $Y \leq 1.2$  is applied, which is required in most cases of an experimental detector.

The productions of the excited states ( $1^3S_1$ ,  $2^1S_0$ ,  $2^3S_1$ ), which lie below the threshold of the  $B$  and  $D$  meson production, are also calculated here, for the same reason as pointed in Ref. [2] that they will eventually decay into the ground state, the  $B_c$  meson, by emitting  $\gamma, \pi, \eta$ , so that they will also contribute a substantial component of the production of the  $B_c$  mesons.

Our results are in the range about  $\frac{1}{5} - \frac{1}{10}$  of those Ref.

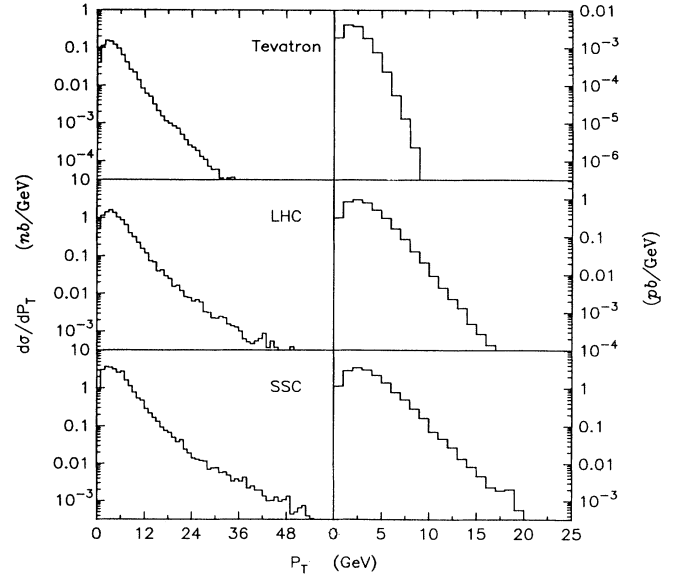


FIG. 2. The  $p_T$  distributions of the production at the various colliders. Those in the left column mean at the collider energies and in nb/GeV; those in the right column mean at the fixed target energies and in pb/GeV.

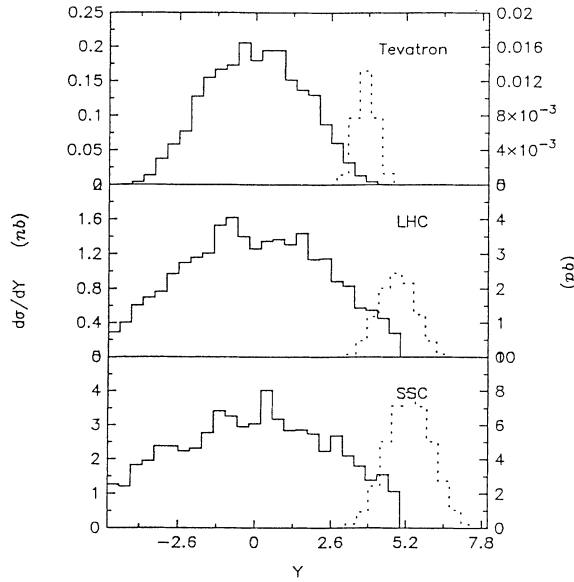


FIG. 3. The rapidity  $Y$  distributions of the production at the various colliders. The solid lines mean at the collider energies and in nb (with left-hand side scale); the dashed ones mean at the fixed target energies and in pb (with right-hand side scale).

[5]; however, the latter were obtained by Monte Carlo simulation of QCD parton showers, and the HERWIG code; thus some uncertainty and free parameters are involved as pointed out. If comparing our results with those of the production of the four-heavy quark jets ( $c\bar{c}b\bar{b}$ ), which were calculated by perturbative QCD [11], the  $B_c$  meson production roughly is about 10% of it; that is reasonable.<sup>1</sup>

The signals of detecting the  $B_c$  meson at hadron-hadron colliders have features similar to those at LEP I [2], but owing to a greater number of hadrons produced, i.e., a dirtier environment, a much stronger ability to reject the backgrounds becomes crucial for the detecting. Knowing the properties of the  $B_c$  or  $\bar{B}_c$  meson [15,16], one may understand that precision microvertex detectors may be helpful for the detecting. It is possible though difficult to identify the mesons for the following reasons: (a) the  $B_c$  or  $\bar{B}_c$  meson is charged; (b) the lifetime of the meson  $B_c$  ( $\bar{B}_c$ ) is not too short. According to the esti-

<sup>1</sup>In general the color matching of a quark-antiquark pair into a colorless meson makes the meson production gain a factor  $\frac{1}{9}$ . While at a very high energy, far above the threshold, the phase space of the four heavy quarks would not be suppressed much; furthermore, owing to the big decay constants of the meson and its excited states, and having all the cross sections of the meson and its excited state productions in a three body ( $\bar{c}, b$  jets and the bound states  $c\bar{b}$  in  $S$  wave) final state manner summed up, the phase space of the total  $B_c$  production, associated with two jets, may become comparable with that of the four heavy quarks roughly. Thus concerning the theoretical and numerical uncertainties for the different calculations, we conclude that the production rate is reasonable.

mate of Ref. [15], the lifetime

$$\tau_{B_c} \simeq 3-4 \times 10^{-13} \text{ s}$$

may be reliable. Moreover there is a Lorentz dilatation effect for a high energy production of the  $B_c$  meson; thus it probably is practically possible to measure the lifetime directly with microvertex detectors. (c) The decays of the  $B_c$  or  $\bar{B}_c$  mesons have a very special characteristic. First, one possible and typical feature is that one component of the  $B_c$  ( $\bar{B}_c$ ) meson, i.e.,  $\bar{b}$  ( $b$ ) or  $c$  ( $\bar{c}$ ), decays and it may make a decay vertex first; then when the other component  $c$  ( $\bar{c}$ ) or  $\bar{b}$  ( $b$ ) decays, it may make the second decay vertex; finally the decay product of the  $\bar{b}$  ( $b$ ), which probably is  $\bar{c}$  ( $c$ ), decays, and may make the third decay vertex; thus such a  $B_c$  (or  $\bar{B}_c$ ) even may manifest three decay vertices: two of them present a cascade one, and the other a single one. Another possible and typical feature is that the  $B_c$  ( $\bar{B}_c$ ) meson may decay into  $J/\psi$  plus something else, and due to a quite large branching ratio [15],

$$B(B_c \rightarrow J/\psi + \dots) \simeq 15\% ,$$

the produced  $J/\psi$  may be used as a trigger of the meson events in experiments. Obviously, in this feature the events may manifest one decay vertex. Especially, the decay mode  $B_c \rightarrow J/\psi + \pi$  with a not very small branching ratio [15],

$$B(B_c \rightarrow J/\psi + \pi) \simeq 0.1-0.2\% ,$$

may be used as one of the best channels to identify the meson from the backgrounds. (d) The mass of the  $B_c$  or  $\bar{B}_c$  meson is about 6.4 GeV. If its decay invariant mass may be measured with enough accuracy, the mass will be considered as an experimental characteristic to identify the meson too.

Concerning the sizable cross sections of the  $B_c$  meson production at the TeV energies obtained by the considered mechanism in this paper and the distinguished experimental characteristic features of the mesons as pointed out above, we think that one may conclude that it is very accessible to have thorough experimental studies of the  $B_c$  physics at Tevatron, LHC and SSC in the near future, despite the fact that the present estimates are of the lowest order and some uncertainties are involved so far. We merely would like to note that the most important uncertainties of the theoretical estimates may be controlled under the framework of the potential model, the perturbative QCD and the Mandelstam formalism, via higher order and more precise calculations [17].

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$$C_{1ij}^{ab} = (\lambda^c \cdot \lambda^c \lambda^a \cdot \lambda^b)_{ij} = \frac{N^2 - 1}{N} (\lambda^a \cdot \lambda^b)_{ij} ,$$

$$C_{2ij}^{ab} = (\lambda^c \cdot \lambda^c \lambda^b \cdot \lambda^a)_{ij} = \frac{N^2 - 1}{N} (\lambda^b \cdot \lambda^a)_{ij} ,$$

$$C_{3ij}^{ab} = (\lambda^c \cdot \lambda^a \cdot \lambda^c \cdot \lambda^b)_{ij} = \frac{-1}{N} (\lambda^a \cdot \lambda^b)_{ij} ,$$

$$C_{4ij}^{ab} = (\lambda^c \cdot \lambda^b \cdot \lambda^c \cdot \lambda^a)_{ij} = \frac{-1}{N} (\lambda^b \cdot \lambda^a)_{ij} ,$$

$$C_{5ij}^{ab} = (\lambda^c \cdot \lambda^a \cdot \lambda^b \cdot \lambda^c)_{ij} = \delta_{ij} \text{tr}(\lambda^a \cdot \lambda^b) - \frac{1}{N} (\lambda^a \lambda^b)_{ij} ,$$

$$C_{6ij}^{ab} = (\lambda^c \cdot \lambda^b \cdot \lambda^a \cdot \lambda^c)_{ij} = \delta_{ij} \text{tr}(\lambda^a \cdot \lambda^b) - \frac{1}{N} (\lambda^b \cdot \lambda^a)_{ij} .$$

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$$D_{1ijkl} = (\lambda^a \cdot \lambda^b \cdot \lambda^a)_{ij} \lambda_{kl}^b ,$$

$$D_{2ijkl} = (\lambda^a \cdot \lambda^a \cdot \lambda^b)_{ij} \lambda_{kl}^b ,$$

$$D_{3ijkl} = (\lambda^a \cdot \lambda^b)_{ij} (\lambda^a \cdot \lambda^b)_{kl} ,$$

$$D_{4ijkl} = (\lambda^a \cdot \lambda^b)_{ij} (\lambda^b \cdot \lambda^a)_{kl} .$$

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