Phase of direct CP violation from threshold pion production

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A recent series of experiments on pion production near threshold provides accurate values of the s-wave π-π isospin scattering lengths. Combined with dispersion relations we find the s-wave phase shift difference at the K mass is $\delta_0 - \delta_2 = 42 \pm 4°$ and the phase of direct CP violation is $\arg\varepsilon' = \frac{\pi}{2} - (\delta_0 - \delta_2) = 48 \pm 4°$. We also evaluate the p-wave scattering length to be $a_1 = 0.035 \pm 0.001\, m_\pi^{-3}$.

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I. INTRODUCTION

Knowledge of the phase of the direct kaon CP-violation parameter ε' is of fundamental importance. In the extraction of the magnitude of ε' and for tests of CPT invariance [1] an accurate knowledge of the phase of ε' plays a key role. Assuming CPT invariance, the phase of ε' is determined by final state π-π scattering as

$$\arg\varepsilon' \equiv \frac{\pi}{2} - (\delta_0 - \delta_2) \; , \tag{1}$$

where δ_I is the isospin I s-wave π-π scattering phase shift at the kaon mass c.m. energy.

The phase of ε' has been previously determined by extrapolating downward using π-π phase shifts from high energy pion production [2] or by a theoretically motivated extrapolation upward in energy using a chiral Lagrangian [3].

In this paper we use new results for s-wave π-π scattering lengths a_0 and a_2 together with forward dispersion relations to evaluate $\delta_0 - \delta_2$. The advantage of our method is that it exploits the self-consistency of the data by using information over the entire low-energy region. The addition of the new scattering length data is crucial to our analysis.

Almost twenty-five years ago it was pointed out [4] that in some generality the $\pi N \to \pi\pi N$ amplitude at threshold factorizes into $\pi\pi \to \pi\pi$ scattering at threshold and the known πNN vertex. Within the last few years an impressive sequence of precision measurements has been performed exploring pion production near threshold. These experiments examine all five charge states resulting from $\pi^\pm$ collisions on protons: $\pi^- p \to \pi^-\pi^+ n$ [5], $\pi^- p \to \pi^0\pi^0 n$ [6], $\pi^- p \to \pi^-\pi^0 p$ [7], $\pi^+ p \to \pi^+\pi^+ n$ [8], $\pi^+ p \to \pi^+\pi^0 p$ [9]. A global analysis has been performed on this data [10] showing that overall consistency is achieved by the five final charge states and extracting the following s-wave π-π isospin scattering lengths:

$$\begin{aligned} a_0 &= 0.197 \pm 0.010 m_\pi^{-1}, \\ a_2 &= -0.032 \pm 0.004 m_\pi^{-1}. \end{aligned} \tag{2}$$

Above π-π scattering threshold there is a considerable body of experimental data. We use the phase shifts and inelasticity parameters shown [11–14] in Figs. 1–3. Most of these phase shifts are determined from $\pi N \to \pi\pi N$ scattering at high-energy in a kinematic region favoring single pion exchange [15]. Close to the π-π threshold, K_{e4} decay ($K \to e\nu\pi\pi$) yields valuable $I = 0$ π-π scattering

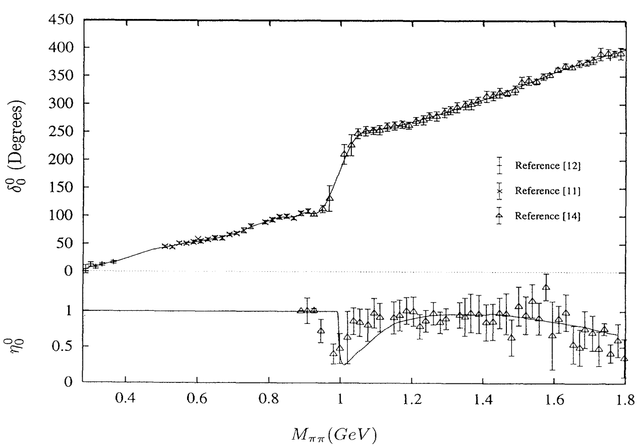

FIG. 1. Isospin zero s-wave scattering phase shift and inelasticity parameter. For the inelasticity parameter η we have used the energy-dependent fit of [14] as shown.

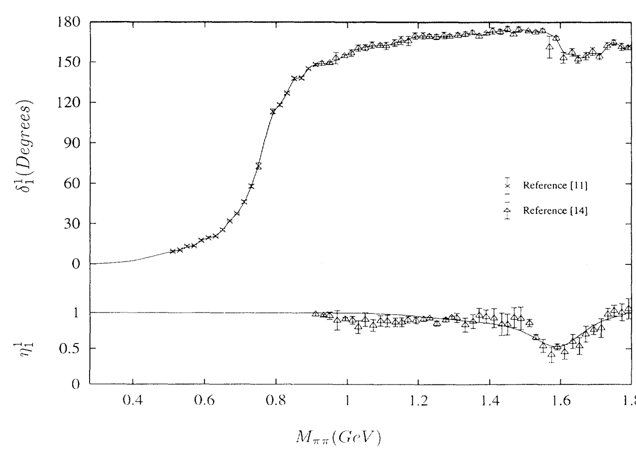

FIG. 2. Isospin one p-wave phase shift and inelasticity parameter. We have used the energy dependent fit of [14] for the inelasticity parameter η.

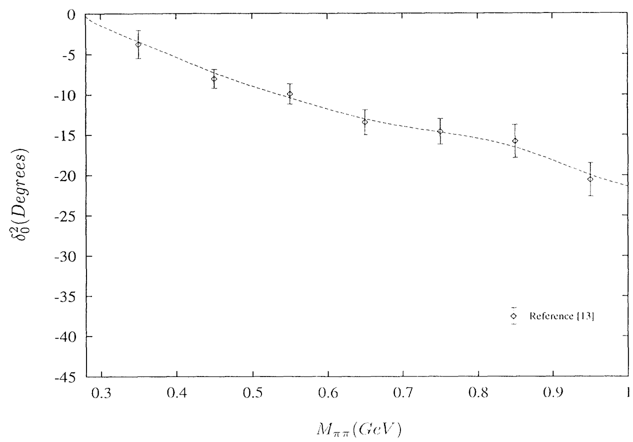

FIG. 3. Isospin two s-wave phase shift from [13].

information [12].

As we see, there is a large amount of π-π scattering data available, although not much specifically at the K-meson mass. This is an ideal situation in which to exploit the analyticity properties of the scattering amplitude in the form of dispersion relations. The dispersion relation can be thought of as a consistency relation involving all scattering energies. By judicious choice of subtraction points the dispersion relation can be engineered to emphasize the low-energy data. Our dispersive sum rules relate π-π scattering at the K-meson mass to the π-π scattering lengths and to π-π scattering between threshold and about 1 GeV.

We have gone to some effort to realistically evaluate the error due to the sum rule integral over experimental data. We have developed a Monte Carlo technique in which the experimental data are "shaken" within their assigned errors and then fitted by a spline interpolation routine. An ensemble distribution of integrals over this data then yields the error in the integral. An attractive aspect of this technique is that the error due to functional combinations of sum rules can be easily evaluated incorporating all data error correlations.

In Sec. II we derive the sum rules used in the analysis. The numerical evaluations are given in Sec. III and our conclusions in Sec. IV.

II. THE DISPERSIVE SUM RULE

We consider the invariant $\pi\pi \to \pi\pi$ forward scattering amplitude $A_I(\omega)$, where ω is the pion lab energy. The fixed-t dispersion relation [16] for t-channel isospin I_t is

$$\mathrm{Re}A_{I_t}(\omega) = \frac{1}{\pi} \int_1^\infty d\omega' \left[\frac{1}{\omega' - \omega} + \frac{(-)^{I_t}}{\omega' + \omega}\right] \mathrm{Im}A_{I_t}(\omega') \ . \tag{3}$$

The isospin definite amplitudes in the s-channel A_I are related to the t-channel amplitudes at the same kinematic point by

$$A_I = \sum_{I_t} C_{I,I_t} A_{I_t} \tag{4}$$

where

$$C_{I,I_t} = \begin{pmatrix} \frac{1}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{1}{2} & -\frac{5}{6} \\ \frac{1}{3} & -\frac{1}{2} & \frac{1}{6} \end{pmatrix} \ .$$

The partial-wave expansion of the invariant amplitude is [17]

$$\begin{aligned} A(\omega) &= \frac{\sqrt{s}}{2q} \sum_{\ell=0}^{\infty} (2\ell+1) f_\ell, \\ f_\ell &= \frac{1}{2i}\left(\eta_\ell e^{2i\delta_\ell} - 1\right) \ . \end{aligned} \tag{5}$$

The ℓth partial-wave scattering length is defined by

$$a_\ell = \lim_{q\to 0} \delta_\ell / q^{2\ell+1}, \tag{6}$$

and from (5) it follows that the elastic s- and p-wave scattering lengths can be expressed as

$$\begin{aligned} a_{\ell=0}^{I=0,2} &= A_{0,2}(\omega = 1), \\ a_{\ell=1}^{I=1} &= \frac{2}{3} \frac{dA_1}{d\omega}\bigg|_{\omega=1} \ . \end{aligned} \tag{7}$$

From the sum rule (3) and the isospin crossing relation (4) we find

$$\begin{aligned} \mathrm{Re}A_I(\omega) &= \frac{1}{\pi} \sum_{I_t} C_{I,I_t} \int_1^\infty d\omega' \left(\frac{1}{\omega' - \omega} + \frac{(-)^{I_t}}{\omega' + \omega}\right) \mathrm{Im}A_{I_t}(\omega') \\ &= \frac{1}{\pi} \sum_{I_t} C_{I,I_t} \int_1^\infty d\omega' \left(\frac{1}{\omega' - \omega} + \frac{1}{\omega' + \omega}\right) \mathrm{Im}A_{I_t}(\omega') - C_{I,1}\frac{2}{\pi} \int_1^\infty \frac{d\omega'}{\omega' + \omega} \mathrm{Im}A_{I_t=1}(\omega') \end{aligned}$$

or

$$\mathrm{Re}A_I(\omega) = \frac{2}{\pi} \int_1^\infty \frac{\omega' d\omega'}{\omega'^2 - \omega^2} \mathrm{Im}A_I(\omega') - \frac{2}{\pi} C_{I,1} \int_1^\infty \frac{d\omega'}{\omega' + \omega} \mathrm{Im}A_{I_t=1}(\omega') \ , \tag{8}$$

where

$$C_{I,1} = \left(1, \ \frac{1}{2}, \ -\frac{1}{2}\right) \ . \tag{9}$$

Defining the real part of $A(\omega)$ as

$$\alpha_I(\omega) \equiv \mathrm{Re}A_I(\omega), \tag{10}$$

we subtract the s-wave scattering length $\alpha_I(1) \equiv a_I$ from the sum rule (8) to give

$$\alpha_I(\omega) = a_I + \frac{2k^2}{\pi} \int_0^\infty \frac{dk'}{k'} \frac{\mathrm{Im}A_I(\omega')}{k'^2 - k^2} + \frac{2(\omega+1)}{\pi} C_{I,1} \int \frac{d\omega' \mathrm{Im}A_{I_t=1}(\omega')}{(\omega'-1)(\omega'+\omega)} . \quad (11)$$

Since we wish to compute phase shifts at the K mass we set $k = k_K$ and drop the primes in (11) to obtain our sum rule for $\alpha_I \equiv \alpha_I(\omega_K)$:

$$\alpha_I = a_I + J_I \quad (12)$$

where

$$J_I = \frac{2k_K^2}{\pi} \int_0^\infty \frac{dk}{k} \frac{\mathrm{Im}A_I(\omega)}{k^2 - k_K^2} + \frac{2(\omega_K+1)}{\pi} C_{I,1} \int_1^\infty \frac{d\omega \mathrm{Im}A_{I_t=1}}{(\omega-1)(\omega+\omega_K)} . \quad (13)$$

By (4) we see explicitly that

$$\mathrm{Im}A_{I_t=1} = \frac{1}{3}\mathrm{Im}A_0 + \frac{1}{2}\mathrm{Im}A_1 - \frac{5}{6}\mathrm{Im}A_2 . \quad (14)$$

Although we could put the above sum rule in a seemingly simpler form by substituting (14) into (13), we retain the expression (13) since the convergence properties are much clearer.

With very little additional effort, the p-wave scattering length a_1 can also be computed. The sum rule is the same as investigated earlier [18]. By differentiating the $I = 1$ sum rule (8) with respect to ω and using (7) we obtain

$$a_1 = \frac{8}{3\pi} \int_0^\infty \frac{dk}{k^3} \mathrm{Im}A_1 + \frac{2}{3\pi} \int_1^\infty \frac{d\omega}{(\omega+1)^2} \mathrm{Im}A_{I_t=1} . \quad (15)$$

The integrand is finite at $k = 0$ since $\mathrm{Im}A_1 \to k^5$ at threshold and has the same convergence properties as (13).

III. SUM RULE EVALUATION

In this section we discuss the evaluation of the sum rules (13) and (15). We consider in turn the principal value singularity, the data input, the convergence, and the assigned error of the sum rule results.

A. Principal value

The evaluation of the principal value integral (13) can be efficiently carried out using the identity

$$\int_0^\infty \frac{dk}{k^2 - k_K^2} = 0 , \quad (16)$$

where $k^2 = \omega^2 - 1$. The integral operator in (13) can be written as

$$\int_0^\infty \frac{dk}{k} \frac{\mathrm{Im}A(\omega)}{k^2 - k_K^2} = \int_0^\infty \frac{dk}{k^2 - k_K^2} \left[\frac{\mathrm{Im}A(\omega)}{k} - \frac{\mathrm{Im}A(\omega_K)}{k_K} \right] , \quad (17)$$

using (16) to subtract a term which integrates to zero.

As we can see, the integrand now has only a removable singularity and can be integrated by standard numerical techniques. At threshold the apparent singularity in the numerator is more than canceled by the threshold zero of ImA.

B. Data Input

We have used the tabulated phase shifts of Estabrooks and Martin [11], Hoogland *et al.* [13], and Rosselet *et al.* [12] below $\sqrt{s} = 0.9$ GeV and the work of Hyams *et al.* [14] between 0.9 GeV and 1.8 GeV. The data are shown in Figs. 1–3. As will be seen, the data above 1 GeV do not make significant contributions except for the $f_2(1270)$ resonance region. Above the region where phase shifts are known we evaluate the various known resonance [19] contributions to the sum rules by the narrow width approximation [20]. As we shall see, the results are quite small due to the rapid convergence of the sum rules. The experimental data entering the integrand of the various dispersive sum rules are interpolated by a standard cubic spline algorithm [21] and the resulting integrands are shown in Figs. 4 and 5. We observe the insensitivity to contributions at high energy. In Table I we show explicitly the contributions of resonances not included or above the point where phase shifts are known.

C. Convergence

Each of the sum rules (13) and (15) are to be integrated out to infinite energy. The phase shift analyses are known

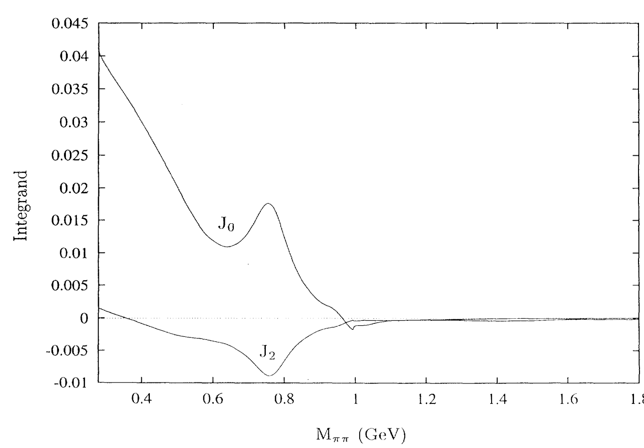

FIG. 4. Integrand for sum rules J_0 and J_2.

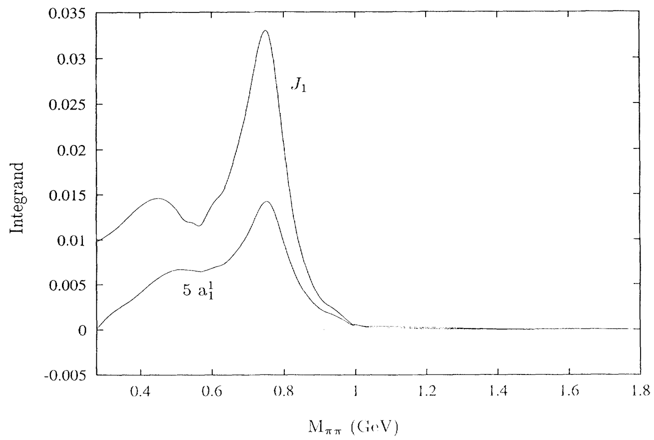

FIG. 5. Integrand for sum rules J_1 and a_1^1.

to roughly $\sqrt{s} = 1.8\,\text{GeV}$ (or $k = 80m_\pi$), so we must at least estimate the contributions above this region. The leading asymptotic amplitudes are

$$A_I(\omega) \underset{k\to\infty}{\sim} \lambda k \; , \tag{18a}$$

$$A_{I_t=1}(\omega) \underset{k\to\infty}{\sim} \beta\sqrt{k} \; . \tag{18b}$$

The first (18a) is a diffractive contribution equivalent to a constant asymptotic cross section by the optical theorem. The integrals in which $A_I = \lambda k$ enter converge as $1/k^2$ and the asymptotic contributions for $k_A \gg k$ are

$$J_I^{\text{asy}} \simeq \frac{2k_K^2}{\pi}\int_{k_A}^{\infty}\frac{\lambda dk}{k^2} = \frac{2k_K^2\lambda}{\pi k_A} \; . \tag{19}$$

By the optical theorem [16]

$$\lambda = 10^{-3}\sigma_A(\pi\pi) \; , \tag{20}$$

where $\sigma_A(\pi\pi)$ is the asymptotic π-π cross section in millibarns. To estimate σ_A we may use factorization [22, 23] with asymptotic πN and NN cross sections which yields $\sigma_A(\pi\pi) = \sigma_A^2(\pi N)/\sigma_A(NN)$, or the independent quark scattering model [24, 23] which gives $\sigma_A = \frac{2}{3}\sigma_A(\pi N)$. The values for $\sigma_A(\pi\pi)$ are nearly the same with an average of

$$\sigma_A(\pi\pi) = 14.5 \pm 1.5\,\text{mb} \tag{21}$$

TABLE I. Contributions to the four sum rules from resonances not included in the phase shift data.

Resonance	α_0	α_1	α_2	a_1
$f_2(1270)$	0.0064	0.0205	−0.0204	0.0035
$\rho_3(1690)$	0.0121	0.0086	−0.0061	0.0011
$f_4(2050)$	0.0047	0.0020	−0.0020	0.0002

and the result with $k_A = 80m_\pi$ is

$$J_I^{\text{asy}} \simeq 0.003 \; . \tag{22}$$

An analogous calculation for the a_1 sum rule gives

$$a_1^{\text{asy}} = \frac{4}{3k_K^2}J_I^{\text{asy}} \simeq 0.00015 \; . \tag{23}$$

The nonleading asymptotic amplitude (18b) is dual to the s-channel resonances, and it will thus suffice to estimate this part by the contributions of those resonances above $k_A = 80$ which have appreciable branching ratios to the π-π channel. These resonance contributions are listed in Table I.

D. Error assignment

A critical step in the evaluation of any integral over experimental data is the estimation of the error in the integral. The procedure we have adopted is the following.

(1) An interpolation scheme such as the cubic spline [21] is fit to the data and the sum rule is evaluated.

(2) Each data point is "shaken" to randomly produce a new point with a Gaussian distribution consistent with the quoted error. A new fit is done and a new sum rule evaluation made.

(3) The procedure is repeated a large number of times and a distribution of sum rule results is made. As an example we show 10^5 evaluations of the J_0 sum rule in Fig. 6.

(4) A standard deviation error is assigned using the conventional definition

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2} \; . \tag{24}$$

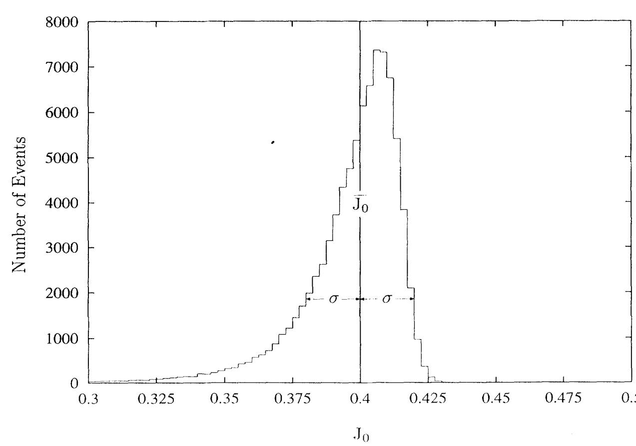

FIG. 6. Histogram resulting from 10^5 calculations of J_0 in which the individual data points have been stochastically generated, weighted by their experimental errors. The mean is $\bar{J}_0 = 0.397$ and the standard deviation is $\sigma = 0.019$.

IV. CONCLUSIONS

By carrying out the procedure outlined in the last section the results for J_I [defined in (13)] and a_1^1 [given in (15)] are

$$J_0 = 0.397 \pm 0.019 m_\pi^{-1} , \tag{25a}$$

$$J_1 = 0.370 \pm 0.006 m_\pi^{-1} , \tag{25b}$$

$$J_2 = -0.114 \pm 0.003 m_\pi^{-1} , \tag{25c}$$

$$a_1^1 = 0.035 \pm 0.001 m_\pi^{-3} . \tag{26}$$

Using the s-wave scattering lengths (2) in the sum rules (13) we find, for the real parts of the isospin amplitudes at the K mass,

$$\begin{aligned} \alpha_0 &= 0.594 \pm 0.029 m_\pi^{-1} , \\ \alpha_1 &= 0.370 \pm 0.006 m_\pi^{-1} , \\ \alpha_2 &= -0.146 \pm 0.007 m_\pi^{-1} , \end{aligned} \tag{27}$$

which imply the phase shifts

$$\begin{aligned} \delta_0(M_K) &= 35.3 \pm 2.7^\circ , \\ \delta_1(M_K) &= 3.5 \pm 0.2^\circ , \\ \delta_2(M_K) &= -7.0 \pm 0.2^\circ . \end{aligned} \tag{28}$$

Comparing the results (28) with the direct local measurements from Figs. 1–3 we observe the general consistency of the data. For example, if the experimental result for the isospin zero scattering length a_0 were $a_0 = 0.10 m_\pi^{-1}$ instead of $0.20 m_\pi^{-1}$ the sum rule would have given $\delta_0(M_K) = 26^\circ$, clearly inconsistent with the measured phase shift. On the other hand if a_0 were greater than $a_0 = 0.25$ unitarity would be violated since α_0 would be so large that no δ_0 could account for its value [25]. Knowledge of low-energy π-π scattering together with dispersion relations can also form the basis of sensitive tests of chiral perturbation theory and the investigation of the pattern of chiral symmetry breaking [26].

The main result of our analysis concerns δ_0-δ_2 at the K mass. Since the error in δ_2 is much smaller than the error in δ_0, a direct subtraction of δ_0 and δ_2 from (28) does not suffer seriously from error correlation between J_0 and J_2. To take into account possible systematic errors we nearly double the statistical error in δ_0-δ_2 giving

$$\delta_0 - \delta_2 = 42 \pm 4^\circ . \tag{29}$$

Assuming CPT invariance (1) then gives

$$\arg \varepsilon' = 48 \pm 4^\circ . \tag{30}$$

It appears that $\arg \varepsilon'$ is still consistent with $\arg \varepsilon = 45.6 \pm 0.4^\circ$ as determined [1] by CP violation due to K_1, K_2 admixture. This remains a curious coincidence.

The sum rule result (29) should be compared with direct (local) phase shifts in the K region of [2]

$$\delta_0 - \delta_2 = 41.4 \pm 8.1^\circ . \tag{31}$$

Alternatively, the expectation from the chiral Lagrangian approach of Gasser and Meissner [3] is

$$\delta_0 - \delta_2 = 45 \pm 6^\circ . \tag{32}$$

A recent analysis by Ochs [27] has compared phase shifts in the K region with the Roy equation analysis of Basdevant *et al.* [28] to obtain

$$\delta_0 - \delta_2 = 44 \pm 5^\circ . \tag{33}$$

And finally our own local fit to the δ_0-δ_2 difference using our Monte Carlo varied fits is

$$\delta_0 - \delta_2 = 42 \pm 6^\circ . \tag{34}$$

The sum rule result (29) is completely consistent with other methods of computing δ_0-δ_2 but with a significantly reduced error. This improvement is directly related to the recent determination of the π-π scattering lengths (2). We see this from (8) since the unsubtracted dispersion relation does not converge. A subtraction at threshold yields a convergent sum rule (13) but now requires knowledge of the scattering lengths.

ACKNOWLEDGMENTS

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