Strong resonances effects and future high precision measurements

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We develop a general formalism where the relevant oblique contributions to a number of observables in future higher energies e^+e^- experiments are expressed in the form of a once-subtracted dispersion integral. The necessary subtraction constants are always provided by model-independent CERN LEP 1 results. This procedure is particularly suitable for calculating possible effects from vector particles that are strongly coupled to the known gauge bosons. As a specific illustration, in the case of technicolorlike resonances we derive limits for their masses that are in the TeV range for a 500 GeV linear collider.

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I. INTRODUCTION

A general conclusion that emerges at the intermediate stage of the projected high precision measurements at various accelerators [1] is that no striking evidence exists for any kind of deviation from the predictions of the Glashow-Salam-Weinberg minimal standard model (MSM) at the level of virtual effects at the one loop accuracy, and that the still possible residual effects of new physics would be in any case extremely weak. This statement is valid, for instance, for a class of models of either extended gauge symmetry [2] or supersymmetric origin [3], or of technicolor type [4], and for a more detailed discussion of these topics we refer to the rich existing literature [5].

A very important point that we want to stress is the fact that the search for new physics effects can (and should) always be performed in a way that is unaffected by the still existing theoretical uncertainties of the MSM, particularly by the lack of knowledge of the exact value of the top mass. This idea, which has been qualitatively stated in previous papers [6–8], has been recently reformulated in a more systematic way by Altarelli and Barbieri [9]. In particular, it has been stressed by these authors that a completely model-independent search for new physics effects can be performed by using purely leptonic variables, i.e., partial widths and asymmetries, measured on top of the Z resonance in e^+e^- annihilation, without introducing any extra relevant unnecessary theoretical uncertainties (the one coming from the running of $\alpha_{\rm QED}$ from zero to M_Z^2 being, at least for the moment, safely negligible [10]). With this aim one introduces two parameters, called ϵ_1 and ϵ_3 in Ref. [9] and related to the leptonic Z width and to the effective angle $s^2_{\rm eff}(M_Z^2)$ (defined by the leptonic asymmetries) through

$$\frac{\Gamma_l}{M_Z}=\frac{G_F M_Z^2}{24\sqrt{2}\pi}[1+\epsilon_1]\{1+[1-4s^2_{\rm eff}(M_Z^2)]^2\} , \qquad (1)$$

$$s^2_{\rm eff}(M_Z^2)\equiv s_1^2[1+\Delta\kappa'(M_Z^2)] , \qquad (2)$$

where

$$\Delta\bar{\kappa}'(M_Z^2)=\frac{-c_1^2}{c_1^2-s_1^2}\epsilon_1+\frac{1}{c_1^2-s_1^2}[\epsilon_3+c_1^2\Delta\alpha(M_Z^2)] , \qquad (3)$$

$$s_1^2\equiv 1-c_1^2=\frac{1}{2}\left[1-\left[1-\frac{3\pi\alpha(0)}{\sqrt{2}G_F M_Z^2}\right]^{1/2}\right]\simeq 0.2172 . \qquad (4)$$

$\Delta\alpha(M_Z^2)=0.0602\pm0.0009$ is taken from Ref. [10] and the effective angle must reproduce the original Kennedy-Lynn [11] definition that relates it to the hadronic longitudinal polarization asymmetry on top of the Z resonance:

$$A(M_Z^2)\equiv A_{{\rm LR},h}(M_Z^2)\equiv\frac{2[1-4s^2_{\rm eff}(M_Z^2)]}{1+[1-4s^2_{\rm eff}(M_Z^2)]^2} . \qquad (5)$$

It should be noticed that the quantities $\epsilon_{1,3}$ are not purely oblique (i.e., due to vacuum polarization effects in the customary definition [12]) corrections. Their complete expression can be found in Ref. [3]; for our specific purposes we shall rather concentrate on the (equivalent) quantities ϵ_1, $\Delta\bar{\kappa}'(M_Z^2)$, and write them in a compact way as

$$\epsilon_1=\Delta_\rho(0)-M_Z^2{\rm Re}\{F'_{ZZ}(M_Z^2)\}+\epsilon_1^{(V,B)} , \qquad (6)$$

$$\Delta\bar{\kappa}'(M_Z^2)=\frac{1}{c_1^2-s_1^2}[c_1^2\Delta\alpha(M_Z^2)-c_1^2\Delta\rho(0)+\Delta_3(M_Z^2)]+\Delta\bar{\kappa}'^{(V,B)} , \qquad (7)$$

where the vertex and box contributions must be retained to ensure the gauge invariance of the previous expressions and the oblique correction components of Eqs. (6) and (7) are

$$\Delta\rho(0)=\frac{A_{ZZ}(0)}{M_Z^2}-\frac{A_{WW}(0)}{M_W^2} , \qquad (8)$$

$$\Delta\alpha(M_Z^2)=F_\gamma(0)-F_\gamma(M_Z^2) , \qquad (9)$$

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$$\Delta_3(M_Z^2)=-c_1^2F_Z(M_Z^2)+c_1^2F_\gamma(M_Z^2)+\frac{c_1}{s_1}(1-2s_1^2)F_{Z\gamma}(M_Z^2)\ , \quad (10)$$

having defined the various transverse components of the vacuum polarizations as

$$A_{ij}\equiv A_{ij}(0)+q^2F_{ij}(q^2)\quad (i,j=W,Z,\gamma)\ , \quad (11)$$

so that

$$F'_{ZZ}(M_Z^2)=\frac{d}{dq^2}F_{ZZ}(q^2)\bigg|_{q^2=M_Z^2}\ .$$

The strategy for the identification of new physics effects proceeds now as follows. From the experimentally measured values of $\Gamma_l, s_{\rm eff}^2(M_Z^2)$ a model-independent determination of $\epsilon_{1,3}$ is provided. These values are then compared to the predictions of the MSM and in this way limits on a number of candidate models can be fixed. To give a particularly relevant example, the parameter ϵ_3 is at the moment experimentally bounded by the pure data from the CERN e^+e^- collider LEP 1

$$-0.01\le\epsilon_3\le 0.01\quad (95\%\ \text{C.L.})\ , \quad (12)$$

and the value of the upper bound already sets strong limitations on possible models of technicolor type, as discussed in Ref. [4].

It is expected that in the next two years the accuracy of several high precision measurements, in particular the accuracy of LEP 1 and SLAC Linear Collider (SLC) results, will still sensibly increase. If no evidence of deviations from the MSM via virtual effects were found, the natural question would then arise whether such virtual effects could be present in future intermediate energy precision experiments, or whether the only possibility would be that of "brute force" production at (very) high energies.

For the specific case of models of new physics containing some extra neutral vector resonance weakly coupled to fermions, to be generically called Z' (whose theoretical origin can be of different sources), this problem has already been considered. In fact it has been shown [13,14] that the expected accuracy of the future "intermediate" energies e^+e^-collider experiments a LEP 2 and, possibly, at a 500 GeV linear collider [Next Linear Collider (NLC)] would be sufficient to detect clean signals for values of M_Z' ranging up to about 1 TeV (LEP 2) and a few TeV (NLC). This happens in spite of the very strong LEP 1 constraint on Z-Z' mixing, because a different kind of "direct" effect is relevant at such higher energies (more precisely, one has now the diagram with Z' exchange, that is no longer kinematically suppressed). The aim of this preliminary paper is that of investigating whether a similar phenomenon might occur for *oblique* corrections coming from the transverse γ, Z, W propagators. For this purpose, we shall not concentrate our attention here on the vertices and box content of our expressions; i.e., we shall only consider the possible effects of models of new physics on the various propagators, much like in the spirit of the original approaches to such problems at lower energies. The discussion of the vertices and also of the (now possibly relevant) box effects will be given in a longer and more detailed forthcoming article. The present paper will be organized as follows. In Sec. II we shall briefly define our preliminary variables and describe our method in a general kinematical configuration of the considered process $e^+e^-\to l^+l^-$. As a first consequence of our approach, we shall show that for a large class of electroweak models the chances of producing virtual oblique effects at future higher energies are rather small once the already existing LEP 1 limits are consistently taken into account. In Sec. III we shall discuss the corresponding estimates for models where strong vector and/or axial-vector resonances appear and show that, even when the LEP 1 constraints are fully incorporated into the analysis, the chance of producing visible signals is remarkable at a 500 GeV NLC, and also not completely negligible at the next coming LEP 2 phase.

II. DESCRIPTION OF THE METHOD

To develop our approach, we first define the *three* independent observables that can be measured in the process $e^+e^-\to l^+l^-$ at a variable total c.m. energy $\sqrt{q^2}$, where $l^\pm$ is a generic charged lepton. The first two observables will be conventionally chosen as the cross section for muon production $\sigma_\mu(q^2)$ and the related forward-backward asymmetry $A_{{\rm FB},\mu}(q^2)$. Different from the LEP 1 situation one now has a *third* independent observable. This can be identified with, e.g., the conventionally defined final τ polarization asymmetry $A_\tau(q^2)$ or, equivalently, with the longitudinal polarization asymmetry for final lepton production $A_{{\rm LR},l}(q^2)$ whose theoretical expressions coincide. Note that the tree level relationship

$$A_{{\rm FB},\mu}=\tfrac{3}{4}A_\tau^2 \quad (13)$$

is now no longer valid since the photon exchange cannot be neglected. Note also that, owing to the same reason, the theoretical expression of the hadronic left-right asymmetry is different from that of the leptonic one. In this preliminary paper we shall try to avoid any source of extra theoretical uncertainty coming from the final state strong interactions, and therefore the hadronic asymmetry [as well as the potentially interesting ratio $R=\sigma_h(q^2)/\sigma_\mu(q^2)$] will not be considered for the moment.

In order to briefly illustrate our procedure, we consider the simplest example of the muon cross section. Using the tree level identity involving bare quantities,

$$\frac{\alpha_0}{s_0^2c_0^2}=\frac{\sqrt{2}}{\pi}G_\mu^0M_Z^{0^2}\ ,$$

this observable can be written to lowest order as

$$\sigma_\mu^{(0)}(q^2)=\frac{4}{3}\pi q^2\left\{\left[\frac{\alpha_0}{q^2}\right]^2+\left[\frac{G_\mu^0\sqrt{2}M_Z^{02}}{4\pi}\frac{(g_{V,l}^0)^2+(g_{A,l}^0)^2}{q^2-M_Z^{02}}\right]+\frac{\alpha_0 G_\mu^0\sqrt{2}M_Z^{02}}{2\pi q^2}g_{V,l}^0\mathrm{Re}\frac{1}{q^2-M_Z^{02}}\right\}, \tag{14}$$

where

$$g_{V,l}^0=-\tfrac{1}{2}+2s_0^2\;,\quad g_{A,l}^0=-\tfrac{1}{2}\;. \tag{15}$$

When one moves to one loop, a certain number of formal replacements must be performed, some of which involve a redefinition of the Fermi coupling, of α_{QED}, of M_Z^0, of the photon and Z propagators, and of the electron couplings $g_{V,A}^0$. To extract the oblique component of these corrections one isolates the corresponding terms in the redefinition of the Fermi coupling $\simeq A_{WW}(0)/M_W^2$ and in the redefinition of the vector coupling g_V,

$$g_V+\Delta g_V(q^2)=-\tfrac{1}{2}+2s_{\mathrm{eff}}^2(q^2)=-\tfrac{1}{2}+2s_1^2[1+\Delta\bar{\kappa}'(q^2)]\;, \tag{16}$$

and the oblique component of $\Delta\bar{\kappa}'(q^2)$ is the analogue of that in Eqs. (7) and (10) with the replacement $F_{Z\gamma}(M_Z^2)\rightarrow F_{Z\gamma}(q^2)$.

By conventional definition of the physical couplings and masses and by a general treatment of the relevant Z propagator in the configuration $q^2\neq M_Z^2$ one is then easily led to the modification of Eq. (14) at one loop that takes into account the overall oblique corrections and *some* of the vertex (and box) corrections (in particular those that would appear at $q^2=M_Z^2$ in the definition of ϵ_1) and reads

$$\begin{aligned}\sigma_\mu\equiv\frac{4}{3}\pi q^2&\left\{\left[\frac{\alpha}{q^2}[1+\Delta\alpha(q^2)]\right]^2+\frac{1}{(q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2}\right.\\&+\left[\frac{\sqrt{2}G_\mu}{4\pi}M_Z^2 g_{A,l}^2\left[1+\epsilon_1-\mathrm{Re}[I_Z(q^2)-I_Z(M_Z^2)]+2\frac{\Delta g_{A,l}(q^2)-\Delta g_{A,l}(M_Z^2)}{g_{A,l}}\right]\right]^2\\&\left.\times(1+\{[1-4s_{\mathrm{eff}}^2(M_Z^2)]+4[s_{\mathrm{eff}}^2(M_Z^2)-s_{\mathrm{eff}}^2(q^2)]\}^2)^2+[O(v_1^2)]\right\},\end{aligned} \tag{17}$$

where we have neglected to write the complete expression of the third contribution coming from Z-γ interference since it turns out to be completely negligible from a numerical point of view, and we have used the definition

$$I_Z(q^2)=q^2\frac{F_Z(q^2)-F_Z(M_Z^2)}{q^2-M_Z^2}\;. \tag{18}$$

From the definitions Eqs. (16) and (1) we see therefore that, at the one loop level, the effect of the oblique (SE=self-energy) corrections on σ_μ can be fully incorporated in the compact notation (we still neglect the irrelevant $Z-\gamma$ interference)

$$\begin{aligned}\sigma_\mu^{\mathrm{SE}}(q^2)=\frac{4\pi q^2}{3}&\left\{\frac{\alpha(M_Z^2)}{q^2}\{1+2[\Delta\alpha(q^2)-\Delta\alpha(M_Z^2)]\}\right.\\&\left.+\frac{1}{(q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2}\left[\frac{3\Gamma_l}{M_Z}\right]^2\left[1-2\,\mathrm{Re}[I_z(q^2)-I_z(M_Z^2)]-\frac{16s_1^2v_1}{1-v_1^2}\mathrm{Re}[\Delta\bar{\kappa}'(q^2)-\Delta\bar{\kappa}'(M_Z^2)]^{\mathrm{SE}}\right]\right\},\end{aligned} \tag{19}$$

where the quantity Γ_l in the square brackets is the leptonic Z width *rigorously* defined by Eq. (1) and we have chosen for purely conventional reasons to use rather than $\alpha(0)$, $\alpha(M_Z^2)$, numerically equal to [10]

$$\alpha(M_Z^2)=\frac{1}{128.87}[1\pm0.001] \tag{20}$$

[the error in Eq. (20) being completely negligible for the practical purposes of this paper].

Before moving to the remaining observables we feel that a short comment is now in order on the gauge invariance of the relevant expressions. In Eq. (19) we have retained three differences of transverse self-energies. In fact, it is well known [15] that to obtain gauge-invariant expressions one must add to transverse self-energies a precisely defined amount of boxes and vertices (we assume that tadpoles have already been incorporated). These quantities could be correctly taken into account by systematically retaining them from the beginning in the definitions of $I_Z(q^2)$, $\Delta\bar{\kappa}'(q^2)$, and $\Delta\alpha(q^2)$ that generalize Eqs. (6), (7), and (9) and appear in the Z propagator, respectively. Once they are properly inserted in the various observables, one can work, e.g., in the 't Hooft gauge $\xi=1$ and perform the various calculations.

With this caveat, our Eq. (19) becomes meaningful and understandable.

Bearing in mind the previous remark, it is now straightforward to perform the analogous calculation for the two remaining observables. Here we shall present the final meaningful results, only retaining those terms that are numerically significant. We find in this realistic situation the simplified expressions

$$A^{\mathrm{SE}}_{\mathrm{FB},\mu}(q^2)=\frac{3}{4}\left[\frac{3q^2\sigma_\mu(q^2)}{4\pi}\right]^{-1}\left\{6\alpha(M_Z^2)\frac{\Gamma_l}{M_Z}\frac{q^2(q^2-M_Z^2)}{(q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2}\{1+[\Delta\alpha(q^2)-\Delta\alpha(M_Z^2)]-\mathrm{Re}[I_Z(q^2)-I_z(M_Z^2)]\}\right\}, \tag{21}$$

$$A^{(\mathrm{SE})}_{\tau}(q^2)\equiv A^{(\mathrm{SE})}_{\mathrm{LR},1}=\left[\frac{3q^2\sigma_\mu(q^2)}{4\pi}\right]^{-1}A(M_Z^2)\left\{\left[6\alpha(M_Z^2)\frac{\Gamma_l}{M_Z}\frac{q^2(q^2-M_Z^2)}{(q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2}+18\left[\frac{\Gamma_l}{M_Z}\right]^2\frac{q^4}{(q^2-M_Z^2)^2+M_Z^2\Gamma_Z^2}\right]\right.$$

$$\left.\times\left[1-\frac{8s_1^2}{A(M_Z^2)}\mathrm{Re}[\Delta\bar{\kappa}'(q^2)-\Delta\bar{\kappa}'(M_Z^2)^{\mathrm{SE}}]\right]\right\}. \tag{22}$$

Here we have defined $A(M_Z^2)$ as

$$A(M_Z^2)\equiv\frac{2[1-4s^2_{\mathrm{eff}}(M_Z^2)]}{1+[1-4s^2_{\mathrm{eff}}(M_Z^2)]^2}. \tag{23}$$

Note that Eq. (23) represents theoretically both the final τ polarization asymmetry and the longitudinal (leptonic or hadronic) polarization on top of the Z resonance, where they all coincide. Its numerical value, though, can be entirely derived form that of $s^2_{\mathrm{eff}}(M_Z^2)$, measured by *all* LEP 1 leptonic asymmetries, including, e.g., the forward-backward muon asymmetry.

Equations (19), (21), and (22) show some properties that, we feel, deserve a short comment. They express the three independent observables of the process $e^+e^-\to$ leptons at any q^2 in terms of three perfectly defined and measurable quantities, i.e., Γ_l, $s^2_{\mathrm{eff}}(M_Z^2)$, and α_{QED} and of three differences of transverse self-energies expressed automatically in a form that, from a mathematical point of view, corresponds to a once-subtracted dispersion relation. As a consequence of this, these differences can be estimated also for models where a perturbative approach would not be allowed, such as the case of technicolorlike schemes. This statement is relatively trivial for the case of the pure photon exchange contribution proportional to $F_\gamma(q^2)$ where the subtraction constant is provided in the dispersive approach by the vanishing of $F_\gamma(0)$. But for the other two ZZ and $Z\gamma$ contributions this would not be the case and one subtraction constant for each quantity would be necessary. In Eq. (22) the introduction of the two LEP 1 observables removes the problem and provides, so to say, the subtraction constants required in the dispersive approach.

From the explicit expression of the various transverse self-energies Eqs. (7)–(9), (11), and (18), it is then straightforward to derive the following formal expressions for the three relevant differences that we shall call from now on D_i $(i=\gamma,Z,\gamma Z)$:

$$D_\gamma(q^2)\equiv\Delta\alpha(q^2)-\Delta\alpha(M_Z^2)$$
$$=-\frac{q^2-M_Z^2}{\pi}\mathcal{P}\int_0^\infty\frac{ds\,\mathrm{Im}F_\gamma(s)}{(s-q^2)(s-M_Z^2)}, \tag{24}$$

$$D_Z(q^2)\equiv\mathrm{Re}[I_Z(q^2)-I_Z(M_Z^2)]$$
$$=\frac{q^2-M_Z^2}{\pi}\mathcal{P}\int_0^\infty\frac{ds\,s\,\mathrm{Im}F_{ZZ}(s)}{(s-q^2)(s-M_Z^2)^2}, \tag{25}$$

$$D_{\gamma Z}(q^2)\equiv\mathrm{Re}[\Delta\bar{\kappa}'(q^2)-\Delta\kappa'(M_Z^2)]$$
$$=\frac{q^2-M_Z^2}{\pi}\mathcal{P}\int_0^\infty\frac{ds\,\mathrm{Im}F_{\kappa'}(s)}{(s-q^2)(s-M_Z^2)}, \tag{26}$$

where

$$F_{\kappa'}=c_1/s_1F_{Z\gamma}.$$

Having at our disposal these compact and simplified expressions for the overall oblique corrections, we have began to investigate in a systematic way the possible effect on the three observables Eqs. (19), (21), and (22) of a number of models, including cases that can be treated perturbatively and cases that cannot. Specifically, we have considered in the perturbative sector the possible contributions coming from extra heavy fermions or sfermions, or from extra heavy (charged or neutral) Higgs bosons and from their supersymmetric partners, using some very general parametrization that includes possible (and reasonable) widths, for several reasonable values of the involved masses always assuming no direct production but only virtual effects. The results of this analysis are rather disappointing, although in a sense predictable, since no realistic enhancement was found coming from $D_{\gamma,Z,\gamma Z}$ for any value of the masses that was not extremely close to the chosen value of $\sqrt{q^2}$ (for example, for $\sqrt{q^2}=500$ GeV, only masses that were a few tens of GeV higher were possibly affecting the observables at the predicted [14] accuracy at that energy[1]).

[1]The same remark applies to the virtual effect of a conventional Higgs boson produced by the Bjorken mechanism, that is practically canceled in the various differences.

Rather than showing the details of these negative results, we shall summarize them by saying that, for heavy particles that are weakly coupled to the photon and to the Z, there is not much difference between their oblique effects on the considered neutral current processes at $q^2=M_Z^2$ and at $q^2=(\text{a few } M_Z)^2$, once the "hard" component of a possible effect has been globally incorporated into ϵ_1 (i.e., into the measured Z leptonic width Γ_l) and into $A(M_Z^2)$. The fact of carrying a coupling of typical electroweak size $\simeq e$ seems therefore to prevent realistic perturbative models from contributing appreciably to the considered class of future high precision measurements via this type of virtual effects, once the existing constraints from LEP 1 physics are systematically taken into account.

III. VIRTUAL EFFECTS OF STRONG RESONANCES

There could be one possible exception to the previous negative statement. If some resonances existed that were strongly coupled to the photon and/or to the Z, the smoothness of the effect could be partially compensated by the strength of the coupling for values of their masses not too far from $\sqrt{q^2}$. This is the subject to which we shall devote the final part of this paper.

To make our investigation more definite, we shall assume that a couple of vector and axial-vector resonances, to be generically called V and A, with unknown (but larger than $\sqrt{q^2}$) masses and unknown but "reasonable" widths, exist and that these particles are strongly coupled to the conventionally defined vector and axial-vector components of the transverse self-energies, exactly like a ρ and a A_1 in the corresponding QCD case, with strengths F_V and F_A of typical strong interaction size. We shall assume in this preliminary stage, although this is by no means necessary, that there are no appreciable effects from ω-like resonance coupled to the "hypercharge" component. With standard decomposition of the various F_{ij} appearing in Eqs. (24)–(26) one is then led to the "effective" representations

$$D_\gamma(q^2)\simeq -\frac{\alpha(q^2-M_Z^2)}{3\pi}\mathcal{P}\int_0^\infty \frac{ds R_{VV}(s)}{(s-q^2)(s-M_Z^2)} , \quad (27)$$

$$D_Z(q^2)\simeq \frac{\alpha(q^2-M_Z^2)}{3\pi}\mathcal{P}\int_0^\infty \frac{s ds}{(s-q^2)(s-M_Z^2)} \times\frac{(1-2s_1^2)^2}{4s_1^2c_1^2}\left[R_{VV}(s)+\frac{R_{AA}(s)}{(1-2s_1^2)^2}\right] , \quad (28)$$

$$D_{\gamma Z}(q^2)\simeq \frac{\alpha(q^2-M_Z^2)}{3\pi}\frac{1-2s_1^2}{2s_1^2}\mathcal{P}\int_0^\infty \frac{ds\, R_{VV}(s)}{(s-q^2)(s-M_Z^2)} , \quad (29)$$

where, following our conventions, we have defined

$$\text{Im}F_{VV,AA}(s)=\frac{1}{12\pi}R_{VV,AA}(s) \quad (30)$$

with $R_{VV,AA}(s)>0$.

To fix the normalization of our search, we remind the reader that the quantity originally called S in Ref. [4] was given by the expression

$$S=\frac{1}{3\pi}\int\frac{ds}{s}[R_{VV}(s)-R_{AA}(s)] . \quad (31)$$

In our analysis of strong resonance effects, we shall exploit the fact that, for the quantity S in Eq. (31), rigorous experimental bounds are provided by the exiting data. These bounds will be then used to determine observability limits for the resonance *masses* in a way that will be, to a certain extent, independent of several extra details of the model.

In order to show the main features of our approach, we shall proceed to an illustration using the following oversimplified representation of the two resonances:

$$R_i=12\pi^2F_i^2\delta(s-m_i^2) . \quad (32)$$

With this "zero-width" choice, we would have to play with four free parameters, i.e., F_i, M_i ($i=VV, AA$). To reduce the number of arbitrary quantities, we decided to impose the validity of the second Weinberg sum rule [16]:

$$\int ds\, s[R_{VV}(s)-R_{AA}(s)]=0 . \quad (33)$$

Our choice was motivated by the fact that Eq. (33) implies a constraint on the asymptotic behavior of the imaginary parts of the vacuum polarizations (generally true in asymptotic free gauge theories [17]) that is, though, independence of extra details of the model. For this reason we decided not to implement from the beginning the validity of the first Weinberg sum rule

$$\int ds[R_{VV}(s)-R_{AA}(s)]=12\pi^2F_\pi^2 , \quad (34)$$

which can be usefully exploited once F_π is determined by more specific features of the model (e.g., in the technicolor case the number of technicolors and techniflavors[2]).

As a consequence of our choice, we obtain the relationship between the parameters:

$$\frac{F_V^2}{F_A^2}=\frac{M_A^2}{M_V^2} . \quad (35)$$

The next (and fundamental) step of our program is to use the available experimental bounds on S, Eq. (31). This has to be done with some care since, as a matter of fact, the available model-independent information can be given in terms of the parameter ϵ_3, such as shown in Eq. (12). The rigorous expression of ϵ_3 in terms of the involved quantities reads

$$\epsilon_3=\frac{e^2}{s_1^2}[F_{33}(M_Z^2)-R_{3Q}(M_Z^2)] +c_1^2M_Z^2F'_{ZZ}(M_Z^2)+\text{vertices}\cdots . \quad (36)$$

[2]Note that, strictly speaking, if the considered model does not foresee the Higgs particle of the MSM, this contribution should be subtracted out from the theoretical expressions. But this is already automatically guaranteed by our approach that only uses LEP 1 data and subtracted quantities (see the previous footnote).

But for the type of heavy strong resonance (HSR) models that we are considering here one can safely neglect the shift in the various self-energies when one moves from $q^2=0$ to $q^2=M_Z^2$ (and possible vertex effects on the electron-Z couplings) and write the approximate equality

$$S^{\rm HSR}\simeq\frac{4s_1^2}{\alpha}\epsilon_3^{\rm HSR}\ . \tag{37}$$

From the experimental bound on ϵ_3, Eq. (12), and from the value of ϵ_3 in the MSM ($\epsilon_3^{\rm MSM}\lesssim 5\times10^{-3}$) we can then derive a bound on $S^{\rm HSR}$ in our considered models that we write as

$$|S^{\rm HSR}|\lesssim 1 \tag{38}$$

and for the next discussion we shall assume the representative value

$$|S^{\rm HSR}|=4\pi\frac{F_V^2}{M_V^2}\left|1-\frac{M_V^4}{M_A^4}\right|\le 1 \tag{39}$$

(possible smaller values of the bound will be considered later) that we interpret as the constraint imposed by the available experimental data upon the parameters of the model that we are considering. At this point we are left with two independent parameters that we shall identify with the vector mass M_V and the ratio M_A/M_V. For each couple of values of these parameters we shall allow the coupling F_V to saturate the bound Eq. (39), and calculate the corresponding effect on the three observables Eqs. (19), (21), and (22). From the request that the effect is not larger than the corresponding expected experimental errors, we shall derive limits for M_V at variable M_A/M_V. This illustrates the gross features of our analysis.

A few points should now be discussed. First of all, although we decided to consider M_A/M_V as a free parameter, we retained the theoretical "prejudice" that it should not differ too much from the approximate value

$$\frac{M_A}{M_V}=\frac{m_{A_1}}{m_\rho}\simeq 1.6 \tag{40}$$

given by the large-N rescaling relation between technicolor and QCD [18]. In our analysis, we actually imposed this ratio to remain larger than one (to avoid catastrophically large values of the coupling F_V/M_V) by a "reasonable" amount, quantitatively stated by the request

$$\frac{M_A}{M_V}\ge 1.1\ . \tag{41}$$

With this choice, the maximum value of the squared effective coupling F_V^2/M_V^2 varies from that of the extreme configuration,

$$\frac{F_V^2}{M_V^2}(M_A\to\infty)\le\frac{1}{4\pi}\simeq 2\frac{f_\rho^2}{m_\rho^2}\ , \tag{42}$$

by no more than about a factor 2, which we consider a tolerable situation. Second, although to illustrate the main points of our approach the oversimplified parametrization Eq. (32) was particularly useful, we feel that a more general description might be requested. We did this by modifying Eq. (32) as

$$R_i(s)=\frac{\beta(s)\chi_i M_i^4}{(s-M_i^2)^2+M_i^2\Gamma_i^2}\quad (i=VV,AA)\ , \tag{43}$$

where $\beta(s)=\sqrt{1-s_{\rm th}/s}$ and the various parameters are related to the zero-width approximation as follows:

$$\pi\beta(M_i^2)\chi_i\frac{M_i^3}{\Gamma_i}\Longrightarrow 12\pi^2F_i^2\ . \tag{44}$$

For the quantity $s_{\rm th}$, that should be interpreted as the threshold for production of possible decay products of V and A, we only assumed that it is larger than $\sqrt{q^2}$ (no direct production). In this way, the number of free parameters becomes apparently equal to seven. But it can be easily realized that the outcome of the analysis depends extremely weakly on the choice of $s_{\rm th}$ in the range $q^2<s_{\rm th}<M_i^2$ and on any "reasonable" choice (i.e., of order $1/6M_{V,A}$) of Γ_V,Γ_A. Thus, the number of effective parameters remains actually still equal to four, and our previous approach can be repeated, although now the constraints between the parameters are less simple to be expressed analytically. In fact, for the region of parameter space that corresponds to values of M_V not too close to $\sqrt{q^2}$ the two methods that use Eqs. (32) and (43) are practically coincident. For M_V close to $\sqrt{q^2}$, though, the results are not identical; here we shall always quote those obtained with the more realistic parametrizations of Eq. (43).

As a first configuration where to apply our method, we have chosen the value $\sqrt{q^2}=500$ GeV that has been rather intensively investigated in recent times [19]. We assumed for the measurements of σ_μ and $A_{{\rm FB},\mu}$ a relative accuracy of 10^{-2} as from the discussion of Ref. [14], and for $A_{{\rm LR},l}$ we assumed a (mildly optimistic) relative accuracy of 5×10^{-2}. With these representative values, we obtained for M_V the detectability limits, at variable M_A/M_V, that we represent in Fig. 1.

As one sees from inspection of the figure, the largest amount of information comes from the longitudinal asymmetry and the muon cross section. We also show the result which follows from combining quadratically these informations. The forward-backward muon asymmetry is, on the contrary, never competitive. This is due to the interplay between the combinations of vector and axial-vector components that appear in the observables, that lead in some cases to strong cancellations (as one can see particularly for values $M_A/M_V=1.2$).

Concerning the detectability limit for M_V, one sees that it is strongly dependent on the value of the ratio M_A/M_V. In particular it ranges from approximately 1.15 TeV when $M_A/M_V\to\infty$ to approximately 2.4 TeV when $M_A/M_V\to 1.1$. For the "canonical" value $M_A/M_V=1.6$ we find a detectability limit of

$$M_V\left[\frac{M_A}{M_V}=\frac{m_{A_1}}{m_\rho}\right]\le 1.2\ \text{TeV}\ . \tag{45}$$

The limits that we just mentioned were obtained under the constraint of Eq. (39) $S^{\rm HSR}<1$. Since the next experi-

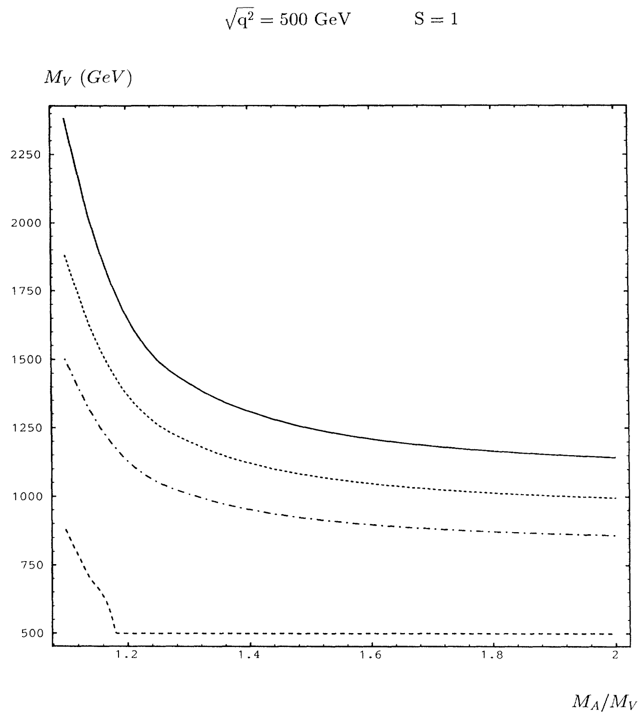

FIG. 1. Limits on M_V at variable M_A/M_V obtained at $\sqrt{q^2}=500$ GeV from σ_μ (dot-dashed), $A_{\mathrm{LR},l}$ (dotted), and $A_{\mathrm{FB},\mu}$ (dashed) and $|S^{\mathrm{HSR}}|<1$. The full line represents the result of combining quadratically the previous limits.

mental data are expected to reduce sensibly the experimental errors on various quantities, one might wonder what would be the impact on our limits of a tighter constraint on S. To produce a reasonable answer, we assumed that, as a consequence of the near future expected LEP 1 accuracies [20], the error on ϵ_3 is reduced by a factor of 2. In agreement with this hypothesis, we reduced correspondingly (although this is probably a drastic attitude) the bound on S by the same factor; i.e., we assumed $|S^{\mathrm{HSR}}|<\frac{1}{2}$. With this new input, we obtained the new limits that are represented in Fig. 2, and that are roughly lowered by a reduction factor 1.3. Therefore, the limit for the canonical value in Eq. (45) would now be reduced to 900 GeV.

To appreciate the meaning of these bounds, one should review the predictions of a number of popular existing technicolor models. For instance, in the so-called "minimal model" [21], one has the prediction $M_V=1.8$ TeV. More elaborate models [22] predict values of about 9000 GeV or less. But more recently, it has been advocated [23] that the lightest techniresonances could be "much lighter than naively supposed," in particular lighter than ≈ 550 GeV. We think, therefore, that our presented limits should be considered as a reasonably unbiased way of investigating whether a relatively light strong resonance exists in a mass region that might be realistic for several respectable models. In particular, the generality of our approach makes it applicable also to a class of models of nontechnicolor origin [24] where no *a priori* prejudice for the values of the resonance masses exists.

As a final example of a possibly "exotic" model, we

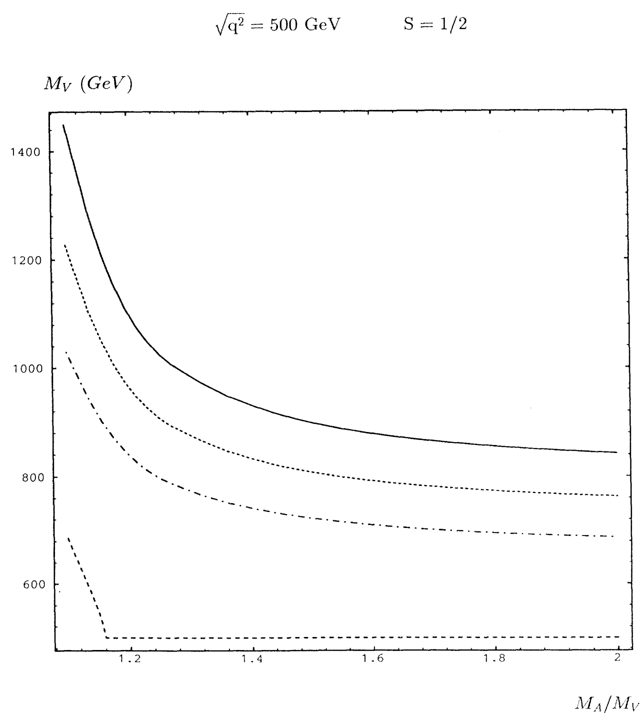

FIG. 2. Same as Fig. 1 for $|S^{\mathrm{HSR}}|<\frac{1}{2}$.

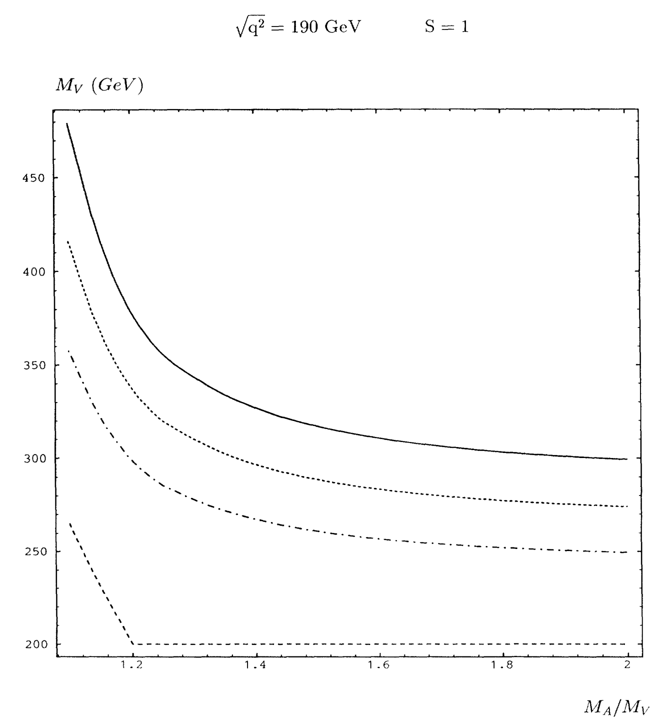

FIG. 3. Same as Fig. 1 for $|S^{\mathrm{HSR}}|<1$ at $\sqrt{q^2}=190$ GeV with accuracies expected at LEP 2.

have considered one where *only* an axial resonance is predicted (the situation with *only* vector resonance can be deduced by Figs. 1 and 2 taking the limit of large M_A). In this case, σ_μ and $A_{LR,l}$ would provide no information and the limits would be entirely provided by $A_{FB,\mu}$. This would lead to a limit of approximately 800 GeV for M_A. As a second example, particularly motivated by the discussion of Ref. 23, we have repeated our analysis for the LEP 2 case, $\sqrt{q^2}=190$ GeV, for $|S^{HSR}|<1$ (in this case, one should consider as a variable A_τ rather than $A_{LR,l}$). The outcome of the analysis is depicted in Fig. 3. It results in a limit on M_V that ranges from about 300 GeV to about 500 GeV, showing that, in principle, this could be a meaningful search for some models.

The conclusions of our preliminary analysis are, we believe, therefore encouraging. We have shown that, if a couple of strong resonances exists in the TeV range, they would be identifiable via specific virtual effects at e^+e^- colliders of a realistically near future, without too many specific assumption son the details of the theoretical origin, and fully exploiting the existing experimental (in particular LEP 1) information. In a forthcoming paper, a more quantitative discussion that takes into account some neglected one loop effects will be given.

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